Section 6.3- The Natural Exponential Function

Name/ Uid:______________________________________ Date:___________

Since the function $y = \ln x$ is one-to-one for $x > 0$, it has an inverse:

Definition. *The inverse of* $\ln x$ *is the* ***natural exponential function*** *denoted* $\exp x$ *or* e^x*.*

The fact that $\ln x$ and e^x are inverses implies that $e^{\ln x} = x$ for $x > 0$ and $\ln e^x = x$ for all x.

Proposition 1 (Properties of the Natural Exponential)**.** *For any* a, b,

1. $e^0 = 1$
2. $e^a e^b = e^{a+b}$
3. $\frac{e^a}{e^b} = e^{a-b}$

Example 1. *Use properties of the natural exponential to evaluate the following:*

1. $e^{\ln 3}$
2. $\ln\left(\frac{e^5}{e^3}\right)$
3. $e^{(\ln 1 - \ln 2)} \ln\left(e^2\right)$

Example 2. *Solve for* x*:*

(a) $\ln\left(5 + x^2\right) = 6$

(b) $e^{4-2x} = 7$.

To find the derivative of e^x, we use logarithmic differentiation. If $y = e^x$, then $\ln y = x$ and so

$$1 = D_x(x) = D_x(\ln y) = \frac{1}{y}\frac{dy}{dx}$$

From this we obtain

$$D_x(e^x) = e^x \iff \int e^x \, dx = e^x + C$$

Example 3. *Differentiate the following functions:*

(a) $D_x(e^{-3x})$

(b) $D_x(xe^{x^2})$

(c) $D_x(e^x \sin(e^x))$

Example 4. *Find the following indefinite integrals:*

(a) $\displaystyle\int e^{-3x}\ dx$

(b) $\displaystyle\int xe^{x^2}\ dx$

(c) $\displaystyle\int e^x \sin(e^x)\ dx$