Sections 6.4-6.5 Worksheet

Name/ Uid: solutions   Date:

Exercise 1. *Compute the following derivatives:*

(a) $y = 6^{3x}$

$\ln y = \ln 6^{3x}$

$\ln y = 3x \ln 6$

$\frac{1}{y}\frac{dy}{dx} = 3\ln 6$

$\frac{dy}{dx} = y(3\ln 6)$

$\frac{dy}{dx} = 6^{3x}(3\ln 6)$

OR

$y = 6^{3x}$

$y = e^{3x\ln 6}$

$\frac{dy}{dx} = e^{3x\ln 6}\cdot 3\ln 6$

$\frac{dy}{dx} = 6^{3x}(3\ln 6)$

(b) $f(x) = \log_3 e^x$

$y = \log_3 e^x$

$\frac{dy}{dx} = \frac{1}{e^x \ln 3}\cdot e^x = \frac{1}{\ln 3}$

OR

change of base

$y = \log_3 e^x = \frac{\ln e^x}{\ln 3}$

$y = \frac{x}{\ln 3}$

$\frac{dy}{dx} = \frac{1}{\ln 3}$

(c) $y = \log_{10}(2x^3 + 6x)$

$y' = \frac{1}{(2x^3+6x)\ln 10}\cdot(6x^2+6)$

OR

change of base

$y = \frac{\ln 2x^3+6x}{\ln 10}$

$y' = \frac{1}{\ln 10}\cdot\frac{1}{2x^3+6x}\cdot 6x^2+6$

(d) $y = (\ln x^2)^{2x+3}$

$\ln y = \ln(\ln x^2)^{2x+3}$

$\ln y = (2x+3)\ln(\ln x^2)$

$\frac{1}{y}\frac{dy}{dx} = 2\ln(\ln x^2) + (2x+3)\frac{1}{\ln x^2}\cdot\frac{1}{x^2}\cdot 2x$

$\frac{dy}{dx} = y\left[2\ln(\ln x^2) + \frac{2x+3}{\ln x^2}\cdot\frac{2}{x}\right]$

$\frac{dy}{dx} = (\ln x^2)^{2x+3}\left[2\ln(\ln x^2) + \frac{2x+3}{\ln x^2}\cdot\frac{2}{x}\right]$

OR

$y = e^{(2x+3)\ln(\ln x^2)}$

$y' = e^{(2x+3)\ln(\ln x^2)}\cdot\left[2\ln\ln x^2 + (2x+3)\frac{1}{\ln x^2}\cdot\frac{1}{x^2}\cdot 2x\right]$

$y = (\ln x^2)^{2x+3}\left[2\ln\ln x^2 + (2x+3)\frac{1}{\ln x^2}\cdot\frac{2}{x}\right]$

(e) $y = x^{\cos x}$

$\ln y = \ln x^{\cos x}$

$\ln y = \cos x \ln x$

$\frac{1}{y}\frac{dy}{dx} = -\sin x\ln x + \frac{\cos x}{x}$

$\frac{dy}{dx} = y\left[-\sin x\ln x + \frac{\cos x}{x}\right]$

$\frac{dy}{dx} = x^{\cos x}\left[-\sin x\ln x + \frac{\cos x}{x}\right]$

OR

$y = x^{\cos x}$

$y = e^{\cos x\ln x}$

$y' = e^{\cos x\ln x}\cdot\left[-\sin x\ln x + \frac{\cos x}{x}\right]$

$y' = x^{\cos x}\left[-\sin x\ln x + \frac{\cos x}{x}\right]$

Exercise 2. *Compute the following integrals:*

(a) $\displaystyle\int x2^{x^2}\,dx$

let $u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \int x\,2^u\,dx$

$= \int \frac{1}{2}\,2^u\,du$

$= \frac{1}{2}\int 2^u\,du = \frac{1}{2}\left[\frac{2^u}{\ln 2} + C\right]$

$= \frac{1}{2}\left[\frac{2^{x^2}}{\ln 2} + C\right]$

(b) $\displaystyle\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx$

let $u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\,dx$
$2du = \frac{1}{\sqrt{x}}\,dx$

$= 2\int 5^u\,du$

$= 2\left[\frac{5^u}{\ln 5} + C\right]$

$= 2\left[\frac{5^{\sqrt{x}}}{\ln 5} + C\right]$

Exercise 3. *The population of a certain country is growing at 3.2% per year. Assuming that it is 4.5 million now, what will it be at the end of 1 year? 10 years?*

$y = Ce^{kt}$

$k = 0.032 \rightarrow y = Ce^{0.032t}$
$y(0) = 4.5 \rightarrow 4.5 = Ce^{0} \Rightarrow C = 4.5$

$y = 4.5e^{0.032t}$

@ 1 year $(t=1)$: $y = 4.5e^{0.032}$
@ 10 years $(t=10)$: $y = 4.5e^{(0.032)(10)}$

Exercise 4. *If a radioactive substance loses* 15% *of its radioactivity in 2 days, what is its half life?*

$y = Ce^{kt}$

$0.15 = e^{2k} \Rightarrow \ln(0.15) = 2k$

$k = \frac{\ln(0.15)}{2}$

$y = e^{\frac{\ln 0.15}{2} \cdot t}$

$\frac{1}{2} = e^{\frac{\ln 0.15}{2} t}$

$\ln(0.5) = \frac{\ln(0.15)}{2} t$

$t = \frac{2\ln(0.5)}{\ln(0.15)}$

Exercise 5. *An object initially at* 26° *C is placed in water having temperature* 90° *C. If the temperature of the object rises to* 70° *C in 5 minutes, what will be the temperature after 5 minutes?*

$T(t) = Ce^{kt} + A$

@ $t = 0$, $T = 26$, $A = 90$

$\Rightarrow 26 = Ce^{0} + 90$

$C = -64$

$T(t) = -64e^{kt} + A$

@ $t = 5$, $T = 70$, $A = 90$

$70 = -64e^{5k} + 90$

$-20 = -64e^{5k}$

$\frac{20}{64} = e^{5k}$

$\ln\left(\frac{20}{64}\right) = 5k$

$k = \frac{\ln\left(\frac{20}{64}\right)}{5}$

After 5 minutes $(t > 5)$

$y = -64e^{\frac{\ln\left(\frac{20}{64}\right)}{5} t} + 90$