Problems

1. If 20 boys are on my school's soccer team, 25 boys are on my school's hockey team, and 11 boys play both sports, then how many boys play soccer or hockey?

2. A true-false test has 10 questions. Suppose that if you answer any five questions "true" and the remaining five questions "false", then your score is guaranteed to be at least four. How many answer keys are there for which this is true?

3. Annie the Ant starts at the lattice point $(0, 0)$ and each minute moves 1 space up or 1 space to the right (with equal probability). Benny the Beetle starts at $(5, 7)$ and each minute moves 1 space down or 1 space to the left (with equal probability). What is the probability that they meet?

4. What is the average number of pairs of consecutive integers in a randomly selected subset of 5 distinct integers chosen from the set $\{1, 2, 3, \ldots, 30\}$? For example the set $\{1, 17, 18, 19, 30\}$ has 2 pairs of consecutive integers.

5. There are K boys placed around a circle. Each of them has an even number of sweets. At a command each boy gives half of his sweets to the boy on his right. If, after that, any boy has an odd number of sweets, someone outside the circle gives him one more sweet to make the number even. This procedure can be repeated indefinitely. Prove that there will be a time at which all boys will have the same number of sweets.