Mowmit lab 8

Imports

In [4]:
```
using Roots
using Plots
using ForwardDiff
using DataFrames
```

Testing functions

In [5]:
```
a(x) = sin(x) - x/2
b(x) = 2x - exp(-x)
c(x) = x*exp(-x)
d(x) = (x+3) * (x-1)^2
e(x) = x^3
f(x) = cos(x) -x
```

Out[5]:
```
f (generic function with 1 method)
```

In [6]:
```
iterations = []
f_calls = []
```

Out[6]:
```
Any[]
```

Using interval and sign change

a

In [7]:
```
a_0 = find_zero(a, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 1.0
* Algorithm: Roots.BisectionExact()
* iterations: 0
* function evaluations: 3
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* Note: Exact zero found

Trace:
(a_0, b_0) = (-1.0000000000000000,  1.0000000000000000)
```

Out[7]:
```
0.0
```

In [8]:
```
push!(iterations, [])
push!(f_calls, [])
append!(iterations[1], 0)
append!(f_calls[1], 3)
a(a_0)
```

Out[8]:
```
0.0
```

b

In [9]:
```
b_0 = find_zero(b, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.3517337112491959
* Algorithm: Roots.BisectionExact()
* iterations: 61
```

```
* function evaluations: 63
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
(a_0, b_0) = ( 0.0000000000000000,  1.0000000000000000)
(a_1, b_1) = ( 0.0000000000000000,  1.0000000000000000)
(a_2, b_2) = ( 0.0000000000000000,  1.0000000000000000)
(a_3, b_3) = ( 0.0000000000000000,  1.0000000000000000)
(a_4, b_4) = ( 0.0000000000000000,  1.0000000000000000)
(a_5, b_5) = ( 0.0000000002401066,  1.0000000000000000)
(a_6, b_6) = ( 0.0000154972076416,  1.0000000000000000)
(a_7, b_7) = ( 0.0039367675781250,  1.0000000000000000)
(a_8, b_8) = ( 0.0627441406250000,  1.0000000000000000)
(a_9, b_9) = ( 0.2504882812500000,  1.0000000000000000)
(a_10, b_10) = ( 0.2504882812500000,  0.5004882812500000)
(a_11, b_11) = ( 0.2504882812500000,  0.3753662109375000)
(a_12, b_12) = ( 0.3129272460937500,  0.3753662109375000)
(a_13, b_13) = ( 0.3441467285156250,  0.3753662109375000)
(a_14, b_14) = ( 0.3441467285156250,  0.3597564697265625)
(a_15, b_15) = ( 0.3441467285156250,  0.3519515991210938)
(a_16, b_16) = ( 0.3480491638183594,  0.3519515991210938)
(a_17, b_17) = ( 0.3500003814697266,  0.3519515991210938)
(a_18, b_18) = ( 0.3509759902954102,  0.3519515991210938)
(a_19, b_19) = ( 0.3514637947082520,  0.3519515991210938)
(a_20, b_20) = ( 0.3517076969146729,  0.3519515991210938)
(a_21, b_21) = ( 0.3517076969146729,  0.3518296480178833)
(a_22, b_22) = ( 0.3517076969146729,  0.3517686724662781)
(a_23, b_23) = ( 0.3517076969146729,  0.3517381846904755)
(a_24, b_24) = ( 0.3517229408025742,  0.3517381846904755)
(a_25, b_25) = ( 0.3517305627465248,  0.3517381846904755)
(a_26, b_26) = ( 0.3517305627465248,  0.3517343737185001)
(a_27, b_27) = ( 0.3517324682325125,  0.3517343737185001)
(a_28, b_28) = ( 0.3517334209755063,  0.3517343737185001)
(a_29, b_29) = ( 0.3517334209755063,  0.3517338973470032)
(a_30, b_30) = ( 0.3517336591612548,  0.3517338973470032)
(a_31, b_31) = ( 0.3517336591612548,  0.3517337782541290)
(a_32, b_32) = ( 0.3517336591612548,  0.3517337187076919)
(a_33, b_33) = ( 0.3517336889344733,  0.3517337187076919)
(a_34, b_34) = ( 0.3517337038210826,  0.3517337187076919)
(a_35, b_35) = ( 0.3517337038210826,  0.3517337112643872)
(a_36, b_36) = ( 0.3517337075427349,  0.3517337112643872)
(a_37, b_37) = ( 0.3517337094035611,  0.3517337112643872)
(a_38, b_38) = ( 0.3517337103339742,  0.3517337112643872)
(a_39, b_39) = ( 0.3517337107991807,  0.3517337112643872)
(a_40, b_40) = ( 0.3517337110317840,  0.3517337112643872)
(a_41, b_41) = ( 0.3517337111480856,  0.3517337112643872)
(a_42, b_42) = ( 0.3517337112062364,  0.3517337112643872)
(a_43, b_43) = ( 0.3517337112353118,  0.3517337112643872)
(a_44, b_44) = ( 0.3517337112353118,  0.3517337112498495)
(a_45, b_45) = ( 0.3517337112425807,  0.3517337112498495)
(a_46, b_46) = ( 0.3517337112462151,  0.3517337112498495)
(a_47, b_47) = ( 0.3517337112480323,  0.3517337112498495)
(a_48, b_48) = ( 0.3517337112489409,  0.3517337112498495)
(a_49, b_49) = ( 0.3517337112489409,  0.3517337112493952)
(a_50, b_50) = ( 0.3517337112491681,  0.3517337112493952)
(a_51, b_51) = ( 0.3517337112491681,  0.3517337112492817)
(a_52, b_52) = ( 0.3517337112491681,  0.3517337112492249)
(a_53, b_53) = ( 0.3517337112491681,  0.3517337112491964)
(a_54, b_54) = ( 0.3517337112491822,  0.3517337112491964)
(a_55, b_55) = ( 0.3517337112491893,  0.3517337112491964)
(a_56, b_56) = ( 0.3517337112491929,  0.3517337112491964)
(a_57, b_57) = ( 0.3517337112491947,  0.3517337112491964)
(a_58, b_58) = ( 0.3517337112491956,  0.3517337112491964)
(a_59, b_59) = ( 0.3517337112491956,  0.3517337112491960)
(a_60, b_60) = ( 0.3517337112491958,  0.3517337112491960)
(a_61, b_61) = ( 0.3517337112491958,  0.3517337112491959)
```

Out[9]: 0.35173371124919584

In [10]:
```
push!(iterations, [])
push!(f_calls, [])
append!(iterations[2], 61)
append!(f_calls[2], 61)
b(b_0)
```

Out[10]: 0.0

c

In [11]:
```
c_0 = find_zero(c, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 1.0
* Algorithm: Roots.BisectionExact()
* iterations: 0
* function evaluations: 3
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* Note: Exact zero found

Trace:
(a_0, b_0) = (-1.0000000000000000,  1.0000000000000000)
```

Out[11]: 0.0

In [12]:
```
push!(iterations, [])
push!(f_calls, [])
append!(iterations[3], 0)
append!(f_calls[3], 3)
c(c_0)
```

Out[12]: 0.0

d

In [13]:
```
d_0 = find_zero(d, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 1.0
* Algorithm: Roots.BisectionExact()
* iterations: 0
* function evaluations: 2
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
(a_0, b_0) = (-1.0000000000000000,  1.0000000000000000)
```

Out[13]: 1.0

In [14]:
```
push!(iterations, [])
push!(f_calls, [])
append!(iterations[4], 0)
append!(f_calls[4], 2)
d(d_0)
```

Out[14]: 0.0

e

In [15]:
```
e_0 = find_zero(e, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 1.0
* Algorithm: Roots.BisectionExact()
* iterations: 0
* function evaluations: 3
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* Note: Exact zero found

Trace:
(a_0, b_0) = (-1.0000000000000000,  1.0000000000000000)
```

Out[15]: 0.0

Out[15]: 0.0

In [16]:
```julia
push!(iterations, [])
push!(f_calls, [])
append!(iterations[5], 0)
append!(f_calls[5], 3)
e(e_0)
```

Out[16]: 0.0

f

In [17]:
```julia
f_0 = find_zero(f, (-1, 1), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.7390851332151609
* Algorithm: Roots.BisectionExact()
* iterations: 60
* function evaluations: 62
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)·ϵ) using δ = atol, ϵ = rtol

Trace:
(a_0, b_0) = ( 0.0000000000000000,  1.0000000000000000)
(a_1, b_1) = ( 0.0000000000000000,  1.0000000000000000)
(a_2, b_2) = ( 0.0000000000000000,  1.0000000000000000)
(a_3, b_3) = ( 0.0000000000000000,  1.0000000000000000)
(a_4, b_4) = ( 0.0000000000000000,  1.0000000000000000)
(a_5, b_5) = ( 0.0000000002401066,  1.0000000000000000)
(a_6, b_6) = ( 0.0000154972076416,  1.0000000000000000)
(a_7, b_7) = ( 0.0039367675781250,  1.0000000000000000)
(a_8, b_8) = ( 0.0627441406250000,  1.0000000000000000)
(a_9, b_9) = ( 0.2504882812500000,  1.0000000000000000)
(a_10, b_10) = ( 0.5004882812500000,  1.0000000000000000)
(a_11, b_11) = ( 0.5004882812500000,  0.7502441406250000)
(a_12, b_12) = ( 0.6253662109375000,  0.7502441406250000)
(a_13, b_13) = ( 0.6878051757812500,  0.7502441406250000)
(a_14, b_14) = ( 0.7190246582031250,  0.7502441406250000)
(a_15, b_15) = ( 0.7346343994140625,  0.7502441406250000)
(a_16, b_16) = ( 0.7346343994140625,  0.7424392700195313)
(a_17, b_17) = ( 0.7385368347167969,  0.7424392700195313)
(a_18, b_18) = ( 0.7385368347167969,  0.7404880523681641)
(a_19, b_19) = ( 0.7385368347167969,  0.7395124435424805)
(a_20, b_20) = ( 0.7390246391296387,  0.7395124435424805)
(a_21, b_21) = ( 0.7390246391296387,  0.7392685413360596)
(a_22, b_22) = ( 0.7390246391296387,  0.7391465902328491)
(a_23, b_23) = ( 0.7390246391296387,  0.7390856146812439)
(a_24, b_24) = ( 0.7390551269054413,  0.7390856146812439)
(a_25, b_25) = ( 0.7390703707933426,  0.7390856146812439)
(a_26, b_26) = ( 0.7390779927372932,  0.7390856146812439)
(a_27, b_27) = ( 0.7390818037092686,  0.7390856146812439)
(a_28, b_28) = ( 0.7390837091952562,  0.7390856146812439)
(a_29, b_29) = ( 0.7390846619382501,  0.7390856146812439)
(a_30, b_30) = ( 0.7390846619382501,  0.7390851383097470)
(a_31, b_31) = ( 0.7390849001239985,  0.7390851383097470)
(a_32, b_32) = ( 0.7390850192168728,  0.7390851383097470)
(a_33, b_33) = ( 0.7390850787633099,  0.7390851383097470)
(a_34, b_34) = ( 0.7390851085365284,  0.7390851383097470)
(a_35, b_35) = ( 0.7390851234231377,  0.7390851383097470)
(a_36, b_36) = ( 0.7390851308664423,  0.7390851383097470)
(a_37, b_37) = ( 0.7390851308664423,  0.7390851345880947)
(a_38, b_38) = ( 0.7390851327272685,  0.7390851345880947)
(a_39, b_39) = ( 0.7390851327272685,  0.7390851336576816)
(a_40, b_40) = ( 0.7390851331924750,  0.7390851336576816)
(a_41, b_41) = ( 0.7390851331924750,  0.7390851334250783)
(a_42, b_42) = ( 0.7390851331924750,  0.7390851333087767)
(a_43, b_43) = ( 0.7390851331924750,  0.7390851332506259)
(a_44, b_44) = ( 0.7390851331924750,  0.7390851332215504)
(a_45, b_45) = ( 0.7390851332070127,  0.7390851332215504)
(a_46, b_46) = ( 0.7390851332142816,  0.7390851332215504)
(a_47, b_47) = ( 0.7390851332142816,  0.7390851332179160)
(a_48, b_48) = ( 0.7390851332142816,  0.7390851332160988)
(a_49, b_49) = ( 0.7390851332142816,  0.7390851332151902)
(a_50, b_50) = ( 0.7390851332147359,  0.7390851332151902)
(a_51, b_51) = ( 0.7390851332149631,  0.7390851332151902)
(a_52, b_52) = ( 0.7390851332150766,  0.7390851332151902)
```

```
(a_53, b_53) = ( 0.7390851332151334,  0.7390851332151902)
(a_54, b_54) = ( 0.7390851332151334,  0.7390851332151618)
(a_55, b_55) = ( 0.7390851332151476,  0.7390851332151618)
(a_56, b_56) = ( 0.7390851332151547,  0.7390851332151618)
(a_57, b_57) = ( 0.7390851332151582,  0.7390851332151618)
(a_58, b_58) = ( 0.7390851332151600,  0.7390851332151618)
(a_59, b_59) = ( 0.7390851332151600,  0.7390851332151609)
(a_60, b_60) = ( 0.7390851332151605,  0.7390851332151609)
```

Out[17]: 0.7390851332151607

In [18]:
```
push!(iterations, [])
push!(f_calls, [])
append!(iterations[6], 60)
append!(f_calls[6], 62)
f(f_0)
```

Out[18]: 0.0

Using derivative

In [19]:
```
D(f) = x->ForwardDiff.derivative(f, float(x))
```

Out[19]: D (generic function with 1 method)

a

In [20]:
```
a_0 = find_zero((a, D(a)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Roots.Newton()
* iterations: 0
* function evaluations: 1
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[20]: 0.0

In [21]:
```
append!(iterations[1], 0)
append!(f_calls[1], 1)
a(a_0)
```

Out[21]: 0.0

b

In [22]:
```
b_0 = find_zero((b, D(b)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.35173371124919584
* Algorithm: Roots.Newton()
* iterations: 4
* function evaluations: 9
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 = -1.0000000000000000
x_1 =  0.3333333333333333,       fx_1 = -0.0498646439071226
x_2 =  0.3516893315554154,       fx_2 = -0.0001199797491258
x_3 =  0.3517337109929426,       fx_3 = -0.0000000006927722
```

```
x_4 =  0.3517337112491958,       fx_4 =  0.0000000000000000
```

Out[22]: 0.35173371124919584

In [23]:
```
append!(iterations[2], 4)
append!(f_calls[2], 9)
b(b_0)
```

Out[23]: 0.0

c

In [24]:
```
c_0 = find_zero((c, D(c)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Roots.Newton()
* iterations: 0
* function evaluations: 1
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[24]: 0.0

In [25]:
```
append!(iterations[3], 0)
append!(f_calls[3], 1)
c(c_0)
```

Out[25]: 0.0

d

In [26]:
```
d_0 = find_zero((d, D(d)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.9999999893171166
* Algorithm: Roots.Newton()
* iterations: 26
* function evaluations: 53
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  3.0000000000000000
x_1 =  0.6000000000000000,       fx_1 =  0.5760000000000002
x_2 =  0.8117647058823529,       fx_2 =  0.1350604518624058
x_3 =  0.9082650781831720,       fx_3 =  0.0328892070132730
x_4 =  0.9546772328747365,       fx_4 =  0.0081235129715387
x_5 =  0.9774692207697649,       fx_5 =  0.0020191066159544
x_6 =  0.9887666079847495,       fx_6 =  0.0005033388530879
x_7 =  0.9943912241748282,       fx_7 =  0.0001256570221040
x_8 =  0.9971975823794025,       fx_8 =  0.0000313921691694
x_9 =  0.9985992825526103,       fx_9 =  0.0000078452892489
x_10 =  0.9992997639663359,       fx_10 =  0.0000019609786643
x_11 =  0.9996499126368736,       fx_11 =  0.0000004902017402
x_12 =  0.9998249639795151,       fx_12 =  0.0000001225450712
x_13 =  0.9999124839047338,       fx_13 =  0.0000000306355974
x_14 =  0.9999562424310743,       fx_14 =  0.0000000076588156
x_15 =  0.9999781213352094,       fx_15 =  0.0000000019146934
x_16 =  0.9999890606975221,       fx_16 =  0.0000000004786720
x_17 =  0.9999945303562404,       fx_17 =  0.0000000001196678
x_18 =  0.9999972651799900,       fx_18 =  0.0000000000299169
x_19 =  0.9999986325904624,       fx_19 =  0.0000000000074792
x_20 =  0.9999993162953481,       fx_20 =  0.0000000000018698
x_21 =  0.9999996581477033,       fx_21 =  0.0000000000004675
x_22 =  0.9999998290738590,       fx_22 =  0.0000000000001169
```

```
x_23 =  0.9999999145369313,       fx_23 =  0.0000000000000292
x_24 =  0.9999999572684661,       fx_24 =  0.0000000000000073
x_25 =  0.9999999786342332,       fx_25 =  0.0000000000000018
x_26 =  0.9999999893171166,       fx_26 =  0.0000000000000005
```

Out[26]:
```
0.9999999893171166
```

In [27]:
```
append!(iterations[4], 26)
append!(f_calls[4], 53)
d(d_0)
```

Out[27]:
```
4.564959859473074e-16
```

e

In [28]:
```
e_0 = find_zero((e, D(e)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Roots.Newton()
* iterations: 0
* function evaluations: 1
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[28]:
```
0.0
```

In [29]:
```
append!(iterations[5], 0)
append!(f_calls[5], 1)
e(e_0)
```

Out[29]:
```
0.0
```

f

In [30]:
```
f_0 = find_zero((f, D(f)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.7390851332151607
* Algorithm: Roots.Newton()
* iterations: 5
* function evaluations: 11
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  1.0000000000000000
x_1 =  1.0000000000000000,       fx_1 = -0.4596976941318602
x_2 =  0.7503638678402439,       fx_2 = -0.0189230738221174
x_3 =  0.7391128909113617,       fx_3 = -0.0000464558989908
x_4 =  0.7390851333852840,       fx_4 = -0.0000000002847206
x_5 =  0.7390851332151607,       fx_5 =  0.0000000000000000
```

Out[30]:
```
0.7390851332151607
```

In [31]:
```
append!(iterations[6], 5)
append!(f_calls[6], 11)
f(f_0)
```

Out[31]:
```
0.0
```

Using derivative approximation

Steffens method

a

In [32]:
```
a_0 = find_zero(a, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Order2()
* iterations: 0
* function evaluations: 2
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[32]:
```
0.0
```

In [33]:
```
append!(iterations[1], 0)
append!(f_calls[1], 2)
a(a_0)
```

Out[33]:
```
0.0
```

b

In [34]:
```
b_0 = find_zero(b, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.35173371124919584
* Algorithm: Order2()
* iterations: 5
* function evaluations: 9
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 = -1.0000000000000000
x_1 =  0.3333336697473488,       fx_1 = -0.0498637300279566
x_2 =  0.3508272237172430,       fx_2 = -0.0024509486253887
x_3 =  0.3517315326564641,       fx_3 = -0.0000058897561466
x_4 =  0.3517337112502477,       fx_4 =  0.0000000000028437
x_5 =  0.3517337112491958,       fx_5 =  0.0000000000000000
```

Out[34]:
```
0.35173371124919584
```

In [35]:
```
append!(iterations[2], 5)
append!(f_calls[2], 9)
b(b_0)
```

Out[35]:
```
0.0
```

c

In [36]:
```
c_0 = find_zero(c, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Order2()
* iterations: 0
```

```
* function evaluations: 2
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)·ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[36]: 0.0

In [37]:
```
append!(iterations[3], 0)
append!(f_calls[3], 2)
c(c_0)
```

Out[37]: 0.0

d

In [38]:
```
d_0 = find_zero(d, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.9999999882918915
* Algorithm: Order2()
* iterations: 28
* function evaluations: 50
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)·ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  3.0000000000000000
x_1 =  0.6000007266572467,       fx_1 =  0.5759980234937676
x_2 =  0.7425745512658876,       fx_2 =  0.2480124126004012
x_3 =  0.8503843973092605,       fx_3 =  0.0861901946566459
x_4 =  0.9078063739355543,       fx_4 =  0.0332150438398251
x_5 =  0.9438095460191374,       fx_5 =  0.0124520545825248
x_6 =  0.9654014978509854,       fx_6 =  0.0047468090470905
x_7 =  0.9787031988582305,       fx_7 =  0.0018045557117044
x_8 =  0.9868614564220237,       fx_8 =  0.0006882173165696
x_9 =  0.9936176755396728,       fx_9 =  0.0001626762840448
x_10 =  0.9968525182799099,      fx_10 =  0.0000395953837412
x_11 =  0.9984368327132128,      fx_11 =  0.0000097701482792
x_12 =  0.9992210184017803,      fx_12 =  0.0000024267766258
x_13 =  0.9996111546600934,      fx_13 =  0.0000006047439998
x_14 =  0.9998057380704235,      fx_14 =  0.0000001509434581
x_15 =  0.9999029091426591,      fx_15 =  0.0000000377056231
x_16 =  0.9999514645885342,      fx_16 =  0.0000000094226303
x_17 =  0.9999757347973264,      fx_17 =  0.0000000023551860
x_18 =  0.9999878680243914,      fx_18 =  0.0000000005887375
x_19 =  0.9999939341689927,      fx_19 =  0.0000000001471770
x_20 =  0.9999969671225958,      fx_20 =  0.0000000000367934
x_21 =  0.9999984835713741,      fx_21 =  0.0000000000091982
x_22 =  0.9999992417899326,      fx_22 =  0.0000000000022995
x_23 =  0.9999996209014563,      fx_23 =  0.0000000000005749
x_24 =  0.9999998104473484,      fx_24 =  0.0000000000001437
x_25 =  0.9999999051887366,      fx_25 =  0.0000000000000360
x_26 =  0.9999999525752368,      fx_26 =  0.0000000000000090
x_27 =  0.9999999762971821,      fx_27 =  0.0000000000000022
x_28 =  0.9999999882918915,      fx_28 =  0.0000000000000005
```

Out[38]: 0.9999999882918915

In [39]:
```
append!(iterations[4], 28)
append!(f_calls[4], 50)
d(d_0)
```

Out[39]: 5.483192153677992e-16

e

In [40]:
```
e_0 = find_zero(e, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.0
* Algorithm: Order2()
* iterations: 0
* function evaluations: 2
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  0.0000000000000000
```

Out[40]: 0.0

In [41]:
```
append!(iterations[5], 0)
append!(f_calls[5], 2)
e(e_0)
```

Out[41]: 0.0

f

In [42]:
```
f_0 = find_zero(f, 0, Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.7390851332151607
* Algorithm: Order2()
* iterations: 6
* function evaluations: 10
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 =  0.0000000000000000,       fx_0 =  1.0000000000000000
x_1 =  0.9999969722835389,       fx_1 = -0.4596921186823234
x_2 =  0.6850738998209052,       fx_2 =  0.0892983907342113
x_3 =  0.7362990541607766,       fx_3 =  0.0046599445167859
x_4 =  0.7391193608624661,       fx_4 = -0.0000572842351875
x_5 =  0.7390851330409081,       fx_5 =  0.0000000002916313
x_6 =  0.7390851332151607,       fx_6 =  0.0000000000000000
```

Out[42]: 0.7390851332151607

In [43]:
```
append!(iterations[6], 6)
append!(f_calls[6], 10)
f(f_0)
```

Out[43]: 0.0

In [44]:
```
df = DataFrame()
functions = ["a", "b", "c", "d", "e", "f"]
methods = ["Bisection", "Newton", "Steffens"]
df[:, :Function] = [functions[j] for i=1:3 for j=1:6]
df[:, :Method] = [methods[j] for j=1:3 for i=1:6]
df[:, :Iterations] = [iterations[i][j] for j=1:3 for i=1:6]
df[:, :Function_calls] = [f_calls[i][j] for j=1:3 for i=1:6]
df
```

Out[44]: 18 rows × 4 columns

	Function	Method	Iterations	Function_calls
	String	String	Int64	Int64
1	a	Bisection	0	3
2	b	Bisection	61	61
3	c	Bisection	0	3
4	d	Bisection	0	2
5	e	Bisection	0	3

6	f	Bisection	60	62
7	a	Newton	0	1
8	b	Newton	4	9
9	c	Newton	0	1
10	d	Newton	26	53
11	e	Newton	0	1
12	f	Newton	5	11
13	a	Steffens	0	2
14	b	Steffens	5	9
15	c	Steffens	0	2
16	d	Steffens	28	50
17	e	Steffens	0	2
18	f	Steffens	6	10

Methods on (x+3) * (x-1)^2

In [45]:
```
find_zero(x->(x+3)*(x-1)^2, (-10, 10), Bisection(),verbose=true)
```

```
Results of univariate zero finding:

* Converged to: -2.9999999999999964
* Algorithm: Roots.BisectionExact()
* iterations: 58
* function evaluations: 60
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
(a_0, b_0) = (-10.0000000000000000,  0.0000000000000000)
(a_1, b_1) = (-10.0000000000000000, -0.0000000000000000)
(a_2, b_2) = (-10.0000000000000000, -0.0000000000000000)
(a_3, b_3) = (-10.0000000000000000, -0.0000000000000000)
(a_4, b_4) = (-10.0000000000000000, -0.0000000000000000)
(a_5, b_5) = (-10.0000000000000000, -0.0000000021973392)
(a_6, b_6) = (-10.0000000000000000, -0.0001482963562012)
(a_7, b_7) = (-10.0000000000000000, -0.0385131835937500)
(a_8, b_8) = (-10.0000000000000000, -0.6206054687500000)
(a_9, b_9) = (-10.0000000000000000, -2.4912109375000000)
(a_10, b_10) = (-4.9912109375000000, -2.4912109375000000)
(a_11, b_11) = (-3.4934082031250000, -2.4912109375000000)
(a_12, b_12) = (-3.4934082031250000, -2.9923095703125000)
(a_13, b_13) = (-3.2428588867187500, -2.9923095703125000)
(a_14, b_14) = (-3.1175842285156250, -2.9923095703125000)
(a_15, b_15) = (-3.0549468994140625, -2.9923095703125000)
(a_16, b_16) = (-3.0236282348632813, -2.9923095703125000)
(a_17, b_17) = (-3.0079689025878906, -2.9923095703125000)
(a_18, b_18) = (-3.0001392364501953, -2.9923095703125000)
(a_19, b_19) = (-3.0001392364501953, -2.9962244033813477)
(a_20, b_20) = (-3.0001392364501953, -2.9981818199157715)
(a_21, b_21) = (-3.0001392364501953, -2.9991605281829834)
(a_22, b_22) = (-3.0001392364501953, -2.9996498823165894)
(a_23, b_23) = (-3.0001392364501953, -2.9998945593833923)
(a_24, b_24) = (-3.0000168979167938, -2.9998945593833923)
(a_25, b_25) = (-3.0000168979167938, -2.9999557286500931)
(a_26, b_26) = (-3.0000168979167938, -2.9999863132834435)
(a_27, b_27) = (-3.0000016056001186, -2.9999863132834435)
(a_28, b_28) = (-3.0000016056001186, -2.9999939594417810)
(a_29, b_29) = (-3.0000016056001186, -2.9999977825209498)
(a_30, b_30) = (-3.0000016056001186, -2.9999996940605342)
(a_31, b_31) = (-3.0000006498303264, -2.9999996940605342)
(a_32, b_32) = (-3.0000001719454303, -2.9999996940605342)
(a_33, b_33) = (-3.0000001719454303, -2.9999999330029823)
(a_34, b_34) = (-3.0000000524742063, -2.9999999330029823)
(a_35, b_35) = (-3.0000000524742063, -2.9999999927385943)
(a_36, b_36) = (-3.0000000226064003, -2.9999999927385943)
(a_37, b_37) = (-3.0000000076724973, -2.9999999927385943)
(a_38, b_38) = (-3.0000000002055458, -2.9999999927385943)
(a_39, b_39) = (-3.0000000002055458, -2.9999999964720701)
(a_40, b_40) = (-3.0000000002055458, -2.9999999983388079)
(a_41, b_41) = (-3.0000000002055458, -2.9999999992721769)
(a_42, b_42) = (-3.0000000002055458, -2.9999999997388613)
(a_43, b_43) = (-3.0000000002055458, -2.9999999999722036)
(a_44, b_44) = (-3.0000000000888747, -2.9999999999722036)
(a_45, b_45) = (-3.0000000000305391, -2.9999999999722036)
```

```
(a_46, b_46) = (-3.0000000000013713, -2.9999999999722036)
(a_47, b_47) = (-3.0000000000013713, -2.9999999999867875)
(a_48, b_48) = (-3.0000000000013713, -2.9999999999940794)
(a_49, b_49) = (-3.0000000000013713, -2.9999999999977254)
(a_50, b_50) = (-3.0000000000013713, -2.9999999999995484)
(a_51, b_51) = (-3.0000000000004596, -2.9999999999995484)
(a_52, b_52) = (-3.0000000000000040, -2.9999999999995484)
(a_53, b_53) = (-3.0000000000000040, -2.9999999999997762)
(a_54, b_54) = (-3.0000000000000040, -2.9999999999998899)
(a_55, b_55) = (-3.0000000000000040, -2.9999999999999467)
(a_56, b_56) = (-3.0000000000000040, -2.9999999999999751)
(a_57, b_57) = (-3.0000000000000040, -2.9999999999999893)
(a_58, b_58) = (-3.0000000000000040, -2.9999999999999964)
```

Out[45]:
```
-3.0
```

In [46]:
```
find_zero((x->(x+3)*(x-1)^2, D(x->(x+3)*(x-1)^2)), -1, Roots.Newton(),verbose=true) # start at -1 / 0
```

```
Results of univariate zero finding:

* Converged to: 1.0
* Algorithm: Roots.Newton()
* iterations: 1
* function evaluations: 3
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 = -1.0000000000000000,       fx_0 =  8.0000000000000000
x_1 =  1.0000000000000000,       fx_1 =  0.0000000000000000
```

Out[46]:
```
1.0
```

In [47]:
```
find_zero(x->(x+3)*(x-1)^2, -1,Order2(), verbose=true)
```

```
Results of univariate zero finding:

* Converged to: 0.9999999881903239
* Algorithm: Order2()
* iterations: 10
* function evaluations: 21
* stopped as |f(x_n)| ≤ max(δ, max(1,|x|)⋅ϵ) using δ = atol, ϵ = rtol

Trace:
x_0 = -1.0000000000000000,       fx_0 =  8.0000000000000000
x_1 =  0.9999939445041808,       fx_1 =  0.0000000001466759
x_2 =  0.9999969722911086,       fx_2 =  0.0000000000366681
x_3 =  0.9999984861575568,       fx_3 =  0.0000000000091669
x_4 =  0.9999992430797958,       fx_4 =  0.0000000000022917
x_5 =  0.9999996215388520,       fx_5 =  0.0000000000005729
x_6 =  0.9999998107514819,       fx_6 =  0.0000000000001433
x_7 =  0.9999999054030192,       fx_7 =  0.0000000000000358
x_8 =  0.9999999527612857,       fx_8 =  0.0000000000000089
x_9 =  0.9999999764981869,       fx_9 =  0.0000000000000022
x_10 =  0.9999999881903239,      fx_10 =  0.0000000000000006
```

Out[47]:
```
0.9999999881903239
```

Method fails

Bisection

In [48]:
```
find_zero(x->x^2, (-1, 1), Bisection(),verbose=true)
```

```
ArgumentError: The interval [a,b] is not a bracketing interval.
You need f(a) and f(b) to have different signs (f(a) * f(b) < 0).
Consider a different bracket or try fzero(f, c) with an initial guess c.
```

```
Stacktrace:
 [1] init_state(::Bisection, ::Roots.DerivativeFree{var"#27#28"}, ::Tuple{Float64,Float64}) at C:\Users\Norbert\.julia\packages\Roots\AV3zx\src\bracketing.jl:83
 [2] find_zero(::Function, ::Tuple{Int64,Int64}, ::Bisection; tracks::Roots.NullTracks, verbose::Bool, kwargs::Base.Iterators.Pairs{Union{},Union{},Tuple{},NamedTuple{(),Tuple{}}}) at C:\Users\Norbert\.julia\packages\Roots\AV3zx\src\bracketing.jl:335
 [3] top-level scope at In[48]:1
 [4] include_string(::Function, ::Module, ::String, ::String) at .\loading.jl:1091
```

Newton

In [49]:

```
find_zero((x->-(x)^2 + 1, D(x->-(x)^2 + 1)),0, Roots.Newton(),verbose=true)
```

```
Results of univariate zero finding:

* Convergence failed: Too many steps taken.
* Algorithm Roots.Newton()

Trace:
x_0 =  0.0000000000000000,       fx_0 =  1.0000000000000000
x_1 =  0.0000000000000000,       fx_1 =  1.0000000000000000
x_2 =  0.0000000000000000,       fx_2 =  1.0000000000000000
x_3 =  0.0000000000000000,       fx_3 =  1.0000000000000000
x_4 =  0.0000000000000000,       fx_4 =  1.0000000000000000
x_5 =  0.0000000000000000,       fx_5 =  1.0000000000000000
x_6 =  0.0000000000000000,       fx_6 =  1.0000000000000000
x_7 =  0.0000000000000000,       fx_7 =  1.0000000000000000
x_8 =  0.0000000000000000,       fx_8 =  1.0000000000000000
x_9 =  0.0000000000000000,       fx_9 =  1.0000000000000000
x_10 =  0.0000000000000000,      fx_10 =  1.0000000000000000
x_11 =  0.0000000000000000,      fx_11 =  1.0000000000000000
x_12 =  0.0000000000000000,      fx_12 =  1.0000000000000000
x_13 =  0.0000000000000000,      fx_13 =  1.0000000000000000
x_14 =  0.0000000000000000,      fx_14 =  1.0000000000000000
x_15 =  0.0000000000000000,      fx_15 =  1.0000000000000000
x_16 =  0.0000000000000000,      fx_16 =  1.0000000000000000
x_17 =  0.0000000000000000,      fx_17 =  1.0000000000000000
x_18 =  0.0000000000000000,      fx_18 =  1.0000000000000000
x_19 =  0.0000000000000000,      fx_19 =  1.0000000000000000
x_20 =  0.0000000000000000,      fx_20 =  1.0000000000000000
x_21 =  0.0000000000000000,      fx_21 =  1.0000000000000000
x_22 =  0.0000000000000000,      fx_22 =  1.0000000000000000
x_23 =  0.0000000000000000,      fx_23 =  1.0000000000000000
x_24 =  0.0000000000000000,      fx_24 =  1.0000000000000000
x_25 =  0.0000000000000000,      fx_25 =  1.0000000000000000
x_26 =  0.0000000000000000,      fx_26 =  1.0000000000000000
x_27 =  0.0000000000000000,      fx_27 =  1.0000000000000000
x_28 =  0.0000000000000000,      fx_28 =  1.0000000000000000
x_29 =  0.0000000000000000,      fx_29 =  1.0000000000000000
x_30 =  0.0000000000000000,      fx_30 =  1.0000000000000000
x_31 =  0.0000000000000000,      fx_31 =  1.0000000000000000
x_32 =  0.0000000000000000,      fx_32 =  1.0000000000000000
x_33 =  0.0000000000000000,      fx_33 =  1.0000000000000000
x_34 =  0.0000000000000000,      fx_34 =  1.0000000000000000
x_35 =  0.0000000000000000,      fx_35 =  1.0000000000000000
x_36 =  0.0000000000000000,      fx_36 =  1.0000000000000000
x_37 =  0.0000000000000000,      fx_37 =  1.0000000000000000
x_38 =  0.0000000000000000,      fx_38 =  1.0000000000000000
x_39 =  0.0000000000000000,      fx_39 =  1.0000000000000000
x_40 =  0.0000000000000000,      fx_40 =  1.0000000000000000
x_41 =  0.0000000000000000,      fx_41 =  1.0000000000000000
```

```
Roots.ConvergenceFailed("Stopped at: xn = 0.0. Too many steps taken. ")

Stacktrace:
 [1] find_zero(::Tuple{var"#29#31",var"#1#2"{var"#30#32"}}, ::Int64, ::Roots.Newton, ::Nothing; tracks::Roots.NullTracks, verbose::Bool, p::Nothing, kwargs::Base.Iterators.Pairs{Union{},Union{},Tuple{},NamedTuple{(),Tuple{}}}) at C:\Users\Norbert\.julia\packages\Roots\AV3zx\src\find_zero.jl:715
 [2] top-level scope at In[49]:1
 [3] include_string(::Function, ::Module, ::String, ::String) at .\loading.jl:1091
```

Steffens

In [50]:

```
find_zero(x->exp(x) * sin(x) - 2, -5,Order2(), verbose=true) # first root at ~ 0.92
```

Results of univariate zero finding:

```
Results of univariate zero finding:

* Converged to: -5.0
* Algorithm: Order2()
* iterations: 3
* function evaluations: 5
* stopped as x_n ≈ x_{n-1} using atol=xatol, rtol=xrtol
* Note: x_n ≈ x_{n-1}.

Trace:
x_0 = -5.0000000000000000,       fx_0 = -1.9935388190611834
x_1 =  233.1057525070502834,     fx_1 =  1012763641080249744192758880062814984717565017328080322332253670686764743804458394335141704358103613344.0000000000000000
x_2 = -5.0000000000000000,       fx_2 = -1.9935388190611834
x_3 = -5.0000000000000000,       fx_3 = -1.9935388190611834
```

```
Roots.ConvergenceFailed("Stopped at: xn = -5.0. x_n ≈ x_{n-1}. ")

Stacktrace:
 [1] find_zero(::Function, ::Int64, ::Order2, ::Nothing; tracks::Roots.NullTracks, verbose::Bool, p::Nothing, kwargs::Base.Iterators.Pairs{Union{},Union{},Tuple{},NamedTuple{(),Tuple{}}}) at C:\Users\Norbert\.julia\packages\Roots\AV3zx\src\find_zero.jl:715
 [2] top-level scope at In[50]:1
 [3] include_string(::Function, ::Module, ::String, ::String) at .\loading.jl:1091
```