ECON500: Problem Set 1

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Problem 2

(16.AA.1)

(a)

Problem 4

Problem 6

We provide a counterexample. Take $K = [0, 1]$, and let $\gamma(0) = (0, 1)$ and $\gamma(k) = \{0.5\}$ for any $k \in (0, 1]$. It is clear that γ is a (constant) continuous function on $(0, 1]$, so we just need to check upper hemicontinuity at 0. Take a sequence $k_n \in K$ converging to 0. Then $\gamma(k_n)$ only contains 0.5 if $k_n \neq 0$, so we can always pick out a subsequence k'_n such that 0.5 is always in the image, and hence the image sequence converges to 0.5, and $0.5 \in \gamma(0)$. So γ is upper hemicontinuous. It is trivial to see that γ is not closed; $\gamma(0)$ is not closed.

Problem 7

Problem 8

We first prove a lemma.

Lemma: Any compact, convex set $K \subseteq \mathbb{R}^n$ is diffeomorphic to a d-simplex for some d.

Proof: TODO

Now, the generalization of Brouwer's follows from this lemma. Suppose $f : K \to K$ is continuous. Since K is diffeomorphic to a d-simplex, let $g : K \to S$ be such a diffeomorphism to some d-simplex S. Consider the composition $h = g \circ f \circ g^{-1}$. Then h takes $S \to S$, and since composition of continuous functions is continuous, h must be continuous. So by Brouwer's theorem on simplices, we have that h has some fixed point $p \in S$. This implies

$$(g \circ f \circ g^{-1})(p) = p$$

$$g^{-1}((g \circ f \circ g^{-1})(p)) = g^{-1}(p)$$

$$f(g^{-1}(p)) = g^{-1}(p)$$

Then since $g^{-1}(p) \in K$, we have that $g^{-1}(p)$ is a fixed point of f. Hence we have extended Brouwer to an arbitrary compact convex domain K.

Problem 9