Angle Chasing (Continued)

Maths Extension Group MHS

Summary

Use Parallel line properties

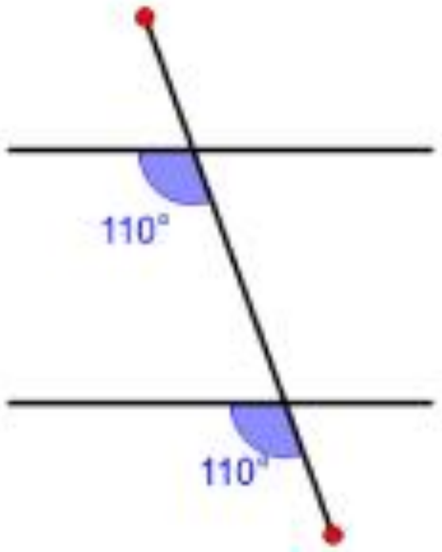
Use Triangle properties (angle sums, different centres)

Circle Properties (Inscribed angle theorem, Thales)

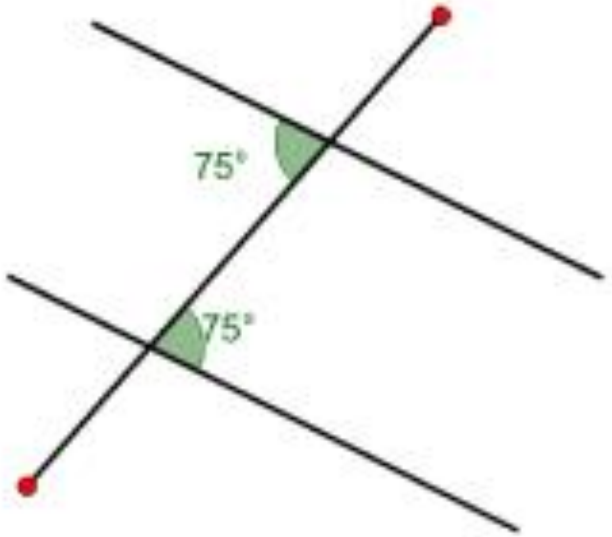
Find cyclic quadrilaterals and use them

Parallel lines

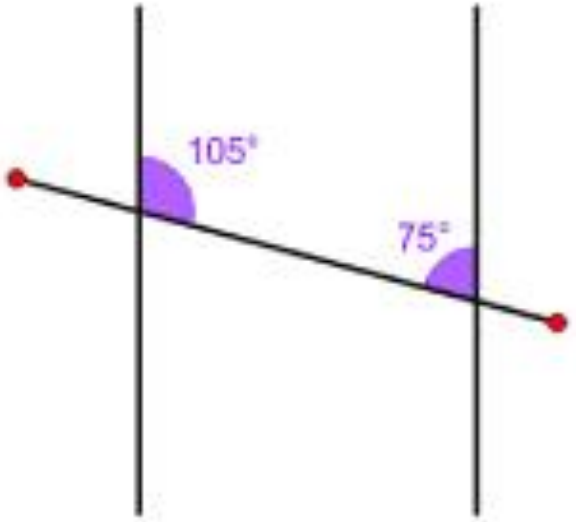
Corresponding Angles



Alternate Angles

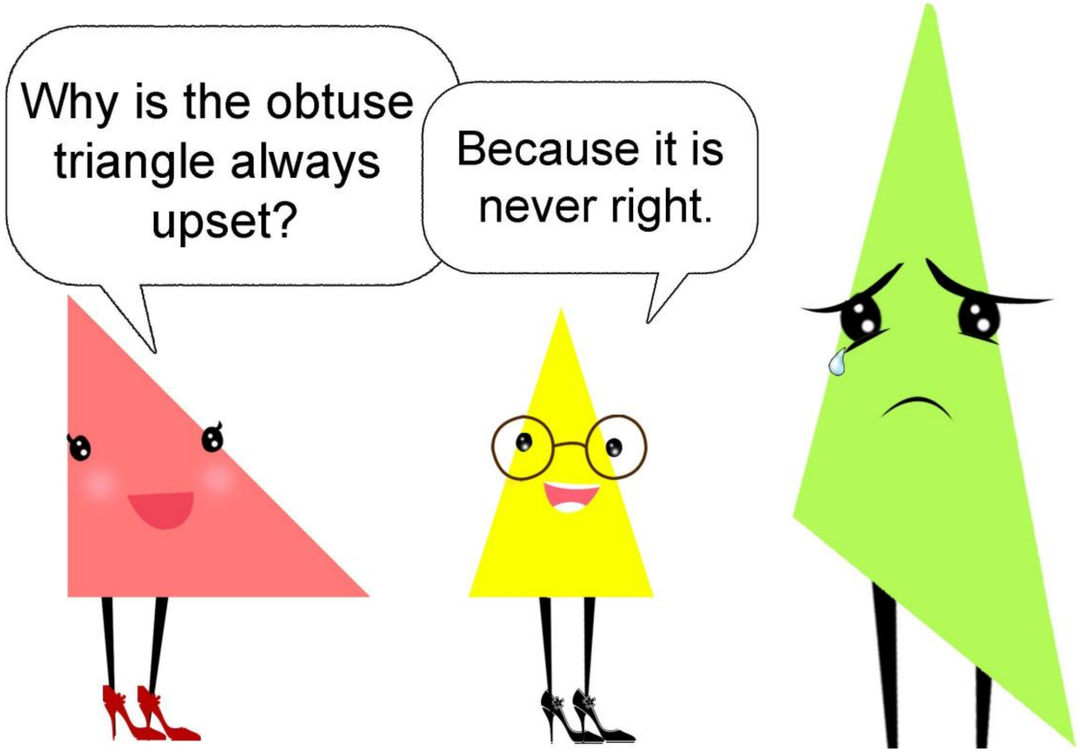


Interior Angles



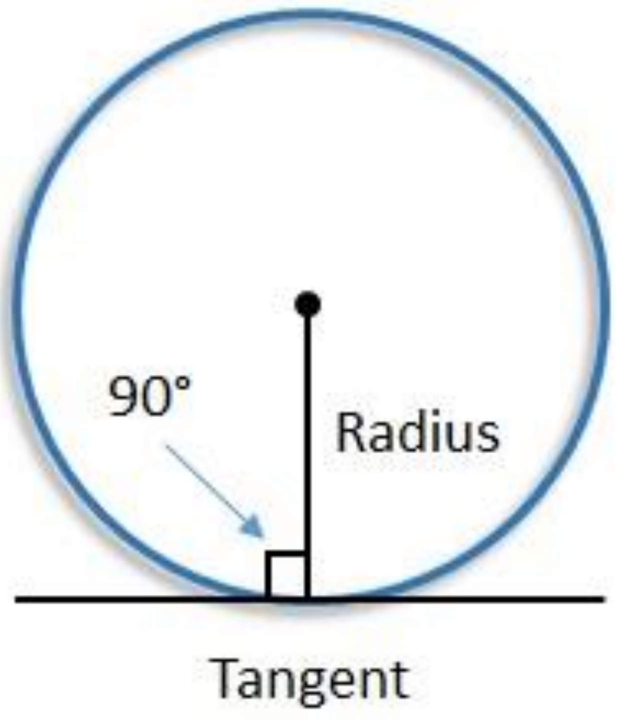
Triangles

- Interior angles sum to 180 degrees
- Properties of iscosceles triangle
- Orthocentre
- Incentre
- Circumcentre
- Centroid

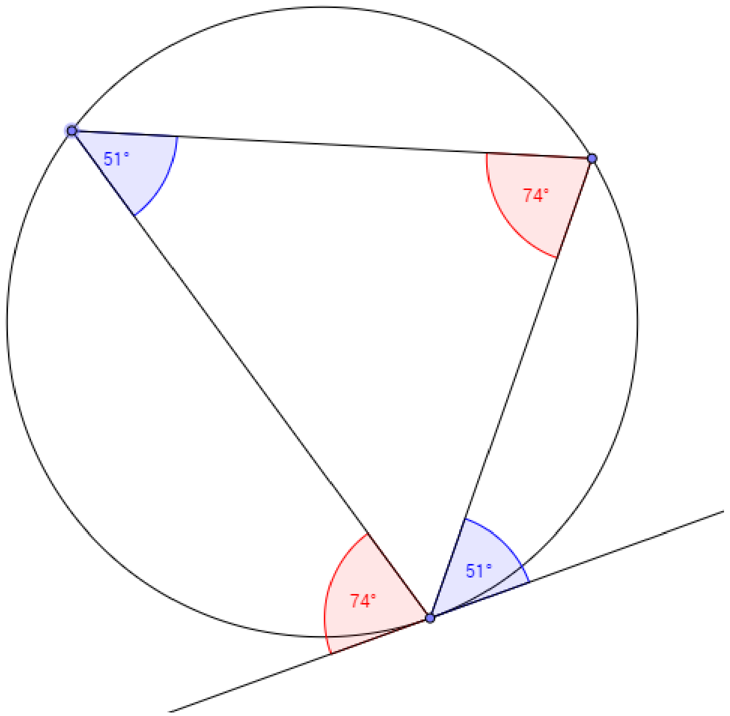


Circle Properties

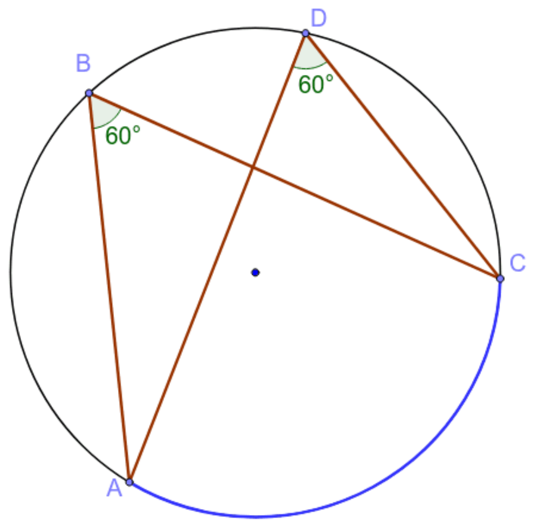
Tangent meets radius at 90 degrees



Alternate Segment Theorem



Angles subtended by the same arc are equal

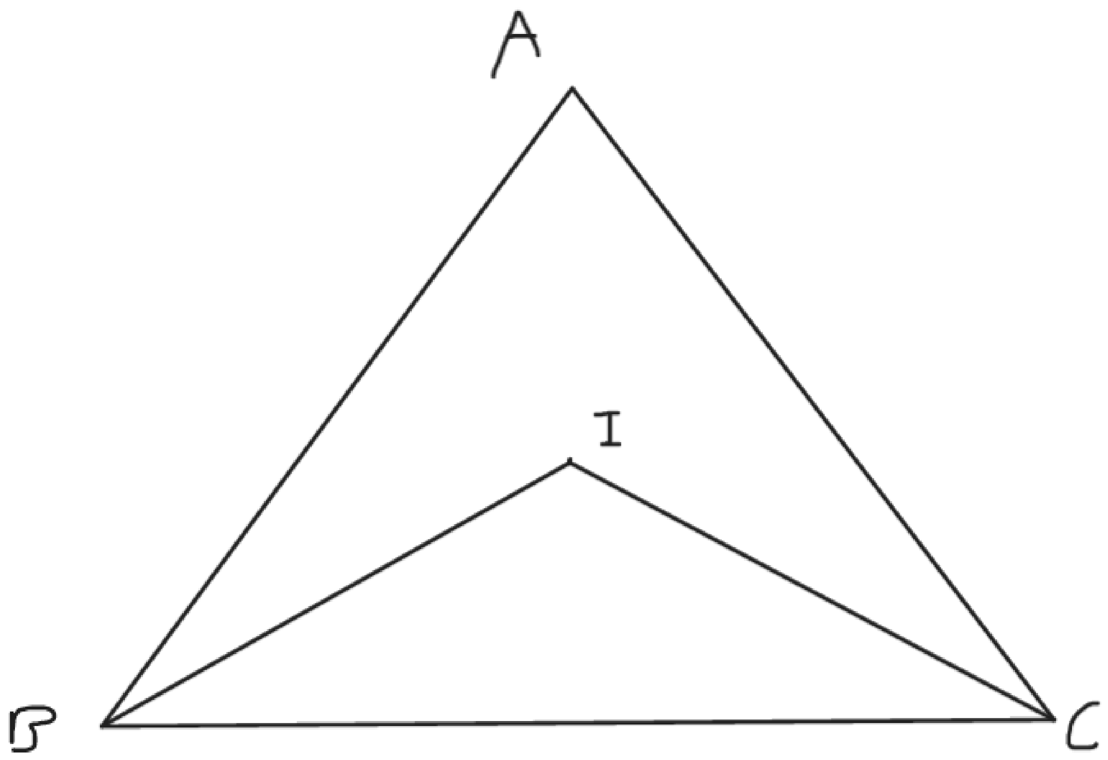


Incentre Formula

Let I be the incentre in triangle ABC. Then,

<BIC = 90 + A/2

Task: Prove it



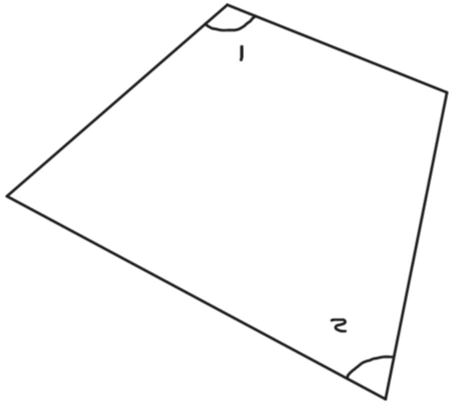
How to spot cyclic quadrilaterals

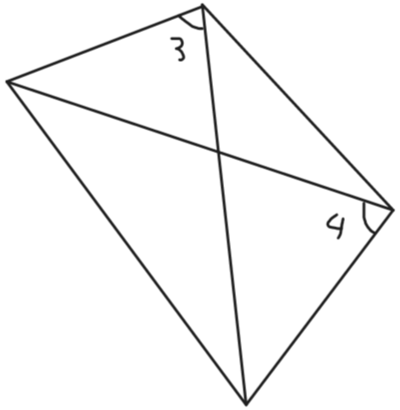
The two main ways:

<1 + <2 = 180

Or

<3 = <4



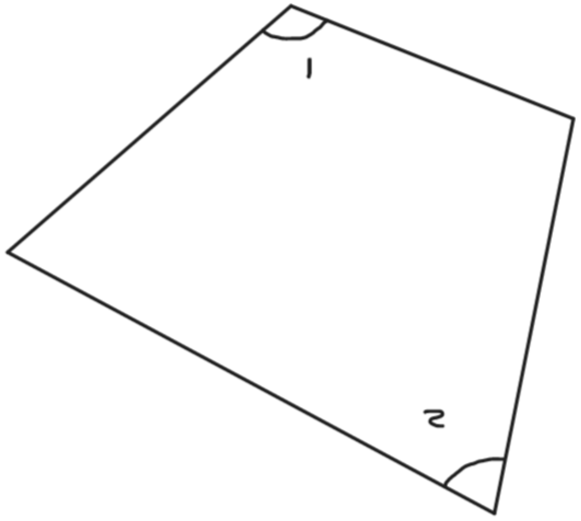


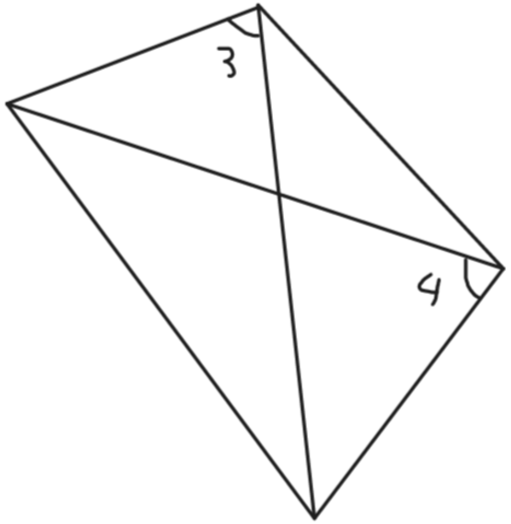
Properties of Cyclic Quadrilaterals

The two useful properties:

<1 + <2 = 180

<3 = <4

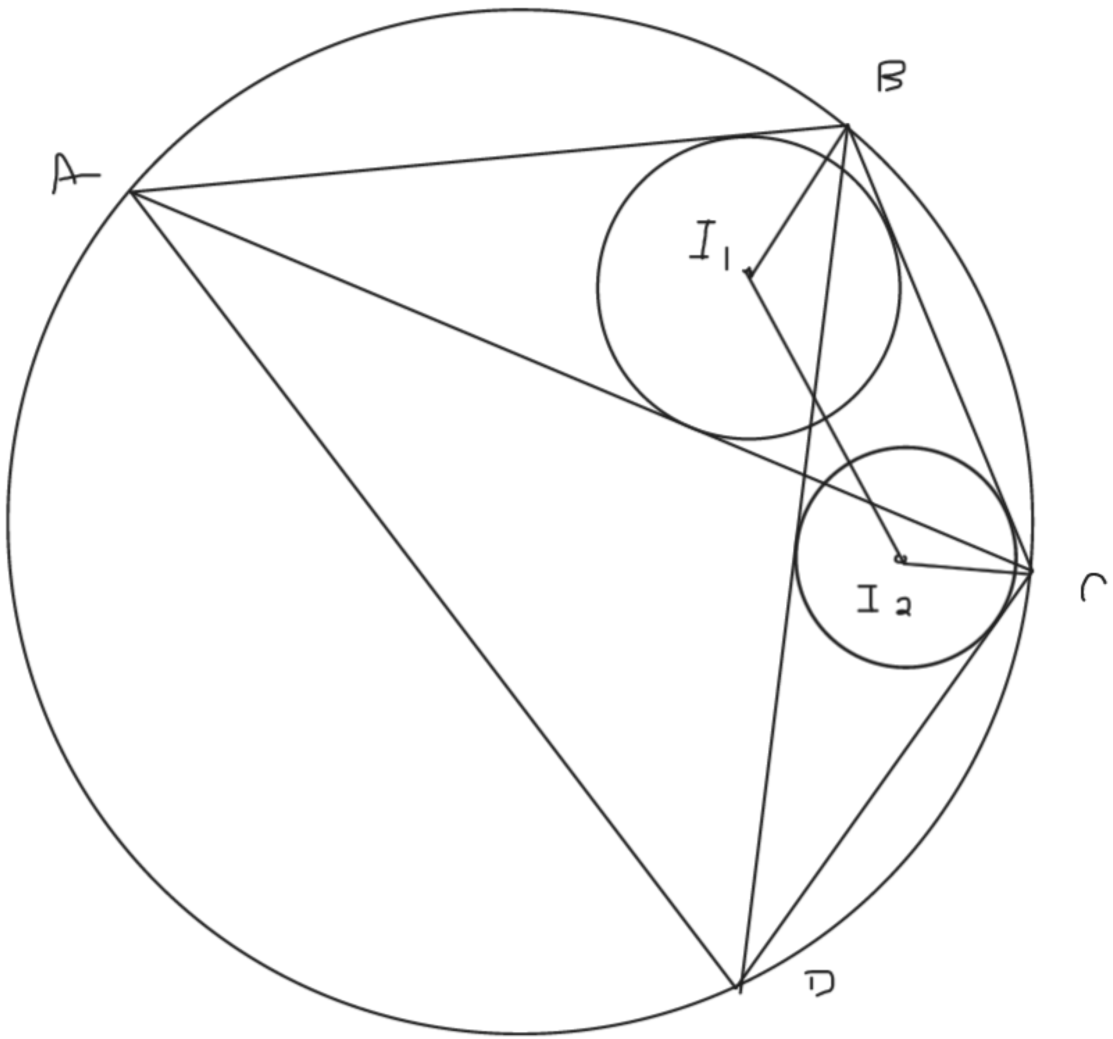




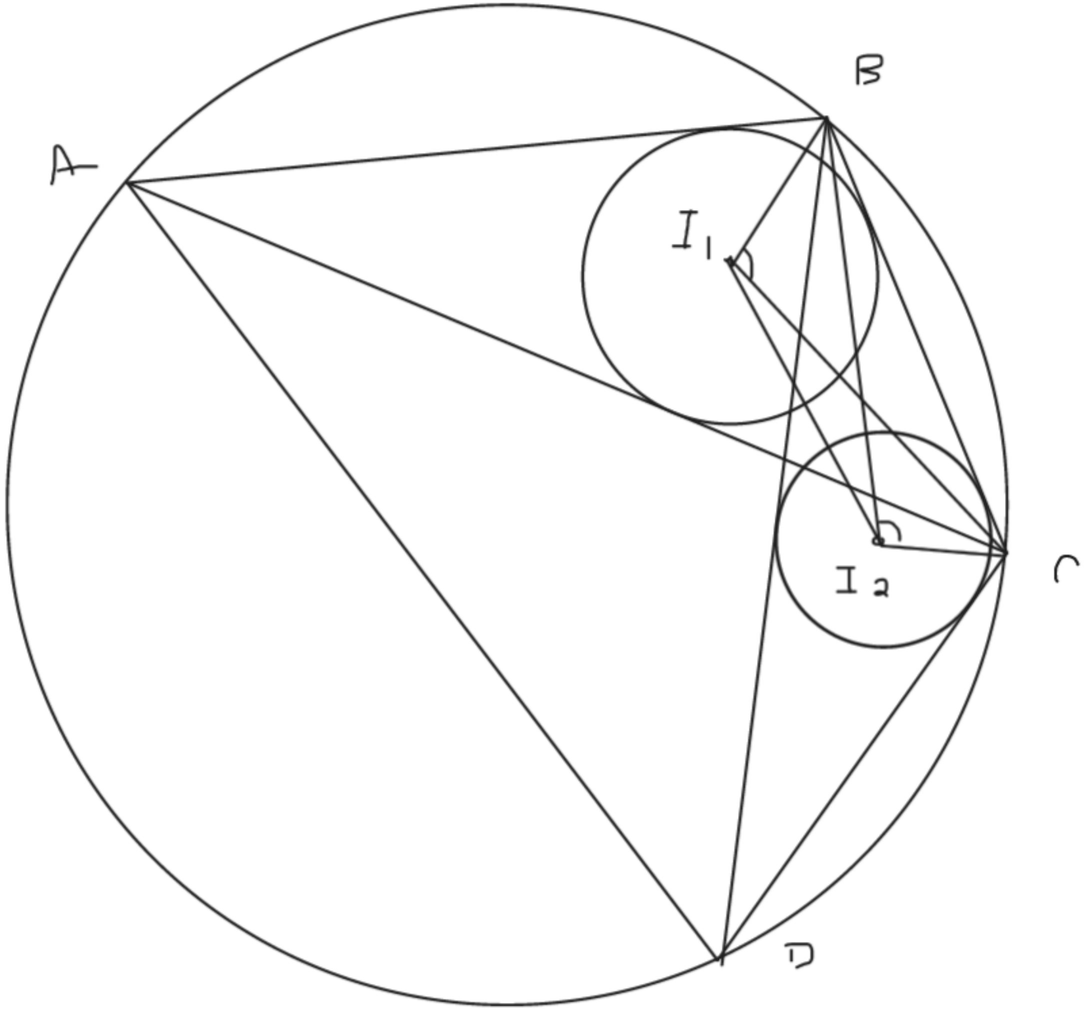
Incentre and Cyclic Quadrilaterals

In cyclic quadrilateral ABCD, let I_1 and I_2 denote the incentres of triangle ABC and triangle DBC respectively.

Prove that I_1 I_2 BC is cyclic.



Incentre and Cyclic Quadrilaterals

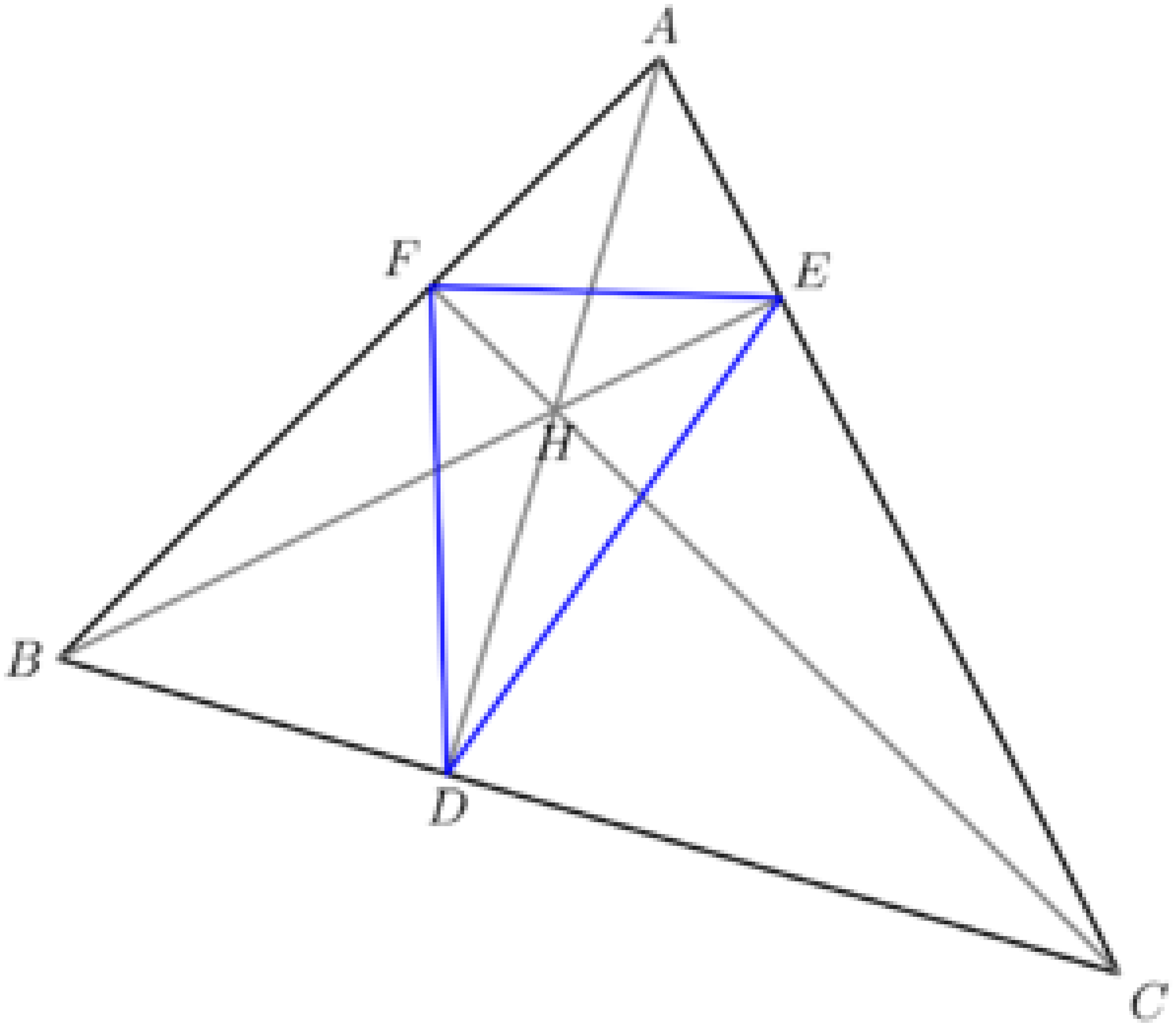


The Orthic Triangle

Triangle FDE is the orthic triangle.

AD, CF, BE are the altitudes.

Try and spot all six cyclic quadrilaterals!



More Tools…

Now we have all the tools to tackle the following upcoming problem.

Some more techniques you can investigate in your own time:

- Circle theorems
- The rest of the triangle centres
- Incentre Excentre Lemma
- Nine point Circle

Let's do p2 EGMO 2023

Problem 2. We are given an acute triangle ABC. Let D be the point on its circumcircle such that AD is a diameter. Suppose that points K and L lie on segments AB and AC, respectively, and that DK and DL are tangent to circle AKL.

Show that line KL passes through the orthocentre of ABC.

The orthocentre of a triangle is the point of intersection of its altitudes.

P2 EGMO 2023

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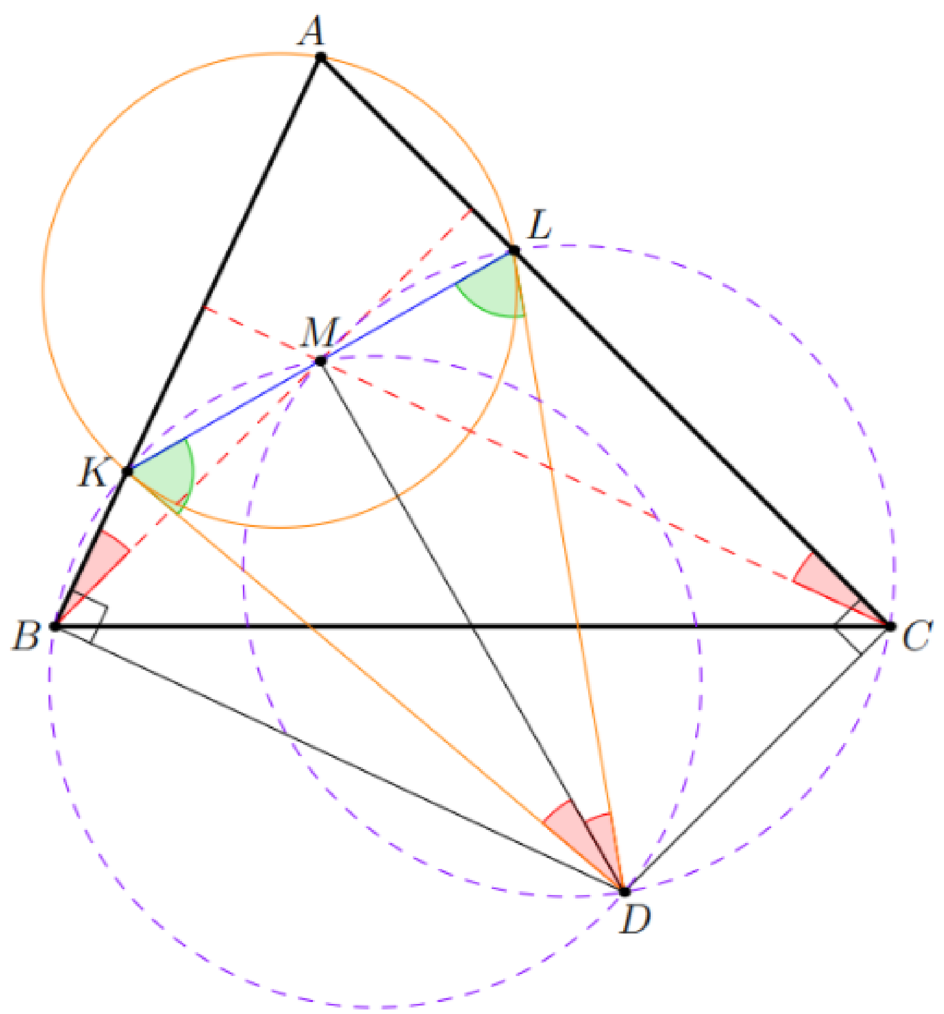

- Let M be the midpoint of KL
- Now lets show M is the orthocentre of ABC
- KL and LD are tangent to circle AKL, so DK = DL and DM is perpendicular to KL
- Anyone know why ABCD is cyclic? (AD is the diameter!)

P2 EGMO 2023

- Alternate segment theorem on circle AKL suggests <MDK = <LDM = 90 - <A
- Why are MDBK and MDCL cyclic?
- <MBK = <LCM = 90 - <A (subtended by the same arc)

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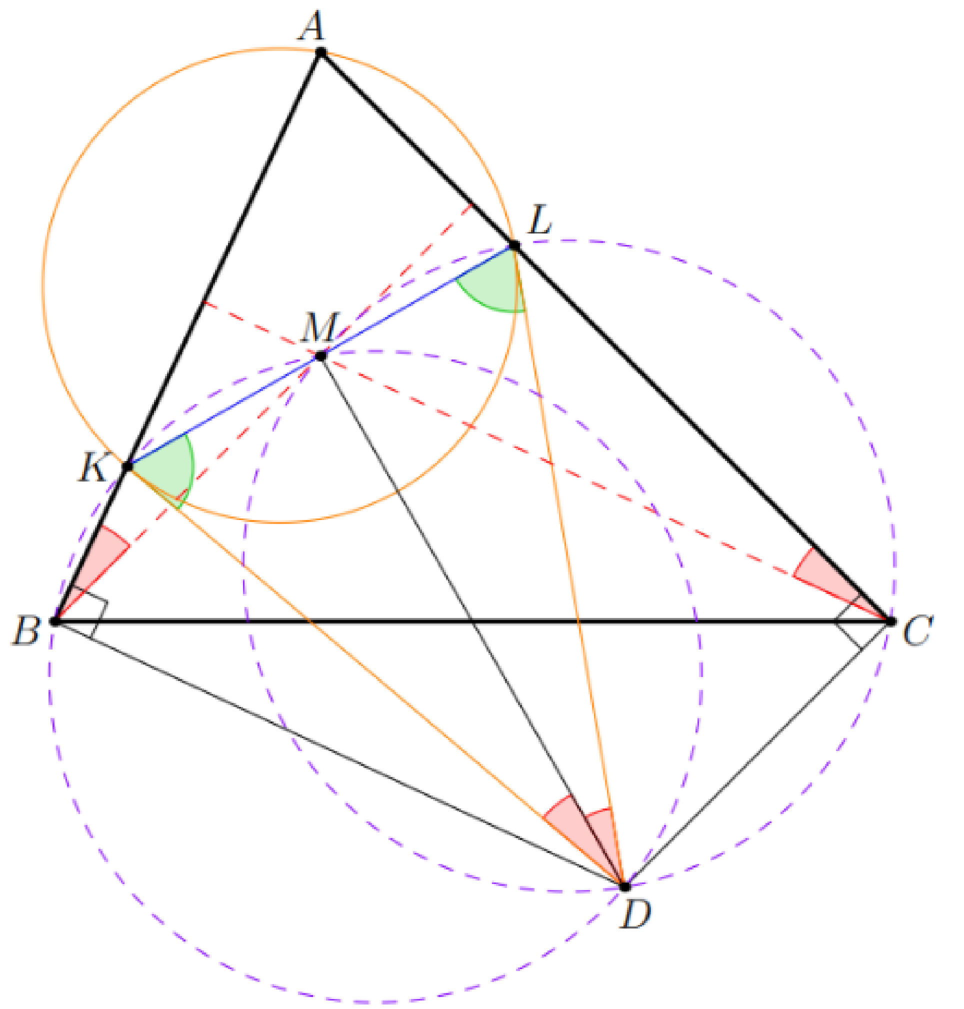

Your Turn!

Try doing the problems on the worksheet! We have learnt all the necessary tools for all of them.

Also try to complete P2 EGMO 2023. (Super close to finishing!)

Any Questions?