Homework 4

Nathan Wolthuis*
Haverford College
(Dated: February 27, 2020)

1. NEWMAN 5.12

1.1.

For this problem, we're going to use the equation for thermal energy as a function of the angular frequency of light, using the following equation.

$$I(\omega) = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}$$

For making this easier to integrate for the purposes of this problem, we're going to use some x-substitution with the following identities.

$$x = \hbar\omega/k_B T$$
$$dx = \hbar d\omega/k_B T$$
$$\omega = k_B T x/\hbar$$

For integrating ω over all wavelengths, we just need to integrate from 0 to ∞ , plugging in x instead of ω makes the computation easier, and results in the following equation, in terms of W:

$$W = \frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^2} \int_0^\infty \frac{x^3}{e^x - 1}$$

1.2.

For part b) I used the z substitution shown in Newman so that I could have the computer evaluate to infinity. For the process of integrating itself I used gaussian quadrature. The integral seems to work nicely, and the energy emitted for 1000K is found to be 56,851 $Jm^{-2}s^{-1}$,which seems to work out.

1.3.

We used the same calculations for the total energy for all wavelengths per second per unit area, eliminating the Temperature dependence. We find our computed value of the constant to be approximately $5.68510 * 10^{-8} Wm^{-2}K^{-4}$. This is pretty close to the actually determined value of $\sigma = 5.670374419...10^8 Wm^{-2}K^{-4}$.

*Electronic address: email; URL: Optionalhomepage

2. NEWMAN 5.14

2.1.

Part a) of this problem mostly just involves deriving an integral for the force of gravity on a mass above a uniform plate of heaviness. Below are some equations we will use:

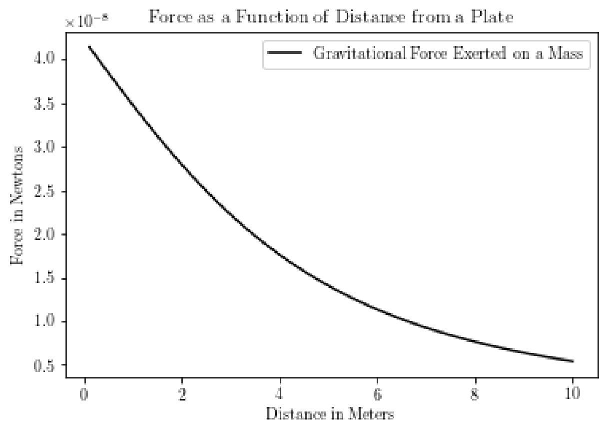
$$F_G = G\frac{Mm}{r^2} r_{vector}$$
$$r = (x^2 + y^2 + z^2)^{1/2}$$
$$r_{vector} = z/r$$

We're going to make this an integrating function, and we're setting $m = 1kg$. We're going to break M into an integrand in terms of x and y, with σ defined as the mass per unit area. Then we plug everything in.

$$M = \sigma dxdy$$
$$F_z = G * \sigma * z \int_0^z \frac{dxdy}{(x^2 + y^2 + z^2)^{3/2}}$$

2.2.

Here we just plot out the function we derived before, using the dblquad tool from scipy, which made this rather manageable.



2.3.

The plot did not have an issue with trailing back when Z is small. So I cannot really discuss an artifact that doesn't exist in my plot. Sorry.

3. NEWMAN 5.21

This problem was one from Hell, and was personally responsible for many hours of frustration. Very Big Sad

3.1.

This part was not so bad. Katharine Bancroft showed me the wonders of using meshgrid stuff for density plots, so this turned out nice. Katharine and I set max and minimum values so that we could see more of a gradient in the potential.

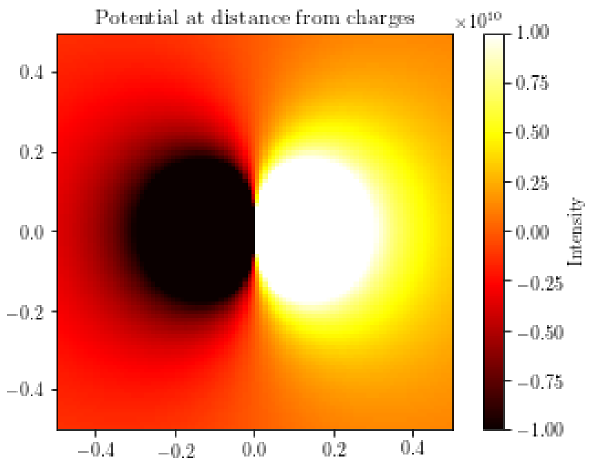
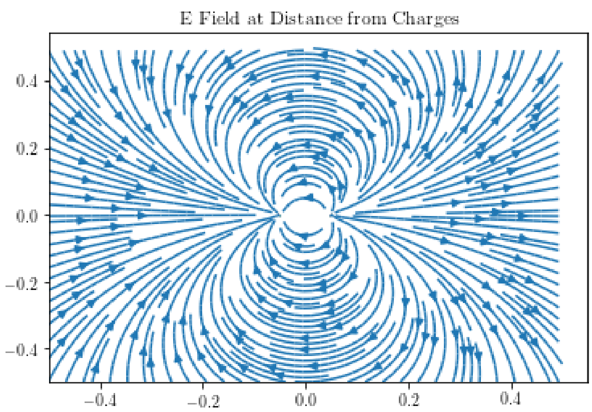


FIG. 1: Force of Gravity on a mass as a function of Height

3.2.

For this part we found the E Field by taking the negative grad of the potential. This wasn't so bad, although it took a bit to find the streamplot option as opposed to quiver.

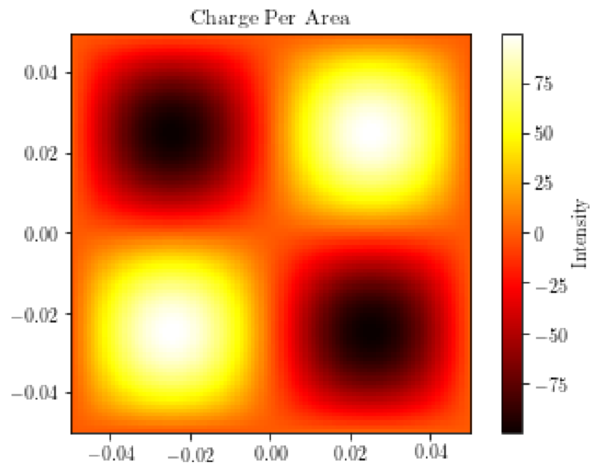


3.3.

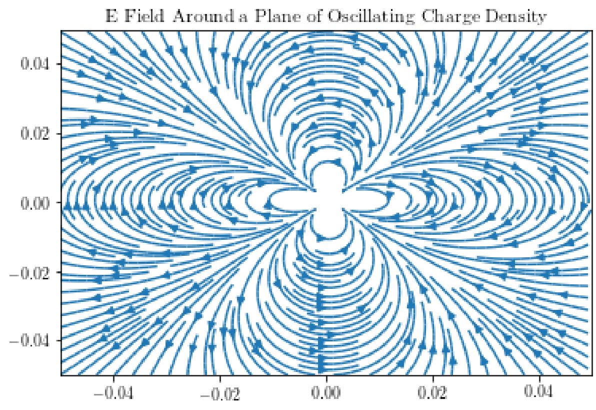
Here is the hellish part. Nathan Merril and Katharine Bancroft are responsible for figuring out most of this stuff, we ended up using the dblquad tool, as well as discarding the mesh approach for nested for-loops. We were tasked with finding the potential surrounding a plate with surface charge density given by the following equation:

$$\sigma = q_0 sin(2\pi x/L) sin(2\pi y/L)$$

I made a plot to look at the surface charge density as shown below A lot of this was confusing, but we inte-



grated the whole 1 sq. meter area's potential due to the pictured plate above. After that we used numpy's gradient function to get the E field for the area we were observing, as shown by the image below.



4. SURVEY RESPONSE

The last problem on this set was really really difficult, and ended up eating tons of time. I learned how to use streamplot and how to use the meshgrid stuff for density functions, however as neither of those were in the book it was really difficult. I feel like problem 1 2 were manageable and were good for learning the tools given in the book and lecture, while problem 3 wasn't super great. Because of that, this problem set was a bit long compared to the more comfortable length of the previous ones.