Hospital Insurance Charges Model

Final for STAT 425

$\mathrm{May}\ 6,\ 2024$

Contents

Section 1: Introduction	2
Section 2: Exploratory Data Analysis	3
Section 3: Methodology	8
Section 3.1: Simple Model	8
Section 3.2: Predictions from Simple Model	9
Section 3.3: Advanced Models	6
Section 4: Discussion and Conclusions	4

Section 1: Introduction

This project focuses on the analysis of medical insurance data, particularly focusing on the medical charges acquired from hospital visits. The response variable is the variable charges from the insurance data. This data is found on Kaggle, as was last updated 6 years ago, in 2018. In this data set, there 6 variables excluding the response variable, charges, which shows the exact charges the patient acquired from their hospital visit. Each variable is defined within the Kaggle (2018) website and explained in this order:

- age: Age of primary beneficiary
- sex: Insurance contractor gender (female or male)
- bmi: Body mass index, providing an understanding of body, weights that are relatively high or low relative to height, objective index of body weight (kg / m ^ 2) using the ratio of height to weight, ideally 18.5 to 24.9
- children: Number of children covered by health insurance / Number of dependents
- smoker: Smoking (yes or no)
- region: Beneficiary's residential area in the US (northeast, southeast, southwest, northwest)
- charges: Individual medical costs billed by health insurance

This report compares multiple models derived from analysis of the data set including a Regression Model, an a Ridge Model, and a Lasso Model. All analyses are performed in the R environment. This is possible to recreate with the code provided in this file. Setting seeds might yield slightly different results; however, we did not use a seed to analyze this data. No function (i.e. set.seed) should change the results concluded from this data, although it is possible to slightly alter the data using said functions. Again, the results would be concluded the same way. This report has 4 sections. Section 1 is this introduction. Section 2 uses modelling assumptions and models to analyze the data and provide insights on the data. Section 3 presents predictive models and shows training from said models. Section 4 compares models and provides a discussion of these results.

Section 2: Exploratory Data Analysis

Here is a brief overview of all of the variables of this dataset:

Numerical Variables:

- age: age of primary beneficiary
- bmi: body mass index
- children: number of children/dependents covered by health insurance
- charges (response variable): medical costs billed by health insurance

Categorical Variables:

• sex: male or female

• smoker: yes or no

• region: residential area in the US, can be northeast, southeast, southwest, or northwest

The first step is to check for unusual observations by performing an outlier test. To do so, we obtained the studentized residuals for our data, and compared the largest values with the Bonferroni critical value.

The Bonferroni critical value we calculated is -4.137174. Any observation with studentized residual higher than the absolute value of it will be considered an outlier.

Next, here are the five largest studentized residuals, which we will be comparing to the Bonferroni critical value:

Observation Number	Studentized Residual
1301	5.009599
578	4.219800
243	4.053228
220	3.998326
517	3.863878

Since the absolute value of their respective studentized residual is greater than the Bonferroni critical value, we can conclude that Observation #1301 and Observation #578 are outliers.

We also checked Cook's distance, however we found that there is no point with Cook's distance greater than 1. We will simply remove the two outliers we found earlier.

After removing these two points, we will be analyzing the predictors to see if there are any variables we should add or remove.

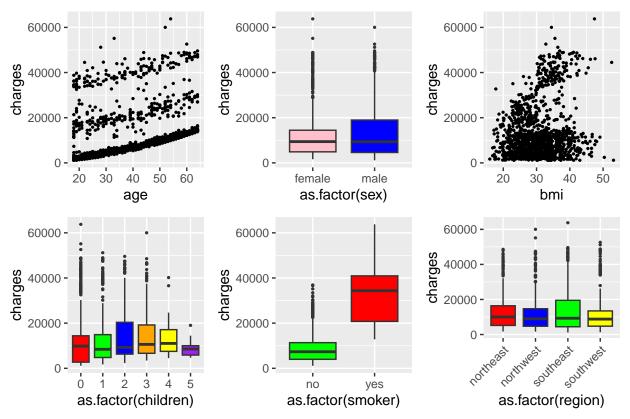


Figure 1: Plot of Charges vs All Predictors

From the plot of charges vs age, we notice a trend that the charges increase as age increases. We will not be removing 'age' as a predictor since it is clear that it has a significant impact on the response variable 'charges'. This makes sense because as people grow older, their health declines, and the insurance costs increase.

The charges vs sex boxplot tells us that males tend to have higher charges than females. We will conduct a t-test to test this difference.

From the t-test, we obtained a p-value of about 0.0346.

Given that the p-value is less than 0.05, we conclude that 'sex' is a significant predictor, with males incurring higher insurance costs than females. A possible explanation for this is that males are usually more at risk of health conditions that result in increased charges compared to females.

From the plot of charges vs bmi, we can see a general trend that as bmi increases, the charges also increase. As a result, we will not be removing 'bmi' as a predictor since it appears to have an impact on the response variable. The graph also makes sense logically since people with a higher bmi tend to be overweight and subsequently, have worse health than people with a lower bmi, causing their insurance charges to be greater as well.

The boxplot of charges vs children conveys to us that charges tend to increase as the number of children grows from 0 or 1 to 2 or 3. While there are some data points for people with 4

or 5 children, it appears that these groups have a very low amount of people, and it may be challenging to draw meaningful conclusions from it. We perform an ANOVA test to test the significance of 'children' as a predictor.

We obtained a p-value of approximately 0.00862.

This p-value is less than 0.05, thus signifying that the predictor 'children' is significant. We will be keeping as a predictor in our model. A larger number of children/dependents covered leads to higher charges, as there are more individuals who need to be covered.

From the boxplot of charges vs smoker, it is evident that smokers generally face significantly higher charges than non-smokers. Consequently, we will not be removing 'smoker' as a predictor. People who smoke typically incur higher insurance costs due to the health risks posed by smoking.

The boxplot of charges vs region shows us that the southeast region seemingly has higher charges than the other three regions. We will conduct an ANOVA test to make sure that the predictor 'region' is significant.

From the ANOVA test, we obtained a p-value of about 0.0453.

The p-value is less than 0.05, indicating that the predictor 'region' is significant, and thus we will retain it as a predictor. This confirms that there are regional differences in the insurance costs.

Now, we will explore whether or not we should include any interactions between categorical variables. We will be analyzing all possible first-order and second-order interactions between our categorical predictors, including:

- 'sex' and 'smoker'
- 'sex' and 'region'
- 'region' and 'smoker'
- 'sex', 'smoker', and 'region'

Here are some plots that show the interactions between these predictors.

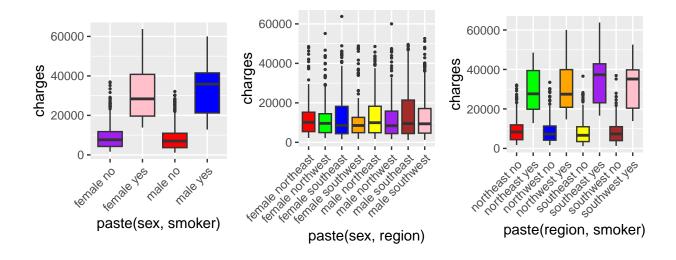


Figure 2: Plot of Charges vs Interactions Between Categorical Variables

From the plots, it appears as though smoking increases the charges more for males than females. Additionally, it seems like smoking has a greater impact on charges in the southeast and southwest, when compared to northeast and northwest. However, it does not appear that sex and region is a significant interaction.

We applied sequential F-tests with the anova function to assess the significance of the interaction terms in our model. Initially, we included all potential interactions among the three variables 'sex', 'smoker', and 'region'. We first found that the second-order interaction sex:smoker:region was not significant, and removed it from the model. Subsequently, we tested the first-order interactions and found that the interaction sex:region was not significant. Finally, after removing sex:region as a predictor, we concluded that only the interac-

tions sex:smoker and smoker:region are statistically significant, and we will retain them in the final model.

Now, we will examine the numerical variables and see if there is evidence of any nonlinear trends.

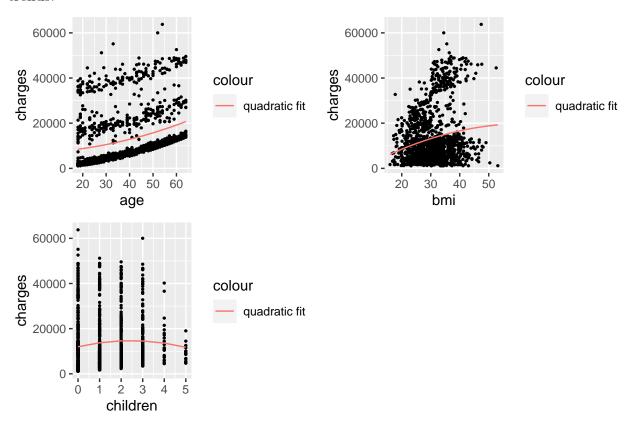


Figure 3: Plot of Quadratic Fit for Charges vs Numerical Predictors

Based on the plot of the quadratic fit for charges vs age, we see there is evidence of non-linearity for the predictor 'age' since the quadratic fit has some upward curvature.

Similarly, based on the plot of the quadratic fit for charges vs bmi, we see there is some evidence of non-linearity for the predictor 'bmi', although maybe not as clear as for 'age'. This time, the quadratic fit curves downwards.

Finally, the plot of charges vs children also suggests some non-linearity. However, the non-linearity is not as evident as for 'age and 'bmi' since although the quadratic fit does appear to curve downwards as the number of children increases, there is not many data points for 4 and 5 children, so we may not be able to draw a strong conclusion.

Section 3: Methodology

Section 3.1: Simple Model

Model Selection

The choice of linear regression for this analysis is based on its efficacy in providing a clear and straightforward interpretation of how various predictors affect insurance charges. Linear regression is particularly valued for its ability to establish a baseline understanding of the relationships between variables, offering insights into the direct impacts of age, BMI, children count, smoking status, sex and regional differences on insurance costs.

Variable Selection and Diagnostics

Age: A continuous variable expected to positively correlate with insurance charges.

BMI: Another continuous variable, which is hypothesized to influence insurance costs due to its association with health risks.

Children: A discrete variable that represents the number of dependents, potentially affecting insurance premiums.

Smoker Status: A categorical variable that significantly impacts insurance costs, as identified in the exploratory analysis.

Sex: A categorical variable, often considered in insurance cost analysis because gender may influence health risks and insurance premiums.

Region: As a categorical variable, the region captures variations in insurance costs across different geographical locations.

Sex and Smoker Interaction: This interaction term examines how gender and smoking status jointly influence insurance costs. Smokers of different genders may face different health risks, which can be reflected in their insurance premiums.

Region and Smoker Interaction: This interaction term was included to examine if the impact of smoking on insurance charges varies by region, which could suggest regional variations in healthcare costs or lifestyle patterns.

	GVIF	Df	$GVIF^{(1/(2*Df))}$
age	1.017141	1	1.008534
bmi	1.108041	1	1.052635
children	1.004025	1	1.002011
smoker	1.011905	1	1.005935
sex	1.009150	1	1.004565
region	1.099632	3	1.015955

Model Diagnostics:

The diagnostics conducted include:

Multicollinearity Check: The Variance Inflation Factor (VIF) was calculated for all predictors, with results indicating minimal multicollinearity among the variables used. VIF values were as follows: age (1.00), BMI(1.05), children (1.00), smoker status (1.01), sex (1.00) and region(1.02).

Model Fit: The linear model was fitted with charges as the dependent variable. The R-squared value of 0.7612 suggests that about 76.12% of the variability in insurance charges is explained by the model, which is quite substantial for a simple linear model.

Residual Analysis: Residuals: The residuals' plot indicated that while there is some spread, major deviations from normality or constant variance (homoscedasticity) were not apparent, suggesting that the model assumptions are reasonably met.

Coefficients:

Intercept: The model's intercept is significantly negative, which might indicate that the baseline insurance charges are negative when all other variables are zero. This could suggest that further adjustments to the model might be necessary.

Age: The coefficient for age is significant and positive, indicating that insurance charges increase with age. This aligns with the general logic of insurance pricing, where increased age is associated with higher health risks.

BMI: The coefficient for BMI is also significantly positive, suggesting that higher BMI is associated with higher insurance charges, likely due to increased health risks associated with higher BMI.

Children: The coefficient for the number of children is significant and positive, indicating that having more children leads to higher insurance charges, possibly reflecting the greater healthcare needs of larger families.

Significant Impact of Smoking Status: The coefficient for smokers is significantly higher than for non-smokers, consistent with previous studies indicating that smoking is a significant factor affecting insurance costs.

Interactions of Region and Gender: Some regional categories, such as southeast and southwest, are significant, suggesting that insurance costs in these regions are higher compared to the baseline region.

Interaction of Smoking Status and Gender: This interaction term is significant, indicating that the combination of smoking status and gender has a specific impact on insurance charges, particularly for male smokers who may face higher charges.

These conclusions suggest that while the overall model performs well, the impact of certain variables like regions and interaction terms may require more complex models to more finely interpret these factors' effects on insurance costs. Further model validation and adjustments are also necessary to ensure the model's generalization capability and predictive accuracy across different datasets.

Section 3.2: Predictions from Simple Model

Model Implementation:

The linear regression model was implemented using the lm() function in R, focusing on charges as the dependent variable. This model incorporates several predictors, including age, BMI, number of children, smoker status, and the interaction between smoker status

and region, which were identified as significant in influencing insurance costs during the exploratory analysis.

Testing and Predictions:

To validate the effectiveness of the linear regression model, the dataset was split into training and testing subsets. Specifically, 80% of the data was used for training the model, and the remaining 20% served as the testing set. This division ensures that the model is tested on unseen data, providing a fair assessment of its predictive accuracy.

	Actual	Predicted
5	3866.855	5801.68030
9	6406.411	8756.86756
14	11090.718	14035.78850
19	10602.385	14611.43699
25	6203.902	7658.89374
28	12268.632	14258.06601
32	2198.190	2238.82489
38	2302.300	297.26102
51	2211.131	5433.91875
55	8059.679	9485.30139
63	30166.618	12937.41709
65	14711.744	23486.45766
72	6799.458	8096.49453
75	7726.854	8015.81054
89	8026.667	9363.20246
91	2026.974	5030.22419
106	17560.380	27346.06055
115	11488.317	13823.08837
128	9634.538	12995.75678
132	13616.359	11696.53016
139	27322.734	12130.06114
140	2166.732	4893.31000
144	18157.876	6213.21931
147	40720.551	35679.76602
151	5125.216	5373.88362
166	10407.086	11623.58727
167	4830.630	6929.14278
168	6128.797	8985.11391
169	2719.280	4345.50310
176	48824.450	39170.60160
181	11735.879	12309.58864
182	1631.821	3852.76679
184	7419.478	8398.62838
188	5325.651	6496.03743
200	14901.517	18392.47404

	Actual	Predicted
001	11000101	
$\frac{201}{205}$	2130.676 7147.105	4003.15701 5893.36623
$\frac{203}{208}$	20984.094	29142.31543
211	1980.070	3399.93119
216	7371.772	10978.38355
$\frac{210}{223}$	5253.524	6574.01717
226	11987.168	12667.80890
227	2689.495	5922.09220
228	24227.337	15261.18657
235	6710.192	6960.71490
236	19444.266	30076.52495
238	4463.205	7677.85304
239	17352.680	27010.47240
243	35160.135	10560.30176
250	4040.558	5990.75708
252	47305.305	38163.36975
255	41097.162	33474.92496
263	24869.837	32727.88544
266	46151.124	40445.35102
270	9282.481	9996.17915
273	7265.703	11772.44281
276	9715.841	10584.85954
278	2150.469	877.99334
300	9249.495	10702.72563
306	19442.354	7452.13325
315	34838.873	27862.13018
337	12142.579	9824.11171
341	18955.220	2518.65066
342	13352.100	13814.89782
346	6184.299	6142.71995
355	14133.038	3852.76679
356	24603.048	7712.27036
359	1837.282	5973.04136
368	8017.061	8281.76147
375	1391.529	2266.27765
381	15006.579	22255.80728
390	4618.080	5941.89255
391	10736.871	14420.68720
392	2138.071	5828.92493
398	16586.498	1727.57606
402 404	8083.920 10269.460	14436.07453
410	4074.454	12614.97638 4622.70014
410	4074.404	4044.70014

	Α	D 11 1
	Actual	Predicted
419	14418.280	15886.13353
424	2727.395	5088.58876
430	18804.752	6769.29670
444	28287.898	14105.61652
460	7682.670	8950.64483
467	13224.693	12482.69199
468	12643.378	14870.76505
469	23288.928	4534.97752
486	4347.023	7171.83348
492	24513.091	10247.50614
496	1967.023	2905.37455
502	6837.369	8096.83376
508	3077.095	2126.70199
512	2498.414	4157.99253
516	11362.755	13540.89758
517	27724.289	3385.94787
520	3857.759	6710.79407
524	5397.617	8745.33361
525	38245.593	32961.65227
532	14043.477	15276.10206
536	6067.127	7951.32155
540	27346.042	9970.94221
541	6196.448	9948.94679
546	23807.241	34404.26216
552	3972.925	4200.87007
558	3935.180	6125.20909
564	9058.730	14254.50167
571	3761.292	4811.89720
579	9724.530	10569.11434
580	3206.491	3039.19730
583	6356.271	12126.43524
585	1242.816	-1515.52028
596	8823.986	12335.86329
603	11070.535	9859.42854
606	9283.562	10802.41454
620	10713.644	13654.99809
625	12129.614	12628.86069
629	11365.952	14330.23334
631	10085.846	12847.99820
633	3366.670	5703.72234
635	9391.346	13575.34814
646	10141.136	12317.77919
653	8280.623	9020.94370

	Actual	Predicted
659	26392.260	13593.86436
662	22192.437	9293.88862
667	8703.456	9292.13200
682	1242.260	-1652.79606
690	34806.468	30887.20016
693	2362.229	4399.35372
697	29186.482	14065.43576
715	2457.502	802.70337
717	9566.991	8804.99378
720	12231.614	14368.17498
721	9875.680	15546.05736
723	12979.358	15140.58387
724	1263.249	3529.36475
726	40932.429	33751.89810
727	6664.686	8365.41486
740	44585.456	32153.52013
741	8604.484	8800.61306
745	8827.210	9527.19126
748	1627.282	53.59675
752	1906.358	3039.55674
753	14210.536	17029.07880
755	17128.426	7638.43901
757	7985.815	7688.36628
758	23065.421	32899.96999
762	2416.955	4917.14615
765	9095.068	9587.68246
768	7050.642	8012.45990
769	14319.031	16833.88514
771	27941.288	15760.83678
774	17748.506	25449.39551
777	6986.697	9888.22683
787	12741.167	16165.30145
794	21195.818	33541.93425
795	7209.492	9899.58221
807	28476.735	12973.79302
809	1131.507	663.37118
812	6360.994	12597.28881
814	4428.888	3959.84222
816	1877.929	2020.34679
818	3597.596	6449.51629
827	43813.866	38921.77577
830	6117.494	5607.67586
831	13393.756	13918.93525

	Actual	Predicted
834	11743.934	14298.37155
843	36021.011	29381.48275
854	11729.680	11131.16590
859	18218.161	3995.08801
868	11576.130	16472.50228
871	8457.818	11679.96390
872	3392.365	3305.38305
882	2789.057	3433.59295
885	4877.981	5033.96932
891	29330.983	36197.70484
894	44202.654	39088.98594
898	2221.564	1797.87178
902	48673.559	42191.04956
903	4661.286	5522.01512
904	8125.784	10842.22003
916	2473.334	3904.88032
923	5488.262	7286.74119
929	13470.804	15333.67976
931	2927.065	8760.92989
933	10096.970	9295.30476
940	9487.644	9329.17793
955	20009.634	30837.77927
958	12609.887	3492.04101
959	41034.221	33212.51664
966	4746.344	5186.10415
972	4992.376	5358.39476
973	2527.819	689.98035
982	4500.339	4212.69237
984	16796.412	6432.70937
996	7986.475	7851.38127
997	7418.522	9880.83816
998	13887.969	14794.97817
1006	4433.388	6897.88228
1008	24915.221	35151.36010
1011	8269.044	6968.92649
1022	35595.590	28963.55834
1024	1711.027	-1237.02584
1027	16450.895	26407.33153
1036	12094.478	9882.42594
1042	1704.700	734.48952
1043	33475.817	25461.70745
1050	39727.614	34775.85947
1065	5708.867	4863.22234

	Λ -41	Duc diet 1
	Actual	Predicted
1069	14349.854	11640.05509
1071	39871.704	35466.40606
1072	13974.456	15036.52569
1076	4562.842	4867.53020
1077	8551.347	10312.35853
1085	15019.760	15254.05640
1092	11286.539	12845.57973
1097	44641.197	34189.32924
1099	23045.566	12442.01456
1100	3227.121	4599.10146
1102	11253.421	11154.38743
1111	11512.405	13879.34673
1115	2396.096	2493.86476
1133	20709.020	16544.20825
1141	9048.027	11611.94284
1144	6338.076	7634.08523
1150	5979.731	8926.73763
1151	2203.736	3608.15083
1165	7153.554	8728.64528
1167	10982.501	14082.77533
1173	11093.623	14753.05446
1177	23887.663	32643.58035
1183	2632.992	3699.32824
1190	13126.677	2401.86039
1195	4134.082	3498.22279
1196	18838.704	3889.09033
1210	12347.172	14715.58396
1211	5373.364	7942.76256
1212	23563.016	8238.09868
1213	1702.455	180.23855
1214	10806.839	12468.99017
1220	7537.164	9498.81727
1222	6593.508	5358.84498
1225	6858.480	7311.59077
1229	10594.226	11790.16390
1233	12479.709	11640.29717
1239	6985.507	6751.79776
1240	3238.436	7392.66365
1242	49577.662	42728.59368
1246	5615.369	4207.33166
1249	1633.962	4381.27855
1250	37607.528	29961.34810
1258	11305.935	11803.28296

	Actual	Predicted
1261	4544.235	3806.96670
1262	3277.161	6839.53638
1279	22462.044	30468.29351
1282	24535.699	33019.71613
1283	14283.459	20360.41959
1291	7133.903	6016.05519
1295	11931.125	11614.39754
1310	6875.961	8900.92178
1311	6940.910	7902.21494
1315	18765.875	27771.42241
1317	1731.677	-1154.00580
1319	19496.719	12519.75747
1331	12629.166	10338.05715
1337	2007.945	1138.71148

Predictive Performance Evaluation:

The model was fitted on the training data and then used to make predictions on the testing set. Here are the results of the predictions for the first few observations:

As observed, the model provides reasonable estimates for the charges, although there are noticeable differences between the actual and predicted values, suggesting areas for improvement.

The initial testing indicates that while the linear regression model captures general trends and is able to provide ballpark estimates of insurance charges, discrepancies remain between predicted and actual charges. These differences highlight the potential need for more complex models that could better account for nuances in the data not captured by this simple linear model. The following sections will explore such models, comparing their predictions with those of the baseline linear model to identify the most effective approach for predicting insurance charges.

Section 3.3: Advanced Models

Model Types:

Regularized Regression: To enhance the predictive accuracy and manage potential issues of multicollinearity and overfitting present in our linear regression model, we employed advanced modeling techniques such as Ridge and Lasso regression. These methods are forms of regularized regression that include a penalty term to the loss function:

Ridge Regression (L2 regularization): Adds a penalty equal to the square of the magnitude of coefficients. This method is particularly effective in reducing the model complexity while still allowing the use of all variables. It helps in handling multicollinearity, preventing overfitting by shrinking the coefficients.

Lasso Regression (L1 regularization): Adds a penalty equivalent to the absolute value of the magnitude of coefficients. Unlike Ridge, Lasso can completely eliminate the weight of less important variables by setting their coefficients to zero. This results in feature selection within the model, which is beneficial when we need a sparse model with fewer variables.

Implementation and Predictions: Both models were implemented using the glmnet package in R, which efficiently handles large datasets and complex models with its capabilities for both Lasso and Ridge regression. The matrix of predictors x was derived from the training data, excluding the intercept to allow glmnet to handle regularization properly. The regularization parameter s was set to 0.01 for both models. This choice is typically subject to tuning through methods like cross-validation, but for simplicity, we've chosen a small but non-zero value, which demonstrates regularization without overly constraining the models.

Predictive Performance Evaluation: The predictions made by the Ridge and Lasso models on the testing set were compared with the actual charges and those predicted by the simple linear model.

	Actual	Linear	Ridge	Lasso
5	3866.855	5801.68030	5976.57916	5894.9576
9	6406.411	8756.86756	8931.15582	8638.0897
14	11090.718	14035.78850	14047.50003	14047.1576
19	10602.385	14611.43699	14110.49305	14658.8922
25	6203.902	7658.89374	7695.46121	7725.3471
28	12268.632	14258.06601	14087.68250	14248.7236
32	2198.190	2238.82489	3236.13877	2123.1090
38	2302.300	297.26102	1005.95371	424.4736
51	2211.131	5433.91875	6113.48593	5291.8664
55	8059.679	9485.30139	9687.97827	9487.2407
63	30166.618	12937.41709	12636.18093	13003.7124
65	14711.744	23486.45766	22902.94922	23478.3777
72	6799.458	8096.49453	8309.75598	7948.6830
75	7726.854	8015.81054	8095.95492	8084.4524
89	8026.667	9363.20246	9604.72165	9400.6252
91	2026.974	5030.22419	5687.99254	5071.2211
106	17560.380	27346.06055	28252.19776	27292.1037
115	11488.317	13823.08837	13605.85054	13672.7228
128	9634.538	12995.75678	12947.96680	13007.0933
132	13616.359	11696.53016	12059.85830	11557.6545
139	27322.734	12130.06114	12333.11213	12127.7592
140	2166.732	4893.31000	5437.41067	4933.2020
144	18157.876	6213.21931	6336.52092	6281.3941
147	40720.551	35679.76602	35953.21444	35572.3628
151	5125.216	5373.88362	5618.07354	5465.5667
166	10407.086	11623.58727	11594.79777	11476.0362
167	4830.630	6929.14278	7283.31941	6904.9871

	Actual	Linear	Ridge	Lasso
168	6128.797	8985.11391	9180.49785	8980.9811
169	2719.280	4345.50310	4898.88742	4380.9340
176	48824.450	39170.60160	36916.21664	39092.0411
181	11735.879	12309.58864	12022.65314	12382.3564
182	1631.821	3852.76679	4606.24289	3899.6461
184	7419.478	8398.62838	8721.81390	8441.4531
188	5325.651	6496.03743	6953.86231	6506.0827
200	14901.517	18392.47404	18111.29026	18202.2284
201	2130.676	4003.15701	4585.22806	4050.3533
205	7147.105	5893.36623	6188.12504	5999.9389
208	20984.094	29142.31543	27392.85442	29225.6372
211	1980.070	3399.93119	3762.65077	3484.9794
216	7371.772	10978.38355	11063.51181	10974.4435
223	5253.524	6574.01717	6717.35087	6630.3073
226	11987.168	12667.80890	12503.51608	12706.6878
227	2689.495	5922.09220	6220.07689	6008.9972
228	24227.337	15261.18657	15165.29327	15265.0037
235	6710.192	6960.71490	7080.97401	7035.4720
236	19444.266	30076.52495	28819.33951	30050.8122
238	4463.205	7677.85304	7833.33670	7736.2217
239	17352.680	27010.47240	27937.49481	26966.9509
243	35160.135	10560.30176	10781.46108	10586.5927
250	4040.558	5990.75708	6377.73561	5893.5283
252	47305.305	38163.36975	36019.87031	38075.2244
255	41097.162	33474.92496	31390.81724	33559.5368
263	24869.837	32727.88544	30748.40549	32794.3729
266	46151.124	40445.35102	40073.25677	40327.7535
270	9282.481	9996.17915	10127.41456	9891.4220
273	7265.703	11772.44281	11428.44263	11810.0866
276	9715.841	10584.85954	10969.64692	10419.4188
278	2150.469	877.99334	1821.41047	950.9827
300	9249.495	10702.72563	10830.64945	10722.7121
306	19442.354	7452.13325	7452.22696	7510.0959
315	34838.873	27862.13018	26475.69280	27830.9055
337	12142.579	9824.11171	9962.19010	9919.6332
341	18955.220	2518.65066	3313.16612	2580.6652
342	13352.100	13814.89782	13406.77974	13880.3551
346	6184.299	6142.71995	6798.61257	6164.2829
355	14133.038	3852.76679	4606.24289	3899.6461
356	24603.048	7712.27036	7826.14223	7803.8502
359	1837.282	5973.04136	6230.31190	6053.1546
368	8017.061	8281.76147	8613.02829	8305.1136
375	1391.529	2266.27765	2870.79119	2373.1218

381 15006.579 22255.80728 19266.04451 22390.8175 390 4618.080 5941.89255 6382.88398 5952.6494 391 10736.871 14420.68720 14120.86031 14251.3547 392 2138.071 5828.92493 6229.42644 5861.0719 398 16586.498 1727.57606 2392.79203 1840.1349 402 8083.920 14436.07453 14028.17693 14468.0702 404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 466 7682.670 8950.64483 9370.07086 8956.7089 467 13224.6919 12548.32018<		Actual	Linear	Ridge	Lasso
390 4618.080 5941.89255 6382.88398 5952.6494 391 10736.871 14420.68720 14120.86031 14251.3547 392 2138.071 5828.92493 6229.42644 5861.0719 398 16586.498 1727.57606 2392.79203 1840.1349 402 8083.920 14436.07453 14028.17693 14468.0702 404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 466 7682.670 8950.64483 9370.0706 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12633.791 1487.076505 <td>381</td> <td>15006.579</td> <td>22255.80728</td> <td></td> <td>22390.8175</td>	381	15006.579	22255.80728		22390.8175
392 2138.071 5828.92493 6229.42644 5861.0719 398 16586.498 1727.57606 2392.79203 1840.1349 402 8083.920 14436.07453 14028.17693 14468.0702 404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 496 1967.023 2905.37455 <td>390</td> <td></td> <td></td> <td></td> <td></td>	390				
398 16586.498 1727.57606 2392.79203 1840.1349 402 8083.920 14436.07453 14028.17693 14468.0702 404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 <td>391</td> <td>10736.871</td> <td>14420.68720</td> <td>14120.86031</td> <td>14251.3547</td>	391	10736.871	14420.68720	14120.86031	14251.3547
402 8083.920 14436.07453 14028.17693 14468.0702 404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376	392	2138.071	5828.92493	6229.42644	5861.0719
404 10269.460 12614.97638 12249.58240 12646.9267 410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 <td>398</td> <td>16586.498</td> <td>1727.57606</td> <td>2392.79203</td> <td>1840.1349</td>	398	16586.498	1727.57606	2392.79203	1840.1349
410 4074.454 4622.70014 5083.78405 4716.4697 419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199	402	8083.920	14436.07453	14028.17693	14468.0702
419 14418.280 15886.13353 15455.23095 15927.8410 424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 80090.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253	404	10269.460	12614.97638	12249.58240	12646.9267
424 2727.395 5088.58876 5531.40954 5002.6425 430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758	410	4074.454	4622.70014	5083.78405	4716.4697
430 18804.752 6769.29670 7149.39259 6777.0566 444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758 13160.67592 13599.7258 517 27724.289 3385.94787	419	14418.280	15886.13353	15455.23095	15927.8410
444 28287.898 14105.61652 14137.13119 14111.2896 460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758 13160.67592 13599.7258 517 27724.289 3385.94787 3884.47110 3474.6192 529 3857.599 6710.79407	424	2727.395	5088.58876	5531.40954	5002.6425
460 7682.670 8950.64483 9370.07086 8956.7089 467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758 13160.67592 13599.7258 517 27724.289 3385.94787 3884.47110 3474.6192 520 3857.59 6710.79407 7035.06606 6619.1225 524 5397.617 8745.33361	430	18804.752	6769.29670	7149.39259	6777.0566
467 13224.693 12482.69199 12548.32018 12499.5089 468 12643.378 14870.76505 14646.57912 14857.6467 469 23288.928 4534.97752 5380.59657 4404.1351 486 4347.023 7171.83348 7771.25105 7032.0227 492 24513.091 10247.50614 10671.60330 10296.4731 496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758 13160.67592 13599.7258 517 27724.289 3385.94787 3884.47110 3474.6192 520 3857.759 6710.79407 7035.06606 6619.1225 524 5397.617 8745.33361 9154.84154 8777.3721 525 38245.593 32961.65227	444	28287.898	14105.61652	14137.13119	14111.2896
46812643.37814870.7650514646.5791214857.646746923288.9284534.977525380.596574404.13514864347.0237171.833487771.251057032.022749224513.09110247.5061410671.6033010296.47314961967.0232905.374553529.664962831.05215026837.3698096.833768368.820058009.02995083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.292 <td>460</td> <td>7682.670</td> <td>8950.64483</td> <td>9370.07086</td> <td>8956.7089</td>	460	7682.670	8950.64483	9370.07086	8956.7089
46923288.9284534.977525380.596574404.13514864347.0237171.833487771.251057032.022749224513.09110247.5061410671.6033010296.47314961967.0232905.374553529.664962831.05215026837.3698096.833768368.820058009.02995083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.530 <t< td=""><td>467</td><td>13224.693</td><td>12482.69199</td><td>12548.32018</td><td>12499.5089</td></t<>	467	13224.693	12482.69199	12548.32018	12499.5089
4864347.0237171.833487771.251057032.022749224513.09110247.5061410671.6033010296.47314961967.0232905.374553529.664962831.05215026837.3698096.833768368.820058009.02995083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.491	468	12643.378	14870.76505	14646.57912	14857.6467
49224513.09110247.5061410671.6033010296.47314961967.0232905.374553529.664962831.05215026837.3698096.833768368.820058009.02995083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.271	469	23288.928	4534.97752	5380.59657	4404.1351
496 1967.023 2905.37455 3529.66496 2831.0521 502 6837.369 8096.83376 8368.82005 8009.0299 508 3077.095 2126.70199 2599.36511 2218.3660 512 2498.414 4157.99253 4624.28449 4258.1643 516 11362.755 13540.89758 13160.67592 13599.7258 517 27724.289 3385.94787 3884.47110 3474.6192 520 3857.759 6710.79407 7035.06606 6619.1225 524 5397.617 8745.33361 9154.84154 8777.3721 525 38245.593 32961.65227 33294.56335 32918.5309 532 14043.477 15276.10206 15290.57728 15108.9952 536 6067.127 7951.32155 8207.49046 7849.4016 540 27346.042 9970.94221 10044.51229 10056.3328 541 6196.448 9948.94679 10091.90353 9935.6280 542 3972.925 4200.87007	486	4347.023	7171.83348	7771.25105	7032.0227
5026837.3698096.833768368.820058009.02995083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.986 <td>492</td> <td>24513.091</td> <td>10247.50614</td> <td>10671.60330</td> <td>10296.4731</td>	492	24513.091	10247.50614	10671.60330	10296.4731
5083077.0952126.701992599.365112218.36605122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.535	496	1967.023	2905.37455	3529.66496	2831.0521
5122498.4144157.992534624.284494258.164351611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	502	6837.369	8096.83376	8368.82005	8009.0299
51611362.75513540.8975813160.6759213599.725851727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	508	3077.095	2126.70199	2599.36511	2218.3660
51727724.2893385.947873884.471103474.61925203857.7596710.794077035.066066619.12255245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	512	2498.414	4157.99253	4624.28449	4258.1643
520 3857.759 6710.79407 7035.06606 6619.1225 524 5397.617 8745.33361 9154.84154 8777.3721 525 38245.593 32961.65227 33294.56335 32918.5309 532 14043.477 15276.10206 15290.57728 15108.9952 536 6067.127 7951.32155 8207.49046 7849.4016 540 27346.042 9970.94221 10044.51229 10056.3328 541 6196.448 9948.94679 10091.90353 9935.6280 546 23807.241 34404.26216 34820.58668 34320.1393 552 3972.925 4200.87007 5019.54043 4262.7213 558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730	516	11362.755	13540.89758	13160.67592	13599.7258
5245397.6178745.333619154.841548777.372152538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	517	27724.289	3385.94787	3884.47110	3474.6192
52538245.59332961.6522733294.5633532918.530953214043.47715276.1020615290.5772815108.99525366067.1277951.321558207.490467849.401654027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	520	3857.759	6710.79407	7035.06606	6619.1225
532 14043.477 15276.10206 15290.57728 15108.9952 536 6067.127 7951.32155 8207.49046 7849.4016 540 27346.042 9970.94221 10044.51229 10056.3328 541 6196.448 9948.94679 10091.90353 9935.6280 546 23807.241 34404.26216 34820.58668 34320.1393 552 3972.925 4200.87007 5019.54043 4262.7213 558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329	524	5397.617	8745.33361	9154.84154	8777.3721
536 6067.127 7951.32155 8207.49046 7849.4016 540 27346.042 9970.94221 10044.51229 10056.3328 541 6196.448 9948.94679 10091.90353 9935.6280 546 23807.241 34404.26216 34820.58668 34320.1393 552 3972.925 4200.87007 5019.54043 4262.7213 558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854	525		32961.65227	33294.56335	32918.5309
54027346.0429970.9422110044.5122910056.33285416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	532	14043.477	15276.10206	15290.57728	15108.9952
5416196.4489948.9467910091.903539935.628054623807.24134404.2621634820.5866834320.13935523972.9254200.870075019.540434262.72135583935.1806125.209096445.770956218.08615649058.73014254.5016713886.0495914291.81715713761.2924811.897205428.259444863.92965799724.53010569.1143410447.0137110635.84855803206.4913039.197304006.821762925.80475836356.27112126.4352411896.5506412158.33025851242.816-1515.52028-676.45582-1382.28595968823.98612335.8632912534.0278312163.658660311070.5359859.4285410152.058989901.7146	536	6067.127	7951.32155	8207.49046	7849.4016
546 23807.241 34404.26216 34820.58668 34320.1393 552 3972.925 4200.87007 5019.54043 4262.7213 558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	540	27346.042	9970.94221	10044.51229	10056.3328
552 3972.925 4200.87007 5019.54043 4262.7213 558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	541		9948.94679	10091.90353	
558 3935.180 6125.20909 6445.77095 6218.0861 564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146		23807.241		34820.58668	34320.1393
564 9058.730 14254.50167 13886.04959 14291.8171 571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	552	3972.925	4200.87007	5019.54043	4262.7213
571 3761.292 4811.89720 5428.25944 4863.9296 579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	558		6125.20909	6445.77095	6218.0861
579 9724.530 10569.11434 10447.01371 10635.8485 580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	564	9058.730		13886.04959	14291.8171
580 3206.491 3039.19730 4006.82176 2925.8047 583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146	571		4811.89720	5428.25944	4863.9296
583 6356.271 12126.43524 11896.55064 12158.3302 585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146			10569.11434		10635.8485
585 1242.816 -1515.52028 -676.45582 -1382.2859 596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146					
596 8823.986 12335.86329 12534.02783 12163.6586 603 11070.535 9859.42854 10152.05898 9901.7146					
603 11070.535 9859.42854 10152.05898 9901.7146					
606 9283.562 10802.41454 11100.03232 10834.0638					
	606	9283.562	10802.41454	11100.03232	10834.0638

	Actual	Linear	Ridge	Lasso
620	10713.644	13654.99809	13563.03608	13664.7237
625	12129.614	12628.86069	12317.30359	12700.2711
629	11365.952	14330.23334	13871.51356	14382.5552
631	10085.846	12847.99820	12506.39587	12897.2223
633	3366.670	5703.72234	6351.54818	5749.3628
635	9391.346	13575.34814	13147.15316	13616.0282
646	10141.136	12317.77919	12221.72394	12174.7241
653	8280.623	9020.94370	9474.33735	9063.4541
659	26392.260	13593.86436	13681.18175	13413.8390
662	22192.437	9293.88862	9789.66257	9336.6749
667	8703.456	9292.13200	9275.63670	9365.5689
682	1242.260	-1652.79606	-800.07976	-1518.4302
690	34806.468	30887.20016	31319.47000	30842.0625
693	2362.229	4399.35372	4633.51699	4479.9540
697	29186.482	14065.43576	14146.86251	13878.9516
715	2457.502	802.70337	1767.86689	878.8620
717	9566.991	8804.99378	9128.77329	8841.8972
720	12231.614	14368.17498	14197.51171	14379.6353
721	9875.680	15546.05736	15455.26184	15362.7072
723	12979.358	15140.58387	14629.79397	15191.3237
724	1263.249	3529.36475	3866.72391	3621.0152
726	40932.429	33751.89810	32063.26734	33674.2028
727	6664.686	8365.41486	8354.87885	8440.0856
740	44585.456	32153.52013	32233.40583	32066.4645
741	8604.484	8800.61306	9027.58664	8691.6696
745	8827.210	9527.19126	9459.92475	9612.6987
748	1627.282	53.59675	707.44974	177.6874
752	1906.358	3039.55674	3410.71999	3141.5835
753	14210.536	17029.07880	16315.57478	17070.5911
755	17128.426	7638.43901	7841.98803	7494.4261
757	7985.815	7688.36628	8309.52829	7527.6621
758	23065.421	32899.96999	31401.18597	32877.7920
762	2416.955	4917.14615	5150.36966	4993.5114
765	9095.068	9587.68246	10057.37848	9427.9124
768	7050.642	8012.45990	8394.31473	8043.1853
769	14319.031	16833.88514	16489.95339	16828.8773
771	27941.288	15760.83678	15197.31132	15778.3588
774	17748.506	25449.39551	24663.54675	25423.8611
777	6986.697	9888.22683	9724.47928	9940.1274
787	12741.167	16165.30145	15756.09320	16032.6553
794	21195.818	33541.93425	33890.20240	33516.9915
795	7209.492	9899.58221	10048.57108	9905.7759
807	28476.735	12973.79302	12818.82728	12964.8646

	Actual	Linear	Ridge	Lasso
809	1131.507	663.37118	1413.03213	780.8789
812	6360.994	12597.28881	12440.57752	12564.6561
814	4428.888	3959.84222	4547.02134	3869.1358
816	1877.929	2020.34679	2970.31290	2084.8791
818	3597.596	6449.51629	6541.06619	6495.3633
827	43813.866	38921.77577	38767.13418	38838.4241
830	6117.494	5607.67586	5857.13341	5702.5293
831	13393.756	13918.93525	13536.76571	13981.0192
834	11743.934	14298.37155	13813.65495	14354.7462
843	36021.011	29381.48275	28072.21715	29339.8348
854	11729.680	11131.16590	11504.40083	10968.8683
859	18218.161	3995.08801	4789.67578	4040.7712
868	11576.130	16472.50228	15798.96701	16496.9479
871	8457.818	11679.96390	11427.77321	11743.9364
872	3392.365	3305.38305	4175.69029	3358.0257
882	2789.057	3433.59295	3785.18790	3525.9769
885	4877.981	5033.96932	5256.74549	5088.8797
891	29330.983	36197.70484	34663.78088	36140.9213
894	44202.654	39088.98594	38853.55037	38992.7866
898	2221.564	1797.87178	2283.61857	1898.6411
902	48673.559	42191.04956	41729.08406	42103.7347
903	4661.286	5522.01512	5944.93886	5406.9398
904	8125.784	10842.22003	10800.62512	10915.3314
916	2473.334	3904.88032	4695.95062	3958.9763
923	5488.262	7286.74119	7391.25449	7362.6900
929	13470.804	15333.67976	15259.09500	15341.9969
931	2927.065	8760.92989	8767.69704	8812.9427
933	10096.970	9295.30476	9278.54156	9329.1180
940	9487.644	9329.17793	9466.57037	9419.8584
955	20009.634	30837.77927	31496.48349	30772.8822
958	12609.887	3492.04101	3844.95287	3585.2176
959	41034.221	33212.51664	31109.95772	33281.4273
966	4746.344	5186.10415	5478.13385	5275.5449
972	4992.376	5358.39476	6159.54415	5237.3549
973	2527.819	689.98035	1622.92963	768.3096
982	4500.339	4212.69237	4806.78477	4145.4352
984	16796.412	6432.70937	7082.47272	6284.9500
996	7986.475	7851.38127	8456.33171	7689.3334
997	7418.522	9880.83816	10066.21553	9874.4526
998	13887.969	14794.97817	14781.09584	14809.0101
1006	4433.388	6897.88228	6953.09531	6960.4135
1008	24915.221	35151.36010	35484.48707	35049.5868
1011	8269.044	6968.92649	7491.97674	7024.8433

	Actual	Linear	Ridge	Lasso
1022	35595.590	28963.55834	27694.08415	28915.1372
1024	1711.027	-1237.02584	-293.01461	-1112.7971
1027	16450.895	26407.33153	27399.69429	26359.8380
1036	12094.478	9882.42594	10174.58741	9895.1429
1042	1704.700	734.48952	1560.41185	675.5125
1043	33475.817	25461.70745	23960.61666	25574.1393
1050	39727.614	34775.85947	34726.82929	34710.5631
1065	5708.867	4863.22234	5481.65746	4876.5085
1069	14349.854	11640.05509	11460.70992	11715.7699
1071	39871.704	35466.40606	35514.57626	35396.2667
1072	13974.456	15036.52569	14760.96068	14917.0069
1076	4562.842	4867.53020	5625.26134	4914.9427
1077	8551.347	10312.35853	10501.13965	10330.4982
1085	15019.760	15254.05640	15034.52960	15245.4249
1092	11286.539	12845.57973	13051.86002	12689.5865
1097	44641.197	34189.32924	30183.90506	34256.5591
1099	23045.566	12442.01456	12667.04033	12285.5247
1100	3227.121	4599.10146	5333.62111	4639.8059
1102	11253.421	11154.38743	10991.92985	11199.6844
1111	11512.405	13879.34673	13981.05018	13704.6144
1115	2396.096	2493.86476	3180.46752	2426.7574
1133	20709.020	16544.20825	16075.92995	16404.6157
1141	9048.027	11611.94284	11506.28878	11671.0403
1144	6338.076	7634.08523	7850.95726	7703.0096
1150	5979.731	8926.73763	8891.31491	9003.2094
1151	2203.736	3608.15083	4469.28755	3481.1479
1165	7153.554	8728.64528	9002.98178	8755.9831
1167	10982.501	14082.77533	13775.94841	14139.3705
1173	11093.623	14753.05446	14693.43511	14758.5113
1177	23887.663	32643.58035	31382.91348	32591.8566
1183	2632.992	3699.32824	4383.55678	3752.8851
1190	13126.677	2401.86039	3200.86098	2463.5632
1195	4134.082	3498.22279	4216.06230	3564.8737
1196	18838.704	3889.09033	4498.58214	3910.3961
1210	12347.172	14715.58396	14231.03019	14757.0603
1211	5373.364	7942.76256	7938.61128	8014.5453
1212	23563.016	8238.09868	8394.90258	8302.0443
1213	1702.455	180.23855	1061.28020	125.8301
1214	10806.839	12468.99017	12484.30426	12466.7815
1220	7537.164	9498.81727	9685.89108	9498.0964
1222	6593.508	5358.84498	5809.11527	5447.7980
1225	6858.480	7311.59077	7652.76823	7218.7671
1229	10594.226	11790.16390	11704.20384	11864.3822

	Actual	Linear	Ridge	Lasso
1233	12479.709	11640.29717	11728.47590	11642.3145
1239	6985.507	6751.79776	7130.84583	6640.6037
1240	3238.436	7392.66365	7849.36826	7410.3414
1242	49577.662	42728.59368	42252.40621	42624.0582
1246	5615.369	4207.33166	4568.22838	4260.1452
1249	1633.962	4381.27855	5082.19505	4423.8015
1250	37607.528	29961.34810	28103.68826	30043.0393
1258	11305.935	11803.28296	11864.53501	11821.8440
1261	4544.235	3806.96670	4748.14406	3696.1662
1262	3277.161	6839.53638	6917.22876	6906.4276
1279	22462.044	30468.29351	28610.12508	30554.7267
1282	24535.699	33019.71613	31691.35485	32949.5765
1283	14283.459	20360.41959	17484.26671	20517.4709
1291	7133.903	6016.05519	6791.03724	5876.8047
1295	11931.125	11614.39754	11643.51006	11516.7147
1310	6875.961	8900.92178	8871.65495	8958.4447
1311	6940.910	7902.21494	7944.87285	7981.9781
1315	18765.875	27771.42241	26849.15315	27713.9354
1317	1731.677	-1154.00580	-29.89819	-1068.0903
1319	19496.719	12519.75747	12316.29176	12349.5270
1331	12629.166	10338.05715	10735.34930	10363.2930
1337	2007.945	1138.71148	2049.07116	1208.2774

Comparison and Conclusions:

The Ridge and Lasso models both adjusted the predictions closer to actual values in some cases, compared to the basic linear model. However, they also introduced some variability, likely due to the influence of the regularization term balancing bias and variance differently:

Ridge Model: It generally increased the estimates, potentially due to its nature of shrinking coefficients but not setting them to zero, thus still considering all input features.

Lasso Model: It showed a trend similar to the linear model but with slight adjustments, indicating that it might be dropping some less influential variables from the model.

These results suggest that while both Ridge and Lasso offer advantages in terms of handling overfitting and multicollinearity, the choice between them would depend on the specific characteristics of the dataset and the goal of the analysis, with Lasso providing a sparser solution and Ridge offering stability across a broader range of data conditions. Further model tuning and cross-validation would be necessary to optimize their performances and fully assess their effectiveness compared to the simple linear model.

Section 4: Discussion and Conclusions

Summary of Results

In this analysis, we employed various statistical models to predict health insurance costs using a dataset that included variables such as age, BMI, number of children, smoker status, sex and regional interactions. The results can be summarized as follows:

Linear Regression Model: Provided a solid baseline for understanding how different variables impact insurance costs. Significant predictors included age, BMI, and especially smoker status, which greatly increased predicted charges. However, the model sometimes overestimated or underestimated charges significantly.

Ridge Regression Model: Adjusted predictions closer to actual values in some instances, suggesting its effectiveness in handling multicollinearity by shrinking coefficients but considering all variables.

Lasso Regression Model: Offered predictions similar to the linear model but with adjustments likely due to its feature selection capability, which dropped less influential predictors.

For the predictions, we apply to the test data set all the transformations made to the train data set. For each model, the predictions are obtained using the same command predict, and we evaluate the residual sum of squares, RSS.

Model	RSS
Linear Regression	10162756224
Ridge Regression	10163886527
Lasso Regression	10159739488

Comparative analysis showed that while the linear model was useful for understanding direct relationships and setting a baseline, both Ridge and Lasso provided nuanced insights by addressing overfitting and highlighting essential predictors, but the Lasso regression model performs the best on the testing dataset. However, specific model selection should also consider other factors such as model interpretability, computational complexity, and so on.

Impact of Analysis

This analysis significantly enhances our understanding of factors influencing health insurance costs:

Smoker Status: Emerged as a critical determinant of insurance costs, with smokers incurring significantly higher charges. This highlights the potential for insurance companies to adjust premiums or offer programs encouraging smoking cessation.

Age and BMI: Both variables were also strong predictors of costs, aligning with expected health risk increases with age and higher BMI.

Regional Differences: The interaction terms suggested that regional factors might also play a role in insurance costs, potentially due to environmental, policy, or lifestyle differences.

Conclusions

Smoking status is the most potent predictor of health insurance charges, suggesting targeted

health initiatives could be beneficial. Age and BMI are important factors in predicting insurance costs, indicating that personalized insurance plans could be more effective. Regularized regression models (Ridge and Lasso) are valuable for refining predictions and identifying key predictors, particularly in datasets prone to multicollinearity.