Problems

1. Find all natural numbers n such that $2^{2^n} + 5$ is prime.

2. Let N be a regular nonagon, having O as the centre of its circumcircle, and let PQ and QR be adjacent edges of N. The midpoint of PQ is A and the midpoint of the radius perpendicular to QR is B. Determine the angle between AO and AB.

3. Let $0 < a < 1$. Solve

$$x^{a^x} = a^{x^a}$$

for positive numbers x.

4. Show that we can colour the elements of the set $S = \{1, 2, \ldots, 2007\}$ with 4 colours such that any subset of S with 10 elements, whose elements form an arithmetic sequence, is not all one colour.

5. Two points K and L are chosen inside triangle ABC and a point D is chosen on the side AB. Suppose that B, K, L, C are concyclic, $\angle AKD = \angle BCK$ and $\angle ALD = \angle BCL$. Prove that $AK = AL$.