Section 7.2- Integration by Parts

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Integration by parts is a technique of integration which arises from the product rule. Recall that if f and g are differentiable functions, then the product rule says

$$D_x\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x).$$

If we now integrate both sides, using the FTC on the left, we get

$$f(x)g(x) = \int D_x\left(f(x)g(x)\right)\ dx = \int \left(f'(x)g(x) + f(x)g'(x)\right)\ dx$$

We can rewrite this integral in the form

$$\boxed{\int f(x)g'(x)\ dx = f(x)g(x) - \int g(x)f'(x)\ dx} \tag{1}$$

It is common to use differential notation with $u(x) = f(x)$ and $v(x) = g(x)$ (in which case $du = f'(x)\ dx$ and $dv = g'(x)\ dx$) to rewrite this as

$$\boxed{\int u\ dv = uv - \int v\ du}$$

In practice, we are asked to integrate an integrand which is the product of two functions. We pick one to be u (which we will have to differentiate to get du) and the other to be dv (which we will have to integrate to get v). We then use the right-hand side of the equation above and hopefully get another integral that is easier to evaluate.
If we replace the indefinite integral in Eq. (1) with a definite integral, we get the formula

$$\boxed{\int_a^b f(x)g'(x)\ dx = \left(f(x)g(x)\right|_a^b - \int_a^b g(x)f'(x)\ dx} \tag{2}$$

or

$$\boxed{\int_a^b u\ dv = \left(uv\right|_a^b - \int_a^b v\ du}$$

Remark 1. *Choosing which function in our integrand to be u and which to be dv is a bit of an art. One rule of thumb is to remember the acronym **LIATE**, which stands for "**L**ogarithms, **I**nverse trig functions, **A**lgebraic functions (think polynomials), **T**rig functions, and **E**xponentials. The premise is that the function of the type which occurs earlier in this list should be chosen to be the 'u' term. There are exceptions to this rule, however.*

Example 1. *Use integration by part to evaluate the following definite and indefinite integrals:*

1. $\int xe^{-x}\ dx$

2. $\int_0^{\pi} \theta \cos(2\theta)\ d\theta$

3. $\int \ln x\ dx$

4. $\int_0^{\pi} x^2 \sin x\ dx$

5. $\int e^{2x} \cos x\ dx$

6. $\int_0^{1} \tan^{-1} x\ dx$