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Diffusion and Topology

Note that for the k-nearest neighbors, the adjacency matrix will be asymmetric. To illustrate this, consider a case with 4 agents, where we are finding the 2 nearest neighbors. Imagine that 2 of the agents are clustered tightly, and the fourth is a distance away. Each of the 3 in the cluster will set the other two as their two nearest neighbors, and thus have a link coming from those neighbors. The fourth, however, will connect to two of the original 3.

For asymmetric adjacency matrices, we need to make a decision on how to compute the incidence for each agent, whether it is computed by the in-degree or the out-degree. The in-degree is inherently uninteresting since, by definition, each agent looks at—or in other words, receives a connection from—its k nearest neighbors. Thus if the in-degree is used, the incidence, and thus every entry on the diagonal of the Laplacian, is equal to . For this reason, we have chosen to use the out-degree to compute incidence.

# Problem 1

For problem 1, we run the simulations in part 1 using a board with , meaning each agent will always sit in a region with . We simulate each point with 30 agents and 100 repetitions.

## Part 1: Connectivity of Fixed Metric Distance

In this part, we compute the probability that a graph of 30 agents is connected when each agent is connected to all agents within a distance of . The graph below shows the results of this simulation as increases.

## Part 2: Connectivity of K-Nearest-Neighbors Metric

In this part, we compute the probability that a graph of 30 agents is connected when each agent receives a connection from its nearest neighbors. The graph below shows the results of this simulation as 𝑘 increases.

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## Part 3: Summary

In summary, the K-nearest-neighbors metric will be more likely to be connected than the radius metric. To see this, In order to have a high probability for the K-nearest-neighbors metric to be connected, each agent only needs to be connected to 6 others (an aside, does this have anything to do with the 6-degrees of separation?)

The radius metric, on the other hand, must have m. This covers an area of portion of the entire board. With 30 agents distributed evenly across this board, this means that each agent, on average, is connected to 13.5 other agents, much greater than the 6 for the K-nearest-neighbors metric.