WID2002 Computing Mathematics 2

FOLLOWING THE FLOW: UNDERSTANDING INFECTIOUS DISEASE

Group Binary Wizards

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OVERVIEW

Presentation Outline

- Problem Statement
- Proposed Solution and Method
- Results and Discussion: Results Analysis
- Strengths and Limitations
- Future Work
- Conclusion

PROBLEM STATEMENT

How do the susceptible, transmission, infectious and recovery rates of disease affect society?

How effective are safety measures in reducing disease transmission and controlling outbreaks?

What is the impact of maternally derived immunity on the dynamics of infectious diseases?

PROPOSED SOLUTION AND METHOD

COMPARTMENTAL MODEL

Equation Based Model

Ordinary Differentials Equations (ODE)

- Forward Euler Method
- K = compartment states
- t = time
- c = constant / time interval

$$\frac{\partial K}{\partial t} = \lim_{c \to 0} \frac{K(t + c) - K(t)}{c}$$

$$K(t + c) = K(t) + c\frac{\partial K}{\partial t}$$

$$c = 1$$

$$K(t + 1) = K(t) + \frac{\partial K}{\partial t}$$

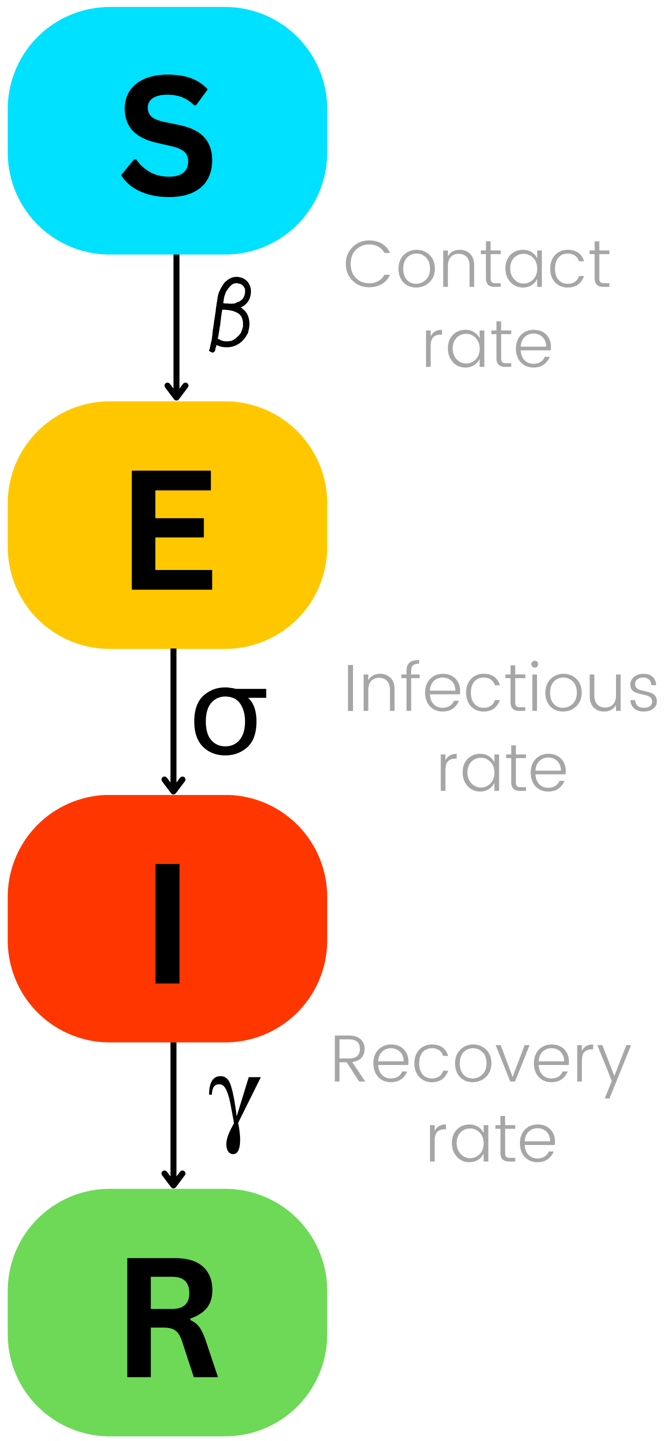
SEIR

$$S \xrightarrow{\beta} E \xrightarrow{\sigma} I \xrightarrow{\gamma} R$$

Assumptions:

- No flow back to the Susceptible (S)
- Population are perfectly mixed
- Individuals can only be infected once

SEIR



Susceptible, S

- Individuals who are yet to be exposed to the disease
- Transmitted through contact rate, beta

Exposed, E

- Representing individuals exposed to the disease but not yet infectious
- Rate of exposed individuals becoming infectious (infectious rate), sigma

Infected, I

- Individuals are currently infected
- Capable of transmitting the disease to susceptible individuals
- Influenced by the recovery rate, gamma

Recovered, R

- Recovered and gain permanent immunity
- No longer transits to the susceptible state

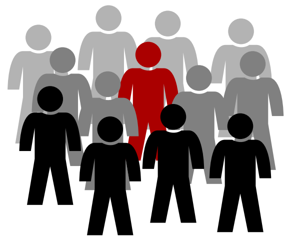
SEIR

$$S \xrightarrow{\beta} E$$

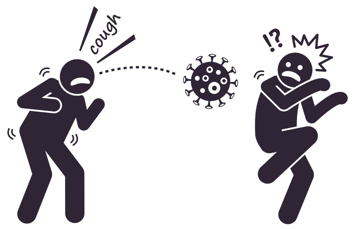
Since no flow back to S, a negative sign is employed in the equation to indicate the decrement

$$\text{Rate of change of Susceptible} = \frac{\partial S}{\partial t} = -\beta \boxed{\frac{SI}{N}}$$

The proportion of infected individuals, I/N is perfectly mixed with susceptible individuals, S



Contact rate (Mixing) is determined by the transmission rate, beta



SEIR

$$\xrightarrow{\beta} \boxed{E} \xrightarrow{\sigma}$$

$$\text{Rate of change of Exposed} = \frac{\partial E}{\partial t} = \frac{\beta S I}{N} \boxed{- \sigma E}$$

The rate of individuals leaving E is determined by 2 factors:

- Infectious rate, sigma, σ
- The current number of exposed individuals, E

The rate of individuals entering E will be equal to the rate of individuals leaving the S compartment

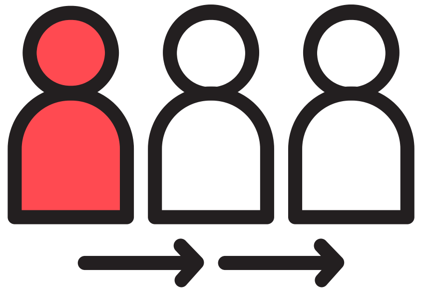
SEIR

$$\xrightarrow{\sigma} \boxed{I} \xrightarrow{\gamma}$$

Exposed individuals enter the infectious compartment, I when they become infectious

$$\text{Rate of change of Infectious} = \frac{\partial I}{\partial t} = \sigma E \boxed{- \gamma I}$$

To leave compartment I (recovered), infected individuals will transition to the recovered compartment, R at a rate determined by the recovery rate, gamma



$$\xrightarrow{\gamma} \boxed{R}$$

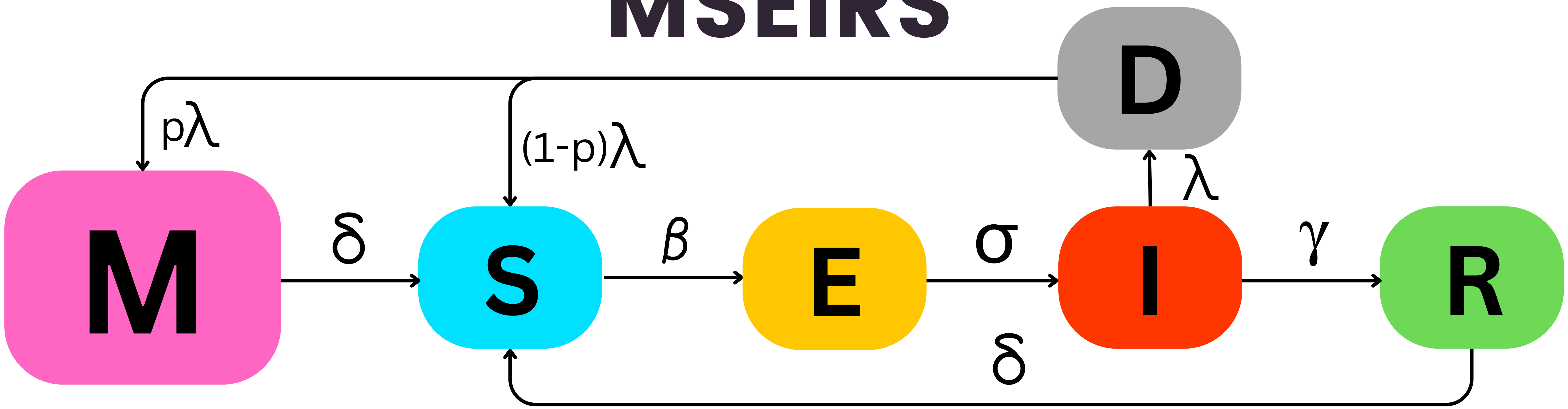
$$\text{Rate of change of Recovered} = \frac{\partial R}{\partial t} = \gamma I$$

- Recovery of infected individuals is determined by the recovery rate, gamma
- No individuals will leave the R compartment as they can only be infected once

MSEIRS

Modelling Maternally-Derived Immunity

MSEIRS



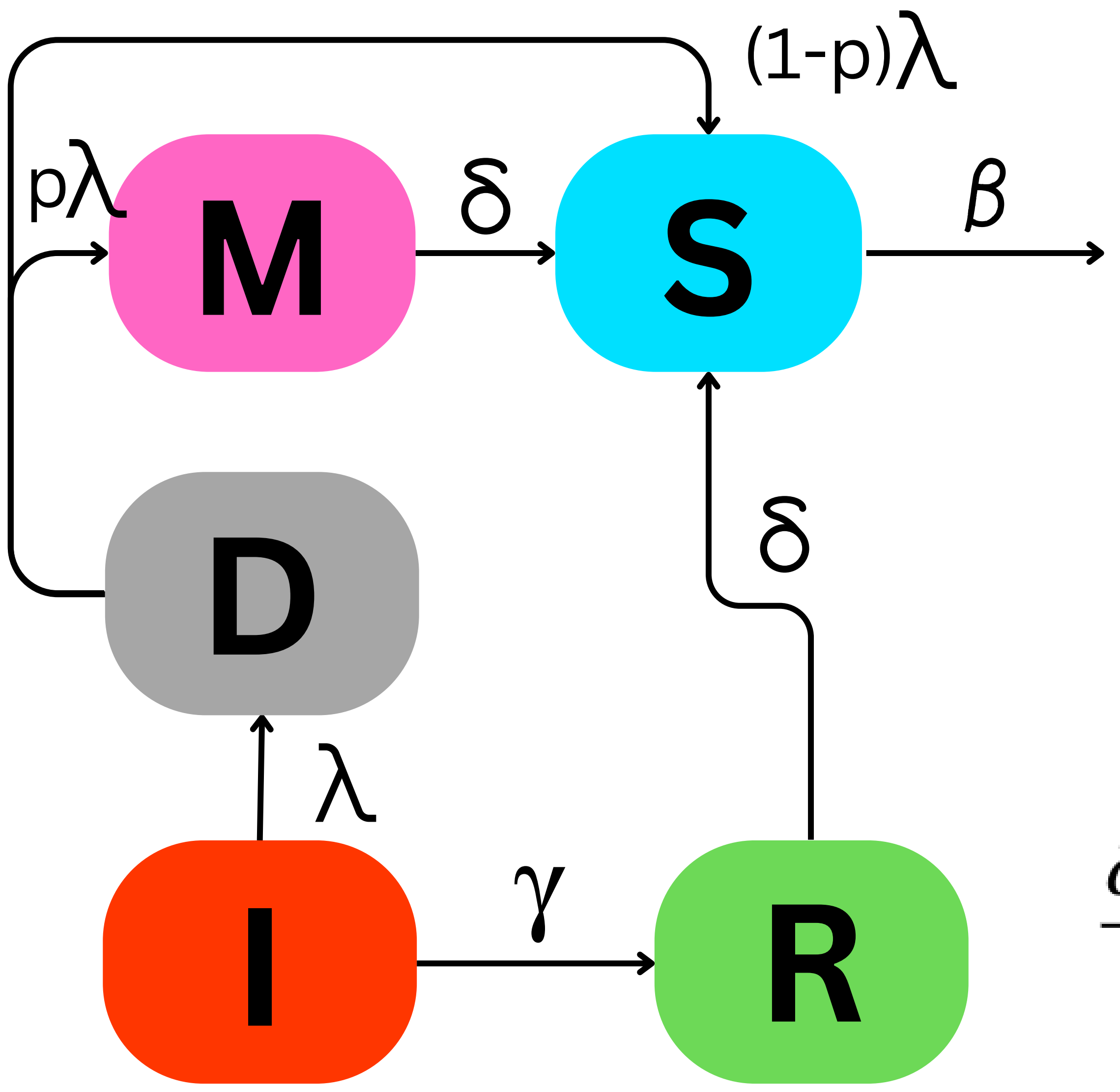
Maternally-derived Immunity, M:

- Representing individuals with maternally-derived immunity
- lambda: rate of new-born
- p: the probability of babies with maternally-derived immunity
- delta: immunity weakened rate

Additional Assumptions:

- Birth rate == death rate
- Maternally-derived immunity's weakened rate == recovery-acquired's
- Probability of babies born with maternally-derived immunity == fraction of recovered individuals in the population

MSEIRS



$$\frac{\partial M}{\partial t} = \boxed{-\delta M} + \boxed{\frac{R}{N}\lambda I}$$

- Individuals in M lose the immunity at the rate of delta, hence the -delta*M
- New individuals are added at the rate of lambda, and among them, p=R/N (ratio of the recovered population) will have immunity from their mothers

$$\frac{\partial S}{\partial t} = \boxed{-\frac{\beta SI}{N}} + \boxed{\delta M} + \boxed{\frac{N-R}{N}\lambda I} + \boxed{\delta R}$$

- S increases with:
  - Babies from Maternally-derived immunity losing the immunity
  - New-born babies without immunity (1-R/N)
  - Recovered individuals losing immunity

$$\frac{\partial D}{\partial t} = \lambda I$$

- Infectious individuals die at rate of lambda

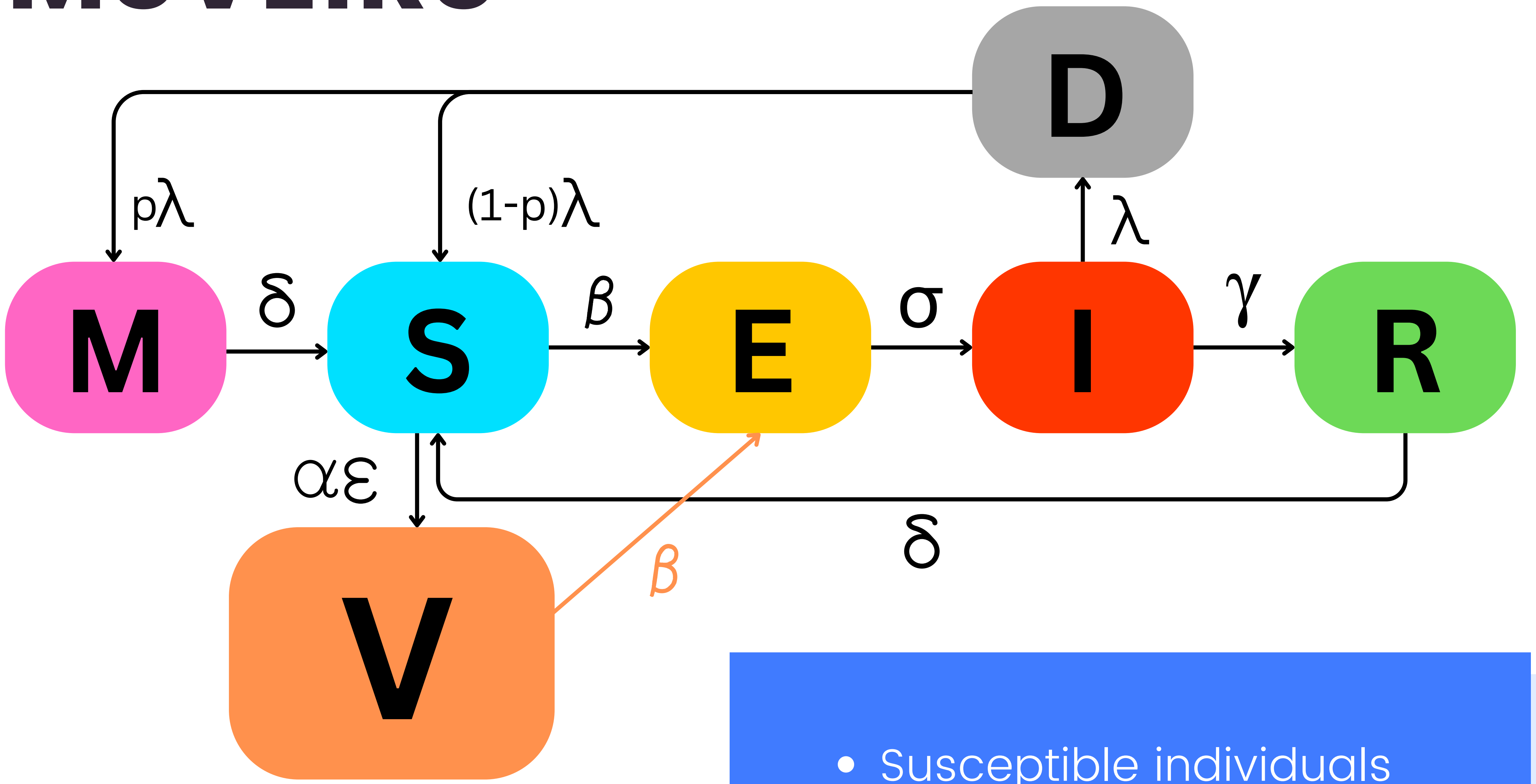
$$\frac{\partial I}{\partial t} = \boxed{\sigma E} \boxed{-\gamma I} \boxed{-\lambda I}$$

- I decreases with:
  - Infectious individuals dying at rate of lambda
  - Infectious individuals recovering at rate of gamma

$$\frac{\partial R}{\partial t} = \boxed{\gamma I} \boxed{-\delta R}$$

- R increases as infectious individuals recovered
- R decreases as the individuals lose the immunity

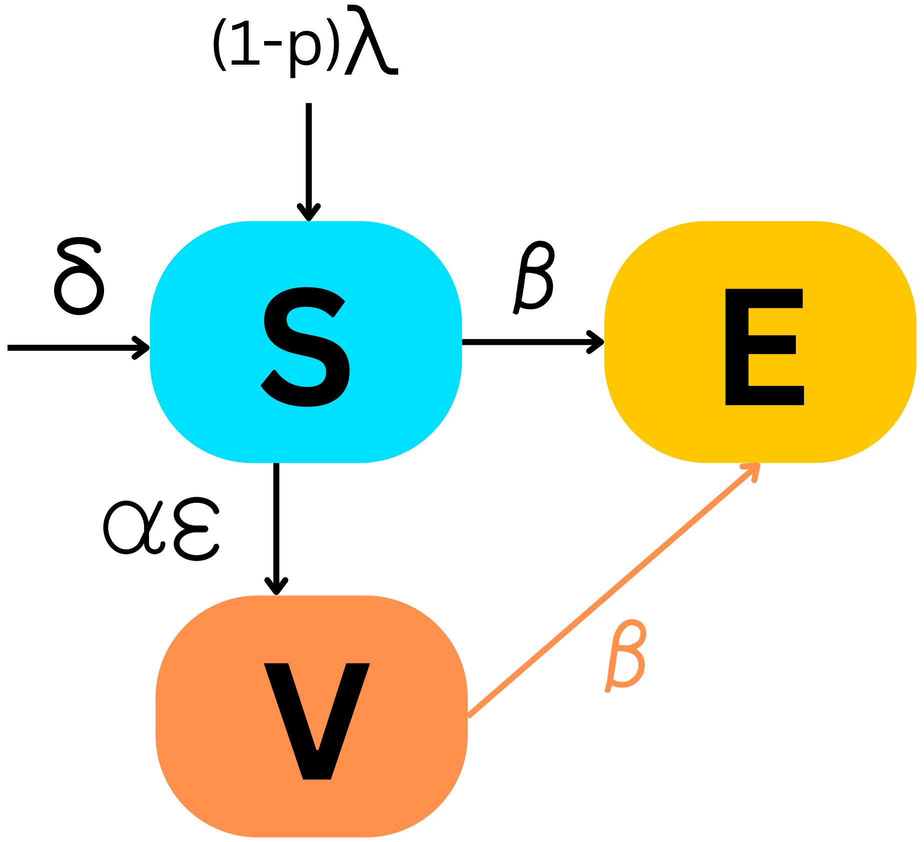
MSVEIRS



Vaccinated, V

- Susceptible individuals receive vaccination
- Vaccination rate, alpha
- Vaccine effectiveness, epsilon

MSVEIRS



$$\frac{\partial S}{\partial t} = -\frac{\beta SI}{N} + \delta M + \frac{\lambda I(N-R)}{N} + \delta R \boxed{-\alpha\epsilon S}$$

- Susceptible individuals may receive vaccinations at a rate determined by vaccination rate, alpha
- The effectiveness of vaccines is represented by epsilon

$$\frac{\partial V}{\partial t} = \alpha\epsilon S - \frac{\beta VI}{N}$$

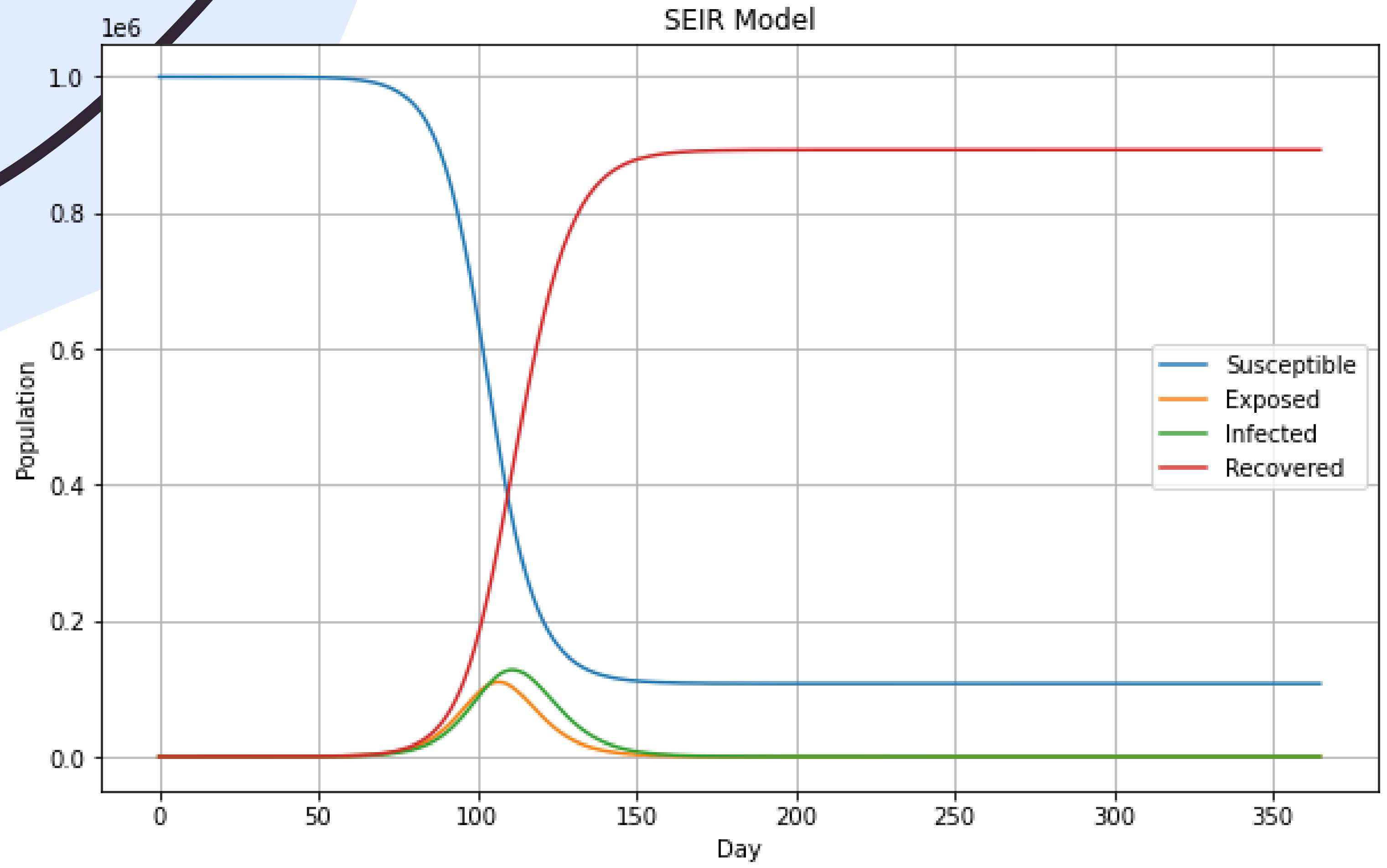
- Susceptible individuals receive vaccination to become vaccinated individuals
- Vaccinated individuals may be exposed to the disease and leave the compartment V and enter compartment E

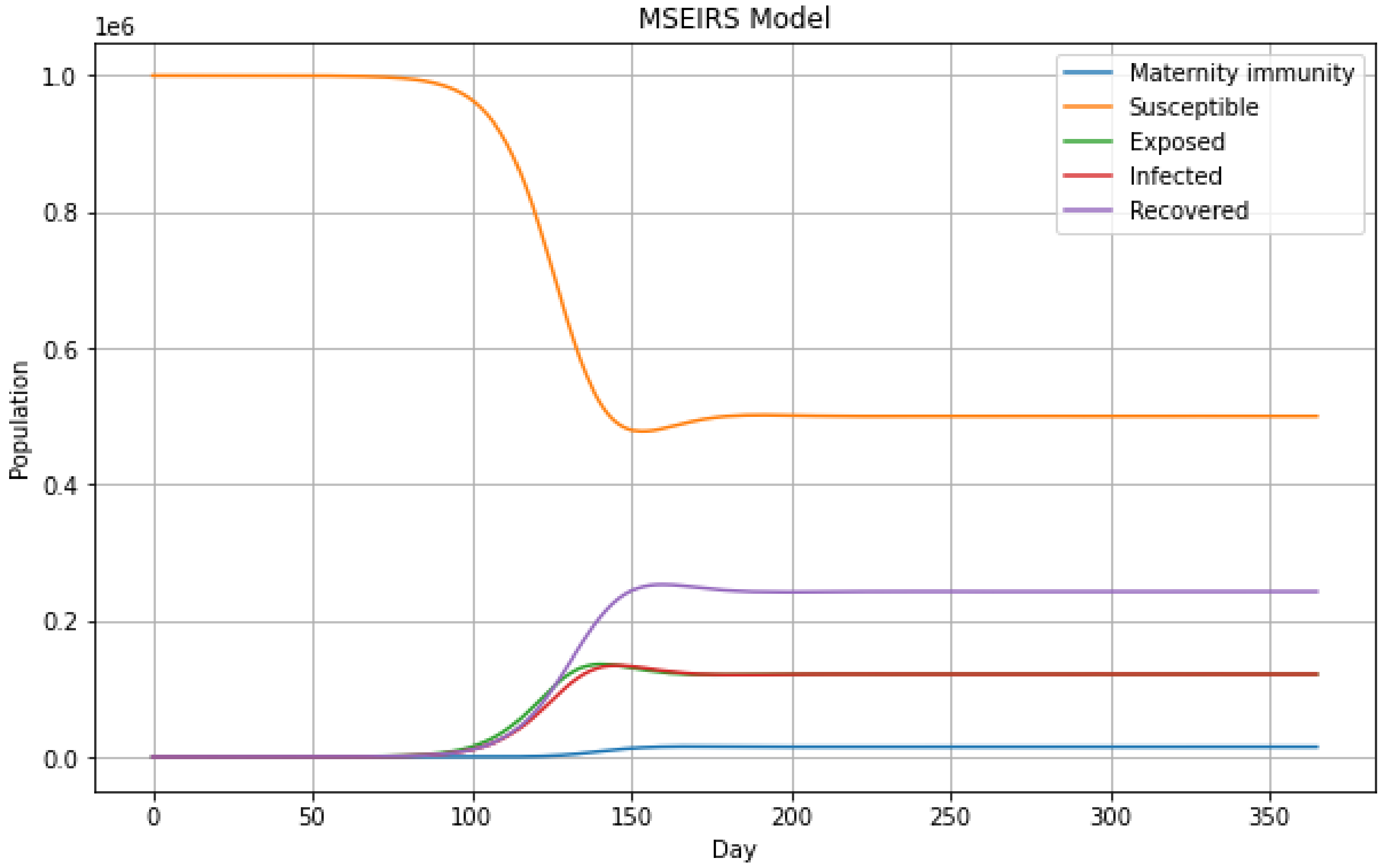
$$\frac{\partial E}{\partial t} = \frac{\beta SI}{N} - \sigma E + \boxed{\frac{\beta VI}{N}}$$

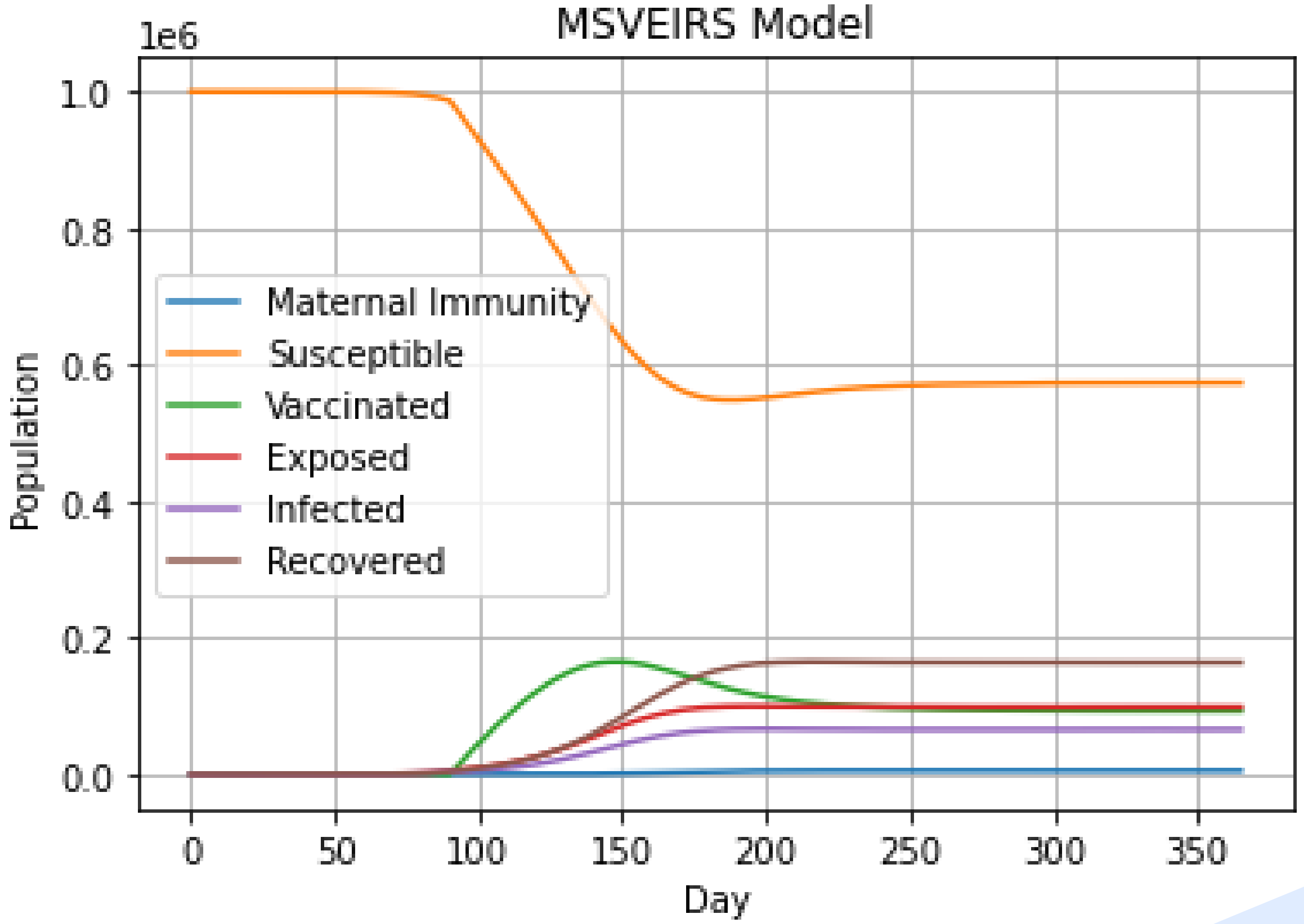
The proportion of vaccinated individuals, V/N who may transition to the Exposed compartment, E due to having contact with infectious individuals, I is determined by the transmission rate, beta

RESULTS AND DISCUSSION

GRAPH OF SEIR, MSEIRS & MSEIRS







CHANGING RATES IN REDUCING MORTALITY

Parameter	Rate Change	Number of Death
$\delta \downarrow$	0.05	1,036,742
	0.15	1,800,412
$\beta \downarrow$	0.45	1,217,903
	0.55	1,752,636
$\sigma \downarrow$	0.20	1,336,456
	0.30	1,642,023
$\gamma \uparrow$	0.15	2,364,798
	0.25	902,859

Table showing the number of deaths for different rates (±5 of the control set). Number of death of control set is 1, 511, 541.

Objective

Reduce the total number of deaths

Method

Changing the respective rates except for the birth and death rate

Results

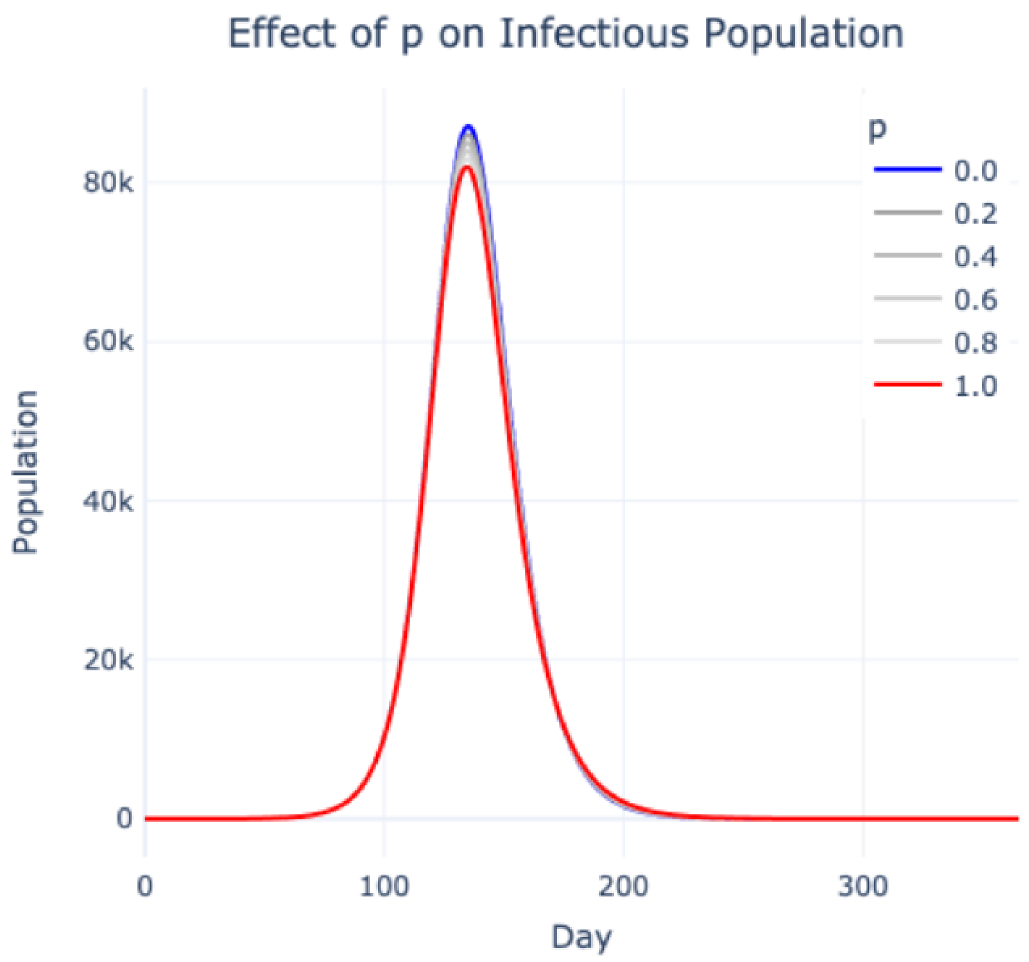
To obtain the lowest number of deaths:

- Increase the recovery rate (γ)
- Decrease the transmission (β), weakened immunity (δ) and infectious rate (σ)

EFFECT OF MATERNALLY-DERIVED IMMUNITY

State, p	0.0	0.2	0.4	0.6	0.8	1.0	Change (%)
M	0.00	7.54	14.90	22.07	29.07	35.91	100.00
S	231.92	238.75	245.39	251.84	258.04	263.86	13.77
E	89.87	88.72	87.65	86.59	85.54	84.52	-5.95
I	87.07	86.02	84.98	83.94	82.92	81.91	-5.93
R	768.08	761.25	754.61	748.16	741.90	735.83	-4.20

Table showing the maximum population for each state (except Susceptible with the minimum population) in thousands (K), when varying the probabilities of babies acquiring immunity.



Graph of the effect of p on Infectious population

- As **p(M) increases**, the maximum population of **E, I, and R decreases** while the minimum population of **S increases**
- Maternally-derived immunity **affects Susceptible the most(+13.77%)**, followed by Exposed(-5.98%), Infected(-5.93%) and Recovered(-4.20%)
- Diseases affect lesser individuals
- R decreases as there are lesser infected individuals

EFFECTS OF INTERVENTION METHODS

Social Distancing

Reduced contact, less likely to be exposed to and contact the virus.

Lockdowns

Significant reduction in interpersonal contact among individuals. Transmission and infection rates are greatly reduced.

	Death $\lambda \downarrow$	Trans. $\beta \downarrow$	Infect. $\sigma \downarrow$	Immunity $\delta \uparrow$	Recovery $\gamma \uparrow$
No Intervention	0.05	0.500	0.2500	0.1	0.20
Social Distancing	0.05	0.375	0.1875	0.1	0.20
Mask	0.05	0.125	0.0625	0.1	0.20
Lockdown	0.05	0.100	0.0500	0.1	0.20
Vaccination	0.05	0.450	0.2000	0.1	0.25

Table with overview of rates used for each intervention method. No intervention is the control set where the settings are constant throughout the experiment. Up arrows and down arrows represent the best direction in changing the rates to control the disease.

Masks

Reduce spreading of disease from the air, reduce transmission and infection rates.

Vaccinations

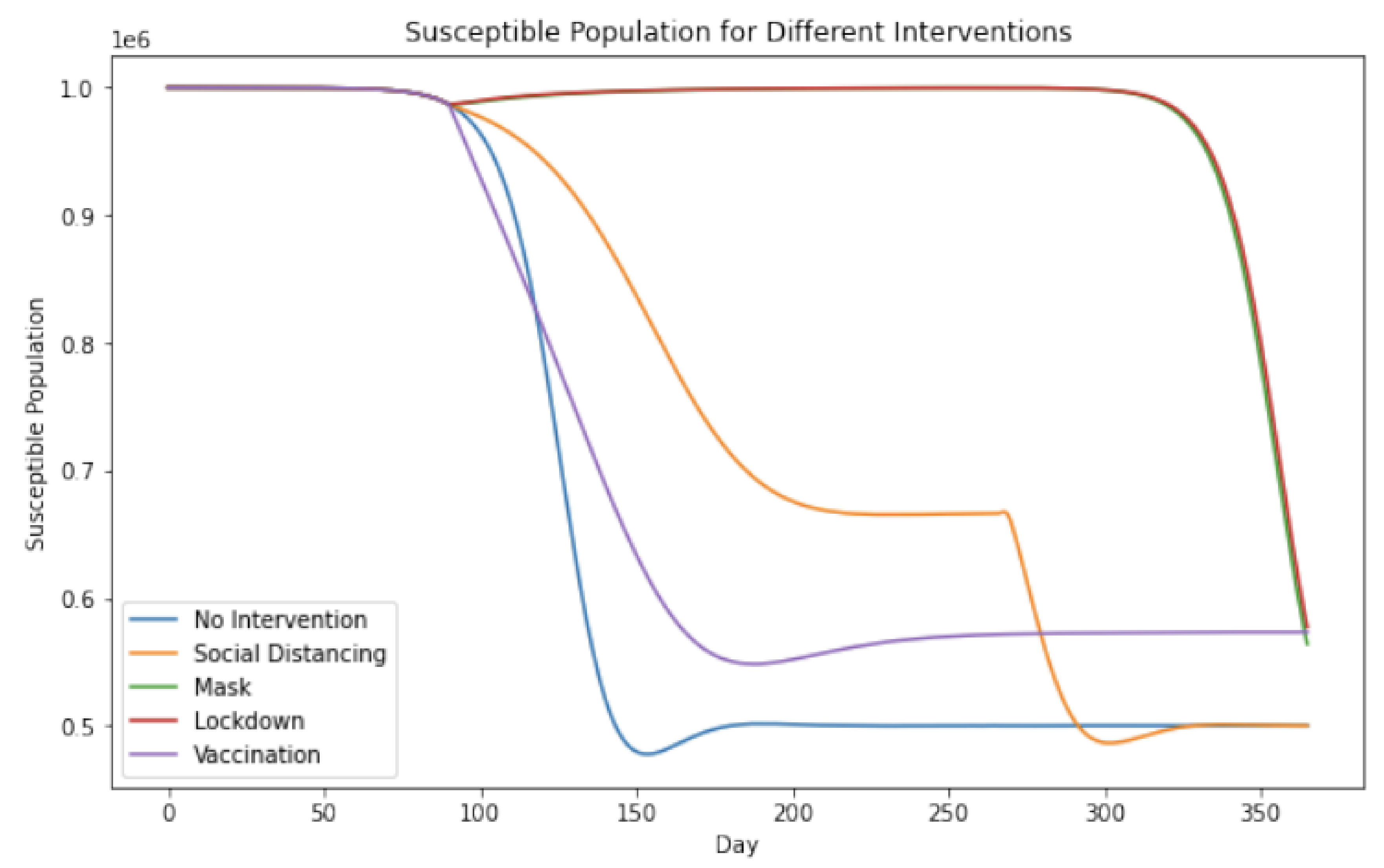
Reduces transmission and infectious rates. Contributes to milder symptoms, which leads to an elevated recovery rate.

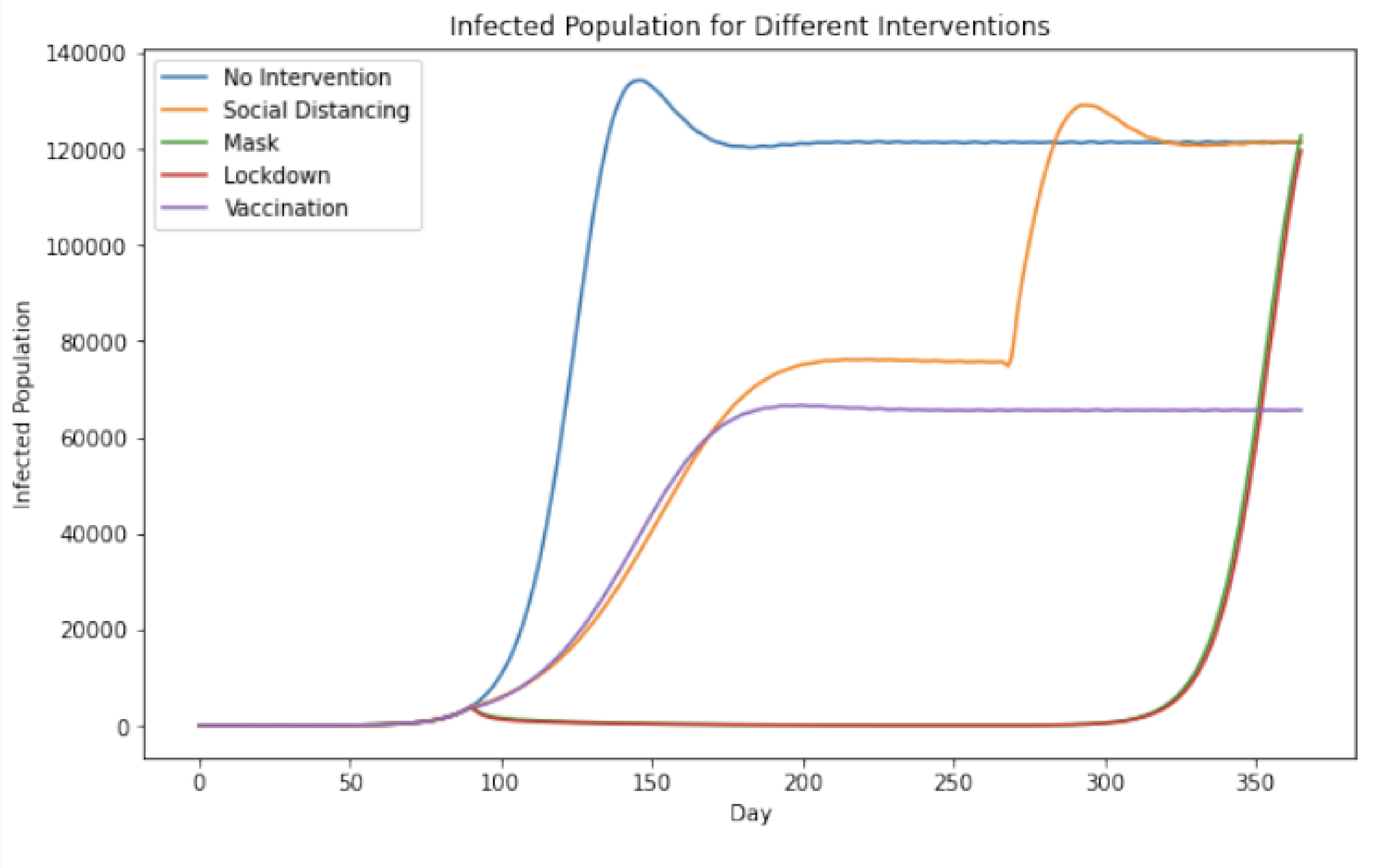
EFFECTS OF INTERVENTION METHODS

$$\text{Percentage Change (\%)} = \frac{\text{Intervention X - Intervention Control}}{\text{Intervention Control}} * 100$$

Intervention	Susceptible		Infected	
	# Individuals	Change (%)	# Individuals	Change (%)
No intervention	477,812	0	134,301	0
Social Distancing	486,395	1.80	129,090	-3.89
Mask	564,029	18.04	122,767	-8.59
Lockdown	577,355	20.83	119,691	-10.88
Vaccination	548,357	14.76	66,596	-50.41

Table showing results of comparison between interventions.





STRENGTHS AND LIMITATIONS

Strengths

- Flexible model structure, easy incorporate new compartments or splits existing compartments
- Can benefit from well-established approaches
- Can be implemented with novel approaches and modified based on specific needs

Limitations

- Can only predict a single peak when employed by itself
- Over-parameterization can bring challenges in model fitting due to the need to estimate numerous parameters
- Time-invariant formulation of hyperparameters restricts the model from accurately capturing the evolving nature of the epidemiological phenomenon

FUTURE WORKS

Suggestions on current models

Additional compartments

Additional compartments adding to the SEIR model (stages of infection, modes of transmission, age-based transmission rates and vaccination).

Integrate machine learning algorithms

Neural networks provide flexibility and adaptability, allowing for various modifications and functions to be incorporated into the models.

Collaborations between medical field and technology companies

Lead to development of epidemiologic model extensions using popular machine learning frameworks.
Enable researchers to experiment, reproduce results and contribute to advancements in healthcare research.

CONCLUSION

Mathematical Models

SEIR, MSEIRS, MSVEIRS

Essential for simulating disease outbreaks and analysing their dynamics.

Manipulate the parameters to observe how changes in the susceptible rate, transmission rate, infectious rate, and recovery rate affect mortality.

- Increasing the recovery rate (γ) is most effective in reducing number of deaths.
- Lockdown measures are most effective in reducing transmission rate (β).
- Vaccination is most effective in reducing the infectious rate (σ).
- Maternally derived immunity has an impact on the dynamics of infectious diseases.
- Simple model may overestimate the number of infectious individuals by not including maternally derived immunity.
- The epidemiological models need to be updated continuously with consideration of various factors to reflect the real-world situation.

THANK YOU