8.2- Other Indeterminate Forms

Name/ Uid:____ Date:___________

Theorem 1 (L'Hôpital's Rule, type ∞/∞)**.** *Suppose f and g are differentiable functions. If*

$$\lim_{x\to a} |f(x)| = \infty \qquad \text{and} \qquad \lim_{x\to a} |g(x)| = \infty$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right hand side exists (or is equal to $\pm\infty$).

Remark 1. *Again, this version of L'Hôpital's rule applies to one-sided limits and limits at infinity as well ($a = \pm\infty$).*

Example 1. *Evaluate the following limits using L'Hôpital's Rule:*

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x}$

(b) $\displaystyle\lim_{x\to 1^+} \frac{\ln x}{(x-1)^2}$

(c) $\displaystyle\lim_{x\to\infty} \frac{x^n}{e^x}$*, where n is any positive integer.*

There are other indeterminate forms, which can be transformed into type $0/0$ or ∞/∞ by some algebraic manipulation.

- (Type $0 \cdot \infty$) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$, then to compute $\lim_{x\to a} f(x)g(x)$, we can apply L'Hospital's Rule to either

$$\lim_{x\to a} \frac{f(x)}{\frac{1}{g(x)}} \qquad \text{or} \qquad \lim_{x\to a} \frac{g(x)}{\frac{1}{f(x)}},$$

  which are of indeterminate form $\frac{0}{0}$ and $\frac{\infty}{\infty}$ respectively.

- (Type $\infty - \infty$) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then to compute

$$\lim_{x\to a} \left(f(x) - g(x)\right),$$

  we can sometimes apply L'Hospital's Rule if we can write the difference as a fraction (either by using a common denominator, rationalization, or by factoring out some term).

- (Type 0^0, ∞^0, and 1^∞) To compute

$$\lim_{x\to a} f(x)^{g(x)},$$

  we can compute the logarithm of the above limit. That is, if $\lim_{x\to a} f(x)^{g(x)} = L$, then we can find L by noting that $\ln L = \lim_{x\to a} g(x) \ln\left(f(x)\right)$.

Example 2. *Evaluate the following limits using L'Hôpital's Rule.*

(a) $\displaystyle\lim_{x\to 0^+} x \ln x$

(b) $\displaystyle\lim_{x\to \frac{\pi}{2}} (\sec x - \tan x)$

(c) $\displaystyle\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x$