Lotka–Volterra Models (Ecology)

A **discrete-time logistic Lotka–Volterra predator-prey model** is a kind of one-patch, two-population model used in computational ecology. It is defined by two difference equations:

$$x_{t+1} = \max\left(x_t\left(1 + r\frac{K - x_t}{K} - by_t\right), 0\right)$$
$$y_{t+1} = \max\left(y_t\left(1 - s + \frac{b}{C}x_t\right), 0\right)$$

where

- t is time,
- x_t is the number of prey at time t,
- y_t is the number of predators at time t,
- K is the patch's carrying capacity for prey in the absence of predation,
- r is the prey population's logistic growth rate,
- b is the predation rate (the fraction of the prey population that one predator kills in one unit of time),
- s is the predator death rate, and
- C is the number of kills that a predator must make to produce one offspring.

`lotka_volterra.py` simulates this model. Download and run it.

Data should appear in the terminal. At the top will be a header row and then the values of the five model parameters. Below that will be another header and then 1000 rows of simulation results. Each row will show a day number, the prey population, and the predator population, in that order. Don't be surprised to see noninteger values; most Lotka–Volterra models suffer from the "atto-fox" problem, where nothing forbids fractional individuals or the unrealism that may result.

Answer the following questions by making comments in your recording:

1. Scroll through the data and give a qualitative description of what is happening to the two populations.

2. Experiment with different C values while keeping the other parameters the same. What values, if any, lead to extinctions? To equilibria? To oscillations?

(Note: All species have a minimum viable population [MVP] below which extinction becomes inevitable, but Lotka–Volterra models do not account for MVPs or related bottlenecking effects. If you see either of these populations drop below ten individuals, you can probably consider that an extinction.)

3. Try to find math expressions that predict the equilibrium populations given a suitable value of C. Hypothesize, based on these formulae, whether oscillations under this model are stable, convergent, or divergent.

4. One would suspect that a patch that is more amenable to the prey species would have higher equilibrium populations. Design and conduct an experiment to investigate this hypothesis. Summarize your results. If you refute the hypothesis, explain why the intuition is wrong.

The same model can be formulated in **continuous time**, which is appropriate when there is no periodicity to the underlying predator-prey interactions (such as a night-day hunting cycle) or when the time scale of these interactions is negligible as compared to that of the population dynamics. The differential equations:

$$\dot{x} = x\left(r\frac{K-x}{K} - by\right)$$
$$\dot{y} = y\left(-s + \frac{b}{C}x\right)$$

where dotted variables, per convention, indicated derivatives with respect to time. (In particular, $\dot{x} = \frac{dx}{dt}$ and $\dot{y} = \frac{dy}{dt}$.)

`lotka_volterra_continuous.py` simulates this model. Use it to answer the following questions:

5. Describe what happens to the two populations under the original parameters. Contrast what you see with the behavior of the discrete-time model.

6. Design and conduct an experiment to determine whether this model allows oscillatory populations, and, if so, of what sort(s): stable, convergent, or divergent.