

1. The Fibonacci sequence is the sequence

$$1, 1, 2, 3, 5, 8 \dots$$

Find the sum of the first 10 terms.

2. The sum of the first  $n$  odd numbers is 841. Find  $n$ .
3. Today (Tuesday) I start taking lectures on potions. If these lectures happen every second day, which lecture will be the first to fall on a Sunday?
4. If the sum of two positive real numbers is 4 times their product, what is the sum of the reciprocals of the two numbers?
5. What is the area of the equilateral triangle circumscribed by a circle of radius 1?
6. My car averages 40 kilometres per litre of petrol, while my friend's car averages 10 kilometres per litre of petrol. If we both drive the same distance, what is the combined rate of kilometres per litre of petrol?
7. If the side lengths of a rectangle are in the ratio 4 : 3 and  $d$  is the length of the diagonal, it can be shown that the area of the rectangle is given by  $kd^2$ . Find  $k$ .
8. The vertices of an isosceles triangle lie on the graph of  $y = x^2$ . If the area of the triangle is 64, what is the length of the shortest side?
9. A palindromic number is one that is the same read forwards and backwards. For example, 292 is palindromic. How many three-digit palindromic numbers are there less than 1000?
10. Three of a rectangular prism's faces have areas of 12, 28, and 21. What is the volume of the rectangular prism?
11. What is the last digit of  $9^{2023}$ ?
12. Two hoses can be used to fill a swimming pool. Hose B takes twice as long as Hose A to fill the pool. If Hose A takes 6 minutes to fill the pool, how long will it take for both hoses together to fill the pool?
13. A normal coin is flipped 3 times. Given that at least one coin landed tails, what is the probability that there are two consecutive heads?
14. There is a tennis tournament with 1025 players. Each round, every player is paired with another, and if there are an odd number of players one player sits out. A loss immediately knocks out a player from the tournament. The tournament continues until only one player remains. How many matches are played in total?

15. For how many integers  $n$ , where  $1 \leq n \leq 100$ , is  $n^n$  a square number?

16. What is the smallest positive integer  $n$  such that

$$(2^2 - 1)(3^2 - 1)(4^2 - 1) \cdots (n^2 - 1)$$

is a square number?

17. In the nine digit number

$$347 * 47 * 64$$

two digits are missing, as indicated by the asterisks. If two digits are randomly chosen for the two missing spots, what is the probability that the number is divisible by 36?

18. Given an arithmetic sequence  $a_1, a_2, \dots, a_n$  where  $a_{k+1} - a_k = d$  for  $k = 0, 1, \dots, n - 1$ , the sum is given by

$$a_1 + a_2 + \cdots + a_n = \frac{n}{2} (2a + (n - 1)d).$$

Find

$$1 - 4 + 9 - 16 + \cdots + 99^2 - 100^2.$$

19. Positive numbers are written in a  $3 \times 3$  grid such that the product of the numbers in every row and column is 1 and the product of the numbers in every possible  $2 \times 2$  grid is 2. Which number is in the centre of the grid?

20. In  $\triangle ABC$ ,  $\angle C$  is a right angle and  $AB = 12$ . Squares  $ACWZ$  and  $ABXY$  are constructed so that points  $X$ ,  $Y$ ,  $W$ , and  $Z$  lie outside the triangle. If those four points also lie on a circle, what is the perimeter of the triangle?

1. 143
2. 29
3. 7th lecture
4. 4
5.  $3\sqrt{3}/4$
6. 16 kilometres per litre
7. 12/25
8. 8
9. 90
10. 84
11. 9
12. 4 minutes
13. 2/7
14. 1024
15. 55
16. 8
17. 11/100
18. -5050
19. 16
20.  $12 + 12\sqrt{2}$