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Diffusion and Topology

# Problem 1

For problem 1, we run the simulations in part 1 using a board with , meaning each agent will always sit in a region with . We simulate each point with 30 agents and 100 repetitions.

## Part 1: Connectivity of Fixed Metric Distance

In this part, we compute the probability that a graph of 30 agents is connected when each agent is connected to all agents within a distance of . The graph below shows the results of this simulation as increases.

## Part 2: Connectivity of K-Nearest-Neighbors Metric

In this part, we compute the probability that a graph of 30 agents is connected when each agent receives a connection from its nearest neighbors. The graph below shows the results of this simulation as 𝑘 increases.

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## Part 3: Summary

In summary, the K-nearest-neighbors metric will be more likely to be connected than the radius metric. To see this, In order to have a high probability for the K-nearest-neighbors metric to be connected, each agent only needs to be connected to 6 others (an aside, does this have anything to do with the 6-degrees of separation?)

The radius metric, on the other hand, must have m. This covers an area of portion of the entire board. With 30 agents distributed evenly across this board, this means that each agent, on average, is connected to 13.5 other agents, much greater than the 6 for the K-nearest-neighbors metric.

# Problem 2

We again set up our initial conditions with , but this time, we used 45 agents to test out our system. Additionally, for this problem, we are using a zone of attraction and dynamics to simulate how agents would act. We used the following update equation:

Since this equation is for continuous time, we used to estimate the derivative. For the noise, we used a normal distribution, with mean 0 and standard deviation 1. In both experiments we also use 5 as the zone of repulsion radius and simulate for 100 time steps.

## Part 1: Distance Measurement

The first format for the zone of attraction is distance measurement. We gave the zone a minimum radius of 5.1 and a maximum radius of 10. Simulating with random initial conditions yielded the following results:

Essentially, both values bounce around quite a bit, but the average Fielder eigenvalue is about -15 and the connectivity is about 35.

## Part 2: Nearest Neighbors

Next we set the zone of attraction to be the 5th-closest through 10th-closest neighbors, while leaving the zone of repulsion the same. The interesting thing about doing this is that strictly following these rules will lead to an asymmetric Laplacian. One approach to fixing this could be to make all directed edges undirected. However, we decided to leave it as is, to see what happened. Also, because we are mixing dynamics, some neighbors will be in both zones. We decided this is fine, since the effects from the two zones will just cancel each other out. We got the following results:

As can be seen here, there is much less variance on both graphs and the values are smaller in magnitude. One interesting thing about having an asymmetric Laplacian is that the matrix is no longer Hermitian, so we are not guaranteed that the eigenvalues will be real. The above graph shows the real values of the Fiedler eigenvalues observed, since most imaginary values were negligible.

## Part 3: Summary

We see some differences in the behavior of both the Fiedler eigenvalues and the average connectivity. Basically, using a nearest-neighbor approach causes more stability in both. This is not surprising, since the nearest neighbor approach is fixing the connectivity of the attraction Laplacian. Therefore, the fluctuations merely come from other nodes entering or leaving the repulsion zone.

This is an interesting result given our initial conditions and parameters. Because the Fielder eigenvalue is higher with the distance measurement, we would anticipate that such swarms would be able to congregate more compactly. The connectivity is higher and convergence occurs faster. However, if initial conditions were such that the agents were sparse, this measurement might never lead to a swarm at all. For this reason, we might consider the nearest neighbor approach to be more robust, since it can gather agents together from any distance. Therefore, there is probably value in either approach.