Math 1220-001
Spring 2020
Exam 1
1/31/20
Time Limit: 50 Minutes

Fancy Number: ______________________

This exam contains 5 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, phone or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	12	
3	26	
4	12	
5	20	
6	10	
Total:	100	

1. Differentiate the following:

   (a) (5 points) $D_x(e^{100x+x^3})$

   $= e^{100x+x^3} \cdot Dx(100x+x^3)$

   $= e^{100x+x^3} \cdot (100+3x^2)$

   (b) (5 points) $D_x(\tan^{-1}(\ln(x^2)))$

   $= \frac{1}{1+(\ln x^2)^2} \cdot Dx(\ln(x^2))$

   $= \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x^2} \cdot Dx(x^2) = \frac{1}{1+(\ln x^2)^2} \cdot \frac{1}{x^2} \cdot 2x$

   (c) (5 points) $D_x(\log_4(\cos^{-1}(x)))$

   $= \frac{1}{\cos^{-1}x \cdot \ln 4} \cdot Dx(\cos^{-1}x)$

   $= \frac{1}{\cos^{-1}x \cdot \ln 4} \cdot \frac{-1}{\sqrt{1-x^2}}$

   (d) (5 points) $D_x(x^{5x})$

   $= Dx(e^{5x\ln x})$

   $= e^{5x\ln x} \cdot Dx(5x\ln x)$

   $= e^{5x\ln x} \cdot [Dx(5x)\ln x + Dx(\ln x)5x] = e^{5x\ln x} \cdot [5\ln x + \frac{5x}{x}]$

2. Evaluate the following:

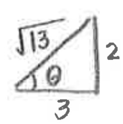
   (a) (1 point) $\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

   (b) (1 point) $\sin^{-1}(0) = 0$

   (c) (2 points) $\sin(\sin^{-1}(1)) = \sin(\frac{\pi}{2}) = 1$

   (d) (4 points) $\cos(\underbrace{\sin^{-1}(\tfrac{1}{x})}_{\theta})$ $= \cos(\theta) = \frac{\sqrt{x^2-1}}{x}$

   (e) (4 points) $\sin(\underbrace{\tan^{-1}(\tfrac{2}{3})}_{\theta})$ $= \sin\theta = \frac{2}{\sqrt{13}}$



3. (10 points) Consider the function

$$f(x) = x^3 + 2x + 1$$

(a) (8 points) Show that $f(x)$ has an inverse.

we use inverse function thm:

$f'(x) = 3x^2 + 2$ for $x > 0 \Rightarrow f'(x) > 0$

$x < 0 \Rightarrow f'(x) > 0$

thus, strictly increasing $\Rightarrow f^{-1}(x)$ exists

(b) (2 points) Find $(f^{-1})(1)$.

$x^3 + 2x + 1 = 1$

$x = 0$

(c) (6 points) Compute $(f^{-1})'(1)$.

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{3(0)+2} = \frac{1}{2}$$

4. A particular radioactive isotope decays from 12 grams to 8 grams in 36 hours. Let $A(t)$ denote the amount (in grams) of the isotope present after t hours.

(a) (8 points) Using $A(t) = Ce^{kt}$, find C and k.

$t = 0,\ A = 12$

· $12 = Ce^0 \Rightarrow C = 12$

$t = 36,\ A = 8$

· $8 = 12e^{36k} \Rightarrow \frac{8}{12} = e^{36k} \Rightarrow \ln\left(\frac{8}{12}\right) = 36k$

$$k = \frac{\ln(8/12)}{36}$$

(b) (4 points) How many hours until there are 3 grams of the isotope present?

$$3 = 12e^{\frac{\ln(8/12)}{36}t}$$

$$\frac{3}{12} = e^{\frac{\ln(8/12)}{3}t}$$

$$\ln\left(\frac{3}{12}\right) = \frac{\ln(8/12)}{3}t$$

$$t = \frac{3\ln(3/12)}{\ln(8/12)}$$

5. Evaluate the following indefinite integral. Do not forget your $+C$!

(a) (5 points) $\displaystyle\int \frac{2}{4x+2}\,dx$

$u = 4x+2$

$du = 4\,dx \Rightarrow \frac{1}{4}du = dx$

$= \int \frac{2}{u} \cdot \frac{1}{4}\,du$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c$

$= \frac{1}{2}\ln|4x+2| + c$

(b) (5 points) $\displaystyle\int \frac{1}{\sqrt{16-x^2}}\,dx$

$= \int \frac{1}{\sqrt{16\left(\frac{16}{16} - \frac{x^2}{16}\right)}}\,dx$

$= \int \frac{1}{4\sqrt{1-\left(\frac{x}{4}\right)^2}}\,dx \qquad u = \frac{x}{4} \Rightarrow du = \frac{1}{4}dx$

$4du = dx$

$= \int \frac{1}{4}\cdot 4\,\frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + c = \sin^{-1}\left(\frac{x}{4}\right) + c$

(c) (5 points) $\displaystyle\int xe^{3x^2}\,dx$

$u = 3x^2$

$du = 6x\,dx \Rightarrow \frac{1}{6}du = x\,dx$

$= \int \frac{1}{6}e^u\,du$

$= \frac{1}{6}e^u + C = \frac{1}{6}e^{3x^2} + c$

(d) (5 points) $\displaystyle\int \frac{e^x}{4+9e^{2x}}\,dx$

$u = e^x$

$du = e^x\,dx$

$= \int \frac{du}{4+9u^2}$

$= \int \frac{du}{4+(3u)^2} = \int \frac{du}{4\left(\frac{4}{4} + \frac{(3u)^2}{4}\right)}$

$= \frac{1}{4}\int \frac{du}{1+\left(\frac{3u}{2}\right)^2} \qquad g = \frac{3u}{2}$

$dg = \frac{3}{2}du$

$2/3\,dg = du$

$= \frac{1}{4}\int \frac{2}{3}\,\frac{dg}{1+g^2}$

$= \frac{1}{4}\cdot\frac{2}{3}\tan^{-1}(g) + c$

$= \frac{1}{4}\cdot\frac{2}{3}\tan^{-1}\left(\frac{3e^x}{2}\right) + c$

6. (10 points) Consider the function

$$y = \frac{(4-3x)^2}{(x^2+5x+6)\sqrt{x}}$$

Use logarithmic differentiation to find the derivative of the function.

$$\ln y = \ln\left(\frac{(4-3x)^2}{(x^2+5x+6)\sqrt{x}}\right)$$

$$\ln y = \ln\left(\frac{(4-3x)^2}{(x+3)(x+2)x^{\frac{1}{2}}}\right)$$

$$\ln y = \ln(4-3x)^2 - \ln(x+3) - \ln(x+2) - \ln x^{\frac{1}{2}}$$

$$\ln y = 2\ln(4-3x) - \ln(x+3) - \ln(x+2) - \tfrac{1}{2}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{4-3x}\cdot(-3) - \frac{1}{x+3} - \frac{1}{x+2} - \frac{1}{2x}$$

$$\frac{dy}{dx} = y\left[\frac{-6}{4-3x} - \frac{1}{x+3} - \frac{1}{x+2} - \frac{1}{2x}\right]$$

$$= \frac{(4-3x)^2}{(x^2+5x+6)\sqrt{x}}\left[\frac{-6}{4-3x} - \frac{1}{x+3} - \frac{1}{x+2} - \frac{1}{2x}\right]$$