8.1- Indeterminate Form of Type 0/0

Name/ Uid:____ Date:___________

In this section, we learn a technique which will allow us to evaluate certain limits which are inaccessible using the standard limit laws. These limits are described as being of **indeterminate form**. There are different types of indeterminate form; we address the first type in this section and others in Section 8.2.

Theorem 1 (L'Hôpital's Rule, type 0/0)**.** *Suppose f and g are differentiable functions with $g'(x) \neq 0$ for x in some open neighborhood of a (but possibly $g'(a) = 0$). If*

$$\lim_{x\to a} f(x) = 0 \qquad \textit{and} \qquad \lim_{x\to a} g(x) = 0$$

then

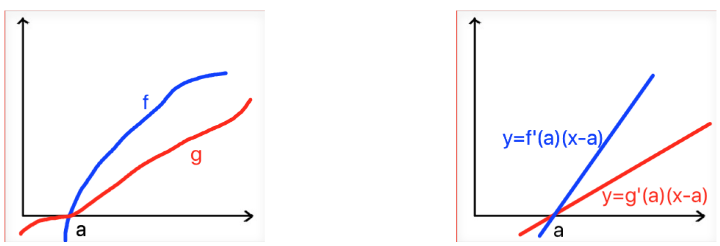
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right hand side exists (or is equal to $\pm\infty$).

Remark 1. *L'Hôpital's rule applies to one-sided limits and limits at infinity as well ($a = \pm\infty$).*

Remark 2. *When $f'(x)$ and $g'(x)$ are continuous, we can understand why L'Hôpital's Rule is true with a picture. If we 'zoom in' close enough near the point $x = a$ on the graphs of f and g, the functions should look like linear functions. Recall the the linearization of f at $x = a$ is given by $L(x) = f(a) + f'(a)(x - a) = f'(a)(x - a)$ (and similarly for g). It follows that*

$$\lim_{x\to a} \frac{f(x)}{g(x)} \approx \lim_{x\to a} \frac{f'(a)(x-a)}{g'(a)(x-a)} = \frac{f'(a)}{g'(a)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$



Example 1. *Evaluate the following limits using L'Hôpital's Rule:*

(a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$

(b) $\displaystyle\lim_{x\to \pi} \frac{\tan(2x)}{x - \pi}$

(c) $\displaystyle\lim_{x\to 1^+} \frac{\ln x}{(x-1)^2}$

Example 2. *Evaluate the following limits using L'Hôpital's Rule more than once.*

(a) $\displaystyle\lim_{x\to \pi} \frac{1+\cos x}{(x-\pi)^2}$

(b) $\displaystyle\lim_{x\to 0} \frac{e^x-1-x}{x^2\cos x}$

(c) $\displaystyle\lim_{x\to 0} \frac{\sin(x^2)\tan x}{x^2}$