Pi Day

Maths Extension Group

Melbourne High School

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Madhva-Leibniz

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \cdots$$

Viéte's Formula

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

Wallis (1655)

$$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdots}$$

Nilakantha (15th century)

$$\pi = 3 + \frac{4}{2 \cdot 3 \cdot 4} - \frac{4}{4 \cdot 5 \cdot 6} + \frac{4}{6 \cdot 7 \cdot 8} - \frac{4}{8 \cdot 9 \cdot 10} + \cdots$$

Basel Problem (Euler)

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

A variant of the Basel problem

$$\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$$

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$$\frac{\pi}{2} = \arctan\frac{1}{1} + \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{13} + \cdots$$

Ramanujan-Sato

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \sum_{k=0}^{\infty} \frac{(4k!)}{k!^4} \cdot \frac{26390k + 1103}{396^{4k}}$$

???

Chudnovsky formula (the craziest one)

$$\frac{1}{\pi} = \frac{\sqrt{10005}}{4270934400} \sum_{k=0}^{\infty} \frac{(6k)!}{(3k)!k!^3} \cdot \frac{13591409 + 545140134k}{(-640320)^{3k}}$$

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Factorial

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Binomial Coefficients

$$\binom{2n}{n} \sim \frac{4^n}{\sqrt{\pi n}}$$

Your Problem (Warning: very hard)

If two random real numbers x and y are chosen on the interval $[0, 1]$, what is the probability that the closest integer to x/y is even?