Section 7.3- Trigonometric Integrals

Name/ Uid:______________________________________ Date:___________

In this section, we learn a few techniques which help in evaluating integrals of trig functions. Some identities which will be used in this section:

$$\sin^2 x + \cos^2 x = 1 \qquad (1)$$
$$\tan^2 x + 1 = \sec^2 x \qquad (2)$$
$$\sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right) \qquad (3)$$
$$\cos^2 x = \frac{1}{2}\left(1 + \cos(2x)\right) \qquad (4)$$
$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right) \qquad (5)$$
$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right) \qquad (6)$$
$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right) \qquad (7)$$

Integrals of the form $\int \sin^m x \cos^n x \; dx$

- **Either m or n is odd-** If, for example, m is odd, then we write $\sin^m x = \sin x \sin^{m-1} x$ and use identity (1) to write $\sin^{m-1} x$ in terms of $\cos x$. We then make the substitution $u = \cos x$. If m is even, but n is odd, we perform the same steps with the roles of $\sin x$ and $\cos x$ reversed.

  Example 1. *Evaluate* $\displaystyle\int \sin^3 x \cos^2 x \; dx$

- **Both m and n are even-** In this case, we use the double angle formulas (identities (3) and (4)).

  Example 2. *Evaluate* $\displaystyle\int \sin^2 x \cos^2 x \; dx$

Integrals of the form $\int \tan^m x \sec^n x \; dx$

- **If n is even-** Pull out a factor of $\sec^2 x$ and then use identity (2) to rewrite the remaining integrand as a polynomial in $\tan x$. Then make the substitution $u = \tan x$.

  Example 3. *Evaluate* $\displaystyle\int \sec^4 x \tan x \; dx$

- **If m is odd-** Pull out a factor of $\sec x \tan x$, and then use identity (2) to rewrite the remaining integrand as a polynomial in $\sec x$. Then make the substitution $u = \sec x$.

  Example 4. *Evaluate* $\int \sec^3 x \tan^3 x \; dx$

- **Otherwise-** No one method. Try using substitution or integration by parts using the facts

$$\boxed{\begin{array}{c} \int \tan x \; dx = \ln|\sec x| + C \\ \int \sec x \; dx = \ln|\sec x + \tan x| + C \end{array}}$$

Integrals of the form $\int \sin(Ax)\cos(Bx) \; dx$ and the like

These are relatively straightforward uses of identities $(5), (6)$, and (7).

Example 5. *Evaluate* $\displaystyle\int \sin(5x)\cos(2x) \; dx$