ECON 600: Merger Homework

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All code is in Python.

3.1

```
# x_jt, w_jt are absolute value of iid standard normal variables
x = np.absolute(np.random.standard_normal(size=(J,T)))
w = np.absolute(np.random.standard_normal(size=(J,T)))

unobservable_mean = [0,0]
unobservable_cov = [[1,0.25],[0.25,1]]
unobservables = np.random.multivariate_normal(unobservable_mean, unobservable_cov, size=(J,T))
xi = unobservables[:,:,0]
omega = unobservables[:,:,1]
```

3.2

(a) (i) We first note that in the parameter specification,

$$\overline{\beta^{(2)}} = 4$$
$$\overline{\beta^{(3)}} = 4$$

Hence, defining, $\sigma^{(2)} = \sigma^{(3)} = 1$, we have that

$$\beta^{(2)} = \overline{\beta^{(2)}} + \sigma^{(2)}\nu_i^{(2)}$$

$$\beta^{(3)} = \overline{\beta^{(3)}} + \sigma^{(3)}\nu_i^{(3)}$$

where $\nu_i^{(2)}$ and $\nu_i^{(3)}$ are i.i.d standard normal.

The multinomial logit choice probabilities are, conditional on all realized coefficients,

$$s_{0t} = \int \frac{1}{Z}\, d\Phi(\nu)$$

$$s_{1t} = \int \frac{\exp(x_{1t} + \overline{\beta^{(2)}} + \alpha p_{1t} + \xi_{1t} + \nu^{(2)}\sigma^{(2)})}{Z} \, d\Phi(\nu)$$

$$s_{2t} = \int \frac{\exp(x_{2t} + \overline{\beta^{(2)}} + \alpha p_{2t} + \xi_{2t} + \nu^{(2)}\sigma^{(2)})}{Z} \, d\Phi(\nu)$$

$$s_{3t} = \int \frac{\exp(x_{3t} + \overline{\beta^{(3)}} + \alpha p_{3t} + \xi_{3t} + \nu^{(3)}\sigma^{(3)})}{Z} \, d\Phi(\nu)$$

$$s_{4t} = \int \frac{\exp(x_{4t} + \overline{\beta^{(3)}} + \alpha p_{4t} + \xi_{4t} + \nu^{(3)}\sigma^{(3)})}{Z} \, d\Phi(\nu)$$

where

$$Z = 1 + \exp(x_{1t} + \overline{\beta^{(2)}} + \alpha p_{1t} + \xi_{1t} + \nu^{(2)}\sigma^{(2)}) + \exp(x_{2t} + \overline{\beta^{(2)}} + \alpha p_{2t} + \xi_{2t} + \nu^{(2)}\sigma^{(2)})$$

$$+ \exp(x_{3t} + \overline{\beta^{(3)}} + \alpha p_{3t} + \xi_{3t} + \nu^{(3)}\sigma^{(3)}) + \exp(x_{4t} + \overline{\beta^{(3)}} + \alpha p_{4t} + \xi_{4t} + \nu^{(3)}\sigma^{(3)})$$