Year 9

1. Find the number whose double is 60 more than its half.

2. The internal angles of a heptagon sum to 900°. Find the sum of the internal angles of a heptadecagon.

3. Given that
$$6^{2x+y} = 36^7$$
and
$$6^{x+4y} = 216^7$$
find xy.

4. Two vertices of an equilateral triangle are $(1, 1)$ and $(7, 1)$. Find the coordinates of the third vertex.

5. What is the thousandth digit after the decimal place of $1/7$?

6. Evaluate
$$3 + 6 + 9 + \cdots + 300.$$

7. How many four-digit numbers can be formed using the digits $3, 1, 4, 1$?

8. A right-angled triangle has vertices $(0, 0)$, (a, b), $(a, 0)$ with hypotenuse $(a+b)/(a-b)$. The point (a, b) lies on a circle of radius 4. Find ab.

9. In a set of 50 numbers, the average of the first 20 is 30 while the average of the other 30 is 20. What is the average of all 50 numbers?

10. Find the first date in this millennium to have 8 unique digits when written in the form DD/MM/YYYY.

11. Find the probability that a randomly chosen number between 2 and 100 inclusive is prime.

12. A family consists of a mother, a father, and a number of children. The average age of the family is 20. The average age of just the mother and the children is 16. If the father's age is 48, how many children are there in the family?

13. An equilateral triangle with area 1 is inscribed inside a circle. This circle is itself inscribed inside a larger equilateral triangle. What is the area of the larger triangle?

14. What is the smallest positive integer that leaves a remainder of 1 when divided by each of the numbers $2, 3, \ldots, 10$?

15. At exactly midnight the hour hand of a clock begins to move twice its normal speed while the minute hand begins to move at half its normal speed. What is the correct time the next time the two hands meet?

16. A regular decagon has an area of 100 square units. What is the length of one of its sides to three decimal places?

17. If

$$A = 1^{-3} + 2^{-3} + 3^{-3} + \cdots$$

find

$$1^{-3} + 3^{-3} + 5^{-3} + \cdots$$

in terms of A.

18. Today, the 18th of July, 2023 is a Tuesday. What was the day of the week on the 18th of July 1823?

19. The numbers $1, 2, \ldots, 1000$ are written on the whiteboard. On each turn, two numbers x and y erased from the board and replaced by $xy - x - y + 2$. This continues until only one number remains. What is this number?

20. Two real numbers x and y, where $x > y$ have the property that they are equal to 1 plus their reciprocal and $x > y$. What is

$$\frac{x^2 - y^2}{\sqrt{5}}?$$

1. 40
2. $2700°$
3. 20
4. $(4, 1 + 3\sqrt{3})$
5. 8
6. 15150
7. 12
8. 24/5
9. 24
10. 17/06/2345
11. 25/99
12. 6
13. 4
14. 2521
15. 3 am
16. 3.605
17. $(1 - 2^{-3})A$
18. Friday
19. 1
20. 1

Year 10

1. The Fibonacci sequence is the sequence

$$1, 1, 2, 3, 5, 8 \ldots .$$

   Find the sum of the first 10 terms.

2. The sum of the first n odd numbers is 841. Find n.

3. Today (Tuesday) I start taking lectures on potions. If these lectures happen every second day, which lecture will be the first to fall on a Sunday?

4. If the sum of two positive real numbers is 4 times their product, what is the sum of the reciprocals of the two numbers?

5. What is the area of the equilateral triangle circumscribed by a circle of radius 1?

6. My car averages 40 kilometres per litre of petrol, while my friend's car averages 10 kilometres per litre of petrol. If we both drive the same distance, what is the combined rate of kilometres per litre of petrol?

7. If the side lengths of a rectangle are in the ratio $4 : 3$ and d is the length of the diagonal, it can be shown that the area of the rectangle is given by kd^2. Find k.

8. The vertices of an isosceles triangle lie on the graph of $y = x^2$. If the area of the triangle is 64, what is the length of the shortest side?

9. A palindromic number is one that is the same read forwards and backwards. For example, 292 is palindromic. How many three-digit palindromic numbers are there less than 1000?

10. Three of a rectangular prism's faces have areas of 12, 28, and 21. What is the volume of the rectangular prism?

11. What is the last digit of 9^{2023}?

12. Two hoses can be used to fill a swimming pool. Hose B takes twice as long as Hose A to fill the pool. If Hose A takes 6 minutes to fill the pool, how long will it take for both hoses together to fill the pool?

13. A normal coin is flipped 3 times. Given that at least one coin landed tails, what is the probability that there are two consecutive heads?

14. There is a tennis tournament with 1025 players. Each round, every player is paired with another, and if there are an odd number of players one player sits out. A loss immediately knocks out a player from the tournament. The tournament continues until only one player remains. How many matches are played in total?

15. For how many integers n, where $1 \leq n \leq 100$, is n^n a square number?

16. What is the smallest positive integer n such that

$$(2^2 - 1)(3^2 - 1)(4^2 - 1) \cdots (n^2 - 1)$$

is a square number?

17. In the nine digit number

$$347 * 47 * 64$$

two digits are missing, as indicated by the asterisks. If two digits are randomly chosen for the two missing spots, what is the probability that the number is divisible by 36?

18. Given an arithmetic sequence $a_1, a_2, \ldots, a_n$ where $a_{k+1} - a_k = d$ for $k = 0, 1, \ldots, n-1$, the sum is given by

$$a_1 + a_2 + \cdots + a_n = \frac{n}{2} (2a + (n-1)d) .$$

Find

$$1 - 4 + 9 - 16 + \cdots + 99^2 - 100^2.$$

19. The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

20. In $\triangle ABC$, $\angle C$ is a right angle and $AB = 12$. Squares $ACWZ$ and $ABXY$ are constructed so that points X, Y, W, and Z lie outside the triangle. If those four points also lie on a circle, what is the perimeter of the triangle?

1. 143
2. 29
3. 7th lecture
4. 4
5. $3\sqrt{3}/4$
6. 16 kilometres per litre
7. 12/25
8. 8
9. 90
10. 84
11. 9
12. 4 minutes
13. 2/7
14. 1024
15. 55
16. 8
17. 11/100
18. -5050
19. 18
20. $12 + 12\sqrt{2}$

Year 11/12

1. Find the next number in the sequence
$$1, 5, 12, 22, 35, \ldots$$

2. Twenty percent less than 60 is one-third more than what number?

3. What is the lowest common multiple of 18 and 35?

4. Find the cube root of 3^{3^3}. Note that $a^{b^c} = a^{(b^c)}$.

5. A triangle is inscribed in a circle of radius 3. Find the area of the triangle if its sides are in the ratio $3 : 4 : 5$.

6. In the plane, a lattice point is a point where both the x and y coordinates are whole numbers. The line $y = 55x/12$ is sketched for $0 < x < 12$. How many lattice points are there on the line?

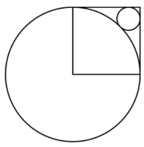
7. Which number is the first number greater than 1 that is the cube of the sum of its digits?

8. Evaluate
$$\sqrt{(2023 + 2023) + (2023 - 2023) + (2023 \times 2023) + (2023 \div 2023)}.$$

9. How many digits does $(2^2 2)^5 \times (5^5 5)^2$ have?

10. In how many ways can 10 objects be put into 3 bins such that one of the bins gets 5 objects, another gets 3 objects, and the third bin gets 2 objects?

11. In the diagram below the side length of the square is 1. What is the radius of the smaller circle?



12. Josh writes the numbers $1, 2, 3, \ldots, 99, 100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

13. Solve the equation
$$7\sqrt{x} - 22 = \frac{24\sqrt{x}}{x}.$$

14. Five points are randomly placed in a square with side length 2. The distance between every pair of points is measured, and the smallest distance between a pair of points is called the *minimum* distance. What is the largest possible minimum distance?

15. For how many integer values of n (positive or negative) is the value of

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

an integer?

16. If $\sin a + \sin b = \sqrt{5/3}$ and $\cos a + \cos b = 1$ what is $\cos(a - b)$?

17. How many ways are there to write 6 as the sum of any number of positive integers?

18. The number

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}$$

is the root of a quadratic with rational coefficients. Find the sum of the roots of this quadratic.

19. We have 5 disks of distinct sizes, and 3 pegs in a line. Let the left peg be A and the right-most peg be B. The disks start off on A in descending order of size; the largest disk is at the bottom. What is the minimum number of moves required to transfer the entire stack of disks from A to B if a larger disk is not allowed to be placed on top of a smaller disk and *direct transfers from A to B are not allowed?* This means every move must be made to or from the middle peg.

20. Find

$$1^3 + 2^3 + 3^3 + \cdots + 50^3$$

given the following four equations.

$$1 = 0 + 1$$

$$2 + 3 + 4 = 1 + 8$$

$$5 + 6 + 7 + 8 + 9 = 8 + 27$$

$$10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64$$

1. 51
2. 36
3. 3^{3^2}
4. 630
5. 8.64
6. 0
7. 512
8. 2024
9. 111
10. 2520
11. $(\sqrt{2}-1)^2$
12. 64
13. 16
14. $\sqrt{2}$
15. 9
16. $1/3$
17. 11
18. 0
19. 242
20. 1625625