

Lab3 interpolacja

```
In [1]: using Plots
using Polynomials
using DataFrames
using Statistics
using Interpolations
```

Zad1

```
In [2]: function Counter(X, i, x)
    n = size(X)[1]
    differences = []
    for k=1:n
        if k != i
            append!(differences, x - X[k])
        end
    end
    result = 1
    for diff in differences
        result *= diff
    end
    return result
end

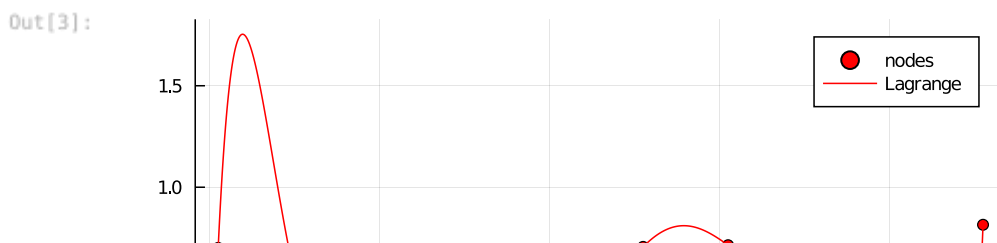
function Denominator(X, i)
    differences = []
    for x in X
        if x != X[i]
            append!(differences, X[i] - x)
        end
    end
    result = 1
    for diff in differences
        result *= diff
    end
    return result
end

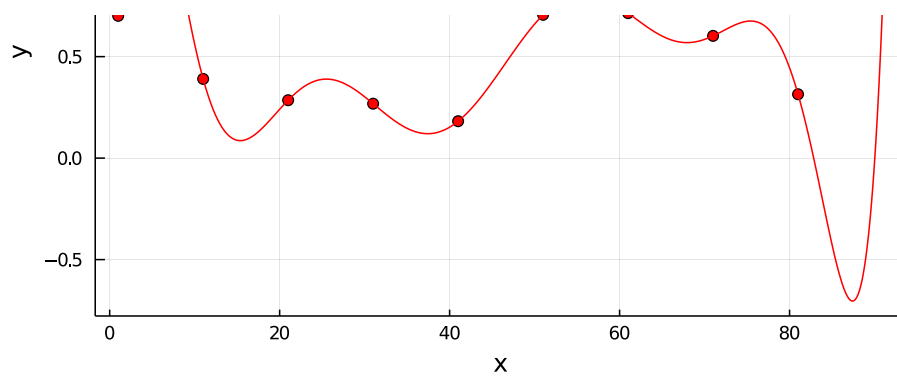
function LagrangeInterpolation(X, Y)
    n = size(X)[1]

    denominators_arr = [Denominator(X, i) for i=1:n]
    x_interpolation = [x for x=X[1]:0.01:X[n]]
    y_interpolation = []
    for x in x_interpolation
        Counters_arr = []
        for i=1:n
            append!(Counters_arr, Counter(X, i, x))
        end
        y_value = 0
        for i=1:n
            y_value += Y[i] * Counters_arr[i] / denominators_arr[i]
        end
        append!(y_interpolation, y_value)
    end
    return x_interpolation, y_interpolation
end
```

Out[2]: LagrangeInterpolation (generic function with 1 method)

```
In [3]: X = 1:10:100
Y = [rand() for x in X]
plot(X, Y, seriestype = :scatter, color = :red, label = "nodes")
plot!(LagrangeInterpolation(X, Y), color = :red, label = "Lagrange", xaxis = "x", yaxis = "y")
```





Zad2

```
In [4]: struct newt
        NewtonDifferences::Array{Any, 1}
        X::Array{Float64, 1}
        Y::Array{Float64, 1}
    end
```

```
In [5]: function NewtonDifferences(X, Y)
        n = size(X)[1]
        differences = []

        for i=1:n
            for j=1:i
                if j == 1
                    push!(differences, [])
                    append!(differences[i], Y[i])
                else
                    append!(differences[i], (differences[i][j-1] - differences[i-1][j-1]) / (X[i] - X[i-j+1]))
                end
            end
        end
        return differences
    end

    function NewtonInterpolation(X, Y)
        return newt(NewtonDifferences(X,Y), X, Y)
    end

    function Horner_schema_Newton(newt, x)
        NewtonDifferences = newt.NewtonDifferences

        X = newt.X
        n = size(X)[1]

        value = NewtonDifferences[n][n]

        for i=(n-1):-1:1
            value = value * (x - X[i]) + NewtonDifferences[i][i]
        end

        return value
    end

    function N_Interpolation(nodes, Y)
        n = size(nodes)[1]
        x_interpolation = [x for x=nodes[1]:0.01:nodes[n]]
        y_interpolation = []

        newt = NewtonInterpolation(nodes, Y)

        for x in x_interpolation
            append!(y_interpolation, Horner_schema_Newton(newt, x))
        end

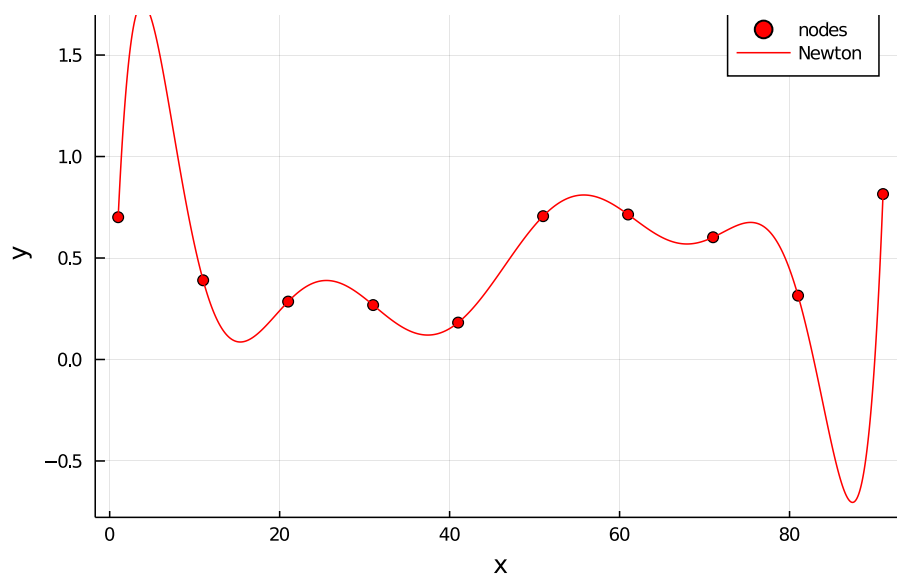
        return x_interpolation, y_interpolation
    end
```

Out[5]: N_Interpolation (generic function with 1 method)

```
In [6]: plot(X, Y, seriestype = :scatter, color = :red, label = "nodes")
        plot!(N_Interpolation(X,Y), color = :red, label = "Newton", xaxis = "x", yaxis = "y")
```

Out[6]:



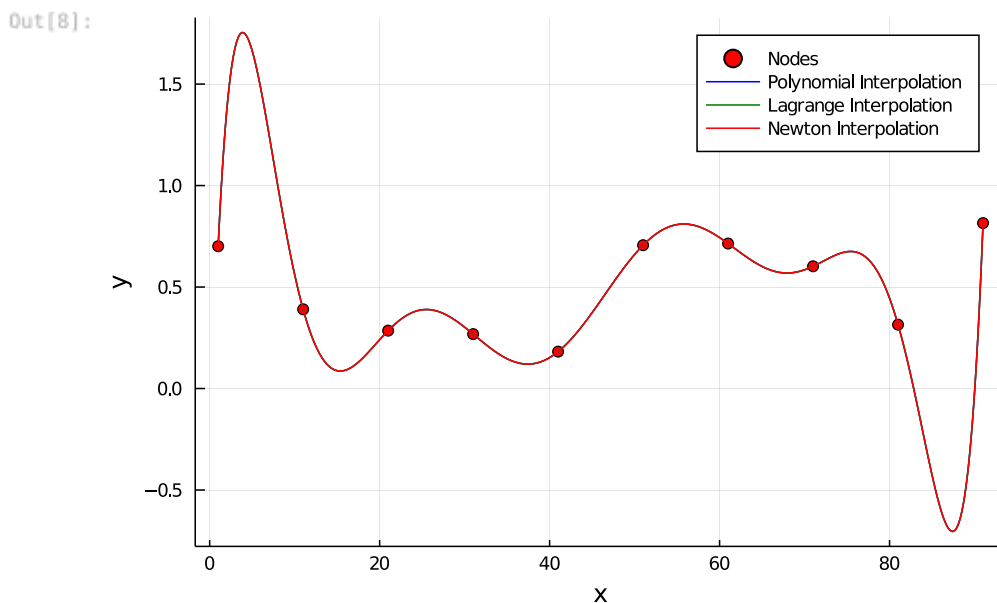


Zad3

```
In [7]: function PolynomialInterpolation(X, Y)
    n = size(X)[1]
    itp = fit(X, Y)
    x_interpolation = [i for i=X[1]:0.01:X[n]]
    y_interpolation = []
    for x in x_interpolation
        append!(y_interpolation, itp(x))
    end
    return x_interpolation, y_interpolation
end
```

Out[7]: PolynomialInterpolation (generic function with 1 method)

```
In [8]: plot(X,Y, color = :red, seriestype = :scatter, label = "Nodes", xaxis = "x", yaxis = "y")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial Interpolation")
plot!(LagrangeInterpolation(X, Y), color = :green, label = "Lagrange Interpolation")
plot!(N_Interpolation(X,Y), color = :red, label = "Newton Interpolation")
```



Zad 4

```
In [9]: times = []
for j=1:5
    push!(times, [])
    for i=10:10:200
        X = 1:i
```

```

Y = [rand() for x in X]
for k=1:10
    if j == 1
        append!(times[j], @elapsed PolynomialInterpolation(X,Y))
    elseif j == 2
        append!(times[j], @elapsed LagrangeInterpolation(X,Y))
    elseif j == 3
        append!(times[j], @elapsed N_Interpolation(X,Y))
    elseif j == 4
        if i == 10 && k == 1
            push!(times, [])
        end
        newt_obj = NewtonInterpolation(X,Y)
        append!(times[j], @elapsed NewtonInterpolation(X,Y))
        append!(times[j+1], @elapsed [Horner_schema_Newton(newt_obj, x) for x in X[1]:0.01:X[i]])
    elseif j == 5
        if i == 10 && k == 1
            push!(times, [])
        end
        itp = fit(X,Y)
        append!(times[j+1], @elapsed fit(X,Y))
        append!(times[j+2], @elapsed [itp(x) for x in X[1]:0.01:X[i]])
    end
end
end
end
end

```

Polynomials Lagrange Newton Interpolation times

```

In [10]: df = DataFrame()
types = ["Polynomials" "Lagrange" "Newton"]
df[:, :Type] = [types[i] for i=1:3 for j=1:200]
df[:, :Size] = [i for j=1:3 for i=10:10:200 for k=1:10]
df[:, :Time] = [times[j][i] for j=1:3 for i=1:200]
df

```

Out[10]: 600 rows × 3 columns

	Type	Size	Time
	String	Int64	Float64
1	Polynomials	10	0.0516305
2	Polynomials	10	3.9899e-5
3	Polynomials	10	4.21e-5
4	Polynomials	10	3.7999e-5
5	Polynomials	10	3.85e-5
6	Polynomials	10	3.94e-5
7	Polynomials	10	4.15e-5
8	Polynomials	10	4.0401e-5
9	Polynomials	10	3.9599e-5
10	Polynomials	10	4.2399e-5
11	Polynomials	20	9.65e-5
12	Polynomials	20	9.3199e-5
13	Polynomials	20	0.0001027
14	Polynomials	20	9.9101e-5
15	Polynomials	20	9.27e-5
16	Polynomials	20	0.0001052
17	Polynomials	20	9.83e-5
18	Polynomials	20	9.3301e-5
19	Polynomials	20	9.3201e-5
20	Polynomials	20	9.75e-5
21	Polynomials	30	0.000164799
22	Polynomials	30	0.0001653
23	Polynomials	30	0.000165801
24	Polynomials	30	0.0001624
25	Polynomials	30	0.000161701
26	Polynomials	30	0.000193399
27	Polynomials	30	0.0001606

28	Polynomials	30	0.0001607
29	Polynomials	30	0.0001594
30	Polynomials	30	0.0001614
:	:	:	:

Polynomials Newton differences and values times

```
In [11]: df_2 = DataFrame()
types_2 = ["Newton" "Polynomials"]
df_2[:, :Type] = [types_2[i] for i=1:2 for j=1:200]
df_2[:, :Size] = [i for j=1:2 for i=10:10:200 for k=1:10]
df_2[:, :Diff_time] = [times[j][i] for j=4:2:6 for i=1:200]
df_2[:, :Val_time] = [times[j][i] for j=5:2:7 for i=1:200]
df_2
```

Out[11]: 400 rows × 4 columns

	Type	Size	Diff_time	Val_time
	String	Int64	Float64	Float64
1	Newton	10	9.401e-6	0.0661144
2	Newton	10	7.0e-6	0.0013418
3	Newton	10	1.9899e-5	0.0009139
4	Newton	10	7.8e-6	0.0006868
5	Newton	10	7.0e-6	0.0008901
6	Newton	10	1.34e-5	0.0015347
7	Newton	10	1.78e-5	0.0011571
8	Newton	10	1.26e-5	0.000799101
9	Newton	10	6.7e-6	0.0005472
10	Newton	10	7.0e-6	0.0005351
11	Newton	20	2.4201e-5	0.0557142
12	Newton	20	2.51e-5	0.0030451
13	Newton	20	6.2901e-5	0.0056353
14	Newton	20	2.4601e-5	0.0028229
15	Newton	20	2.4399e-5	0.0023274
16	Newton	20	2.42e-5	0.0022566
17	Newton	20	2.4201e-5	0.0024594
18	Newton	20	4.5499e-5	0.0042213
19	Newton	20	5.92e-5	0.0028098
20	Newton	20	2.4e-5	0.0028235
21	Newton	30	5.1901e-5	0.0058972
22	Newton	30	5.3701e-5	0.0078524
23	Newton	30	7.2899e-5	0.0059929
24	Newton	30	5.4599e-5	0.0096381
25	Newton	30	9.96e-5	0.0063963
26	Newton	30	5.4701e-5	0.008899
27	Newton	30	0.0001005	0.0067772
28	Newton	30	5.83e-5	0.0106063
29	Newton	30	0.000102	0.0064094
30	Newton	30	5.5001e-5	0.0080143
:	:	:	:	:

Statistics

```
In [12]: df_grouped = groupby(df, [:Type, :Size])
df_stats = combine(df_grouped, "Time" => mean, "Time" => std)
```

Out[12]: 60 rows × 4 columns

	Type	Size	Time_mean	Time_std
	String	Int64	Float64	Float64
1	Polynomials	10	0.00519923	0.0163143
2	Polynomials	20	9.71702e-5	4.30887e-6
3	Polynomials	30	0.00016555	1.00226e-5
4	Polynomials	40	0.00030051	9.56702e-6
5	Polynomials	50	0.00053536	0.000217286
6	Polynomials	60	0.00164456	0.00326946
7	Polynomials	70	0.0006678	2.49893e-5
8	Polynomials	80	0.00095037	0.000241634
9	Polynomials	90	0.00112235	4.95579e-5
10	Polynomials	100	0.00152197	0.00045959
11	Polynomials	110	0.00175651	0.000379228
12	Polynomials	120	0.00221246	0.000446468
13	Polynomials	130	0.00227341	0.00023123
14	Polynomials	140	0.00267164	0.000462139
15	Polynomials	150	0.00409408	0.00371634
16	Polynomials	160	0.00336308	0.000415606
17	Polynomials	170	0.00391856	0.00048112
18	Polynomials	180	0.00452808	0.000697914
19	Polynomials	190	0.00463997	0.00046613
20	Polynomials	200	0.00777299	0.00264348
21	Lagrange	10	0.0167868	0.0321269
22	Lagrange	20	0.0555901	0.00515628
23	Lagrange	30	0.186713	0.0298955
24	Lagrange	40	0.548725	0.0694794
25	Lagrange	50	0.767327	0.0317881
26	Lagrange	60	1.23605	0.00983517
27	Lagrange	70	2.34258	0.0533412
28	Lagrange	80	3.34547	0.0263944
29	Lagrange	90	4.70849	0.121274
30	Lagrange	100	6.29273	0.0568486
:	:	:	:	:

```
In [13]: df_2_grouped = groupby(df_2, [:Type, :Size])
df_2_stats = combine(df_2_grouped, "Diff_time"=>mean , "Val_time"=>mean, "Diff_time"=>std, "Val_time"=>std)
```

Out[13]: 40 rows × 6 columns

	Type	Size	Diff_time_mean	Val_time_mean	Diff_time_std	Val_time_std
	String	Int64	Float64	Float64	Float64	Float64
1	Newton	10	1.086e-5	0.00745202	4.86305e-6	0.0206145
2	Newton	20	3.38302e-5	0.00841155	1.58116e-5	0.0166526
3	Newton	30	7.03202e-5	0.00764831	2.17668e-5	0.00163696
4	Newton	40	0.00010556	0.0183793	2.83474e-5	0.0163522
5	Newton	50	0.0001492	0.030695	1.33675e-5	0.021512
6	Newton	60	0.0002867	0.0419408	0.000103973	0.019774
7	Newton	70	0.00040598	0.0557511	0.000153489	0.0203899
8	Newton	80	0.00054653	0.0816488	0.000327804	0.0277105
9	Newton	90	0.00056728	0.0930716	0.000178611	0.0288786
10	Newton	100	0.00090965	0.109373	0.000383787	0.0244169
11	Newton	110	0.00080861	0.140708	0.000201519	0.0291621
12	Newton	120	0.001308	0.172718	0.000840913	0.0226628
13	Newton	130	0.00111267	0.202242	0.000377066	0.0129412
14	Newton	140	0.00165132	0.230497	0.00046233	0.014369
15	Newton	150	0.00637081	0.25566	0.0150161	0.0213122

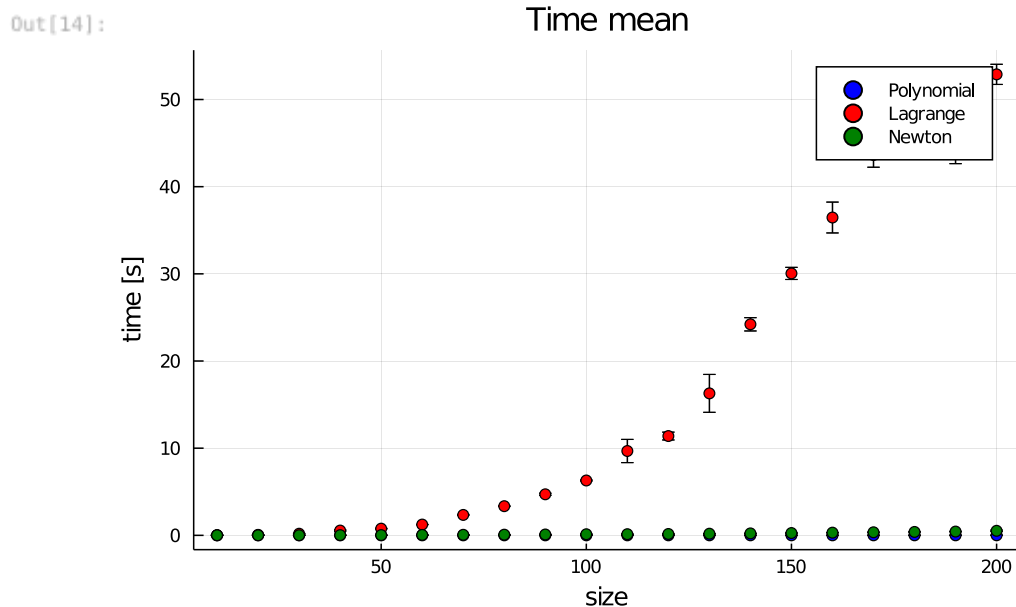
16	Newton	160	0.00184728	0.2932	0.000578907	0.028483
17	Newton	170	0.00237291	0.344249	0.00104914	0.0393247
18	Newton	180	0.0081971	0.365459	0.016621	0.0272996
19	Newton	190	0.0080042	0.411028	0.0164248	0.0256505
20	Newton	200	0.00369178	0.559345	0.0018483	0.097186
21	Polynomials	10	1.6199e-6	0.00462146	1.17016e-6	0.0145399
22	Polynomials	20	1.8901e-6	5.33298e-5	5.48577e-7	9.03534e-6
23	Polynomials	30	3.07e-6	0.0219506	1.0688e-6	0.0690617
24	Polynomials	40	4.87e-6	0.00017545	1.73397e-6	1.48788e-5
25	Polynomials	50	6.4e-6	0.00028693	1.53993e-6	2.55265e-5
26	Polynomials	60	1.03598e-5	0.00047793	2.52399e-6	9.20273e-5
27	Polynomials	70	1.21601e-5	0.00053641	3.87635e-6	3.10805e-5
28	Polynomials	80	2.153e-5	0.00100622	1.28185e-5	0.000599
29	Polynomials	90	1.70701e-5	0.00093622	4.56095e-6	7.02975e-5
30	Polynomials	100	1.87901e-5	0.00112806	2.93134e-6	0.000123014
:	:	:	:	:	:	:

Plots

```
In [14]: df_polynomials = df_stats[1:20, :]
df_lagrange = df_stats[21:40, :]
df_newton = df_stats[41:60, :]
plot(df_polynomials.Size, df_polynomials.Time_mean, colour = :blue, label="Polynomial",
      yerr=df_polynomials.Time_std, seriestype=:scatter, ylabel = "time [s]", xlabel = "size", layout = 1)

plot!(df_lagrange.Size, df_lagrange.Time_mean, colour = :red, label="Lagrange",
      yerr=df_lagrange.Time_std, seriestype=:scatter, ylabel = "time [s]", xlabel = "size", layout = 1)

plot!(df_newton.Size, df_newton.Time_mean, colour = :green, title="Time mean", label="Newton",
      yerr=df_newton.Time_std, seriestype=:scatter, ylabel = "time [s]", xlabel = "size", layout = 1)
```



```
In [15]: df_2_polynomials = df_2_stats[21:40, :]
df_2_newton = df_2_stats[1:20, :]

plot(df_2_polynomials.Size, df_2_polynomials.Diff_time_mean, colour = :blue, label="Polynomial",
      yerr=df_2_polynomials.Diff_time_std, seriestype=:scatter, layout = 1)

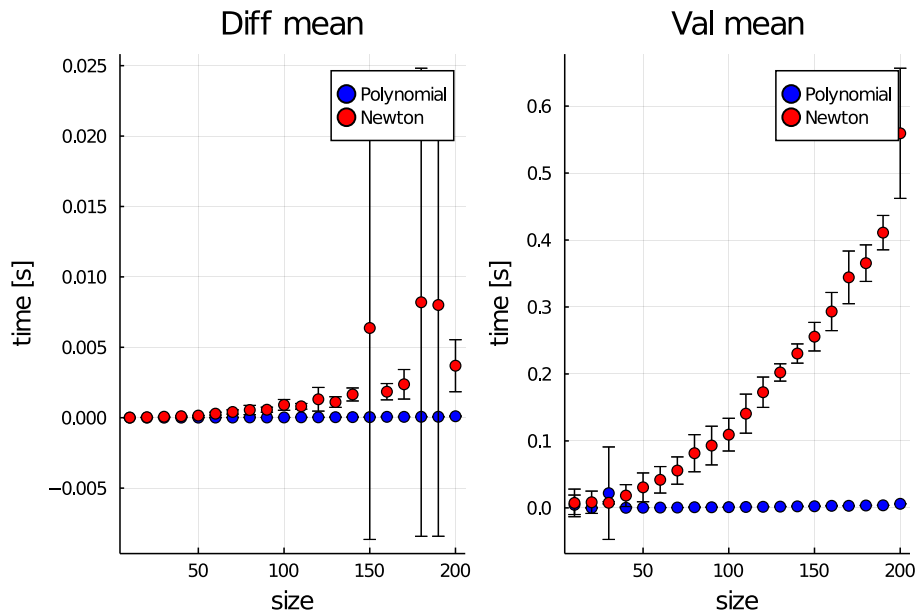
p1 = plot!(df_2_newton.Size, df_2_newton.Diff_time_mean, colour = :red, title="Diff mean", label="Newton",
            yerr=df_2_newton.Diff_time_std, seriestype=:scatter, layout = 1)

plot(df_2_polynomials.Size, df_2_polynomials.Val_time_mean, colour = :blue, label="Polynomial",
      yerr=df_2_polynomials.Val_time_std, seriestype=:scatter, layout = 1)

p2 = plot!(df_2_newton.Size, df_2_newton.Val_time_mean, colour = :red, title="Val mean", label="Newton",
            yerr=df_2_newton.Val_time_std, seriestype=:scatter, layout = 1)
```

```
plot(p1, p2, ylabel = ["time [s]" "time [s]"], xlabel = ["size" "size"],
     layout = 2)
```

Out[15]:



Zad5 Neville's algorithm

In [16]:

```
function neville_polynomial_value(X,Y,x)
    n = size(X)[1]
    t = []

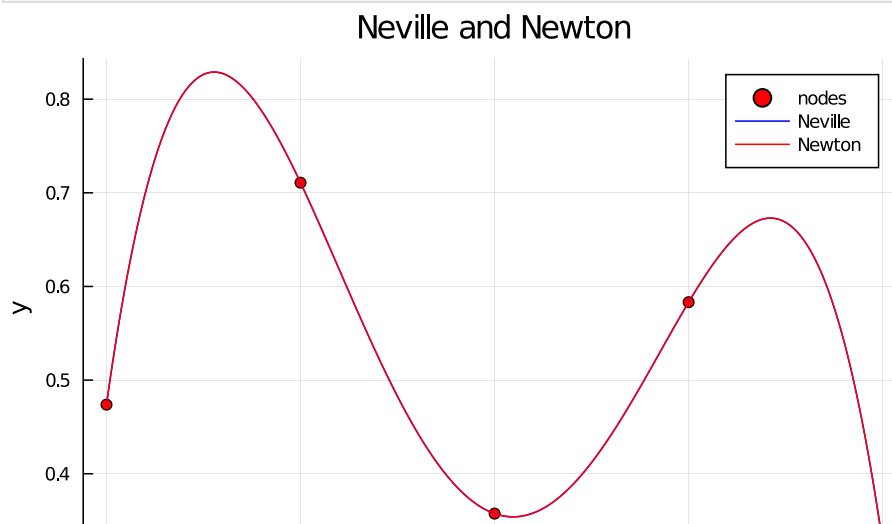
    for i = 1:n
        push!(t, [])
        append!(t[i], Y[i])
    end
    for i = 2:n
        for j = 2:i
            append!(t[i], ((x - X[i-j+1])*t[i][j-1] - (x - X[i])*t[i-1][j-1]) / (X[i] - X[i-j+1]))
        end
    end
    return t[n][n]
end

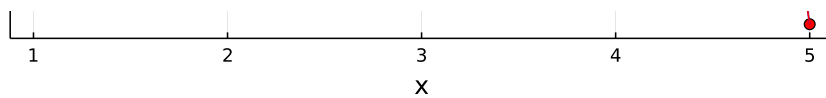
function neville_algorithm(X,Y)
    n = size(X)[1]
    x_interpolation = [x for x=X[1]:0.01:X[n]]
    y_interpolation = []
    for x in x_interpolation
        append!(y_interpolation, neville_polynomial_value(X,Y,x))
    end
    return x_interpolation, y_interpolation
end

X = [i for i=1:5]
Y = [rand() for x in X]

plot(X, Y, seriestype = :scatter, color = :red, label = "nodes")
plot!(neville_algorithm(X,Y), color = :blue, label = "Neville")
plot!(N_Interpolation(X,Y), color = :red, label = "Newton", xaxis = "x", yaxis = "y", title = "Neville and Newton")
```

Out[16]:





```
In [17]: x_neville, y_neville = neville_algorithm(X,Y)
x_newton, y_newton = N_Interpolation(X,Y)
df = DataFrame()
df[:, :x] = [x for x in x_neville]
df[:, :y_neville] = y_neville
df[:, :y_newton] = y_newton
df
```

Out[17]: 401 rows × 3 columns

	x	y_neville	y_newton
	Float64	Any	Any
1	1.0	0.473817	0.473817
2	1.01	0.488426	0.488426
3	1.02	0.502658	0.502658
4	1.03	0.516519	0.516519
5	1.04	0.530013	0.530013
6	1.05	0.543145	0.543145
7	1.06	0.555918	0.555918
8	1.07	0.568337	0.568337
9	1.08	0.580407	0.580407
10	1.09	0.592132	0.592132
11	1.1	0.603516	0.603516
12	1.11	0.614563	0.614563
13	1.12	0.625279	0.625279
14	1.13	0.635666	0.635666
15	1.14	0.64573	0.64573
16	1.15	0.655474	0.655474
17	1.16	0.664903	0.664903
18	1.17	0.67402	0.67402
19	1.18	0.682831	0.682831
20	1.19	0.691339	0.691339
21	1.2	0.699548	0.699548
22	1.21	0.707463	0.707463
23	1.22	0.715086	0.715086
24	1.23	0.722424	0.722424
25	1.24	0.729479	0.729479
26	1.25	0.736255	0.736255
27	1.26	0.742756	0.742756
28	1.27	0.748987	0.748987
29	1.28	0.754952	0.754952
30	1.29	0.760653	0.760653
⋮	⋮	⋮	⋮

Zad6

Bspline Constant and Polynomial

```
In [18]: X = 1:10
Y = [rand() for x in X]
x_itp = X[1]:0.01:X[10]

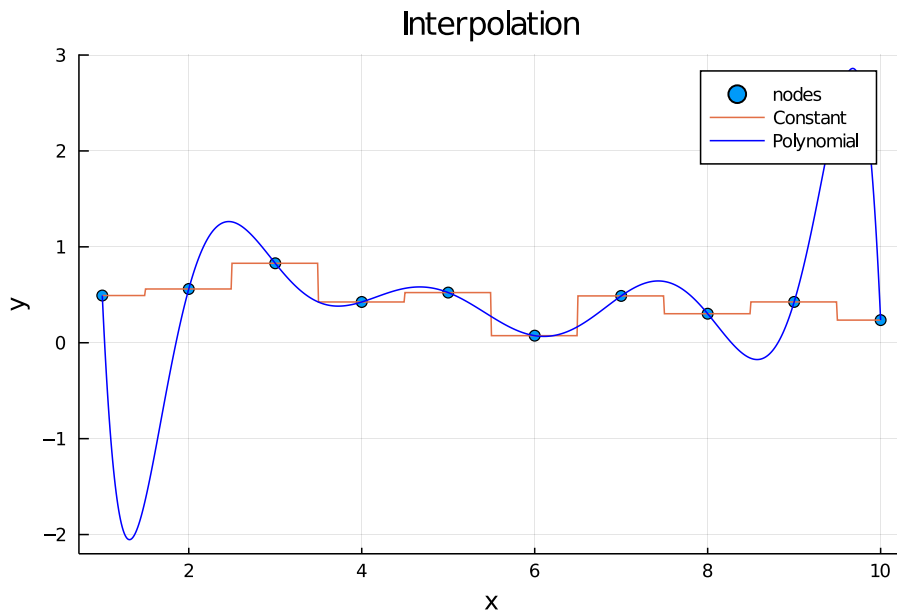
itp = interpolate(Y, BSpline(Constant()))
y1 = [itp(x) for x in x_itp]
```

```

plot(X, Y, seriestype=:scatter, label="nodes")
plot!(x_itp, y1, label="Constant")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial", title = "Interpolation", xaxis = "x", yaxis = "y")

```

Out[18]:



Bspline Linear and Polynomial

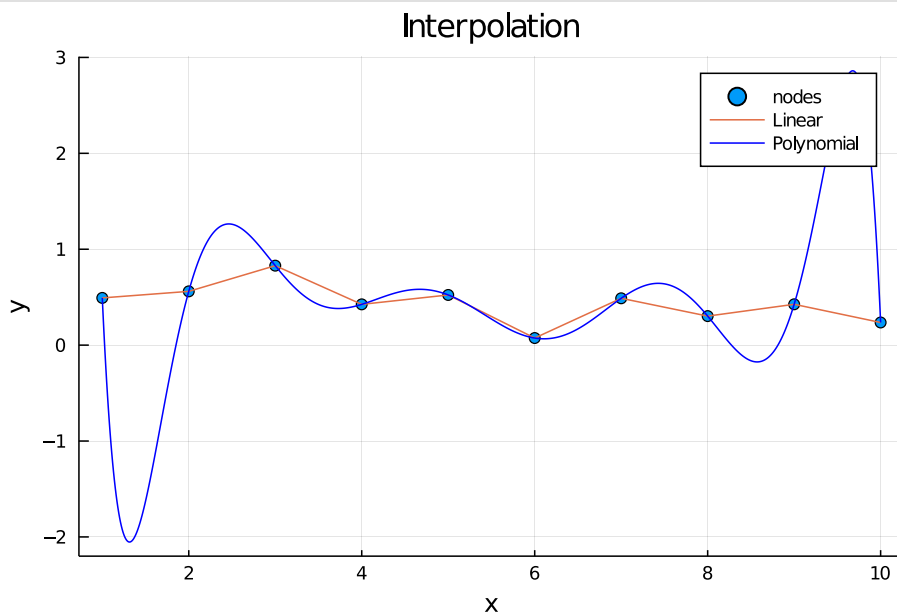
```

In [19]: itp = interpolate(Y, BSpline{Linear}())
y1= [itp(x) for x in x_itp]

plot(X, Y, seriestype=:scatter, label="nodes")
plot!(x_itp, y1, label="Linear")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial", title = "Interpolation", xaxis = "x", yaxis = "y")

```

Out[19]:



Bspline Quadratic and Polynomial

```

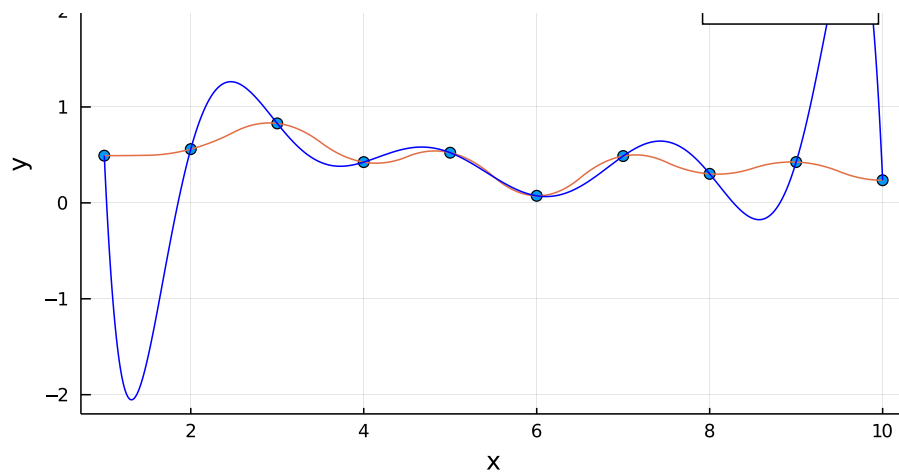
In [20]: itp = interpolate(Y, BSpline{Quadratic}(Flat{OnGrid}()))
y1= [itp(x) for x in x_itp]

plot(X, Y, seriestype=:scatter, label="nodes")
plot!(x_itp, y1, label="Quadratic")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial", title = "Interpolation", xaxis = "x", yaxis = "y")

```

Out[20]:

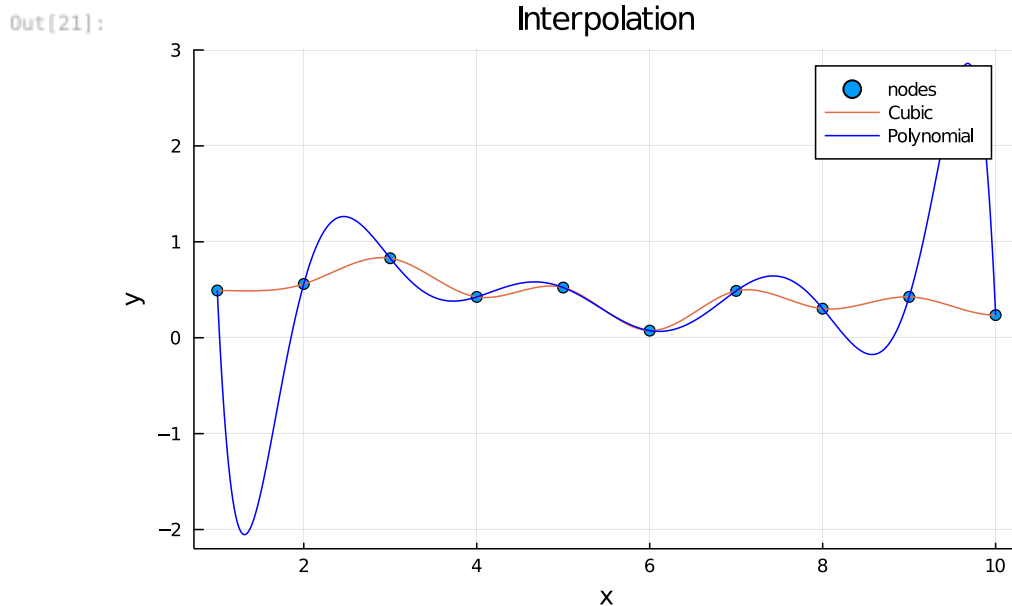




BSpline Cubic and Polynomial

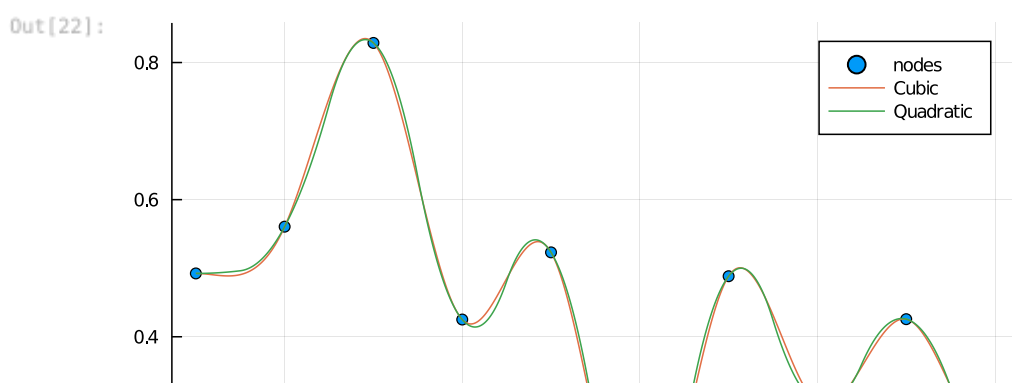
```
In [21]: itp = interpolate(Y, BSpline(Cubic(Flat(OnGrid()))))
y1= [itp(x) for x in x_itp]

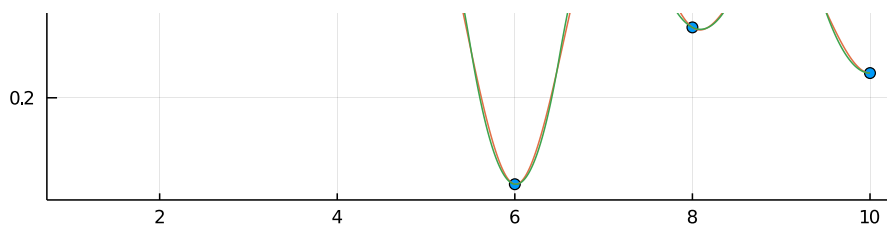
plot(X, Y, seriestype=:scatter, label="nodes")
plot!(x_itp, y1, label="Cubic")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial", title = "Interpolation", xaxis = "x", yaxis = "y")
```



Quadratic and Cubic

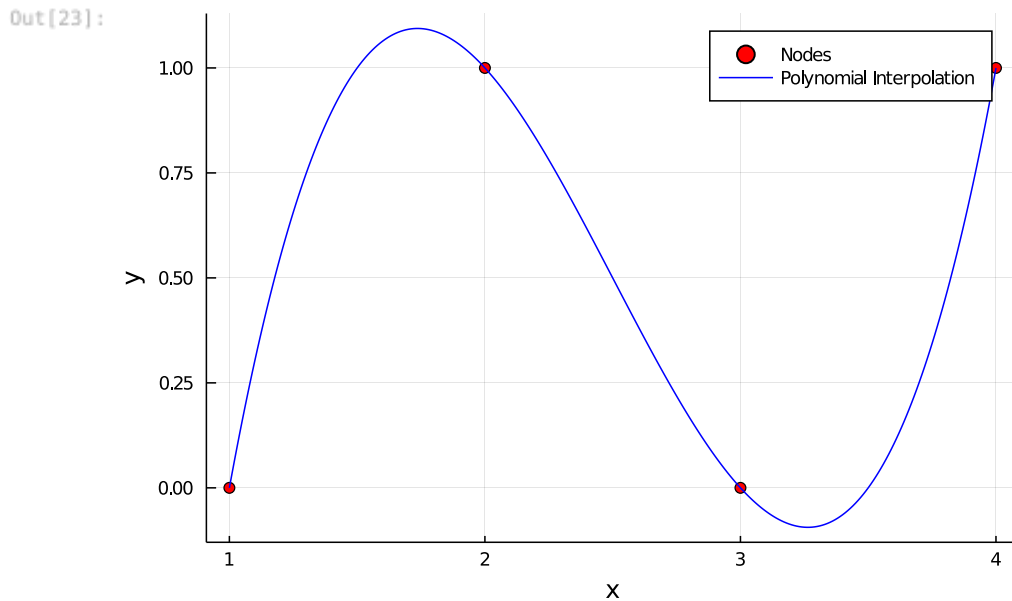
```
In [22]: plot(X, Y, seriestype=:scatter, label="nodes")
itp = interpolate(Y, BSpline(Cubic(Flat(OnGrid()))))
y1= [itp(x) for x in x_itp]
plot!(x_itp, y1, label="Cubic")
itp = interpolate(Y, BSpline(Quadratic(Flat(OnGrid()))))
y1= [itp(x) for x in x_itp]
plot!(x_itp, y1, label="Quadratic")
```



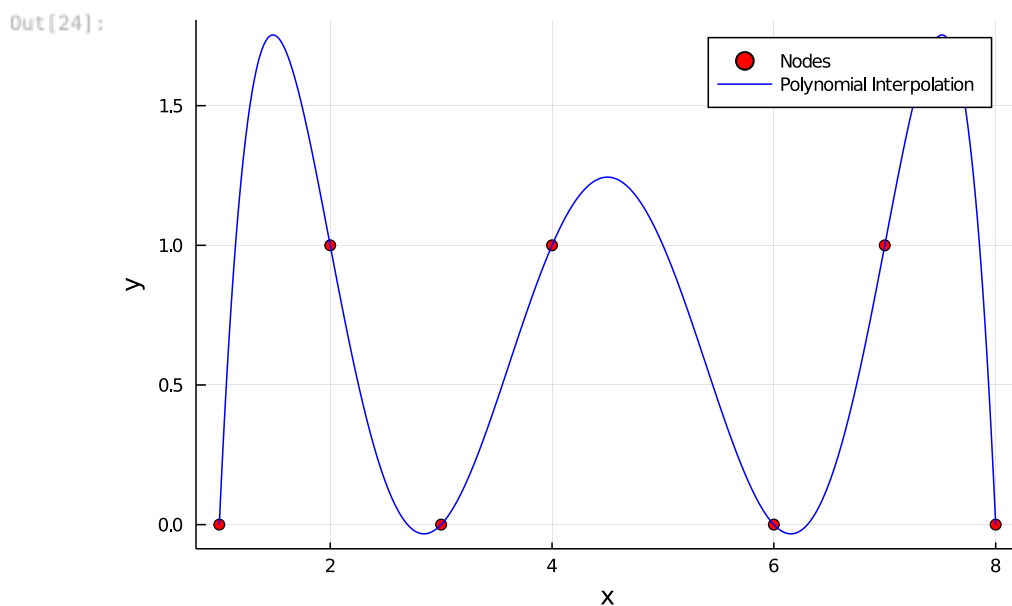


Efekt Rungego

```
In [23]: X = [i for i=1:4]
Y = [rand(0:1) for x in X]
plot(X,Y, color = :red, seriestype = :scatter, label = "Nodes", xaxis = "x", yaxis = "y")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial Interpolation")
```

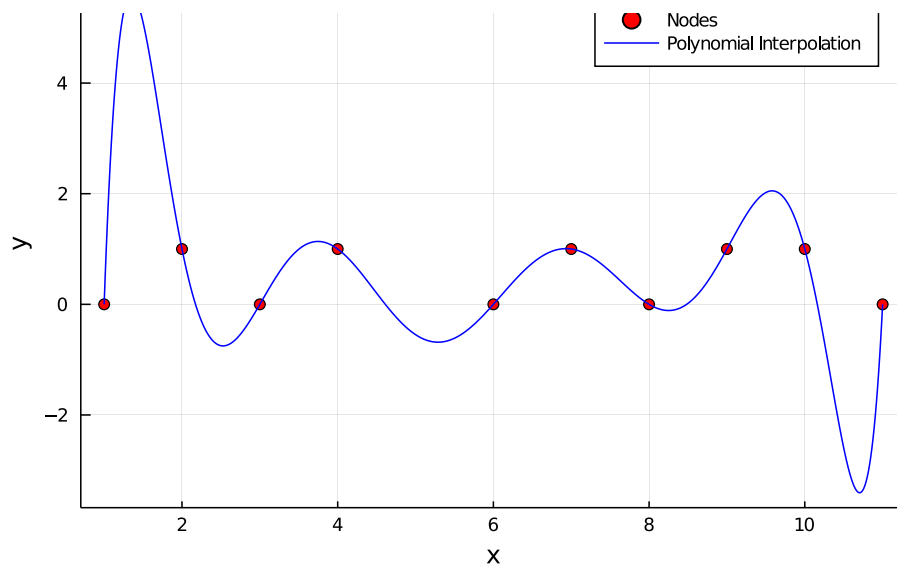


```
In [24]: append!(X, [6 7 8])
append!(Y, [rand(0:1) rand(0:1) rand(0:1)])
plot(X,Y, color = :red, seriestype = :scatter, label = "Nodes", xaxis = "x", yaxis = "y")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial Interpolation")
```

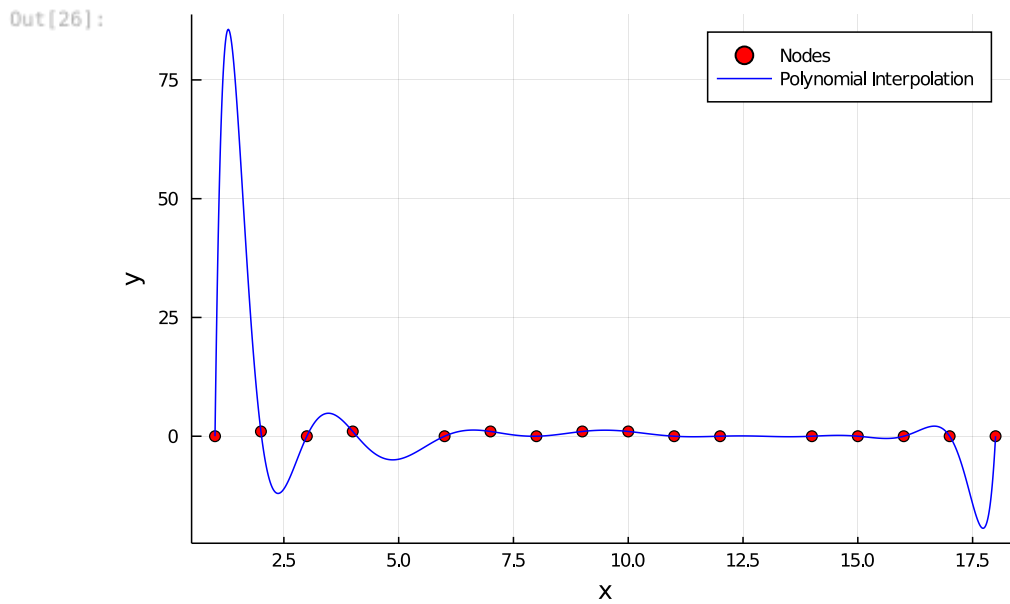


```
In [25]: append!(X, [9 10 11])
append!(Y, [rand(0:1) rand(0:1) rand(0:1)])
plot(X,Y, color = :red, seriestype = :scatter, label = "Nodes", xaxis = "x", yaxis = "y")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial Interpolation")
```





```
In [26]: append(X, [12 14 15 16 17 18])
append!(Y, [rand(0:1) rand(0:1) rand(0:1)])
append!(Y, [rand(0:1) rand(0:1) rand(0:1)])
plot(X,Y, color = :red, seriestype = :scatter, label = "Nodes", xaxis = "x", yaxis = "y")
plot!(PolynomialInterpolation(X,Y), color = :blue, label = "Polynomial Interpolation")
```



In []:

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