MEG angle chasing + interesting geo problems

Tom Yan

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1 Introduction

1. What is the measure of an angle, in degrees, if its supplement is six times its complement?

2. The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$. What is the degree measure of $\angle ABC$?

3. (Inscribed angle theorem) Prove that if $\angle ACB$ is inscribed in a circle, then it subtends an arc with measure $2\angle ACB$.

4. (Russia 1996) Points E and F are on side $\overline{BC}$ of convex quadrilateral $ABCD$ (with E closer to B). It is known that $\angle BAE = \angle CDF$ and $\angle EAF = \angle FDE$. Prove that $\angle FAC = \angle EDB$

5. (USAJMO 2011/5) Points A, B, C, D, E lie on a circle ω and point P lies outside the circle. The given points are such that (i) lines PB and PD are tangent to ω, (ii) P, A, C are collinear, and (iii) $\overline{DE} \parallel \overline{AC}$. Prove that $\overline{BE}$ bisects $\overline{AC}$.