Problem Set 4

Due Sunday, May 28th, at 11:59 PM

CS - 171 Spring 2017

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Problem 2. If there are m items (or features), there are $3^m - 2^{m+1} + 1$ different association rules possible. Prove this. Submit your answer in the file q2.txt or q2.pdf

Hint: Think about how items can be divided up to make rules. Also consider what divisions result in invalid rules (rule X $\rightarrow$ Y must have non-empty X and Y).

Suppose there are m items or features in our set. If we then take n of the items to form the left- hand side. This then gives us $\binom{m}{n}$ (m choose n). After we are now left with $\binom{m-n}{k}$ ((m minus n) choose k) items remaining to choose from for the other side. Thus giving us the rule of:

$$= \sum_{n=1}^{m} \binom{m}{n} \sum_{k=1}^{m-n} \binom{m-n}{k}$$

$$= \sum_{n=1}^{m} \binom{m}{n} \left(2^{m-n} - 1\right)$$

$$= \sum_{n=1}^{m} \binom{m}{n} \left(2^{m-n}\right) - \sum_{n=1}^{m} \binom{m}{n}$$

$$= \sum_{n=1}^{m} \binom{m}{n} \left(2^{m-n}\right) - \left(2^{m} + 1\right)$$

If we use the Binomial theorem of:

$$= \sum_{i=1}^{n} \binom{n}{i} = \left(2^{n} - 1\right)$$

After applying the Binomial theorem we have:

$$\left(1 + x\right)^{n} = \sum_{i=1}^{m} \binom{m}{i} = \left(x^{m-i} + X^{m}\right)$$

Replacing x with 2 in the result from above gives us:

$$\left(3^{n}\right) = \sum_{i=1}^{m} \binom{m}{i} \left(2^{m-i} + 2^{m}\right)$$

$$\therefore$$

$$= \left(3^m\right) - \left(2^m\right) - \left(2^m + 1\right)$$

$$= \left(3^m\right) - \left(2^{m+1}\right) + 1$$

With the line above showing the same equation as asked to prove, show that there are in fact $3^m - 2^{m+1} + 1$ different association rules possible. Thus proving our assupmtion. $\blacksquare$