HW 3 Writeup

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(Dated: February 18, 2020)

1. PROBLEM 1

The first problem was about the use of numerical integration through the use of python. I chose to use Simpson's Rule, and developed it through the use of a "for - if" loop to build the lists of even and odd bins, then summed each bin through the use of lambda functions. I used the routine to compute the integral of the following function:

$$f(x) = e^{-t^2} \tag{1}$$

In order to check the error based on the number of bins for Simpson's Rule, I used the math library's erf function, shown below:

$$Erf(x) = \frac{1}{\pi^{1/2}} \int_{-x}^{x} e^{-t^2} dt \tag{2}$$

By setting up a loop to take the absolute value of the difference between the Simpson Integral and the Erf divided by the Erf, I was able to graph the fractional error for a given number of bins. The below is a plot of the fractional error

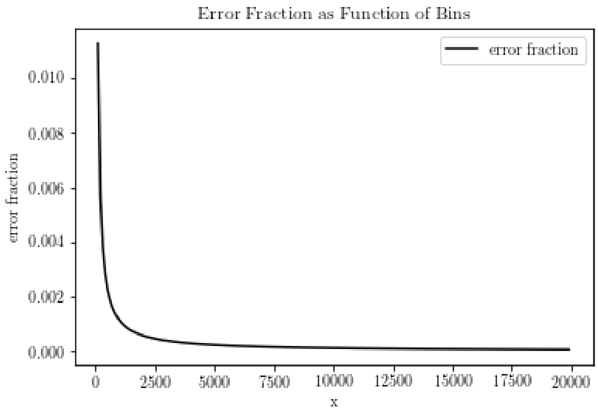

FIG. 1: Error Fraction as a Function of N Bins

Using a number for N where the fractional error falls flat, I graphed my Simpson integral against the function integrated and the Erf function. Below is the plot.

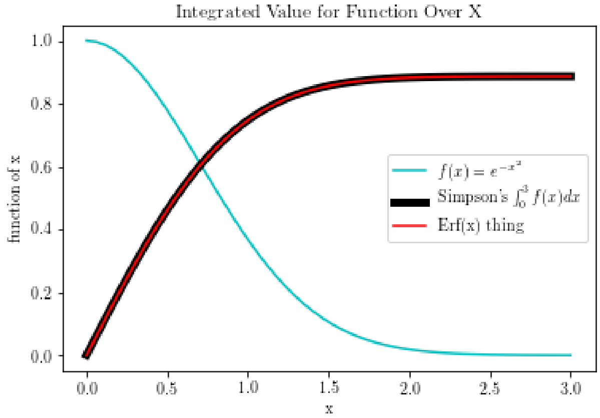

FIG. 2: Simpson Int vs Actual Int w/ f(x) included

2. PROBLEM 2

Problem 2 involved the creation and analysis of computing Bessel Functions, with an application for the Diffraction of light through a point. The First part of the problem involved using the Simpson integral from last problem to compute the Bessel Function by integrating the equation below:

$$J_m(x) = \frac{1}{\pi} \int_0^{\pi} cos(m\theta - xsin(\theta))d\theta \tag{3}$$

Using this, we plotted out the functions for J_1, J_2 and J_3, as shown below: Following this, we used the Bessel

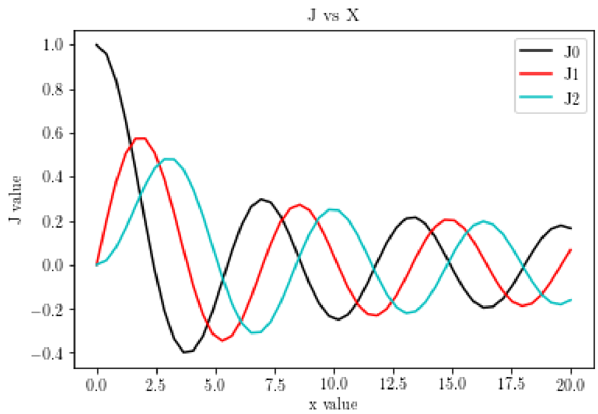

FIG. 3: Bessel Functions as Function of x

Function to get the intensity of light scattering through

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a point source, using the following equation:

$$I(r) = (\frac{J_1(kr)}{kr})^2 \tag{4}$$

This involved using heat plots, and a lot of fiddling with parameters in order to get the plot to function correctly. This was frustrating, and took up several hours of time. Below is the finished and beautiful plot: Next up for this

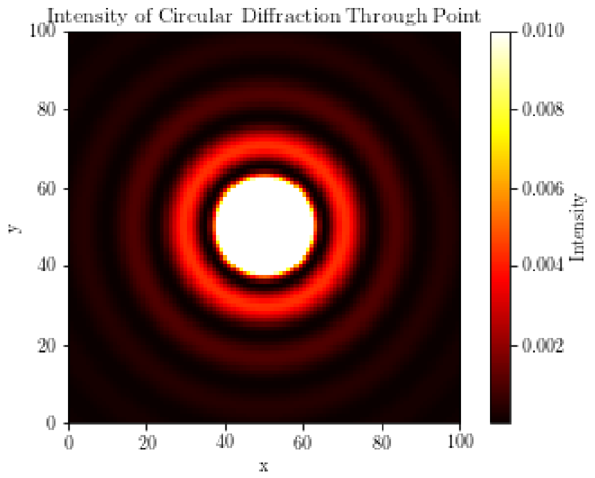

FIG. 4: Light Intensity as a Function of Radius

problem, we decided to check the error function of the integrated definition of the Bessel Function. We used the following recursive definition to check our work:

$$J_n(x) = \frac{2n-1}{x}J_{n-1}(x) - J_{n-2}(x) \tag{5}$$

Using the error checking methods for Problem 1, we plotted the error fraction for the integrated definition of J_2, J_3, and J_4 vs the recursive definitions, which are believed to be more accurate.

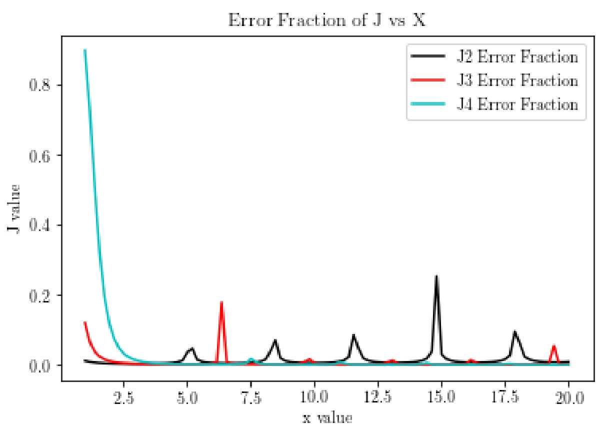

FIG. 5: Error Fraction of Simpson Integral vs Recursive Bessel Function

3. PROBLEM 3

This problem involved using Gaussian Quadrature to determine the Specific Heat Capacity of a Solid. Using the tools provided by the book, I was able to evaluate the following integral, as well as make a function to have the computer calculate it:

$$C_V = 9V\rho k_b(\frac{T}{\theta_D})^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x-1)^2}dx \tag{6}$$

Using the Gaussian integral, we then graph the heat capacity as a function of Temperature, as shown by the following graph:

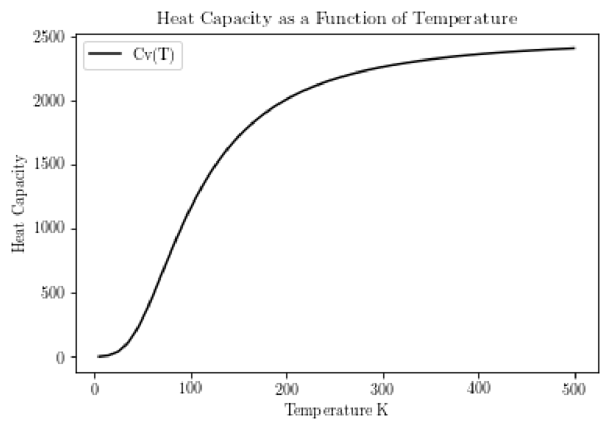

FIG. 6: Heat Capacity of a Solid as a Function of Temperature

4. SURVEY QUESTIONS

The homework took a lot of time, with the Density Plot for Question 2 taking hours, despite my understanding the methodology behind the process. I now have the gaussian integration in my toolkit for future problem sets, which I imagine will be quite useful.