ECON550: Problem Set 5

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Problem 1

(a) We have

$$P(|(\hat{\theta}-\theta)| > 2/\sqrt{n}) = P(|\sqrt{n}(\hat{\theta}-\theta)| > 2)$$

Now, since $\sqrt{n}(\hat{\theta}-\theta) \to_d N(0,\theta^2)$, by symmetry of the normal distribution we can approximate this then as

$$P(|\sqrt{n}(\hat{\theta}-\theta)| > 2) \approx 2P(\sqrt{n}(\hat{\theta}-\theta) > 2)$$

$$= 2(1-\Phi(2/\sqrt{\theta^2})) = 2(1-\Phi(2/|\theta|))$$

(b) Since $\sqrt{n}(\hat{\theta}-\theta) \to_d N(0,\theta^2)$ and $\hat{\theta} \to_p \theta$, we have $\sqrt{n}(\hat{\theta}-\theta)/|\hat{\theta}| \to_d N(0,\theta^2/|\theta|^2) = N(0,1)$. Hence, we get

$$P(|(\hat{\theta}-\theta)| > 2/\sqrt{n}) = P(|\sqrt{n}(\hat{\theta}-\theta)| > 2)$$

$$= P(|(\sqrt{n}(\hat{\theta}-\theta))/\hat{\theta}| > 2/|\hat{\theta}|)$$

$$\approx 2(1-\Phi(2/|\hat{\theta}|))$$

Problem 2

As in PS5, we define

$$\hat{\lambda}_n = \frac{1}{\bar{X}_n}$$

The variance of X_i is given by

$$\int_0^\infty \left(x - \frac{1}{\lambda}\right)^2 \lambda e^{-\lambda x}\, dx = \frac{1}{\lambda^2}$$

Then by the CLT $\sqrt{n}(\bar{X}_n - 1/\lambda) \to_d N(0, 1/\lambda^2)$

Problem 3

The objective can be rewritten as

$$(Y - X\hat{\beta})'(Y - X\hat{\beta}) = Y'Y - Y'X\hat{\beta} - X'Y\hat{\beta} + X'X\hat{\beta}^2 = Y'Y - 2X'Y\hat{\beta} + X'X\hat{\beta}^2$$

The FOC:

$$-2X'Y + 2X'X\hat{\beta} = 0$$

$$X'Y = X'X\hat{\beta}$$

Since $X'X$ is nonsingular,

$$\hat{\beta} = (X'X)^{-1}(X'Y)$$

Problem 4

We can use the delta method. By Slutsky's theorem,

$$g(\hat{\theta}) \to g(\theta) = \begin{bmatrix} \theta_1 - \theta_2 \\ \theta_1\theta_3 \end{bmatrix}$$

The $G(\theta)$ is

$$\begin{bmatrix} 1 & -1 & 0 \\ \theta_3 & 0 & \theta_1 \end{bmatrix}$$

So by the delta method,

$$\sqrt{n}(g(\hat{\theta}) - g(\theta)) \to_d N(0, G(\theta)\Sigma G(\theta)')$$

Problem 5

Problem 6

(a)

Problem 7

(a)

Problem 8

(a)