2. Black-Scholes Model of Pricing Options

A study of how the Black-Scholes model of pricing options was done by deriving the associated formulas presented as follow:

Given ***k*** = strike price; ***S*** = value of underlying asset; ***r*** = risk-free interest rate, ***T*** = time of maturity, ***t*** = current time, ***N(x)*** = cumulative normal distribution

2.1. Derivative of the Cumulative density function of the normal distribution, *N(x)* :

|  |  |
| --- | --- |
|  | (1) |

Show that:

|  |  |
| --- | --- |
|  | (2) |

Black scholes equation

Where

|  |  |
| --- | --- |
|  | (3) |
|  | (4) |
|  | (5) |

From equation (1) we can say that:

|  |  |
| --- | --- |
|  | (6) |

From equation (5) we can also say that , then

|  |  |
| --- | --- |
|  | (7) |

|  |  |
| --- | --- |
|  | (..) |

|  |  |
| --- | --- |
|  | (..) |

|  |  |
| --- | --- |
|  | (..) |

But then

|  |  |
| --- | --- |
|  | (8) |

Substituting from equation (4) into equation (8):

|  |  |
| --- | --- |
|  | (9) |
|  | (..) |
|  | (..) |

Cancelling out the common terms

|  |  |
| --- | --- |
|  | (..) |

Rearrange to get:

|  |  |
| --- | --- |
|  | (..) |

|  |  |
| --- | --- |
|  | (10) |

But

Then

|  |  |
| --- | --- |
|  | (11) |

Hence, introducing S, the value of underlying asset:

|  |  |
| --- | --- |
|  | (…) |
|  | (12) |

Find the derivatives

|  |  |
| --- | --- |
|  | (13) |

|  |  |
| --- | --- |
|  | (13) |

Hence

No. 4.

With the solution for the call option price given by

Show that its derivatives with respect to time is

Show that

The solution for call option price can be rearranged:

|  |  |
| --- | --- |
|  | (14) |

Differentiating with respect to time (t)

|  |  |
| --- | --- |
|  | (15) |

From equation (5) we can as well say that . Taking derivatives of both sides

|  |  |
| --- | --- |
| And substituting equation (16) into (15) we have: | (16) |
|  | (17) |

From equation (14), taking the derivative of c with respect to S will give

|  |  |
| --- | --- |
|  | (18) |

No 5. Differentiate again to get

Break down to:

|  |  |
| --- | --- |
|  | (19) |

And is already derived in equation (13), hence substituting we get:

|  |  |
| --- | --- |
|  | (20) |

Substituting all the relevant expression to show that the expression for the call option price is the solution to the Black-Scholes differential equation.

Partial differential equation:

Where f = c.

Hence, and substituting from equation (17), (18), and (20) :

Cancelling out the common terms, we get:

Rearrange to get

|  |  |
| --- | --- |
|  | (22) |
|  |  |

Thus, it can be seen that Black-Scholes for a call option can derive a close form solution.