Problem Set 7

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Problem 1

(a)

$$E[h(1,u) - h(0,u)] = E[h(1,u)] - E[h(0,u)] = E[y|d=1] - E[y|d=0] = \tau + \alpha - \alpha = \tau$$

(b) The estimator is given by

$$\frac{1}{N}\sum(y_i - \hat{\alpha} - \hat{\tau}d_i) = 0$$

$$\frac{1}{N}\sum(y_i - \hat{\alpha} - \hat{\tau}d_i)d_i = 0$$

Let $D = \sum d_i$. Then

$$\frac{1}{N}\sum y_i = \hat{\alpha} + \hat{\tau}(D/N)$$

$$\frac{1}{N}\sum y_i d_i = \hat{\alpha}(D/N) + \hat{\tau}(D/N)$$

$$\frac{D}{N^2}\sum y_i = \hat{\alpha}(D/N) + \hat{\tau}(D^2/N^2)$$

$$\frac{1}{N}\sum y_i d_i - (D/N)\frac{1}{N}\sum y_i = \hat{\tau}(D/N)(1 - D/N)$$

$$Cov(y,d) = \hat{\tau}Var(d)$$

$$\frac{Cov(y,d)}{Var(d)} = \hat{\tau}$$

Note this is exactly the same as doing OLS on y with d. See code. The estimated value is approx 886.3 with standard error approx 472.086

(c) See code and annotations for work. The estimate is 794.3885736971342 and the standard error is 480.3028362343343.