Discussion 20

Type Checking

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Agenda

1. Preview Type Checking Relation
2. Exercises
3. Midterm Projects

Type Checking Relation

The type checking relation defines what programs are “good” and what programs are “bad”

- *e* is well-typed if

```
T |- e : t
```

- *T* is the typing context (sometimes called Γ)
  - It is a map from variable names → types
  - A lot like the environment we saw when implementing an environment-model interpreter
- *e* is the expression
- *t* is the type of the expression (sometimes called τ)
- Read as “Expression e has type t under context T”

Static Semantics: Integer Addition

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for addition:

```
T |- e1 + e2 : int
     if  ???
```

Static Semantics

Static Semantics: Integer Addition

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for addition:

```
T |- e1 + e2 : int
     if  T |- e1 : int
     and T |- e2 : int
```

Static Semantics

```
<env, e1 + e2> => v
  if <env, e1> => v1
  and <env, e2> => v2
  and v1 + v2 = i
```

Dynamic Semantics
(Environment Model)

Static Semantics: Let Expressions

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for let expressions:

```
T |- let x = e1 in e2 : t
  if ???
```

Static Semantics

```
<env, let x = e1 in e2> => v
  if <env, e1> -->* v1
  and <env[x->v1], e2> => v
```

Dynamic Semantics
(Environment Model)

Static Semantics: Let Expressions

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for let expressions:

```
T |- let x = e1 in e2 : t
  if T |- e1 : t1
  and T[x->t1] |- e2 : t
```

Static Semantics

```
<env, let x = e1 in e2> => v
  if <env, e1> -->* v1
  and <env[x->v1], e2> => v
```

Dynamic Semantics
(Environment Model)

Static Semantics: If-Then-Else

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for if statements:

```
T |- if e1 then e2 else e3 : t
  if ???
```

Static Semantics: If-Then-Else

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for if statements:

```
T |- if e1 then e2 else e3 : t
  if T |- e1 : bool
  and e2 : t
  and e2 : t
```

Type Soundness

What does it mean for a program to be good?

Type Soundness

What does it mean for a program to be good?

Usually we want these two super-useful properties:

- **Progress:** if **e:t**, then **e** is a value or can take a step
- **Preservation:** if **e:t** and **e**→ **e'**, then **e':t**

Type Soundness

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- **Progress:** Well-typed programs always run to completion
- **Preservation:** Evaluation does not change the type of an expression

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If these two properties hold for a type system, we say that type system is “sound”

Type Soundness: Example

Here's an unsound example:

```
T |- if e1 then e2 else e3 : t2
     if  T |- e1 : bool
     and T |- e2 : t2
     and T |- e3 : t3
```

Does this violate Progress or Preservation (or neither)?

- **Progress:** Well-typed programs always run to completion
- **Preservation:** Evaluation does not change the type of an expression