

**How can we determine the optimal target reliability in reliability-based design optimization?
Reliability-based design for market systems (RBDMS) and case study on electric vehicle design**

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Abstract

Reliability-based design optimization (RBDO) allows decision-makers to achieve target reliability in product performance under engineering uncertainties. However, existing RBDO studies assume the target reliability as a given parameter and do not explain how to determine the optimal target reliability. From the perspective of the market, designing a product with high target reliability can satisfy many customers and increase market demand, but it can generate a large cost leading to profit reduction of the company. Therefore, the target reliability should be a decision variable which needs to be found to maximize the company profit. This paper proposes a reliability-based design for market systems (RBDMS) framework by integrating RBDO and design for market systems (DMS) approaches to find the optimal target reliability. The proposed RBDMS framework is applied to electric vehicle (EV) design problems to validate effect of the target reliability on company profit – or market share – and engineering performances of EV. Several observations about the optimal target reliability are presented from the case study with various scenarios. From the EV design case study, it is verified that the proposed RBDMS framework is an effective way of finding the optimal target reliability that maximizes the company profit, and the optimal target reliability varies depending on the situation of market and competitors.

Keywords: Reliability-based design optimization (RBDO), design for market systems (DMS), electric vehicles, target reliability, uncertainty

Nomenclature

SoC: state of charge of battery

DoD: depth of discharge of battery

D: *DoD*

F: additional fraction of nominal capacity

P: penalty factor for deeper *DoD*

A: capacity loss factor

σ : $1 - A$

Π : profits

D : market demands

MC : manufacturing cost

C : compensation costs

\mathbf{X} : deterministic decision variable vector

\mathbf{X}_{power} : powertrain design variable vector

R : target reliability

W : warranted battery lifetime

$Price$: price

P_F^{Target} : target probability of failure for probabilistic constraints

\mathbf{g} : inequality constraint functions

\mathbf{G} : probabilistic constraint functions

N_s : number of battery cells in series

N_p : number of battery cells in parallel

FR : final gear ratio

\mathbf{RP}_e : random parameter vector of engineering model

\mathbf{P} : matrix of probabilistic performances

\mathbf{P}_{MPGe} : vector of probabilistic MPGe

\mathbf{P}_{range} : vector of probabilistic driving range

\mathbf{P}_{speed} : vector of probabilistic top speed

\mathbf{P}_{accel} : vector of probabilistic acceleration

\mathbf{P}_{Batt} : vector of probabilistic battery lifetime

\mathbf{A} : advertised attribute vector

\mathbf{A}_{eng} : vector of advertised attributes determined from engineering model

$f_{engineering}$: engineering model

$f_{attribute}$: attribute model

$f_{marketing}$: marketing model

$f_{\mathbf{X}}(\mathbf{x})$: joint probability density function

Ω_F : failure set

1. Introduction

Engineering design generally aims to maximize functionality of a system while satisfying constraints. To enhance the functionality of an objective system, deterministic optimization has been successfully used in engineering fields as it often provides optimal solutions at the boundaries of design constraints (Lee and Jung 2008). However, small variations in design variables and other parameters are derived from many uncertainties such as geometrical tolerance, physical properties of materials, and operating conditions, often leading to design failure. Currently, the stochastic nature of engineering systems is naturally considered when solving optimization problems (Frangopol and Maute 2003), and the target reliability of a system is significantly considered. Therefore, reliability-based design optimization (RBDO) maximizes the functionality or utility of a system while satisfying the target reliability regardless of inherent uncertainties in the design variables and parameters. In RBDO, the reliability analysis focuses on the evaluation of probabilistic constraints to guarantee that the target reliability is satisfied, whereas optimization focuses on searching for optimal solutions. RBDO has been widely used in various engineering fields such as aerospace (Allen and Maute 2002; Pettit 2004; Lee et al. 2009; Missoum et al. 2010), civil (Ellingwood and Galambos 1982; Nowak 1995), and mechanical engineering (Youn et al. 2004, 2005; Dong et al. 2007; Noh et al. 2009; Lee et al. 2010, 2013; Yoo and Lee 2014; Shin and Lee 2014, 2015; Lim et al. 2015), and in various applications such as composite structures (Qu et al. 2003).

On the other hand, design for market systems (DMS) emerged from the objective of maximizing specific values such as profit or social welfare from the perspective of manufacturers or producers (Lewis et al. 2006; Frischknecht et al. 2010; Kang et al. 2013, 2014). This research area focuses more on selling products or services rather than optimizing products based on their performances. To determine the optimal product design for a market system, an optimization problem that maximizes specific profit or social welfare while satisfying engineering or other constraints is formulated into a mathematical problem. Quantitative market demand models are commonly utilized in the marketing field for estimating customer preferences (market demand) as a function of design attributes and product prices. Therefore, expressing design attributes as functions of decision variables or parameters must be performed first to plug the market demand models into the product design problem. DMS has

been successfully utilized for electric vehicle (EV) and hybrid EV design problems (Kang et al. 2015, 2016, 2017, 2018; Helveston et al. 2015).

However, existing RBDO studies do not suggest how to determine the target reliability, and thus designers use predetermined target reliability for design optimization. In addition, existing DMS studies are more focused on maximizing profit rather than achieving reliable engineering performances. Therefore, a new design framework is needed to overcome the aforementioned disadvantages of each method and to come up with a new product design which can maximize a manufacturer's profit while satisfying required engineering performances. For this purpose, a reliability-based design for a market systems (RBDMS) framework, which is the integration of RBDO and DMS to find the optimum target reliability from the perspective of the market, is proposed in this paper.

The target reliability plays a key role in integrating RBDO and DMS in the RBDMS framework. In an engineering model, the target reliability is used in the probabilistic constraints of RBDO and determines product performances. As product performances improve, market demand for the product increases but the cost increases as well. On the other hand, in a marketing model, product reliability is equal to the target reliability in RBDO, while it affects the market demand because it is one of the attributes evaluated by customers. As the product reliability improves, market demand for the product grows, if the price does not increase. Therefore, the engineering model and marketing model are strongly coupled through the target reliability in RBDMS, and thus it should be identified how the target reliability affects each model. This is why the target reliability is a decision variable in RBDMS to maximize profit and product performances simultaneously. The proposed RBDMS framework is verified through a case study of EV design which shows a new EV design that maximizes profit of an EV manufacturer while ensuring reliability of advertised product performances.

The remainder of this paper is organized as follows. Section 2 introduces the engineering model and uncertain factors. Section 3 presents the marketing model for estimating customers' preferences. Section 4 provides the RBDMS formulation and modeling assumptions. In Section 5, the proposed RBDMS framework is applied to an EV design case, and the optimal results of three design methods are compared. Finally, Section 6 concludes the paper and describes future research directions.

2. Engineering model

2.1 EV simulation model

To understand how uncertainties at the engineering level affect EV performances, two engineering models are presented: 1) an EV performance model that simulates vehicle performances while considering uncertainties in battery and driving characteristics for different mechanical designs, and 2) a battery degradation model that presents the life cycle of a Li-ion battery.

2.1.1. EV performance model

EV performances such as MPGe, driving range, top speed, and acceleration are determined by the design of powertrain, which contains a battery pack and motor that are connected to wheels through a final drive. To simulate such an EV performance model, we adapt the powertrain system of the Nissan Leaf and its specifications listed in Table 1 (Energy Efficiency & Renewable Energy 2011a, b). AMESim software and a battery degradation model explained in Section 2.1.2 are combined to modify our analytical EV performance model (AMESim 2016).

In the battery pack, the cells connected in series form a branch, and several branches are connected in parallel. The battery capacity is determined by the number of cells, which is directly related to the driving range of the EV. Furthermore, the array of cells in the battery pack influences the battery voltage and current limits, which affect the output power of the motor. The weight of the battery pack, which is proportional to the number of cells, also influences the total weight of the EV and in turn affects the EV acceleration and MPGe.

High speed and low torque output from the motor are transformed to low speed and high torque output through the final drive. The final drive ratio, which is one of the decision variables, is the ratio of the input and output speeds. Fuel economy, MPGe, is also related to the final drive ratio in terms of different energy consumptions.

Table 1 Specifications of EV model

Vehicle curb weight	1,631 kg
Frontal area	2.27 m ²
Rim diameter	406.4 mm
Tire width	205 mm
Coefficient of drag	0.29
Motor(s) type	Permanent Magnet AC Synchronous
Max. motor(s) power	80 kW
Max. motor(s) torque	280 Nm
Max. motor(s) speed	10,390 rpm
Rated cell capacity	33.1 Ah
Nominal cell voltage	3.8 V

2.1.2. Battery degradation model

The lifetime of a battery depends highly on the daily driving distance. Li-ion battery capacity decreases owing to increased cell impedance caused by solid-electrolyte interface growth, loss of accessible lithium ions, and degradation of electrical parts because of cycling (Ning et al. 2003; Lawder et al. 2014). The state of charge (*SoC*) is the amount of useful remaining charge compared to its initial fully charged state given by

$$SoC(t) = \frac{\int_{t_0}^t I(\tau) d\tau}{Q_0} \times 100 \quad (1)$$

where I is the charging current, Q_0 is the total charge of the battery, and $\int_{t_0}^t I(\tau) d\tau$ refers to the delivered charge.

The discharged battery capacity, which is the complement of *SoC*, that is, the depth of discharge (*DoD*), is defined as

$$DoD = SoC_{\text{initial}} - SoC_{\text{final}} \quad (2)$$

The capacity fade is related to the number of cycles and *DoD* of the batteries (Peterson et al. 2010). In general, an EV battery should be replaced when its capacity decreases to 80% of its initial capacity (Helveston et al. 2015).

The life cycle, which results from capacity fades with regard to the *DoD* of batteries, was theoretically and experimentally presented by Thaller (1983) as

$$Life\ Cycle = \frac{I + F - D}{(A + 2\sigma)(I + PD)D} \quad (3)$$

where D corresponds to the DoD of the battery, F is the additional fraction of the nominal capacity, P stands for the penalty factor for the deeper DoD , A is the capacity loss factor, and σ represents the standard deviation of $(1 - A)$. In this study, battery life is considered as the life cycle on the assumption that all drivers drive every day and that the battery is recharged once a day. This statement is reasonable in terms of rigorous battery lifetime estimation. Although the battery degradation model depends highly on specific battery chemistry, temperature, and storage conditions, these factors are ignored in this paper.

In this battery degradation model, DoD is calculated using the initial battery capacity and driving distance. By utilizing MPGe, which is predetermined using the EV performance model, the given driving distance of the designed EV can be converted into energy consumption; and using the initial capacity of the battery, DoD is determined by Eqs. (1) and (2).

2.2 Engineering uncertainty

2.2.1. Battery capacity, voltage, and weight

The Li-ion battery is one of the best candidates for EVs owing to its high-energy density, long life span, and relative safety (Gomadam et al. 2002; Millner 2010; Tong et al. 2015). Given the hypersensitivity of Li-ion batteries to uncertainties, uniformity at the component level is highly required (Santhanagopalan and White 2012). However, some deviations of material and physical properties exist between cells and occur during manufacturing (Tong et al. 2015). Dubarry et al. (2010) conducted an experiment with statistical and electrochemical analyses on 100 LiCoO₂ Li-ion battery cells using an equivalent circuit model, and displayed distributions of the capacity, open circuit voltage, and weight of cells. Uncertain cell properties such as solid particle size and porosity may lead to variations in cell characteristics (Hadigol et al. 2015). Properties of these uncertainties are adapted in the engineering model explained in Section 2.1. Table 2 lists statistical properties of all the random parameters including daily driving distance and driving cycle used in the engineering model.

Table 2 Properties of random parameters

	Distribution	Mean	Standard deviation
Cell capacity	Normal	33.1 Ah	0.5 Ah
Cell voltage	Normal	3.8 V	0.02 V
Cell weight	Normal	0.7864 kg	0.0149 kg
Daily driving distance	Log-normal	3.11 mi	0.62 mi
Driving cycles	Combination of the standard driving cycles drawn randomly with the same probability		

2.2.2. Driving distance

Even with the same battery capacity, the *DoD* of the battery differs with energy consumption depending on the driving distance (Lawder et al. 2014). To deal with the uncertainty of daily driving distances of users, we use the daily vehicle miles of travel (VMT) data of 2009 National Household Travel Survey (U.S. Federal Highways Administration 2009). Log-normal distribution is used to describe daily driving distance of drivers as shown in Table 2 (Plötz et al. 2017). The distribution of the daily VMT results in a wide range of battery lifetimes. To determine the actual lifetime of batteries, Eq. (3) is integrated with the distribution of *DoD*.

2.2.3. Driving cycle

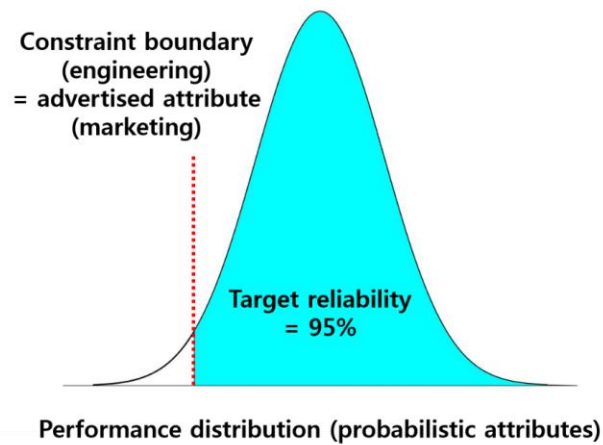
Various driving patterns affect EV performances such as driving range and thus MPGe (Berry 2010). Standard driving cycles, which represent driving patterns as vehicle speed over time, have been used to report fuel consumption of vehicles by the US Environment Protection Agency (EPA). Similarly, to reflect actual driving patterns in the engineering model, representative standard driving cycles provided by EPA are applied when calculating the driving range. Table 3 summarizes the characteristics of the EPA standard driving cycles (EPA website). Given that combinations of different driving cycles are frequent and natural in actual driving conditions, an average driving range calculated from the combination of the standard driving cycles drawn randomly with the same probability is used as the driving range of the designed vehicle in this paper (Kamble et al. 2009).

Table 3 Characteristics of standard driving cycles

Characteristics	UDDS City/ low speed	NYCC City/ frequent stops with low speed	LA92 City/ aggressive driving	HWFET Highway/ under 60 mph	US06 Aggressive driving
Top speed	56.70 mph	27.7 mph	67.20 mph	59.90 mph	80.30 mph
Avg. speed	19.58 mph	7.09 mph	25.92 mph	48.20 mph	47.97 mph
Max. acceleration	1.48 m/s ²	2.68 m/s ²	3.08 m/s ²	1.43 m/s ²	3.76 m/s ²
Avg. acceleration	0.50 m/s ²	0.62 m/s ²	0.64 m/s ²	0.19 m/s ²	0.67 m/s ²
Distance	7.45 mi	1.18 mi	6.99 mi	10.26 mi	8 mi
Time	22.8 min	10 min	16.2 min	12.8 min	10 min

2.3 Target reliability in engineering model

Actual performances fluctuate and vary because of the engineering uncertainties mentioned above, and form performance distributions which can be defined as probabilistic attributes. Therefore, EV performances that are advertised to customers can be described using the target reliability. For example, 95% target reliability implies that vehicles less than 5% among all produced ones will show lower EV performances than the advertised values as shown in Fig. 1. Thus, as the target reliability increases, advertised product attributes will be lowered for the product to satisfy the target reliability. When certain target reliability is used in RBDO, then we assume that the product reliability evaluated by customers and used in a marketing model is identical with the target reliability which will be explained in more detail in Section 3.3.

**Fig. 1** Advertisised attribute determination

3. Marketing model

A marketing model estimates market demand by estimating customer preferences toward price of a designed product and its attributes. This section discusses how to predict the market demand from customer preferences and heterogeneity which influence the optimal product design and company profit.

3.1 Utility model and product attributes

In market systems, a product design problem can be formulated as a mathematical optimization problem that maximizes profit while satisfying various constraints. Such a mathematical optimization problem includes an economic model that is based on market demand and product cost.

To express customer demand as a function of design attributes that are the product properties evaluated by the customers, research on product characteristics assessed by customers, representing the design attributes with respect to decision variables, must be initially performed. As the designer or company chooses the decision variables, product attributes are determined or calculated through simulation. Therefore, after estimating part-worths (customer preferences), a set of weights which indicate the importance of each design attribute perceived by customers, choice probability can be predicted using the logit model. Then, market demand is calculated as the product of the market share and market potential or market size.

Individual-level utility v_{ij} , which is the sum of part-worths of the designed product, can be defined as (Green and Krieger, 1996)

$$v_{ij} = \sum_{k=1}^K \sum_{l=1}^{L_k} \beta_{ikl} z_{jkl} \quad (4)$$

where β_{ikl} represents the part-worth of the l -th level of the k -th attribute for the i -th individual, and z_{jkl} corresponds to a binary dummy number, which is equal to 1 if the level l of the k -th attribute is selected for the j -th alternative, and 0 otherwise. For given utilities of competing products, the choice probability is calculated using

$$P_{ij} = \frac{e^{v_{ij}}}{\sum_{j \in J} e^{v_{ij}}} \quad (5)$$

which is similar to the probability of the i -th individual selecting option j from a set of alternatives J . By using part-worth data of individual i , the predicted market demand for the designed product, which represents the preference of individual i , can be expressed as the product of the market share P_{ij} and potential market size s . Accordingly, the predicted profit is defined as the product of the market demand and margin, which is the price minus unit production and warranty costs. In this paper, the fixed cost for an EV body and its battery cost which is determined by the number of battery cells in series and parallel are included in the unit production cost.

The data needed for the market share estimation explained above can be obtained from customer survey. A method using questionnaires for the survey is more general and suitable for studying customer preferences toward new product concepts such as EVs. Several multiple-choice questions are included in the questionnaire, and a set of designs with combinations of various levels of attributes as listed in Table 4 is presented to respondents. Specifications of general EVs in the real market are used to choose attribute levels. The respondents are asked to answer 16 choice questions, and then select the most preferred design in each question. When no satisfactory design exists, respondents may pick none of the options. Importance in Table 4 means the difference percentage between the maximum and minimum values of the part-worths of the attribute level. The larger the difference between levels is, the more important the attribute is.

Table 4 Attribute levels and their part-worths

Attribute		Part-worth				Importance
Product reliability	Level	5 rating	4 rating	3 rating	2 rating	38.9%
	Mean	2.412	1.515	-0.450	-3.476	
	(Std)	(1.844)	(1.147)	(0.845)	(2.379)	
Warranted battery lifetime	Level	3 years	7 years	11 years	15 years	11.5%
	Mean	-1.089	-0.114	0.563	0.640	
	(Std)	(1.061)	(0.432)	(0.481)	(0.541)	
Range	Level	80 mi	130 mi	180 mi	230 mi	14.1%
	Mean	-1.331	0.038	0.489	0.803	
	(Std)	(1.782)	(0.554)	(0.896)	(1.106)	
MPGe	Level	90	100	110	120	0.9%
	Mean	-0.044	-0.037	-0.008	0.088	
	(Std)	(0.156)	(0.144)	(0.091)	(0.381)	
Top speed	Level	70 mph	90 mph	110 mph	130 mph	4.1%
	Mean	-0.434	0.098	0.154	0.182	
	(Std)	(0.617)	(0.231)	(0.216)	(0.236)	
0–60 mph	Level	6 s	8 s	10 s	12 s	1.4%
	Mean	0.119	0.030	-0.060	-0.090	
	(Std)	(0.266)	(0.192)	(0.189)	(0.243)	
Price	Level	\$15,000	\$25,000	\$35,000	\$45,000	29.1%
	Mean	1.930	0.894	-0.356	-2.468	
	(Std)	(2.093)	(0.871)	(0.836)	(2.294)	

3.2 Hierarchical Bayesian approach

To obtain the individual-level part-worth distribution, actual respondent results collected from a choice-based conjoint (CBC) survey are needed. Given existence of various customer preferences toward product attributes, the part-worths for similar attributes differ. This study uses a hierarchical Bayesian (HB) approach (Train 2001; Rossi et al. 2005; Orme 2009) to build a heterogeneous market. Based on the results of a survey conducted using Mturk (Amazon 2017), which was targeted for the customers in the US, individual-level part-worth distribution is derived. Responses are drawn from 252 subjects living in the US: 49% were male and 51% were female; 9% were 15–24 years of age, 44% were 25–34 years of age, 28% were 35–44 years of age, 12% were 45–54 years of age, and 7% were 55–64 years of age.

CBC analysis is first performed to estimate individual part-worths, and then HB analysis follows. Responses from the survey are utilized in the HB analysis to estimate individual part-worths using the Markov-chain Monte

Carlo. In the HB conjoint, an individual's part-worths β_i are assumed to be derived from a multivariate normal distribution $\beta_i \sim (\theta, \Lambda)$ where θ is a vector of means and Λ is a covariance matrix.

Part-worths can explain a heterogeneous market because an individual-level market demand sP_{ij} is used for calculating profit in system-level optimization. The average profit of all individual market scenarios can then be used as the objective function. Although part-worth coefficients are discrete, the interpolation of intermediate attribute values using a nature cubic spline enables individual-level utility models to describe continuous attributes. As presented in Table 4, the large variance of part-worths demonstrates that heterogeneous preferences should be considered in the market system design.

3.3 Product reliability in marketing model

As product reliability is related to customer-perceived value, a product with high reliability continuously attracts customers through word-of-mouth (Levin and Kalal 2003; Huang et al. 2007; Park et al. 2007). Therefore, as the product reliability influences EV selection of customers, the reliability of EVs should be available to customers for reference when making a purchase. The product reliability used for the questionnaire listed in Table 4 is based on J.D. Power. The predicted reliability provided by J.D. Power, which is a statistically derived formula that uses power circle ratings from the initial quality study (IQS) and vehicle dependability study (VDS), provides consumers with information on a vehicle's reliability over time (J.D. Power website). IQS measures initial vehicle quality during the first 90 days of ownership, whereas VDS measures long-term vehicle quality. To estimate how customers perceive EV reliability, this paper uses the power circle ratings introduced by J.D. Power: 5 = "among the best," 4 = "better than most," 3 = "about average," and 2 = "the rest." Then, each rating perceived by customers is matched to a certain reliability depending on the reliability distribution of EVs in the market.

This product reliability in the marketing model is equal to the target reliability in the engineering model, and it determines product performances (advertised attributes) as explained in Section 2.3. It will be explained in

Section 4 how to integrate the engineering model and the marketing model using the target reliability to propose a reliability-based design for market systems (RBDMS) framework.

4. Reliability-Based Design for Market Systems (RBDMS)

The engineering model explained in Section 2 and the marketing model explained in Section 3 are integrated into a reliability-based design for market systems (RBDMS) framework to find the optimal product design which maximizes a manufacturer's profit and product performances at the same time. Fig. 2 illustrates the information flow of RBDMS for EV design from the viewpoint of the manufacturer. The target reliability, which is one of decision variables, is used in reliability constraints of the engineering model and determines the product reliability, which is one of product attributes and what customers consider when purchasing an EV – that is, customers perceive the product reliability through J.D. Power circle ratings. Other product attributes such as MPGe, range, top speed, and 0 to 60 mph are determined from the engineering model as well. Then, the product attributes determined from the engineering model and two other attributes – warranted battery lifetime and EV price – are passed on to the marketing model. Thus, the product utility can be calculated by the part-worths drawn from survey results. The final product then competes against other conventional EVs, and market share can be estimated from the result of choice probability. Once the predicted market demand is derived from the market share and market size, the profit of the manufacturer will be the product of the market demand and margin. To estimate the feasible range of decision variables, an extensive simulation with a set of constraint functions and specifications of EVs in the real world is performed. The decision variables used in the EV case study and their bounds are listed in Table 5.

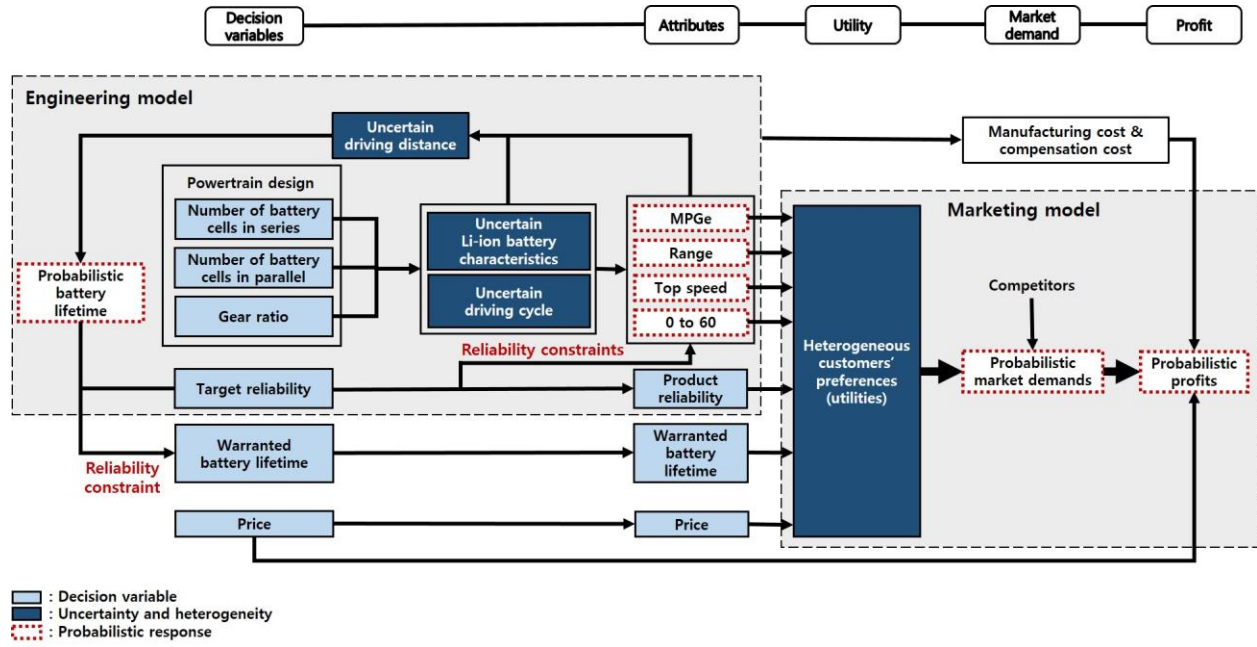


Fig. 2 Information flow of RBDMS for EV design

Table 5 Decision variables and their bounds for EV design

Decision variables	Lower bound	Upper bound
1. Target reliability (R)	10%	100%
2. Price ($Price$)	\$15,000	\$45,000
3. Warranted battery lifetime (W)	3 years	15 years
4. Number of battery cells in series (N_s)	80	250
5. Number of battery cells in parallel (N_p)	1	4
6. Gear ratio (FR)	2	12

4.1 RBDMS formulation

Based on a general formulation of RBDO (Lee et al. 2011), RBDMS is formulated as

$$\begin{aligned}
& \text{find } \mathbf{X} = [\mathbf{X}_{power}^T, R, W, Price] \\
& \max_{\mathbf{X}} \mu(\Pi) = \mu(\mathbf{D} \times (Price - MC) - \mathbf{C}) \\
& \text{subject to } \mathbf{lb} \leq \mathbf{X} \leq \mathbf{ub} \\
& \quad \mathbf{g}(\mathbf{A}_{eng}) \leq 0 \\
& \quad P[\mathbf{G}(\mathbf{X}, \mathbf{RP}_e) > 0] \leq P_F^{\text{Target}} \\
& \text{where } \mathbf{X}_{power} = [N_s, N_p, FR] \\
& \quad P_F^{\text{Target}} = 1 - R \\
& \quad \mathbf{P} = [\mathbf{P}_{MPGe}, \mathbf{P}_{range}, \mathbf{P}_{speed}, \mathbf{P}_{accel}, \mathbf{P}_{Batt}] \\
& \quad \mathbf{A} = [\mathbf{A}_{eng}^T, R, W, Price] \\
& \quad \mathbf{A}_{eng} = [\mathbf{A}_{MPGe}, \mathbf{A}_{range}, \mathbf{A}_{speed}, \mathbf{A}_{accel}]^T \\
& \quad [MC, \mathbf{P}] = f_{\text{engineering}}(\mathbf{X}_{power}, \mathbf{RP}_e) \\
& \quad [\mathbf{C}, \mathbf{A}_{eng}, W] = f_{\text{attribute}}(\mathbf{P}, R) \\
& \quad \mathbf{D} = f_{\text{marketing}}(\mathbf{A})
\end{aligned} \tag{6}$$

where the objective is to maximize the mean of profits Π ; $\mu(\cdot)$ represents the mean value; \mathbf{D} , MC , and \mathbf{C} correspond to the vector of market demand, manufacturing cost, and compensation cost, respectively; \mathbf{X} is the deterministic decision variable vector; \mathbf{X}_{power} stands for the powertrain design variable vector; R , W , and $Price$ indicate the decision variable of the target reliability, warranted battery lifetime, and price, respectively; $P[\cdot]$ is the probability measure; P_F^{Target} is the target probability of failure; \mathbf{lb} , \mathbf{ub} , \mathbf{g} , and \mathbf{G} indicate the lower bounds, upper bounds, inequality constraints on advertised performances, and probabilistic constraints, respectively; FR represents the decision variable of the final gear ratio; \mathbf{RP}_e denotes random parameter vectors of the engineering model; \mathbf{P} represents the probabilistic performance matrix; \mathbf{A} is the advertised attribute vector; \mathbf{A}_{eng} denotes the vector of advertised attributes determined from the engineering model; $f_{\text{engineering}}$, $f_{\text{attribute}}$, and $f_{\text{marketing}}$ indicate the engineering model, attribute model, and marketing model, respectively.

In Eq. (6), the probabilistic constraint is evaluated through a reliability analysis which calculates the probability of failure defined as

$$P_F = P[G(\mathbf{X}) > 0] = \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (7)$$

where $f_{\mathbf{X}}(\mathbf{x})$ represents the joint probability density function, and Ω_F is the failure set defined as $\{\mathbf{x}: G(\mathbf{X}) > 0\}$.

In this study, Monte Carlo simulation is utilized to perform the reliability analysis since both the engineering and marketing models are computationally efficient.

4.2 Modeling assumption

Four assumptions are made when modeling the entire framework for EV design. First, in computing the market share of the designed EV, we assumed that there are two competitors in the market: 2017 Nissan Leaf and 2017 Chevrolet Bolt. Two competitors' market sizes in the U.S. are used to determine the market demand. Second, to satisfy the minimum performances of an EV to drive in the real world, following constraints are applied to EV performances in all scenarios tested: the driving range should be more than 80 mi, 0 mph to 60 mph acceleration should be shorter than 12 s, and the top speed should be faster than 70 mph. Third, the compensation cost is assumed to compensate for 10% of the battery capacity only for failures within the warranted battery lifetime period. Fourth, it is necessary to map the product reliability and the reliability used in the customer survey – that is, J.D. Power rating. In this study, customers score the reliability power circle with ratings of 5, 4, 3, and 2, which correspond to 100%, 75%, 50%, and 25% reliability, respectively.

4.3 Three design methods

To investigate the importance of RBDMS for EV design, three different design methods shown in Fig. 3 are examined:

- Method 1 (RBDO): maximizing *engineering performances* with *fixed* target reliability
- Method 2 (RBDO + DMS): maximizing *profit* with *fixed* target reliability
- Method 3 (RBDMS, proposed): maximizing *profit* by using target reliability as a *decision variable*

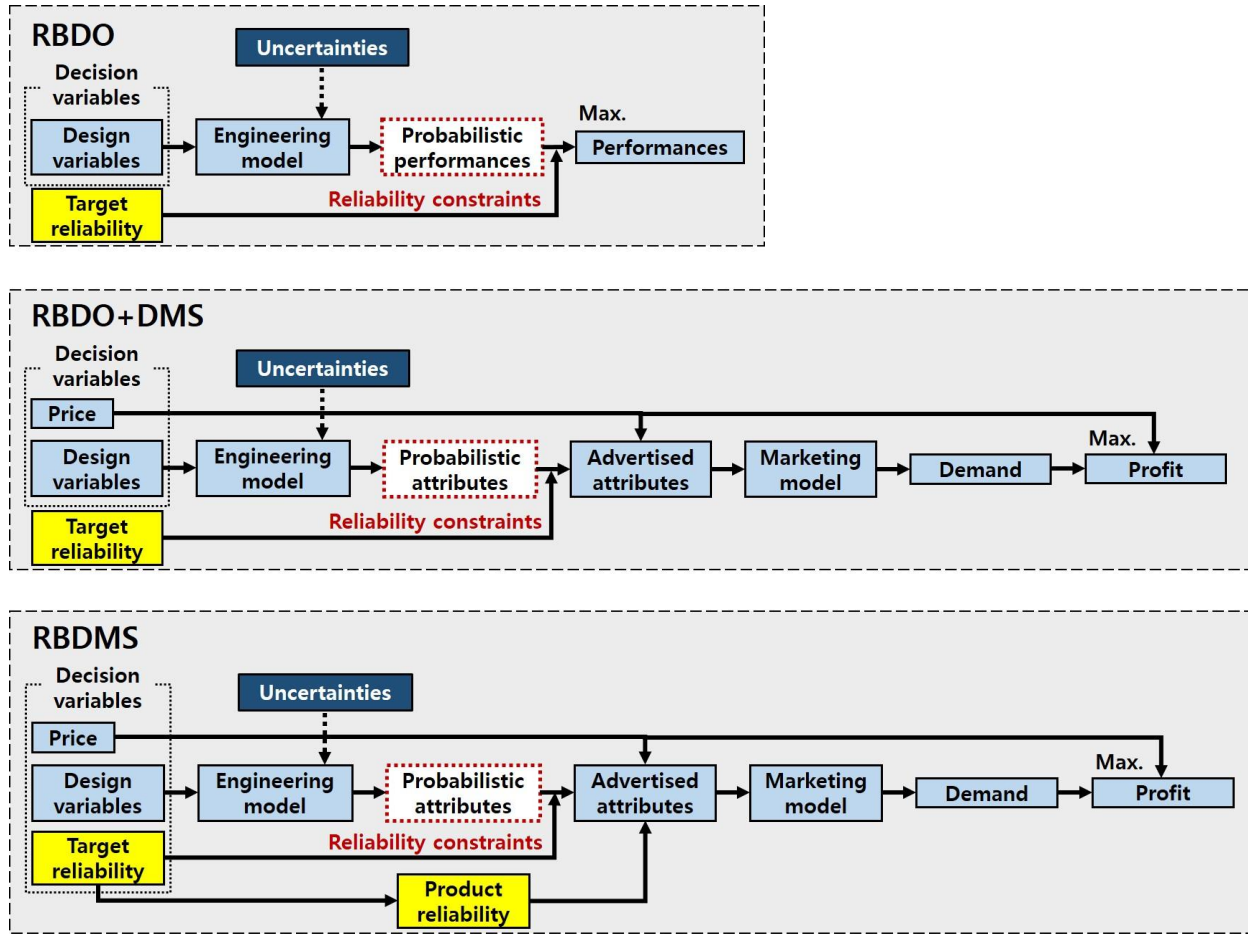


Fig. 3 Comparison among RBDO, RBDO+DMS, and RBDMS

Method 1 utilizes RBDO that maximizes performances with fixed target reliability and does not involve a marketing model for EV design. In Method 1, design variables become decision variables and the target reliability is predetermined before performing RBDO. Since EV has multiple performances (objectives), the weighted sum of the engineering performances is used as the objective function where the weights are determined by the importance of each performance as listed in Table 4. Method 2 simply connects the objective function of RBDO with DMS and the EV price is included in decision variables. In Method 2, the target reliability is still fixed and predetermined as in Method 1 before performing RBDO. Attribute values obtained from RBDO are used as advertised attributes in the marketing model. Method 2 uses profit for the objective function and is an intermediate

scenario to clearly explain benefits of the proposed RBDMS. Method 3 is the proposed RBDMS where the target reliability is optimized as a decision variable to maximize profit as explained in Sections 4.1 and 4.2, and the product reliability is used as an advertised attribute in Method 3.

A target reliability value of 99.87% commonly used in the field of RBDO for vehicles is used for the fixed target reliability in Methods 1 and 2 (Youn et al. 2004). The EV design optimization results obtained using Methods 1, 2, and 3 are shown and compared in the next section.

5. Results and discussion

This section compares optimization results obtained by three methods explained in Section 4.3. In all three methods, we deal with the number of battery cells in parallel and in series as discrete and continuous variables, respectively, and solve the optimization problem in Eq. (6) using sequential quadratic programming with multiple initial points. Optimal values of the number of battery cells in series are then rounded up to natural numbers. Computation time for one optimization is 25 hours on average using a standard desktop (Intel i7 6900 CPU @ 3.20 GHz with 64.0 GB of RAM).

Table 6 summarizes the optimal designs and outcomes obtained using three methods. The table shows the mean and standard deviation of profit and market share for Methods 2 and 3, and the mean and standard deviation of the actual battery lifetime and actual performance for the probabilistic engineering model in all methods. The advertised attributes are the values presented to customers who want to purchase an EV, and actual performances are the performance results obtained from RBDO with the fixed target reliability in Methods 1 and 2 and the optimal target reliability in Method 3.

Table 6 Optimal designs and outcomes obtained using three design methods

		Method 1 (RBDO)	Method 2 (RBDO + DMS)	Method 3 (RBDMS)
Decision variables	Target reliability	99.87%*	99.87%	92.69%
	Warranted battery lifetime	12.1 year	3.28 year	5.63 year
	Price	\$34,658	\$30,654	\$33,154
	Number of battery cells in series	220	112	142
	Number of battery cells in parallel	3	2	2
	Gear ratio	8.58	8.66	9.5
Outcomes	Profit	−\$850M (\$259M)**	\$71.9M (\$46.1M)	\$77.3M (\$47.3M)
	Market share	59.3% (18.1%)	19.7% (16.7%)	24.4% (14.9%)
Cost	Manufacturing cost	\$47,507	\$20,087	\$23,861
	Warranty compensation cost	\$0.11M	\$14,194	\$1.1M
Advertised attributes	Battery lifetime	12.1 year	3.28 year	5.63 year
	MPGe	93.1	101.5	109.1
	Range	229.9 mi	86.2 mi	115.4 mi
	0–60 mph	7.02 s	7.83 s	6.8 s
	Top speed	89.5 mph	83.4 mph	83.7 mph
Probabilistic attributes (Actual performance)	Battery lifetime	16.06 year (1.48 year)	4.92 year (0.57 year)	6.69 year (0.72 year)
	MPGe	105.89 (5.35)	119.21 (6.44)	118.18 (6.37)
	Range	262.07 mi (14.09 mi)	100.04 mi (5.51 mi)	125.76 mi (6.74 mi)
	0–60 mph	6.93 s (0.037 s)	7.74 s (0.031 s)	6.77 s (0.023 s)
	Top speed	89.69 mph (0.087 mph)	83.69 mph (0.099 mph)	83.75 mph (0.062 mph)

* Fixed values, which are not decision variables, are in bold.

** Standard deviations are enclosed in parentheses.

Since Method 1 maximizes performances without considering profit, the total number of battery cells can be extremely high because price and cost are irrelevant to the objective function. In addition, performances such as range and battery lifetime which have a large importance of attributes in Table 4 are extremely high as well. Based on the cost, the profit becomes positive when the price is higher than the manufacturing cost (\$47,507). Assuming that the price is the average price of two competitors, \$30,680 for Leaf and \$37,495 for Bolt, the profit becomes negative. Therefore, from the market's point of view, RBDO without considering DMS yields an infeasible design.

Method 2 results in a feasible design and marketable outcomes. When compared to Method 1, higher profit is achieved since the objective is to maximize profit, and lower overall advertised attributes are obtained because of cost. Method 3 shows an optimum design with the maximum profit since the target reliability is optimized – in this case study, it was lowered from 99.87% to 92.69% – and good product performances simultaneously.

By comparing design optimization results of Methods 2 and 3, following observations can be made:

1. The optimal target reliability in Method 3 is lower than the target reliability in Method 2 to maximize profit which increases from \$71.9M in Method 2 to \$77.3M in Method 3. This is because the improved advertised attributes owing to the lower target reliability have higher effect than the lower product reliability, and thus the market share has increased from 19.7% to 24.4%. In addition, due to the improved advertised attributes, the price can be relatively high (from \$30,654 to \$33,154). When the tradeoff of utility is considered, reduced part-worths due to the lower target reliability and increased price are 0.1496 and 0.3344, respectively. Increased part-worths due to the improved advertised performance and warranted battery lifetime are 0.9282 and 0.5867, respectively. This shows that the increased utility due to improved advertised performance and warranted battery lifetime ($0.9282 + 0.5867 = 1.5149$) is much larger than the decreased utility due to lower target reliability and increased price ($0.1496 + 0.3344 = 0.4840$).
2. The number of battery cells in series in Method 3 increases from 112 in Method 2 to 142. The manufacturing cost will decrease if the number of battery cells decreases. However, instead of reducing the manufacturing cost, Method 3 finds a strategy to improve advertised attributes with more battery capacity, and thus to increase the final price to maximize profit.
3. The warranted battery lifetime in Method 3 increases from 3.28 year in Method 2 to 5.63 year. With the improved battery capacity as mentioned in Observation 2, the battery in the EV design obtained using Method 3 will experience a smaller *DoD* for the same driving distance which leads to longer battery time from Eq. (3). From the increased warranted battery lifetime and as a result significantly increased warranty compensation cost, it can be said that Method 3 finds the optimum value of the warranted battery lifetime and warranty compensation cost by optimizing the target reliability.

4. It is confirmed that the optimization problem can be solved with the target reliability as a decision variable. When performed with 100 initial designs considering tradeoffs between attributes, the optimizations with 87 different initial designs converge to the same optimum design listed in Table 6 which is considered to be the global optimum.

In addition to the comparative study among Methods 1, 2, and 3, parametric studies are performed using Method 3 to see the effect of the target reliability on profit based on different types of the market. For the parametric study, three different reliability markets are considered: high reliability market (HRM), medium reliability market (MRM), and low reliability market (LRM). For example, HRM is a market in which reliability range is narrow compared with MRM and LRM, and a relatively high reliability is required in order to obtain a high reliability rating. The optimization results using Method 3 in Table 6 are based on the MRM case in which market competitors have medium reliability. Matches between customers' perceived and actual reliability are listed in Table 7. Fig. 4 shows parametric study results on reliability matching in three cases of HRM, MRM, and LRM. For HRM, it can be seen that the optimum target reliability should be increased, and profit tends to decrease due to the increased target reliability. For MRM and LRM, the optimum target reliability decreases to maximize profit since competitors' reliabilities are relatively low.

Table 7 Matches between perceived and actual reliability

Perceived reliability	Actual reliability		
Power circle rating	HRM	MRM	LRM
5 rating (among the best)	100%	100%	100%
4 rating (better than most)	80%	75%	70%
3 rating (about average)	60%	50%	40%
2 rating (the rest)	40%	25%	10%



Fig. 4 Optimal target reliability and profit in three reliability markets

To further investigate the effect of the target reliability on profit in three reliability markets, parametric study by changing the target reliability is performed. In this parametric study, the target reliability is not considered as a decision variable but a fixed given parameter for optimization. The optimal target reliability in each market is marked with a red dot in Fig. 5 which is corresponding to the optimal target reliability in Fig 4. In all cases, excessive increase in the target reliability has a very negative impact on profit, that is, the profit drops sharply when the target reliability increases from 98% to 100%. This shows that it is not advantageous for a company to increase the target reliability more than necessary.

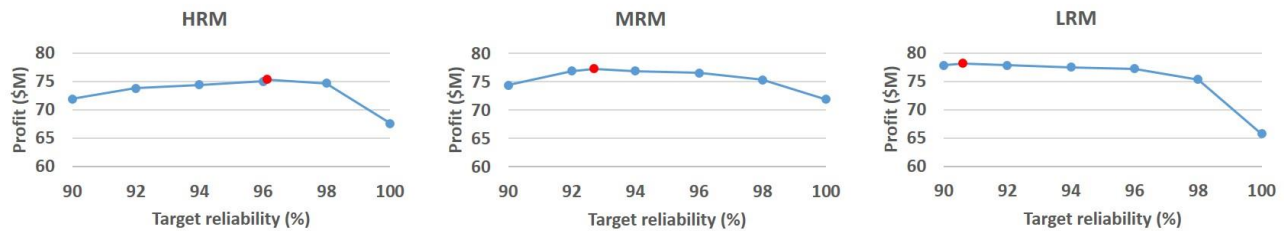


Fig. 5 Effect of target reliability on profit in three markets

From the EV design case study, it is observed that the proposed RBDMS is an effective way of finding the

optimal target reliability that maximizes the company profit by integrating DMS into RBDO. Furthermore, it is also observed that the optimal target reliability varies depending on the situation of market and competitors.

6. Conclusion

In this study, an RBDMS framework that integrates RBDO with DMS to find the optimal target reliability by considering design variables, price, market demand, and cost at the same time is proposed and applied to EV design. The advantages of RBDMS using target reliability as a decision variable are presented by comparing the optimization results of three methods (RBDO, RBDO + DMS, and RBDMS). Although a simple integration of RBDO and DMS with fixed target reliability yields a design with feasibility and marketability, RBDMS can find the optimum target reliability as a decision variable to maximize the profit while satisfying engineering constraints. EV design case study shows that the proposed RBDMS is an effective way of finding the optimal target reliability that maximizes the company profit and the optimal target reliability varies depending on the situation of market and competitors. The proposed approach can be applied to other engineering design problems such as smartphones and notebooks where battery lifetime varies greatly depending on the usage environment, and there is a large gap between real battery lifetime and advertised battery lifetime. There are several limitations in this work. Additional research should focus on measuring the reliability from the perspective of customers, release several assumptions made in this study, and reflect more fidelity in the engineering model and its uncertainties. Market uncertainty and customer preference for a targeted market which considers geographic effects, local regulations, etc. need to be considered in future research as well (Kang et al. 2018).

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