Section 7.1- Basic Integration Rules

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We've now found derivatives of many simple functions, including exponentials, logs, inverse trig function, etc. Each differentiation rule gives a corresponding integral formula, and so we have a fairly long list now of functions that we can integrate. However, these still represent only a small fraction of the integrals we commonly come across. In this chapter, we will study some techniques of integration which will allow us to integrate even more functions. In this section, we review a technique that we already say in Calculus I: **substitution**.

Recall the Chain Rule: if $f(x)$ and $g(x)$ are differentiable functions, then

$$D_x f(g(x)) = f'(g(x))g'(x).$$

It follows that if F is an antiderivative of f, then $F(g(x))$ is an antiderivative of $f(g(x))g'(x)$. In other words,

$$\int f(g(x))g'(x)\ dx = F(g(x)) + C.$$

Now setting $u = g(x)$ we have by the FTC

$$\int f(g(x))g'(x)\ dx = F(g(x)) + C = F(u) + C = \int f(u)\ du.$$

We have just proved the following:

Theorem 1 (The Substitution Rule)**.** *If $g(x)$ is a differentiable function and F is an antiderivative of f. Then if $u = g(x)$,*

$$\int f(g(x))g'(x)\ dx = \int f(u)\ du = F(u) + C = F(g(x)) + C$$

Example 1. *Find the following indefinite integrals:*

(a) $\displaystyle\int \sinh(x-7)\ dx$

(b) $\displaystyle\int e^{7\theta}\ d\theta$

(c) $\displaystyle\int \frac{x^2}{1+x^6}\ dx$

(d) $\int x^5\sqrt{1-x^3}\ dx$

When using substitution to evaluate a definite integral, there are two methods that we can use:

- Use substitution to find the indefinite integral, i.e. the antiderivative, then evaluate this antiderivative at the given endpoints.
- Change the limits of integration using $u = g(x)$, i.e. $\int_a^b f(g(x))g'(x)\ dx = \int_{g(a)}^{g(b)} f(u)\ du$.

Example 2. *Evaluate the following definite integrals:*

(a) $\int_0^1 x^2(2+x^3)^5\ dx$

(b) $\int_0^{1/2} \frac{1}{\sqrt{1-4x^2}}\ dx$

(c) $\int_0^4 \theta e^{\theta^2}\ d\theta$

(d) $\int_{1/6}^{1/2} \cot(\pi t)\ dt$