Homework 5

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1. PROBLEM 1 - 6.4

This problem was largely based around using linear algebra code to determine the potential at various points within a network of resistors. Below is said resistor circuit, with the points where we will try and determine the potential labled as V_1 through V_4. The following equation is given to as as an example of how to set up a series of equalities, given that we can use the Loop law to move back to the same point with a net 0 difference in potential:

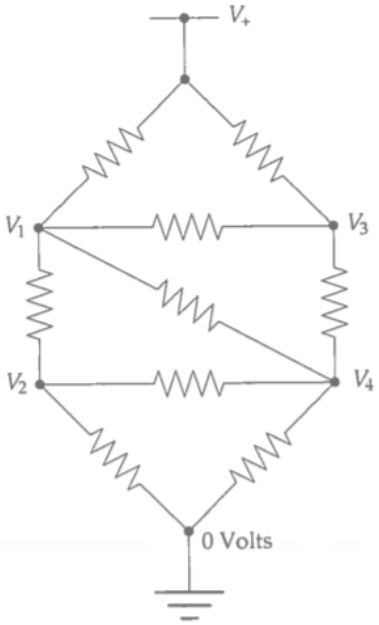

FIG. 1: Circuit of Resistors

$$\frac{V_1 - V_2}{R} + \frac{V_1 - V_3}{R} + \frac{V_1 - V_4}{R} + \frac{V_1 - V_+}{R} = 0$$

Which then turns into an equivalent equation:

$$4V_1 - V_2 - V_3 - V_4 = V_+$$

From here we can derive similar equations for each point, leading us to the following series of equations:

$$\begin{aligned} 4V_1 - V_2 - V_3 - V_4 &= V_+ \\ -V_1 + 3V_2 - 0 - V_4 &= 0 \\ -V_1 - 0 + 3V_3 - V_4 &= V_+ \\ -V_1 - V_2 - V_3 + 4V_4 &= 0 \end{aligned}$$

These equations together can be placed into a 4 by 4 matrix like so, which we will label A:

$$\begin{matrix} 4 & -1 & -1 & -1 \\ -1 & 3 & 0 & -1 \\ -1 & 0 & 3 & -1 \\ -1 & -1 & -1 & 4 \end{matrix}$$

Ax = v, with v being equal to the following vector:

$$\begin{matrix} 1 \\ 0 \\ 1 \\ 0 \end{matrix}$$

Using the np.linalg solve command in python, we were able to determine the following values for V_1 through V_4 in Volts, given that $V_+ = 5$ Volts:

$$\begin{aligned} V_1 &= 3 \\ V_2 &= 1\frac{2}{3} \\ V_3 &= 3\frac{1}{3} \\ V_4 &= 2 \end{aligned}$$

2. PROBLEM 2 - 6.9

2.1. a

$$\hat{H} = -\frac{\hbar^2}{2M}\frac{d^2}{dx^2} + V(x)$$

We're setting out to show that

$$\sum_{n=1}^{\infty} \psi_n \int_0^L Sin(\frac{\pi m x}{L})\hat{H}Sin(\frac{\pi n x}{L})dx = \frac{1}{2}LE\psi_m$$

We will do this with the following relations:

$$\hat{H}\sum_n^{\infty} \psi_n sin(\frac{\pi n x}{L}) = E\sum_m^{\infty} sin(\frac{\pi m x}{L})$$

$$sin(\frac{\pi m x}{L})\hat{H}\sum_n^{\infty} \psi_n sin(\frac{\pi n x}{L}) = E\sum_m^{\infty} sin(\frac{\pi m x}{L})$$

$$\sum_n^{\infty} \psi_n H_{m,n} = E\psi_m$$

2.2. b

This part wasn't too bad, I defined a function $H(m,n)$ which was broken into if, elif, and else statements which made for relatively straightforward computation when m and n were used as inputs.

$$-\frac{8amn}{\pi^2(m^2-n^2)^2} >> even:even/odd:odd$$
$$\frac{a}{2}+\frac{n^2\pi^2\hbar^2}{2ML^2} >> m=n$$
$$0 >> even:odd/odd:even$$

2.3. c

Here we used a nested for loop to run through the values of m and n for a 10 by 10 matrix, and sorted the eigenvalues in ascending order. The eigenvalues are listed as the following, using the eig from the numpy.linalg library: 5.83641932 11.18114227 18.66291636 29.14418658 42.65501838 59.1851465 78.72918429 101.28523358 126.85105129 155.55490412

2.4. d

Part d was much the same as part c, although reassigning the values of n to make it a 100 by 100 matrix rather than a 10 by 10. The first 10 eigenvalues are listed as the following: 5.83641892 11.18114095 18.66291449 29.14417778 42.65500926 59.18509392 78.72913246 101.28460263 126.85021893 155.42527783 While different, they are quite close to the eigenvalues provided by the 10 by 10 matrix.

2.5. e

For part e we just used the first 3 eigenvalues of the 100 by 100 matrix, finding the corresponding eigenvectors through the numpy.where command. We define a psi function adopting the definition given in the book as, making a list and then using a lambda function to take care of the summation,:

$$\psi(x)=\sum_{n=1}^{100}\psi_n Sin(\frac{\pi nx}{L})$$

This allows us to make the following plot for science purposes my people, which upon examining allows us to check and find that it is normalized to have the total area under the curve equal to 1.

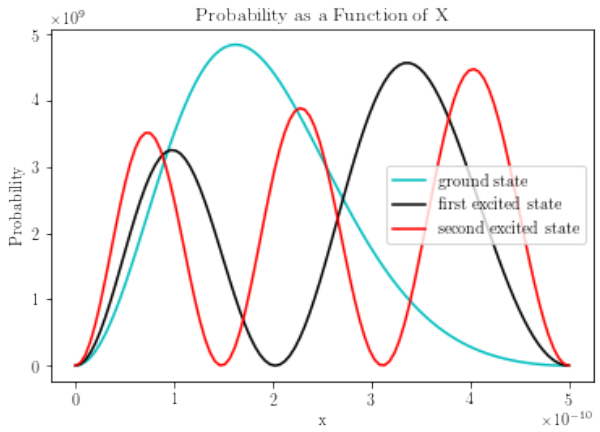

FIG. 2: Probability Wavefunctions For Energy Levels