ECON 600: Merger Homework

Nicholas Wu

Fall 2021

All code is in Python.

3.1

```
# x_jt, w_jt are absolute value of iid standard normal variables
x = np.absolute(np.random.standard_normal(size=(J,T)))
w = np.absolute(np.random.standard_normal(size=(J,T)))

unobservable_mean = [0,0]
unobservable_cov = [[1,0.25],[0.25,1]]
unobservables = np.random.multivariate_normal(unobservable_mean, unobservable_cov, size=(J,T))
xi = unobservables[:,:,0]
omega = unobservables[:,:,1]
```

3.2

(a) (i) We first note that in the parameter specification,

$$\overline{\beta^{(2)}} = 4$$

$$\overline{\beta^{(3)}} = 4$$

Hence, defining, $\sigma^{(2)} = \sigma^{(3)} = 1$, we have that

$$\beta_{it}^{(2)} = \overline{\beta^{(2)}} + \sigma^{(2)}\nu_{it}^{(2)}$$

$$\beta_{it}^{(3)} = \overline{\beta^{(3)}} + \sigma^{(3)}\nu_{it}^{(3)}$$

where $\nu_i^{(2)}$ and $\nu_i^{(3)}$ are i.i.d standard normal.
Define

$$\delta_{jt} = x_{jt} + \overline{\beta^{(2)}} satellite_j + \overline{\beta^{(3)}} wired_j + \alpha p_{jt} + \xi_{jt}$$

$$\mu_{ijt} = \sigma^{(2)} satellite_j \nu_{it}^{(2)} + \sigma^{(3)} wired_j \nu_{it}^{(3)}$$

The multinomial logit choice probabilities are, conditional on all realized coefficients,

$$s_{0t} = \int \frac{1}{Z}\ d\Phi(\nu)$$

$$s_{jt} = \int \frac{\exp(\delta_{jt} + \mu_{ijt})}{Z}\ d\Phi(\nu)$$

for $j > 0$. Where

$$Z = 1 + \sum_{j=1}^{J} \exp(\delta_{jt} + \mu_{ijt})$$

Then the derivatives are

$$\frac{\partial s_{jt}}{\partial p_j} = \int \frac{\alpha \exp(\delta_{jt} + \mu_{ijt})Z - \exp(\delta_{jt} + \mu_{ijt})\left(\alpha \exp(\delta_{jt} + \mu_{ijt})\right)}{Z^2}\ d\Phi(\nu)$$

$$\frac{\partial s_{jt}}{\partial p_k} = \int -\frac{\exp(\delta_{jt} + \mu_{ijt})\left(\alpha \exp(\delta_{kt} + \mu_{ikt})\right)}{Z^2}\ d\Phi(\nu)$$