Section 7.4- Rationalizing Substitution

Name/ Uid:______________________________________ Date:__________

In this section, we examine two techniques that can be used to simplify integrals involving radicals.

Integrands Involving $\sqrt[n]{ax+b}$

The technique here is simply to make a substitution $u = \sqrt[n]{ax+b} = (ax+b)^{1/n}$. The essential observation is that this implies that $u^n = ax + b \Rightarrow x = \frac{u^n - b}{a}$ and $dx = \frac{n}{a}u^{n-1}\ du$.

Example 1. *Find* $\displaystyle\int x\sqrt[3]{x+1}\ dx$

Solution.

Integrands Involving $\sqrt{a^2-x^2}$, $\sqrt{a^2+x^2}$, and $\sqrt{x^2-a^2}$

For these integrals, we perform what might be called a **reverse substitution**: we write x as a function of a new variable (usually t in this section). The chart below gives the terms appearing in our integrand that indicate such a substitution:

term	**substitution**
$\sqrt{a^2-x^2}$	$x = a\sin t$
$\sqrt{a^2+x^2}$	$x = a\tan t$
$\sqrt{x^2-a^2}$	$x = a\sec t$

For example, if we have a $\sqrt{4-x^2}$ in our integrand, the chart says to make the substitution $x = 2\sin t$, in which case $dx = 2\cos t\ dt$. The reason for this is now the term with the radical simplifies:

$$\sqrt{4-x^2} = \sqrt{4-4\sin^2 t} = \sqrt{4\cos^2 t} = 2\cos t$$

The integral then becomes a trigonometric integral which we do by the techniques of the last section. In order to get our answer in terms of x, we usually need to draw a triangle.

Example 2. *Find* $\displaystyle\int \frac{1}{x^2\sqrt{x^2-4}}\ dx.$

Solution.

Example 3. *Evaluate* $\int_0^{\frac{4}{3}} \sqrt{16 - 9x^2}\ dx$

Solution.

Example 4. *Find* $\int \frac{x^3}{\sqrt{x^2 + 100}}\ dx$

Solution.