Community structure in hypergraphs and the emergence of polarization

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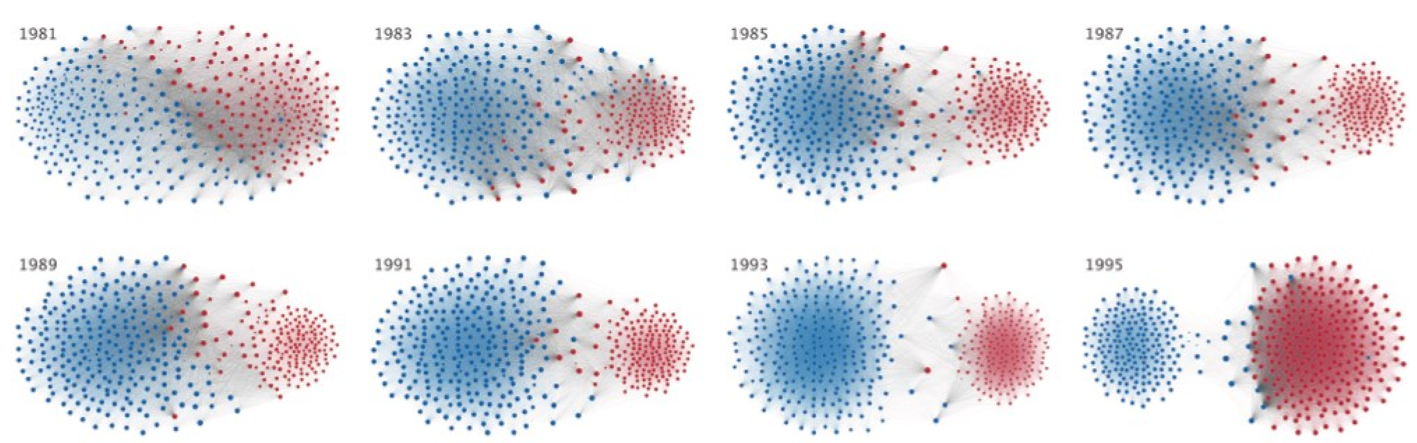
Background

Society has become increasingly divided in politics, social media, and ideology.



Republican (red) and Democrat (blue) representatives with edges drawn between members who agree above a threshold value of votes. Andris et al. 2015

Polarization, the phenomenon where a community reinforces its own beliefs while another community does the same with a different set of beliefs, is often the result of "echo chambers".

A *hypergraph* is an extension of a pairwise network that includes interactions of arbitrary size. An m-uniform hypergraph contains only m-hyperedges, i.e., interactions of size m. The average connection strength or *degree* is denoted $\langle k^{(m)} \rangle$.

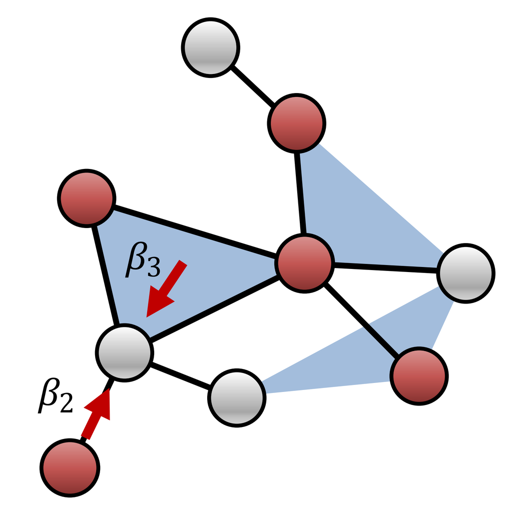


Illustration of a hypergraph. Infected nodes (red) infect a healthy node (gray) via hyperedges of sizes 2 and 3 with rates β_2 and β_3, respectively. Landry and Restrepo 2020.

(Uniform) hypergraph models

Stochastic Block Model (SBM)

m-uniform hypergraph with N nodes, each with a community label g_i.

m-dimensional tensor P specifying the probability that a hyperedge with nodes $i_1, \dots, i_m$ and group memberships $g_{i_1}, \dots, g_{i_m}$ are connected. (Super-symmetric tensor).

Planted Partition Model

SBM with two equally-sized communities and one parameter ϵ_m controlling community strength. ($\epsilon_m = 0$ is no communities, $\epsilon_m = \langle k^{(m)} \rangle$ is completely disconnected communities)

Degree-Corrected Stochastic Block Model (DCSBM)

Nodes $i_1, \dots, i_m$ with degrees $k_{i_1}, \dots, k_{i_m}$ are connected with probability proportional to $k_{i_1} \dots k_{i_m} \omega(g_{i_1}, \dots, g_{i_m})$, where ω is a tensor specifying the connection strengths between communities.

ODE Model

2-variable mean-field equation:

- x_1 is the fraction of community 1 infected and x_2 is the fraction of community 2 infected.
- Denote link degrees as k and triangles as q.

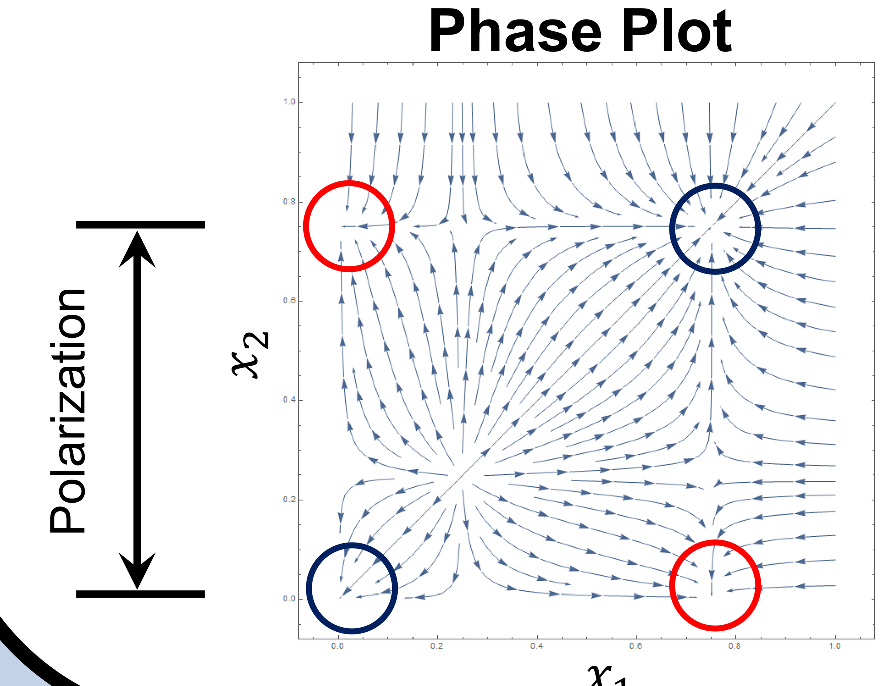
$$\frac{dx_1}{dt} = -\gamma x_1 \quad \text{Healing}$$

$$+\frac{\beta_2}{2}(1-x_1)\big(\langle k \rangle (x_1+x_2) + \epsilon_2 (x_1 - x_2)\big) \quad \text{Pairwise infection}$$

$$+\frac{\beta_3}{4}(1-x_1)\left(\langle q \rangle (x_1+x_2)^2 + \epsilon_3(3x_1^2 - 2x_1x_2 - x_2^2)\right) \quad \text{Triangle infection}$$

For the x_2 equation we substitute $x_2 \to x_1$.

Phase Plot



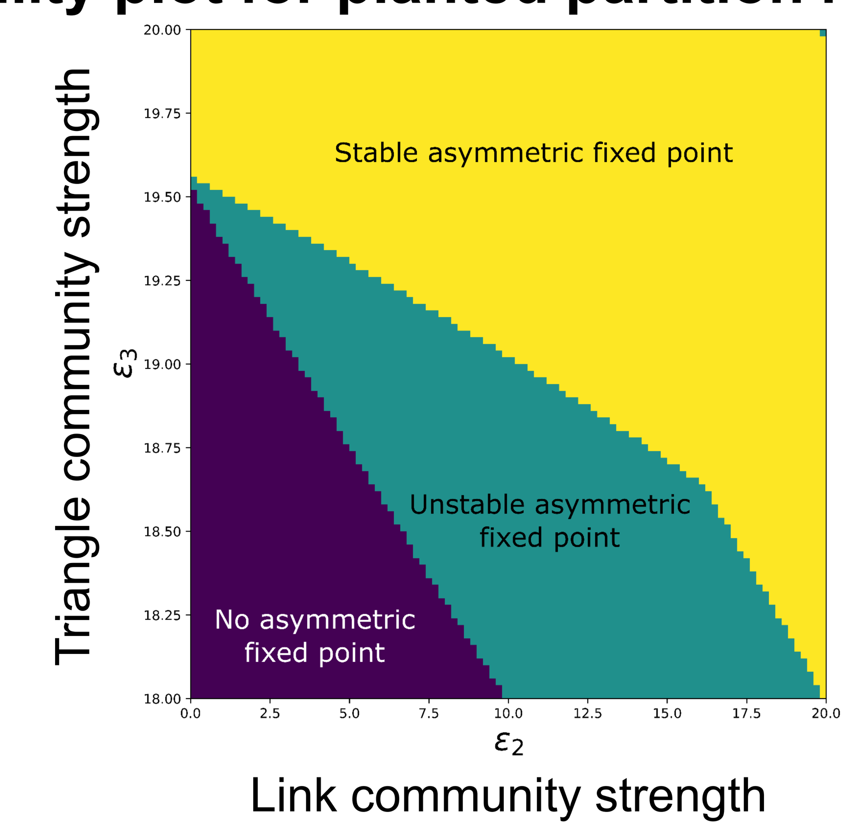
Symmetric fixed points (Erdös-Rényi)

Asymmetric fixed points (Polarization exists)

Stable fixed points with polarization!

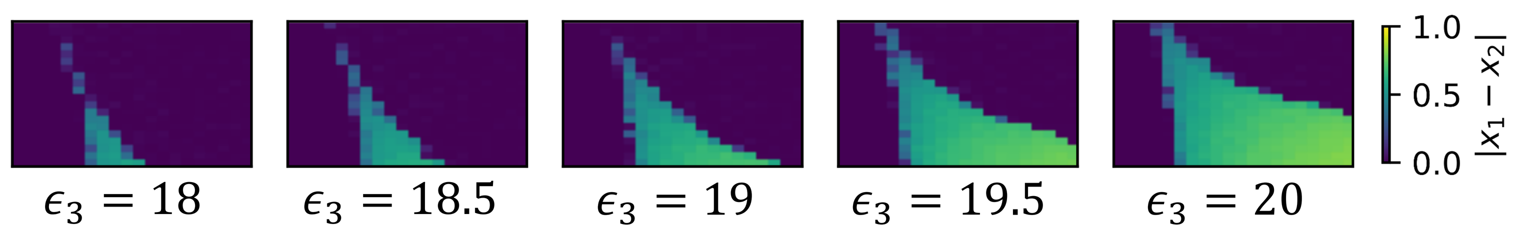
Numerical Results

Stability plot for planted partition model

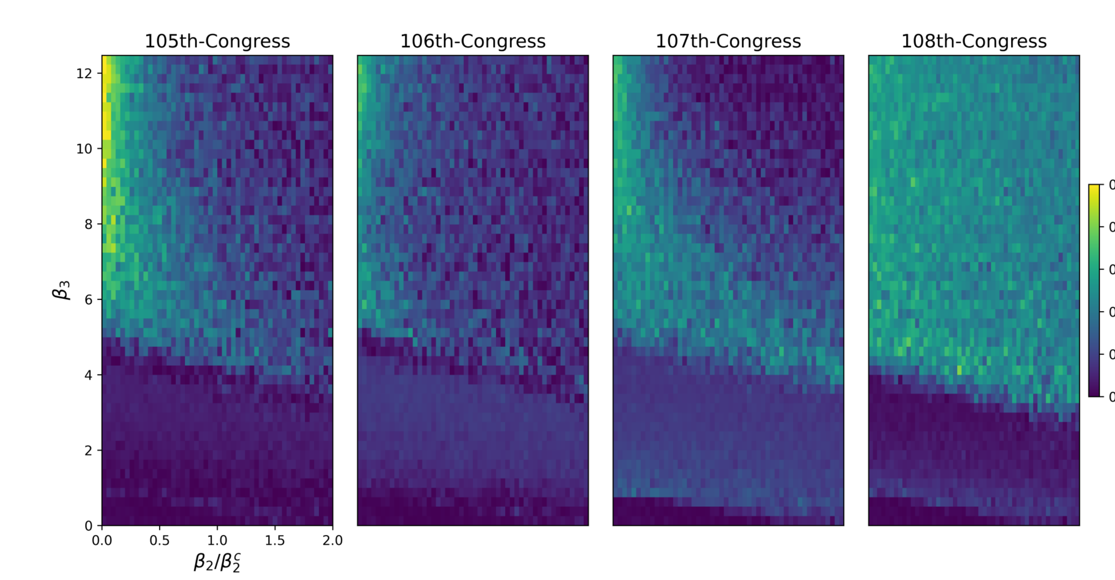


Planted partition model

$\epsilon_2 = 0, \langle k \rangle = \langle q \rangle = 20$



Congress cosponsorship dataset



Conclusions

- When community structure is strong enough, stable polarized states exist.
- Higher-order interactions enable these polarized states.
- Degree heterogeneity strongly influences the magnitude of polarization.

Manuscript in preparation!