Factorisation MEG problems

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1 Introduction

1. Find integers (x, y) such that $x^2 = 12 + y^2$.

2. Find all paris of integers (b, c) such that

$$2b + 3c = bc.$$

3. (INMO) Determine all non negative integral pairs (x, y) for which

$$(xy - 7)^2 = x^2 + y^2.$$

4. (BMO Round 3 2015) The integer n is positive. There are exactly 2005 ordered pairs (x, y) of positive integers satisfying

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n}.$$

Prove that n is a perfect square.

5. (IMO 2006/4) Determine all pairs (x, y) of integers such that

$$1 + 2^x + 2^{2x+1} = y^2.$$