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Dual Chambered Mug Analysis Report
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All of the modeling and analysis was done in Python, and was separated into two parts: thermal and mechanical. The thermal analysis dealt with the flow of heat, and the mechanical analysis dealt with size and mass distribution. In order to perform the numerical analysis of the mug, some design targets had to be chosen. For the most part, these were chosen intuitively using a combination of personal experience and comparisons with existing travel mugs in the market. The targeted features were the hot chamber capacity, the cooling chamber capacity, cooling time in both chambers, outer diameter, tipping angle, and mass. All of the relevant dimensions are labeled in the CAD drawings in Appendix A. Note, these drawings are meant to be illustrative, and may not precisely represent the final product design.

The main capacity of the mug – the volume of the hot chamber – was to be between 20 and 32 oz. The cooling chamber need to have enough capacity for filling it to be worthwhile, while remaining a fraction of the main capacity; 6 - 10 oz was chosen. A good drinkable temperature was considered to be around 60°C (140°F). The hot chamber should, when fully sealed, keep the beverage above this temperature for at least six hours. The cooling chamber on the other hand, should cool the beverage to this temperature in less than two; around the same amount of time as a long commute to work. Since the cooling chamber is placed on top of the hot chamber, top-heaviness is a concern, especially when the cooling chamber is full. For this reason, the tipping angle of $\gtrapprox 20°$ was decided upon for when the cooling chamber is completely full. An outer diameter of 6 - 12 cm was chosen so the bottle could be easily held without great effort. Finally, the mass of the entire mug, when empty of any contents, should be $\lessapprox 500$ g.

Mechanical Analysis

Certain parameters were allowed to change while others were kept fixed. The height of the cooling chamber ($h = 5$ cm), the thickness of the stainless steel ($t = 0.5$ mm), thickness of the separator ($t_{sep} = 1$ cm), and the thickness of the lid ($t_{lid} = 2.5$ cm) were all chosen in advance. Each of these were varied somewhat before settling on a value, but there was either no real benefit to changing these; they either made no difference, or they simply required changing another parameter in a minor way. The hot chamber capacity, V_H, was not varied systematically, but the same analysis was applied to both a 20 and a 32 oz bottle. With all of these parameters fixed, it was possible to constrain all of the other dimensions – inner radius, outer radius, width of the vacuum insulation, and cooling chamber capacity – to the height of the hot chamber, H.

By varying the H, it was possible to see how all other relevant parameters were affected. Figure 1 shows the dimensions and mechanical properties for a 20 oz (top) and 32 oz (bottle for the range $H \in [0.1, 0.3]$ m. For the 20 oz mug, with a hot chamber height of 15 cm, the mass is 480 g, the cooling chamber has a capacity of 7.3 oz, the outer diameter is 7.9 cm, and the tipping angle with a full cooling chamber is 19.7°. All of which meet, or nearly meet the design targets. For the 32 oz mug, meeting all of the design targets is more difficult. With a

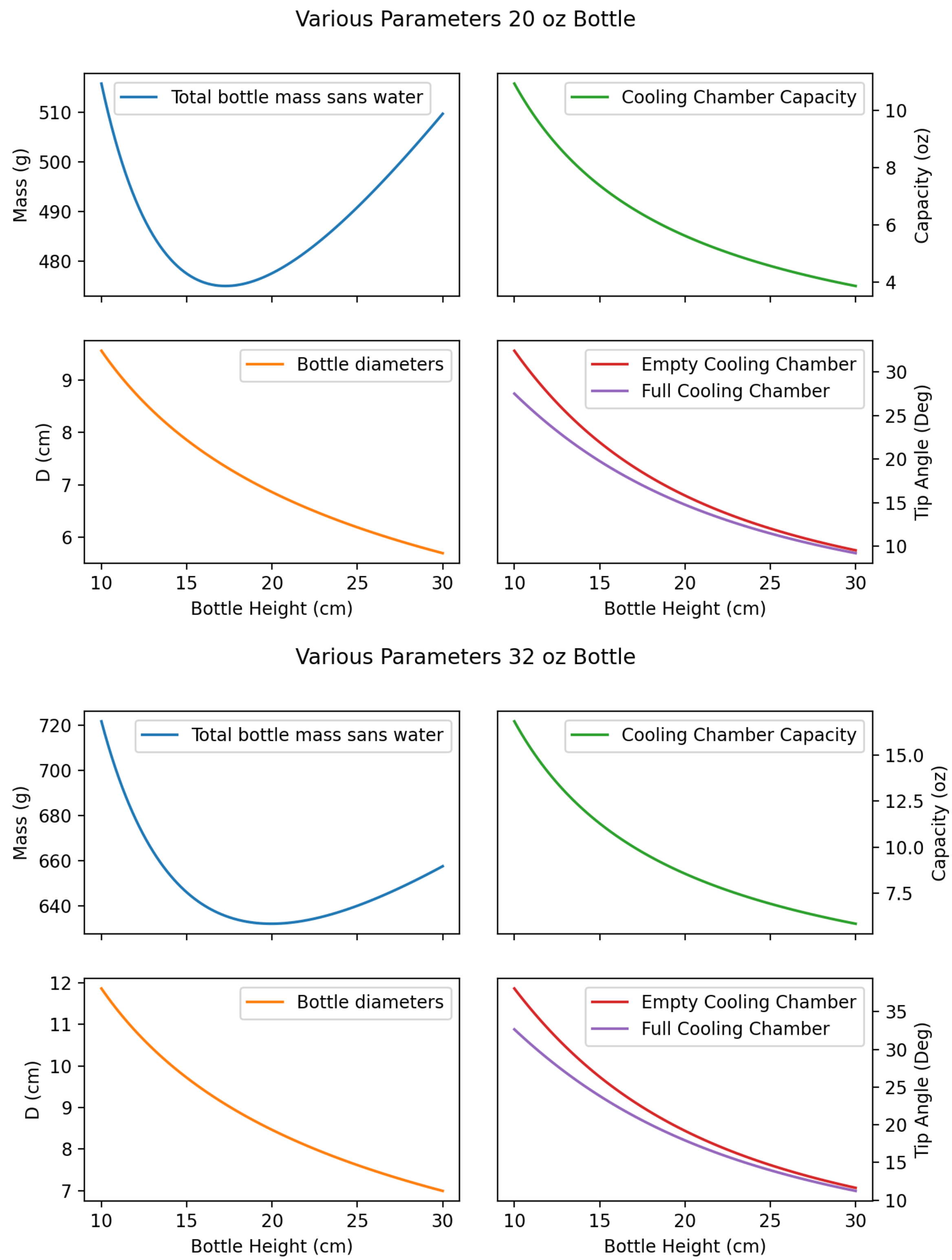

Figure 1: For each figure : Top left: Mass without beverage. Top right: Capacity of the cooling chamber. Bottom left: Outer bottle diameter. Bottom right: Tipping angle for an empty and full cooling chamber.

hot chamber height of 18 cm, the mass is 637 g, the cooling chamber capacity is 9.5 oz, the outer diameter is 8.3 cm, and the tipping angle is 20°. Overall, As the bottle height increases the diameter must decrease to maintain a fixed capacity. Since the cooling chamber height is also fixed, the smaller diameter results in a lower cooling chamber capacity. Also, as both height increases and diameter decreases, the center of mass raises while the base of support shrinks, causing the tipping angle to decrease as well.

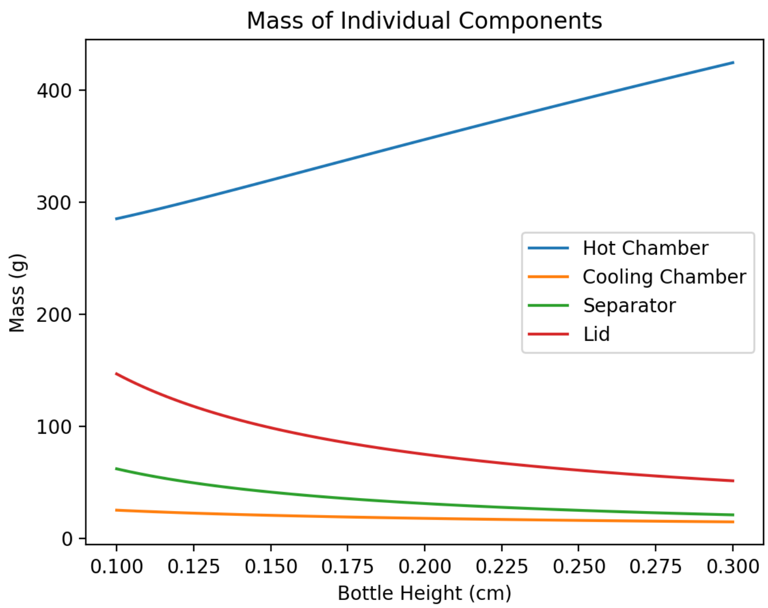

Figure 2: Mass versus bottle height for each component of the bottle.

The behavior of the bottle's mass is more difficult to describe, but can be understood better by examining the effect height has on each component of the bottle, summarized in Figure 2. For the hot chamber, because both the height and diameter are allowed to change, but the capacity is kept fixed, as the height increases, the total amount of material used in the chamber increases as well, resulting in a higher mass. However, all of the other components have a fixed height, so as the diameter of the hot chamber decreases with increasing height, the diameter of these components decreases as well. Since they have a fixed height, their volume simply decreases, along with their mass. The ultimate result is that total mass is highest either when the bottle is very tall (so the mass of the hot chamber dominates) or when the bottle is very wide (so the mass of the other components dominate). There is a certain height where the mass is more balanced between the hot chamber and the other components, which is why there is a minimum in the total mass of the bottle.

Thermal Analysis

For the thermal analysis, two scenarios were considered: an empty cooling chamber, and a full cooling chamber. In either scenario, the total volume of liquid was equal to the capacity of the hot chamber, as though none of the liquid had yet been consumed by the user. When the cooling chamber is empty, there is no gap between the fluid and the separator. Meanwhile, when the cooling chamber is full, there is no gap between the fluid and the lid. These two scenarios involved slightly different heat transfer paths, all of which are summarized schematically in Figure 3. The arrows shown are not meant to represent specific heat transfer modes, but rather the different paths the heat transfer was allowed to take.

In general, the system was modeled using thermal resistances in series. Throughout all of the thermal analysis ambient temperature was set at 20°C, and it was assumed that heat transfer reaches a steady state quickly enough that transient heat flow need not be considered.

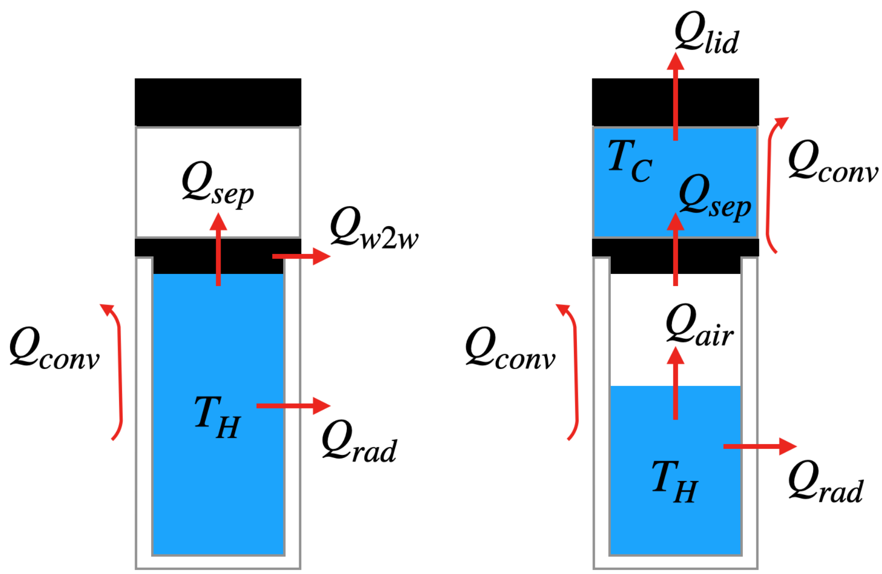

Figure 3: Mass versus bottle height for each component of the bottle.

When the cooling chamber is empty, the hot chamber is entirely full of fluid at a temperature T_H, and heat is able to leave this chamber in three ways: radiation between the inner and outer walls, conduction through the top lip of the chamber where the walls are directly connected, and conduction through the separator. Heat is removed from the system via free convection. The outside of the bottle was modeled as a vertical wall for the purposes of calculating the convective heat transfer coefficient.

For Q_{sep} and Q_{rad}, the typical 1D steady-state heat transfer equations for conduction and radiation were used to find the thermal resistances. For the wall-to-wall conduction, Q_{w2w}, the path of heat conduction was considered to be a thin cylinder with inner radius r, outer radius R, and thickness t. After integrating Fourier's law, the thermal resistance can be written

$$R_{w2w} = \frac{\ln\left(\frac{R}{r}\right)}{2\pi t k_{steel}} \tag{1}$$

When the cooling chamber is full of fluid at temperature T_C, the hot chamber is partially filled with fluid at T_H. In this scenario, heat flow from the hot chamber to the environment, from the cooling chamber to the environment, and from the hot chamber into the cooling chamber must all be accounted for. Most of these use the standard heat transfer equations to find thermal resistances. Free conduction of air inside the hot chamber causes the air to come to thermal equilibrium with the liquid quite rapidly, so the steady-state heat transfer through the air was modeled as conduction alone. Additionally, the effect of free conduction from the top of the cap was accounted for in this scenario, which was modeled as a hot plate.

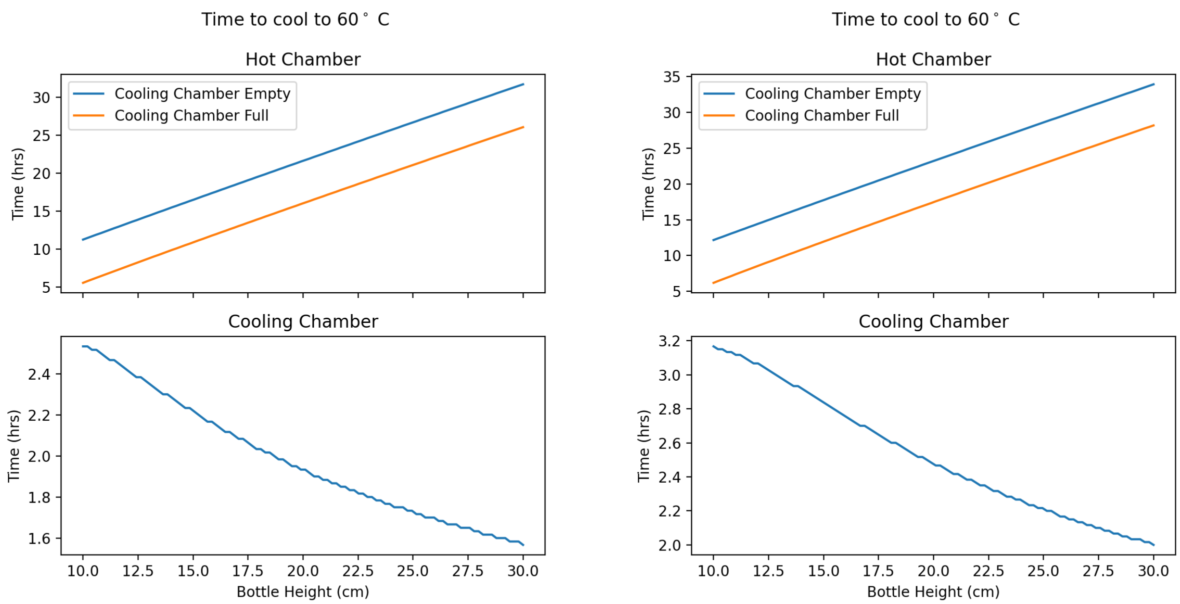

Figure 4: Left : 20 oz hot chamber capacity. Right : 32 oz hot chamber capacity.

The results of the thermal analysis were primarily characterized as the time required for a water-based beverage to cool to a target drinking temperature of 60°C in either chamber. The method for computing this was to compute the rate of heat transfer, and then determine the change in temperature of the volume of water in a given chamber over a certain timestep. This was done iteratively until the temperature of fluid in the chamber reached the target drinking temperature, with the time to cool being the sum of all previous timesteps. The timestep chosen was 60 seconds, temperature variations for smaller timesteps were negligible, resulting in a long computation time. This process was repeated for the same range of hot chamber heights used to generate Fig. 1, and the results are shown in Figure 4.

As the bottle becomes taller, and the diameter decreases, Q_{sep} and Q_{w2w} – normally the most significant terms for heat loss from the hot chamber – also decrease. Thus, the time to cool for the hot chamber increases with increasing height. For the cooling chamber on the other hand, since the height remains constant, a decreasing diameter means a decreasing capacity, so there is less liquid to cool and it cools faster as a result. In order for liquid in the cooling chamber to reach 60°C in two hours or less, the hot chamber height would need to be 18 cm for the 20 oz bottle and nearly 30 cm for the 32 oz bottle. A 30 cm tall chamber would be highly impractical, but for the 18 cm height mentioned earlier the cooling time is 2.5 hours, which is not unreasonably long. For an 18 cm, 20 oz bottle, the tipping angle when the cooling chamber is full reduces to 16°C, and the cooling time for a 15 cm chamber is 2.2 hours. Overall, the 15 cm height is preferable to make the bottle more stable; the cooling time in the cooling chamber is still much less than that of the hot chamber.

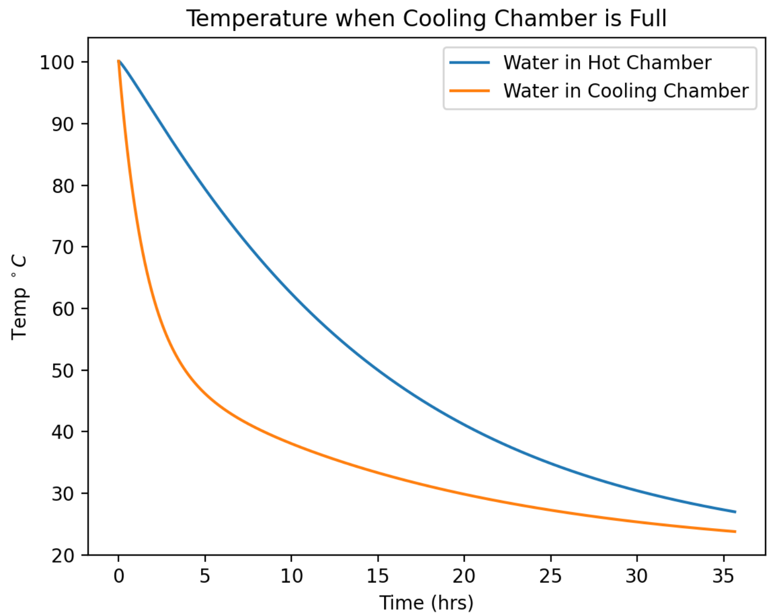

Figure 5: Temperature versus time for a 15 cm tall, 20 oz capacity hot chamber when the cooling chamber is full.

In order to get a full sense of how the heat transfer progresses with time, Figure 5 shows how the temperature in both the hot and cooling chambers change over time for a 20 oz bottle with a 15 cm tall hot chamber. The 32 oz chamber shows the same trend, only the timeframe differs. This image provides an excellent visual summary of the dual-chambered mug concept; the liquid in the cooling chamber drops in temperature very rapidly over a couple of hours while the liquid in the hot chamber drops much more slowly and at a steadier rate.

Material	**Use**	**Amount Needed in Weight (g)**	**Cost per g**	**Cost of Material**
Stainless Steel	Inner and outer walls of the insulated chamber	368	$0.005	$1.84
Polypropylene	Lid and separator	175	$0.002	$0.35
Polyethylene	Walls of the cooling chamber	190	$0.001	$0.19
Silicone	Gaskets for lid and separator	10	$0.003	$0.03
			Total Cost	$2.41

Figure 6: Bill of materials for a single 20 oz bottle.

Materials

Stainless steel, aluminum, polyethelene, and tempered glass were all considered possible materials for both the hot chamber and the cooling chamber. For the hot chamber, the material needed to be thin enough to create a the vacuum-insulated gap while remaining durable. Additionally, since one of the most significant modes of heat transfer from the hot chamber is conduction between the walls, the material should have a relatively low thermal conductivity. Of the materials listed, only stainless steel meets all of these requirements.

For the cooling chamber, it is important for the material to be durable and lightweight. Because increasing heat loss is the objective with the hot chamber, it would seem as though a high thermal conductivity would be essential as well, but heat loss from the cooling chamber is mostly limited by convection, which does not depend on the material. Stainless steel and tempered glass are each relatively dense, causing the bottle to be more top-heavy. Between aluminum and polyethelene, the latter was chosen because of its greater durability, lower density, and because it can be made clear, which could be a more aesthetically pleasing choice. The settled upon bill of materials with corresponding costs are shown in Figure 6.