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Gasoduct.py Report
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I modeled the gasoduct in Python using a compressor map interpolated from the one provided in the assignment. I loaded the image data as an array of RGB values and then searched the dataset for pixels with the color used to draw the efficiency contours in the plot and stored their locations in the image. I then manually chose a set of points along a single contour, converted to polar coordinates, performed a 1D interpolation and converted back to Cartesian.

I performed this process for each contour. The purpose of interpolating the individual contours was to get a smoother dataset for the 2D interpolation. Once I had the interpolated contours, I assigned each one the corresponding efficiency value from the plot. This entire process was highly manual, so it is not incorporated into my final code. Instead, I stored the interpolated curves and their efficiency values as a single array and saved it as a separate NumPy file "Compressor_Map.npy", which is imported and used to perform the 2D interpolation of the compressor map in the final code. The interpolated map is shown below

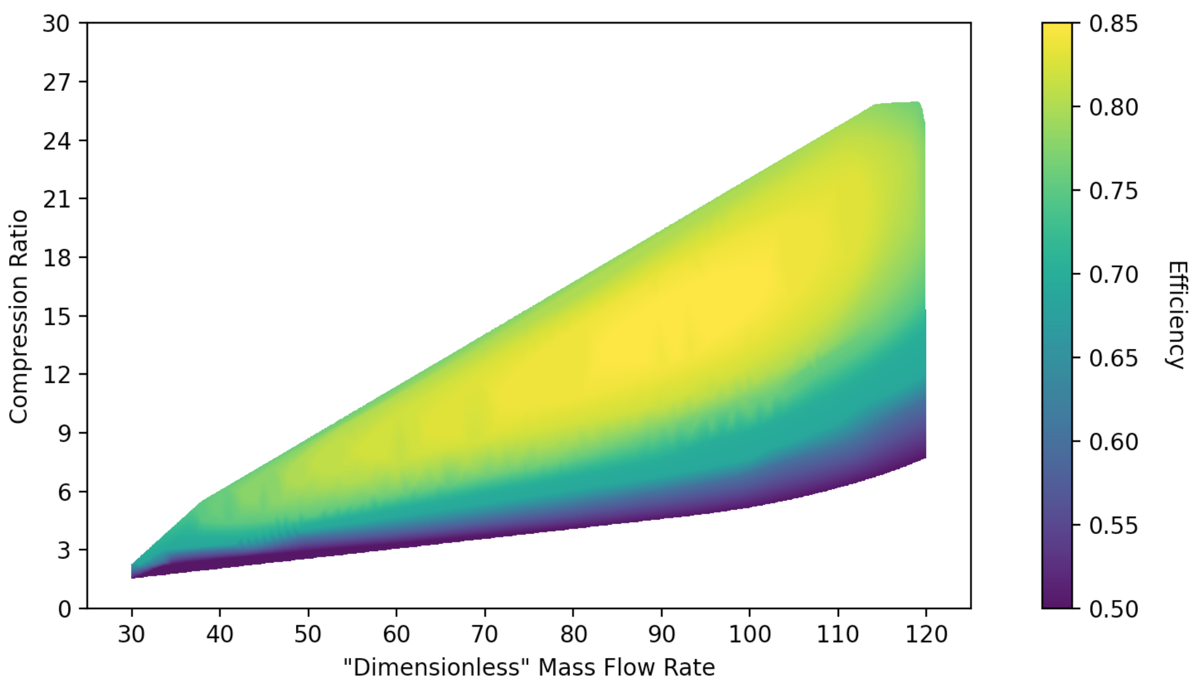

Figure 1: Interpolated compressor map. The map contains some erroneous artifacts from the interpolation process, but the error is very minor.

The map assumes air as the working fluid, so the mass flow was scaled to the properties of methane using the equation provided in the assignment

$$\dot{m} = (70 \text{ lb/s}) \frac{\sqrt{T/518.7 \text{ R}}}{(P/14.7 \text{ psi})(D_{comp}/D_{pipe})^2 \sqrt{(R/k)_{CH_4}/(R/k)_{air}}} \tag{1}$$

Where D_{comp} is the diameter of the compressor and D_{pipe} is the diameter of the pipe. Since the ratio of the two diameters could be chosen, the mass flow rate into the compressor can effectively be chosen as well. So in practice, when running the optimization for the compression ratio, I found the flow rate that would maximize efficiency and solved Eq. 1 for the ratio D_{comp}/D_{pipe} to determine the diameter the compressor needs to have. Note that I assumed the methane would always be at the ambient state when entering the compressor, so the ratios $T/518.7$ R and $P/14.8$ psi were both equal to one and the ratio $((R/k)_{CH_4}/(R/k)_{air}) \approx 1.07$.

I treated the compression as an isentropic process for the purposes of calculating the density (ρ) and kinematic viscosity (μ) of the gas as it exited the compressor, but I took the efficiency into account when calculating the enthalpy change to determine the work done. I numerically determined that, for the range of possible pressures and efficiencies, the density and viscosity of the gas were not meaningfully affected by the entropy loss. I used the PropsSI function from the Python module CoolProp to determine the properties of methane as the pressure changed. However, calling PropsSI significantly affected computation time. To improve computation time I fit the density, viscosity as functions of pressure to polynomial and logarithmic functions respectively. Even so, the code is still somewhat time-consuming; it requires about ten minutes total to find the optimal parameters for all three pipe materials.

Post-compression, the pressure drop along the pipe was determined by calculating the frictional head loss per unit length of a level pipe.

$$h_l = \frac{f_D u^2}{2Dg} \tag{2}$$

Where D is the pipe diameter in m, u is the flow velocity in m/s, $g = 9.81$ m/s^2 is gravitational acceleration. The parameter f_D is the Darcy friction factor, which explicitly requires numerical computations, but standard approximations exist such as the Swamee-Jain equation,

$$f_D = 0.25\left[\log_{10}\left(\frac{\epsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^{-2} \tag{3}$$

where ϵ is the pipe roughness and Re is the Reynolds number. I used this approximation again in the interest of computational efficiency. This equation is only valid for Reynolds numbers about around 4000, which is easily satisfied by the conditions of this problem. The Reynolds number was calculated as

$$\mathrm{Re} = \frac{\rho u D}{\mu} \tag{4}$$

and the flow velocity is given by

$$u = \frac{4\dot{m}}{\rho \pi D^2} \tag{5}$$

where $\dot{m}$ is the mass flow rate converted to SI (≈ 31.8 kg/s).

The effect of pipe diameter on h_l is nontrivial because of how f_D and u each depend on D. However, for diameters of less than about 3 m, the effect of flow velocity dominates over the effect of the friction factor when computing head loss. Since $h_l \propto u^2 D^{-1}$ and $u \propto D^{-2}$, head loss actually becomes proportional to D^{-3}, meaning when $D \lesssim 3$ m, frictional losses in the pipe increase with decreasing diameter regardless of how smooth the pipe is. This fact will become important when discussing the optimization results.

Three parameters were optimized for each pipe material: compression ratio, pipe diameter, and the ratio of diameters, D_{comp}/D_{pipe}. Compression ratio and pipe diameter were optimized to minimize the total cost, and the ratio of diameters was optimized to maximize the compressor efficiency at a given compression ratio. The optimization algorithms were nested with compression ratio being the outermost optimization. For each compression ratio the pipe diameter that minimized cost was found, and for each pipe diameter the diameter ratio was found that maximized efficiency. Total cost over fifteen years includes pipe material cost factor (C_{pipe}), fixed compressor cost (C_{comp} = \$5Mil per compressor), and yearly electricity cost (C_{elec}),

$$C_{tot} = L \cdot D \cdot C_{pipe} + N \cdot C_{comp} + 15C_{elec} \tag{6}$$

Where $L = 900$ mi (1448 km) is the total length of the gasoduct, and N is the number of compressors installed. The yearly cost of electricity was calculated

$$C_{elec} = \dot{W}_{comp} \cdot 8760 \text{ hr/yr} \cdot 0.1 \text{ \$/kWh} \tag{7}$$

where $\dot{W}_{comp}$ is the work done by a single compressor in kW.

There is always one compressor at the beginning of the pipeline to increase the pressure from $P_{amb} = 14.7$ psi to ϕP_{amb}, where ϕ is the compression ratio. The number of additional compressors required depends on how far the gas can travel along the pipe before the total frictional head loss reduces the pressure back to P_{amb}. Dividing the total length, L, by this distance and adding one for the initial compressor gives the total number of compressors, N. However, the amount of head loss per length of pipe, h_l depends on ρ and μ, which in turn depend on pressure. So h_l can be considered a function of pressure, but the exact dependence on which is difficult to discern.

Even with a closed-form equation, solving for the distance at which the pressure drops to P_{amb} would require a numerically solving an integral equation, so I instead chose to formulate the pressure at various locations along the pipe through recursion and iterate until the pressure reached P_{amb}.

$$P_{x+\Delta x} = P_x - h_l(P_x)\Delta x \tag{8}$$

The step size (Δx) needs to be small enough that h_l does not vary significantly along its length. Initially I was using $\Delta x = 1$ m, but this resulted in much longer computational time and an unnecessary level of precision. After a series of tests I found $\Delta x = 100$ m to be a good balance between computational efficiency and numerical precision.

The optimization results are summarized in the table below.

Material	PVC	Steel	Concrete
Comp. Ratio	13.3	19.4	10.3
η_{comp}	0.85	0.84	0.84
N	3	2	2
D Ratio	0.90	0.81	0.96
D_{pipe} (ft)	1.71	1.49	2.31
C_{elec} (\$Mil/yr)	16.9	20.5	15.1
C_{tot} (\$Mil)	1021.2	765.8	571.9

In terms of material choice, concrete was the clear winner with the lowest total minimum cost of \$ 571.9Mil over fifteen years. Pipe material cost was the single largest factor in the total cost of the gasoduct, so it is perhaps unsurprising that the least expensive material resulted in the lowest overall cost.

What was initially surprising to me was that concrete also ended up being the best choice to minimize the required amount of compression and hence minimizing energy costs; I had fully expected the smoother pipe materials to be more expensive overall but with lower energy costs. However, concrete, being the least expensive material, allowed for larger pipe diameters. Since $D < 3$ m, head loss depends chiefly on pipe diameter. The larger diameter of the concrete pipe has a greater impact on reducing pressure loss than the concrete's roughness had on increasing loss. The result is concrete, in spite of having the highest roughness, had the least amount of frictional losses, and thus required the least amount of compression. In addition, the optimal ratio of compressor diameter to pipe diameter was very close to unity with the concrete pipe, so the compressors can be the same size as the pipe without a significant reduction in efficiency.