Year 9

1. In an AFL game, a goal is worth 6 points and a behind is worth 1 point.
   (a) (1 point) How many points does a team have if they score 4 goals and 8 behinds?
   (b) (2 points) How many different ways are there to score some number of goals and some number of behinds such that the score is the product of the number of goals and the number of behinds?
   (c) (2 points) By considering a graph, show that you have found all the ways that this could occur.

   Total for Question 1: 5

2. (a) (2 points) What is the area of the largest square that can fit inside a unit circle?
   (b) (2 points) What is the area of the largest circle that can fit inside a unit square?
   (c) (3 points) A triangle is placed inside a unit square so that of its vertices either lie on the square's edges or inside the square. What areas are possible for this triangle?
   (d) (2 points) What is the side length of the largest square that can fit inside an equilateral triangle that has side length 1 unit?

   Total for Question 2: 9

3. (3 points) What is the volume of a regular tetrahedron with side length 1 unit?

4. (a) (1 point) How many positive integer solutions are there to the equation $2x + 3y = 25$?
   (b) (2 points) How many integer solutions are there to the equation $2x + 3y = 25$?
   (c) (2 points) How many integer solutions are there to the equation $51x+24y = 17$?
   (d) (1 point) Consider the expression $9x + 15y$. Put in as many integer values of x and y as you can. Which number are all the resulting numbers multiples of?
   (e) (2 points) When does the equation $ax + by = c$ always have integer solutions?

   Total for Question 4: 8

5. (a) (2 points) When is the sum of 2 consecutive numbers divisible by 2?
   (b) (2 points) When is the sum of 3 consecutive numbers divisible by 3?
   (c) (2 points) When is the sum of 4 consecutive numbers divisible by 4?
   (d) (2 points) When is the sum of 5 consecutive numbers divisible by 5?
   (e) (2 points) When is the sum of n consecutive numbers divisible by n?

   Total for Question 5: 10

6. (a) (1 point) Two lines can divide the plane into at most how many regions?
   (b) (1 point) Three lines can divide the plane into at most how many regions?
   (c) (1 point) If we add a fourth line, what is the maximum number of intersection points that line can make with the existing lines?

(d) (1 point) Four lines can divide the plane into at most how many regions?

(e) (3 points) What is the maximum number of regions that n lines can divide the plane into?

Total for Question 6: 7

7. (3 points) Prove it possible to pair up the numbers $0, 1, 2, 3, \ldots, 61$ in such a way that when we sum each pair, the product of the 31 numbers we get is a perfect fifth power.

Total for Question 7: 3

Total: 45