Term 2 Holiday problems MEG

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1 Introduction

1. (Tournament of Towns Junior 1981/1) Find all integer solutions to the equation

$$y^k = x^2 + x$$

where k is a natural number greater than 1.

2. Write $(1 + \frac{1}{3})(1 + \frac{1}{3^2})(1 + \frac{1}{3^{2^2}}) \ldots (1 + \frac{1}{3^{2^100}})$ in the form $a(1 - b^c)$, where a, b and c are constants.

3. Find the number of ways to colour each square of a 2007×2007 square grid black or white such that each row and each column has an even number of black squares.

4. Let ABC be an acute triangle. Let BE and CF be altitudes of $\triangle ABC$, and denote by M the midpoint of BC. Prove that ME, MF, and the line through A parallel to BC are all tangents to circle AEF.