

Discussion 12

Balanced Trees

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Key Ideas

- ▶ Red-Black Trees

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 - ▶ Invariants

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 - ▶ Insertion

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 - ▶ Invariants
 - ▶ Insertion
 - ▶ Deletion

Type

```
type color =
```

```
| R
```

```
| B
```

```
type 'a tree =
```

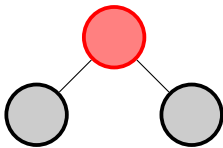
```
| Leaf
```

```
| Node of color * 'a * 'a tree * 'a tree
```

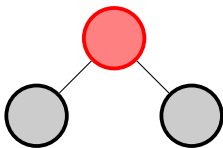
Invariants

- ▶ Convention: root node is black

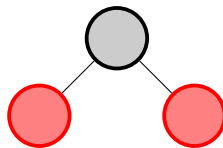
BAD



BAD



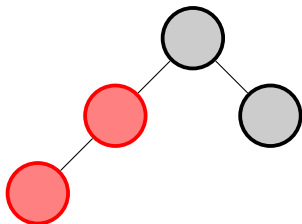
GOOD



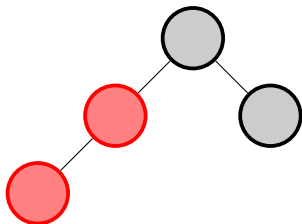
Invariants

- ▶ There are no two adjacent red nodes along any path

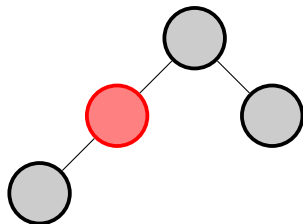
BAD



BAD



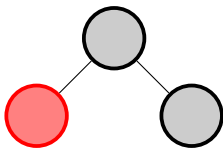
GOOD



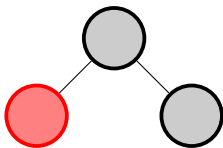
Invariants

- ▶ Every path from root to leaf has equal number of **black** nodes

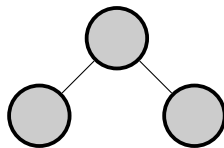
BAD



BAD



GOOD



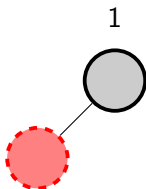
Insertion

- ▶ Assume invariants hold before insertion

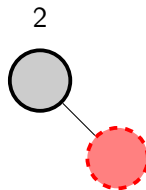
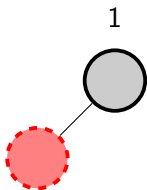
Insertion

- ▶ Assume invariants hold before insertion
- ▶ Fix invariants after insertion

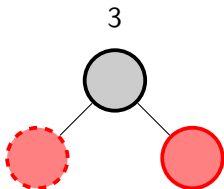
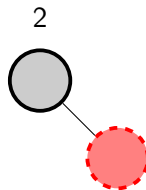
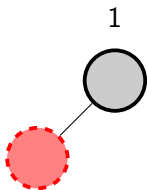
Insertion: Easy Cases



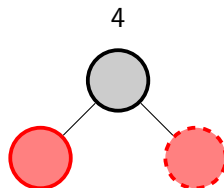
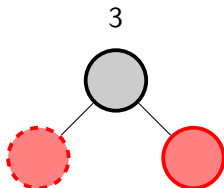
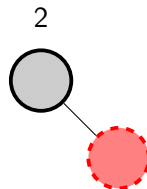
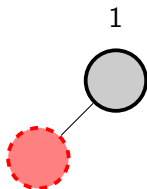
Insertion: Easy Cases



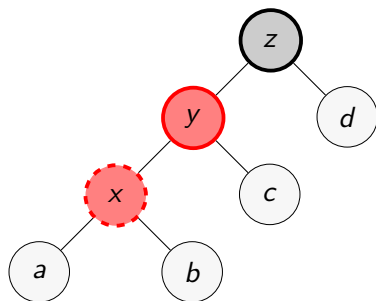
Insertion: Easy Cases



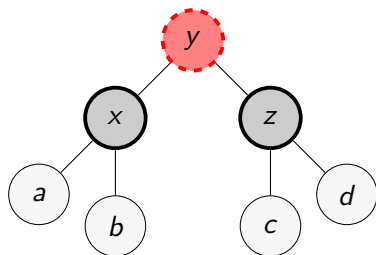
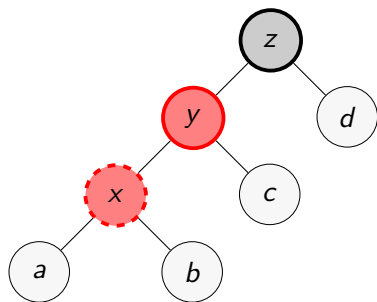
Insertion: Easy Cases



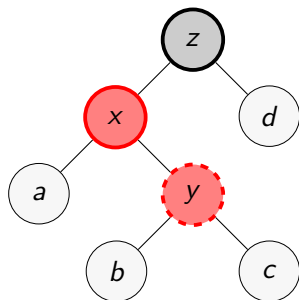
Insertion: Hard Cases



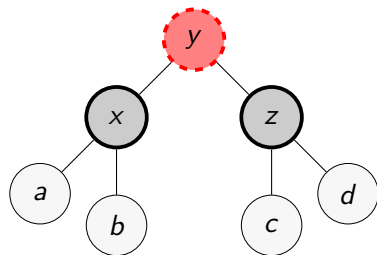
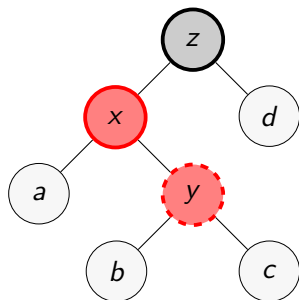
Insertion: Hard Cases



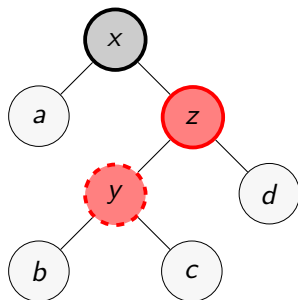
Insertion: Hard Cases



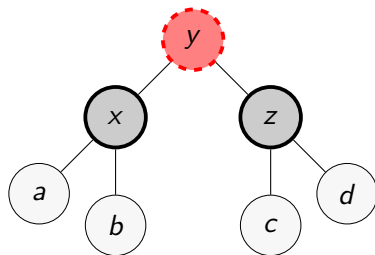
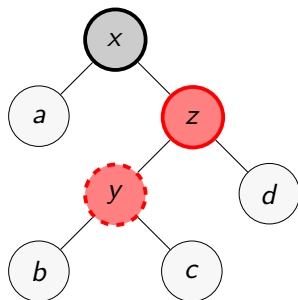
Insertion: Hard Cases



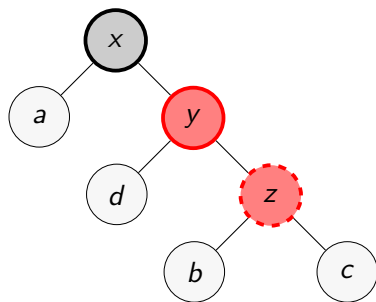
Insertion: Hard Cases



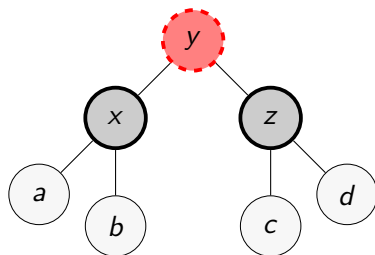
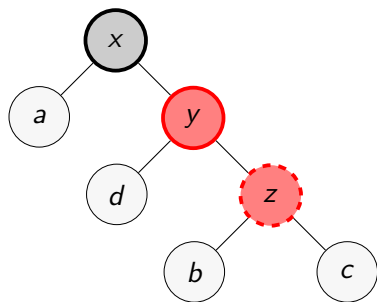
Insertion: Hard Cases



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Insertion: Hard Cases



Insertion: Summary

- ▶ Similar to regular binary tree insertion

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- ▶ Reconstruct balanced tree as we insert

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- ▶ Reconstruct balanced tree as we insert
- ▶ `insert l v |> balance |> rep_ok`

Deletion

- ▶ See paper by Germane, Might

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- ▶ Central idea: introduce **double-black** node during deletion

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- ▶ Central idea: introduce **double-black** node during deletion
- ▶ `type` color = R | B | BB