ECON500: Problem Set 1

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Problem 2

(15.B.4)

(a)

(b)

(c)

(d)

(e)

(f)

(15.B.8) Let the two utility functions be

$$u_1(x, y) = x + f(y)$$

$$u_2(x, y) = x + g(y)$$

By the second welfare theorem, any Pareto optimal allocation is supported by some prices and endowment choices. Normalize the numeraire price to 1. Let some interior Pareto optimal allocation be supported by the price vector $(1, p)$. By the optimization FOCs, since the allocation is interior, we require

$$1 - \lambda = 0$$

$$f'(y_1) - \lambda p = 0$$

or

$$f'(y_1) = p$$

Similarly,

$$g'(y_2) = p$$

Let the aggregate endowment of y be ω_y. Then at any Pareto optimal allocation, we must have

$$f'(y_1) = g'(\omega_y - y_1) = p$$

$$f'(y_1) - g'(\omega_y - y_1) = 0$$

Define

$$h(y) = f'(y) - g'(\omega_y - y)$$

Then

$$h'(y) = f''(y) + g''(\omega_y - y)$$

Due to continuity and strictly convex preferences, both f'' and g'' are nonzero and have the same sign, and hence h is a strictly monotonic function. Therefore, $h(y) = 0$ for at most one unique value of y, and since y_1 satisfies $h(y_1) = 0$, we know that since any interior Pareto optimal allocation must satisfy $h(y) = 0$, any interior Pareto optimal allocation always assigns y_1 to consumer 1 and $\omega_y - y_1$ to consumer 2.

Problem 3

.... TODO smoothness ....

To show x_i is a diffeomorphism, it suffices to argue the preimage map x_i^{-1} is a well-defined function and smooth.

Problem 4

The offer curve is $\mathcal{O}$