Angle Chasing problems

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1 Introduction

1. Quadrilateral $WXYZ$ is a quadrilateral with perpendicular diagonals. If $\angle WZX = 30°$, $\angle XWY = 40°$, and $\angle WYZ = 50°$, find $\angle WZY$.

2. Two parallel lines are tangent to a circle with centre O. A third line, also tangent to the circle, meets the two parallel lines at A and B. Prove that AO is perpendicular to OB

3. (BAMO 1999/2) Let $O = (0, 0)$, $A = (0, a)$, and $B = (0, b)$, where $0 < a < b$ are reals. Let τ be a circle with diameter AB and let P be any other point on τ. Line PA meets the x-axis again at Q. Prove that $\angle BQP = \angle BOP$.

4. (AMO 2008) Let $ABCD$ be a convex quadrilateral. Suppose there is a point P on the segment AB with $\angle APD = \angle BPC = 45°$. If Q is the intersection of the line AB with the perpendicular bisector of CD, prove that $CQD = 90°$.

5. (EGMO 2023/2) We are given an acute triangle ABC. Let D be the point on its circumcircle such that AD is a diameter. Suppose that points K and L lie on segments AB and AC, respectively, and that DK and DL are tangent to circle AKL. Show that line KL passes through the orthocentre of triangle ABC.