

Problem Set 5

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12.1

The estimator is

$$\hat{\beta}_{IV} = \frac{\sum_i y_i d_i}{\sum_i z_i d_i} = \frac{\sum_{i|d_i=1} y_i}{\sum_{i|d_i=1} z_i}$$

12.8

Let $X = \begin{bmatrix} 1 & P & Y \end{bmatrix}'$, $Z = \begin{bmatrix} 1 & W & Y \end{bmatrix}'$, and let $a = \begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}'$. Then we have

$$Q = X'a + e_1$$

$$ZQ = ZX'a + Ze_1$$

$$E[ZQ] = E[ZX']a + E[Ze_1]$$

Since Z is determined outside the market, $E[Ze_1] = 0$, so

$$a = (E[ZX'])^{-1}E[ZQ]$$

so a is identified as long as $E[ZX']$ is invertible. Similarly, if we let $X_2 = \begin{bmatrix} 1 & P & W \end{bmatrix}'$, we get

$$b = (E[ZX_2'])^{-1}E[ZQ]$$

so b is identified as long as $E[ZX_2']$ is invertible.

12.25, 13.28

See code notebook, commentary there.