

Discussion 20

Type Checking

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Agenda

1. Preview Type Checking Relation
2. Exercises
3. Midterm Projects

Type Checking Relation

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 - ▶ It is a map from variable names \rightarrow types
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 - ▶ A lot like the environment we saw when implementing an environment-model interpreter
- ▶ e is the expression
- ▶ t is the type of the expression (sometimes called τ)
- ▶ Read as “Expression e has type t under context T ”

Static Semantics: Integer Addition

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for addition:

```
T |- e1 + e2 : int
    if ???
```

Static Semantics

Static Semantics: Integer Addition

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for addition:

```
T |- e1 + e2 : int
  if T |- e1 : int
  and T |- e2 : int
```

Static Semantics

```
<env, e1 + e2> => v
  if <env, e1> => v1
  and <env, e2> => v2
  and v1 + v2 = v
```

Dynamic Semantics
(Environment Model)

Static Semantics: Let Expressions

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for let expressions:

$T \vdash \text{let } x = e1 \text{ in } e2 : t$
if ???

Static Semantics

$\langle \text{env}, \text{let } x = e1 \text{ in } e2 \rangle \Rightarrow v$
if $\langle \text{env}, e1 \rangle \Rightarrow v1$
and $\langle \text{env}[x \rightarrow v1], e2 \rangle \Rightarrow v$

Dynamic Semantics
(Environment Model)

Static Semantics: Let Expressions

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for let expressions:

$$\begin{array}{l} T \vdash \text{let } x = e1 \text{ in } e2 : t \\ \text{if } T \vdash e1 : t1 \\ \text{and } T[x \rightarrow t1] \vdash e2 : t \end{array}$$

Static Semantics

$$\begin{array}{l} \langle \text{env}, \text{let } x = e1 \text{ in } e2 \rangle \Rightarrow v \\ \text{if } \langle \text{env}, e1 \rangle \Rightarrow^* v1 \\ \text{and } \langle \text{env}[x \rightarrow v1], e2 \rangle \Rightarrow v \end{array}$$

Dynamic Semantics
(Environment Model)

Static Semantics: If-Then-Else

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for if statements:

```
T |- if e1 then e2 else e3 : t
    if ???
```

Static Semantics: If-Then-Else

Suppose we have a Bool Type and an Int Type- here's how we can define the type relation for if statements:

```
T |- if e1 then e2 else e3 : t
  if T |- e1 : bool
  and e2 : t
  and e3 : t
```

Type Soundness

What does it mean for a program to be good?

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Usually we want these two super-useful properties:

- ▶ **Progress:** if $e:t$, then e is a value or can take a step
- ▶ **Preservation:** if $e:t$ and $e \rightarrow e'$, then $e':t$

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- ▶ **Progress:** Well-typed programs always run to completion
- ▶ **Preservation:** Evaluation does not change the type of an expression

If these two properties hold for a type system, we say that type system is “sound”

Type Soundness: Example

Here's an unsound example:

```
T |- if e1 then e2 else e3 : t2
    if T |- e1 : bool
    and T |- e2 : t2
    and T |- e3 : t3
```

Does this violate Progress or Preservation (or neither)?

- ▶ **Progress:** Well-typed programs always run to completion
- ▶ **Preservation:** Evaluation does not change the type of an expression