ECO2704 Lecture Notes: Eaton-Kortum Model

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This paper develops a general equilibrium model of international trade that explains the following stylized facts within a single unified framework

- Trade diminishes dramatically with distance
- Prices vary across locations with greater differences between places further apart
- Factor rewards are far from equal across countries
- Countries relative productivities vary substantially across industries

A good framework for empirical analysis:

- The model developed in this paper can easily be estimated from trade flows data
- The model also provides a way to estimate cross country differences in productivity of tradable sectors

- N countries, a continuum variety of goods produced by competitive firms
- Trade is subject to an iceberg cost: the cost of delivering one unit of good from country i to country n is d_{ni}, $d_{ii} = 1$ and $d_{ni} > 1$ for $n \neq i$
- For simplificaiton, linear production technologies with labour as the only input . (The model in the paper has CRS technology with intermediate input as well)
- Labour can move freely across firms
- Each firm's productivity is randomly drawn from a distribution
- Productivities are independent across firms and countries

Preferences and demand

Identical preferences for consumers in all countries n=1,...N

$$U_n = \left[\int_0^1 Q_n(k)^{\frac{\sigma-1}{\sigma}} dk \right]^{\frac{\sigma}{\sigma-1}}$$

Let X_n be the total expenditures of the representative consumer in country n. Then,

$$Q_n(k) = \frac{P_n(k)^{-\sigma}}{p_n^{1-\sigma}} X_n$$

Here $P_n(k)$ is the price of variety k and p_n is the price index faced by the consumer in country n,

$$p_n = \left[\int_0^1 P_n(k)^{1-\sigma} dk \right]^{\frac{1}{1-\sigma}}$$

Technology

To produce variety k in country i, the firm solves the following problem:

$$max_{L_i(k)} \{P_{ii}(k)Z_i(k)L_i(k) - w_i L_i(k)\}$$

$$P_{ii}(k) = \frac{w_i}{Z_i(k)}$$

Here $Z_i(k)$ is the productivity and w_i is wage in country i.

If a consumer in country n buys variety k from country i, the price she faces is

$$P_{ni}(k) = d_{ni} P_{ii}(k) = \frac{d_{ni} w_i}{Z_i(k)}$$

The actual price of variety k in country i is

$$P_n(k) = min \{P_{ni}(k); i = 1, ..., N\}$$

Productivity distributions

It is assumed that the distribution of productivities in country i is given by a Frechet distribution:

$$Pr\left[k; Z_i(k) \leq z\right] = F_i(z) = exp\left(-\left(\frac{z}{A_i}\right)^{-\theta}\right), \theta > 1$$

which has the following properties:

$$E\left[Z_i\right] = A_i\Gamma(1 - \theta^{-1})$$

$$E\left[log(Z_i)\right] = \gamma\theta^{-1} + log(A_i)$$

(γ is the Euler's constant) and

$$Var\left[log(Z_i)\right] = \frac{\pi}{6\theta^2}$$

Potential price distribution: $P_{ni}(k)$

Cross variety distribution of potential price in country n of goods from country i:

$$G_{ni}(p) = Pr\left[k; P_{ni}(k) \leq p\right] = Pr\left[k; \frac{d_{ni} w_i}{Z_i(k)} \leq p\right]$$

$$G_{ni}(p) = Pr\left[k; Z_i(k) \geq \frac{d_{ni} w_i}{p}\right] = 1 - exp\left(-\left(\frac{d_{ni} w_i}{A_i}\right)^{-\theta} p^{\theta}\right)$$

Actual price distribution: $P_n(k)$

Cross variety distribution of *actual* price of all goods in country n:

$$G_n(p) = Pr\left[k; P_n(k) \leq p\right] = 1 - Pr\left[k; P_n(k) \geq p\right]$$

$$G_n(p) = 1 - \prod_{i=1}^{N} Pr\left[k; P_{ni}(k) \geq p\right] = 1 - exp\left(-\Phi_n p^{\theta}\right)$$

Here

$$\Phi_n = \sum_{i=1}^{N} \left(\frac{d_{ni} w_i}{A_i}\right)^{-\theta}$$

Distribution of import variety by country

The range of varieties that will be imported by country n from country i:

$$\pi_{ni} = Pr\left[k; P_n(k) = P_{ni}(k)\right]$$

$$\pi_{ni} = \int_0^\infty Pr\left[k; P_n(k) = P_{ni}(k) | P_{ni}(k) = p\right] dG_{ni}(p)$$

$$= \int_0^\infty Pr\left[k; P_{nj}(k) \geq p, j = 1, ..., N, j \neq i | P_{ni}(k) = p\right] dG_{ni}(p)$$

$$= \int_0^\infty \prod_{j \neq i} \left(1 - G_{nj}(p)\right) dG_{ni}(p)$$

$$\implies \pi_{ni} = \left(\frac{d_{ni} w_i}{A_i}\right)^{-\theta} \Phi_n^{-1} = \frac{\left(\frac{d_{ni} w_i}{A_i}\right)^{-\theta}}{\sum_{j=1}^{N} \left(\frac{d_{nj} w_j}{A_j}\right)^{-\theta}}$$

Conditional distribution of import prices: P^*_{ni}

Distribution of prices across the range of varieties that country i actually exported to country n:

$$G^*_{ni}(p) = Pr\left[k; P_n(k) \leq p | k; P_n(k) = P_{ni}(k)\right]$$

$$= \frac{Pr\left[k; P_n(k) \leq p, P_n(k) = P_{ni}(k)\right]}{Pr\left[k; P_n(k) = P_{ni}(k)\right]}$$

$$= \frac{1}{\pi_{ni}} \int_0^p Pr\left[k; P_{nj}(k) \geq q, j = 1, ..., N, j \neq i\right] dG_{ni}(q)$$

$$= \frac{1}{\pi_{ni}} \int_0^p \prod_{j \neq i} (1 - G_{nj}(q))\, dG_{ni}(q)$$

$$\implies G^*_{ni}(p) = G_n(p)$$

Price Index

$$p_n = \left[\int_0^1 P_n(k)^{1-\sigma} dk \right]^{\frac{1}{1-\sigma}} = \left[\int_0^\infty p^{1-\sigma} dG_n(p) \right]^{\frac{1}{1-\sigma}}$$

$$p_n = \delta \Phi_n^{-1/\theta}$$

$$\delta = \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{1/(1-\sigma)}$$

Expenditure share of imports from country i

Let $\Omega_i = \{k; P_n(k) = P_{ni}(k)\}$ be the set of varieties that country n imports from country i. Then, country n's expenditures on imports from country i is

$$X_{ni} = \int_{k \in \Omega_i} P_{ni}(k) Q_n(k) dk = \frac{\int_{k \in \Omega_i} P_{ni}^{1-\sigma}(k) dk}{p_n^{1-\sigma}} X_n$$

Note that

$$\int_{k \in \Omega_i} P_{ni}^{1-\sigma}(k) dk = E\left[P_{ni}^{1-\sigma} | \Omega_i\right] Pr\left[\Omega_i\right] = E\left[P_{ni}^{1-\sigma} | \Omega_i\right] \pi_{ni}$$

From the previous slide, however, we know that the average expenditure per good on imports from i is

$$E\left[P_{ni}^{1-\sigma} | \Omega_i\right] = E\left[P_n^{1-\sigma}\right] = \int P_n^{1-\sigma}(k) dk = p_n^{1-\sigma}$$

$$\Longrightarrow \frac{X_{ni}}{X_n} = \pi_{ni}$$

Trade flow

The fraction of goods that country n buys from country i , π_{ni} , is also the fraction of its expenditure on goods from country i:

$$\frac{X_{ni}}{X_n} = \left(\frac{d_{ni} w_i}{A_i}\right)^{-\theta} \Phi_n^{-1} \propto \left(\frac{A_i}{w_i}\right)^{\theta} \left(\frac{d_{ni}}{p_n}\right)^{-\theta}$$

The exporter i's total sales are:

$$R_i = \sum_{m=1}^{N} X_{mi} \propto \left(\frac{A_i}{w_i}\right)^{\theta} \sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m$$

Gravity equation:

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n R_i}{\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m}$$

Real income and welfare

Previous slide shows that

$$\pi_{ni} \propto \left(\frac{A_i}{w_i}\right)^{\theta} \left(\frac{d_{ni}}{p_n}\right)^{-\theta}$$

or

$$w_i \propto A_i \pi_{ni}^{-1/\theta} d_{ni}^{-1} p_n$$

Thus, for i=n, we have

$$\frac{w_i}{p_i} \propto A_i \pi_{ii}^{-1/\theta}$$

That is, the smaller the domestic expenditure share π_{nn} is, the higher the real income and welfare.

For a given vector of total expenditures $(X_1, ..., X_N)$, we have already shown how consumption, prices and trade flows are determined

In equilibrium country n's total expenditures should also equal its total income:$X_n = w_n L_n$, n=1,...,N

So, the only endogenous variables that need to be solved for are the wages $w_1, ..., w_N$

Total income equals total exports and domestic sales:

$$w_i L_i = \sum_{n=1}^{N} X_{ni} = \sum_{n=1}^{N} \pi_{ni} X_n$$

Equation for solving wages

For i=1,...,N:

$$w_i L_i = \sum_{n=1}^{N} \left(\frac{d_{ni} w_i}{A_i} \right)^{-\theta} \Phi_n^{-1} w_n L_n$$

Alveraz and Lucas (2007, JME) showed the existence and uniqueness of the solutions to the equation system above

The case of free trade: $d_{ni} = 1$ for all n and i

$$w_i L_i = \left(\frac{w_i}{A_i} \right)^{-\theta} \sum_{n=1}^{N} \Phi_n^{-1} w_n L_n$$

$$\Longrightarrow w_i \propto \left(\frac{A_i^{\theta}}{L_i} \right)^{1/(1+\theta)}$$

Under free-trade, price levels are identical, but factor prices are different across countries.

Estimating θ

Note that

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

Equivalently,

$$log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right) = -\theta log\left(\frac{p_i d_{ni}}{p_n}\right)$$

This equation can be used to estimate the parameter θ

Estimating competitiveness

Also note that

$$\frac{X_{ni}/X_n}{X_{nn}/X_n} = \left(\frac{A_i}{w_i}\right)^{\theta} \left(\frac{A_n}{w_n}\right)^{-\theta} d_{ni}^{-\theta}$$

or

$$log\left(\frac{X_{ni}/X_n}{X_{nn}/X_n}\right) = \theta logT_i - \theta logT_n - \theta logd_{ni}$$

This equation can be used to estimate the competitiveness measure $T_i = A_i/w_i$

- Alvarez and Lucas (2007) "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," Journal of Monetary Economics, 54(6), 1726-1768.
  - Closes the model without an outside sector to solve for endogenous factor prices
- Rodriguez-Clare, Andres (2009) "Offshoring in a Ricardian World," American Economic Journal: Macroeconomics, forthcoming.
  - Analyzes offshoring within the Eaton-Kortum framework
- Ramondo, Natalia and Andres Rodriguez-Clare (2008) "Trade, Multinational Production and the Gains from Openness," University of Texas, mimeograph, http://www.eco.utexas.edu/nr3353/Trade
  - Analyzes both trade and multinational production within the Eaton-Kortum framework

- David Donaldson (2008) "Railroads of the Raj: Estimating the Economic Impact of Transportation Infrastructure," MIT Economics, mimeograph
    - Uses the Eaton-Kortum framework to evaluate the impact of the construction of the railway network in Colonial India
- Arkolakis, Costas, Arnaud Costinot and Andres Rodriguez-Clare (2009) "New Theories, Same Old Gains?" Yale University, mimeograph
    - Shows that in a class of international trade models, welfare can be expressed in terms of the trade share as in the Eaton-Kortum framework

Multi-Sector Models:

- Costino and Donaldson and Kunmunjer (2010) "What goods do countries trade? A Quantitative Exploration of Ricardo's Ideas," unpublished manuscript, MIT Economics Department
- Kerr, W. R. (2009): "Heterogeneous Technology Diffusion and Ricardian Trade Patterns", unpublished manuscript, Harvard Business School
- Yi and Zhang (2010) "Structural Change in an Open Economy," unpublished manuscript, University of Michigan
- Tombe, Trevor (2010) "The Missing Food Problem," job market paper, University of Toronto