Learning Goal: Construct a confidence interval to estimate a population proportion when conditions are met. Interpret the confidence interval in context.

Specific Learning Objectives:

- Calculate confidence intervals using a sample proportion (instead of p from a previous study) to estimate the margin of error
- Calculate confidence intervals for different levels of confidence
- Describe the effect of increasing sample size on the margin of error

Introduction

We will continue our discussion of confidence intervals in this activity.

1) What is the purpose of a confidence interval?

2) Which of the following questions would be answered by calculating a confidence interval?

   - What is the average amount of money that community college students receive in financial aid?
   - Do the majority of community college students qualify for federal student loans?
   - What proportion of CA community college students qualify for the BOG fee waiver?

3) When we want to estimate a population proportion with a 95% confidence interval, we used the formula: *Sample proportion ± margin of error*

$$\hat{p} \pm 2\sqrt{\frac{p(1-p)}{n}}$$

   a) This interval is estimating a population proportion, p. Yet p appears in the formula. How did we handle this previously?

   b) The margin of error is based on the standard error. What does the standard error tell us?

   c) Why is the margin of error 2 times the standard error, instead of 3 times or something else?

   d) What conditions have to be met before we can use this formula?

So what's new in this Module? We will not have a previous study to estimate a value for the population proportion in order to calculate the standard error. Instead we will estimate the standard error using the sample proportion.

Examples:

4) In an evaluation of academic counseling services on campus, the Chair of the Counseling Department randomly selects 100 students and emails them a survey. Of the 65 who responded, 12 rated their counseling experience as "unsatisfactory".

   Based on these results, estimate the proportion of the entire student body that has had "unsatisfactory" academic counseling.

   a) Notice that we do not have a previous study that gives us an estimate for p. Therefore, we will estimate p using the sample proportion $\hat{p}$. This changes the way that we verify that the normal model is a good fit for the distribution of sample proportions.

      Verify that conditions are met for use of the normal model using $\hat{p}$ as an estimate for p.

   b) Construct the confidence interval: $\hat{p} \pm 2\sqrt{\frac{p(1-p)}{n}}$ becomes $\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

   c) Interpret the confidence interval in the context of counseling.

5) While 95% confidence intervals are the most common in the media, it is possible to adjust the formula to calculate confidence intervals with other levels of confidence.

   a) What would change in our previous example if we calculated the 99.7% confidence interval?

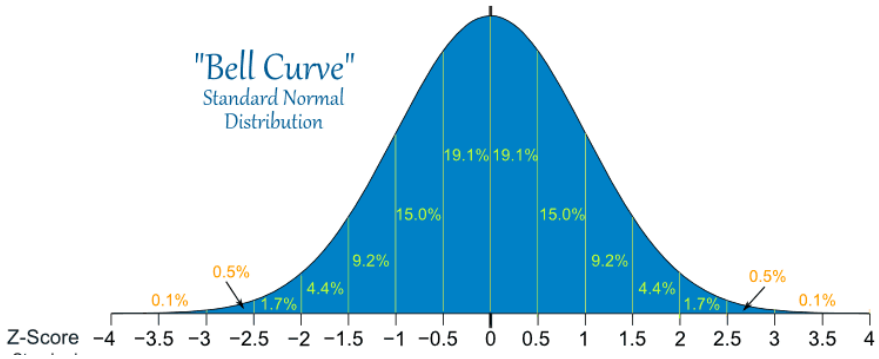
   b) Calculate the 99.7% confidence interval.

   c) Does the error get larger or smaller? Why does this make sense?

   d) Why might someone want to calculate a 99.7% confidence interval instead of a 95% confidence interval?

6) In real life statistical practice, occasionally, you might see a 90% confidence interval or 99% confidence interval.

   a) In general, what is formula for the margin of error for a 90% confidence interval? Use the standard normal curve to estimate it.



Summary:
The general formula for a confidence interval for a population proportion is $\hat{p} \pm Z_c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. The notation Z_c stands for "critical z-score". The critical z-score is determined by the confidence level.

Confidence level	68%	90%	95%	99%	99.7%
Critical z-score	1	1.645	2 or 1.96	2.576	3

Group work: Information in these problems was obtained from Gallup's *Confidence in Institutions* poll at http://www.gallup.com/poll/1597/confidence-institutions.aspx. Gallup's polling methodology uses random-digit-dial methods, so we can treat these samples as random samples. From information given in the methodology section, assume that 450 U.S. adults were surveyed in June 2003 and in June 2012.

7) "Americans' confidence in the U.S. military, the institution with the highest confidence rating in June 2012 (75% of adults surveyed reported a great deal or quite a lot of confidence in the U.S. military), is down only seven percentage points from an all-time high of 85% in June 2003."

   a) Use a 90% confidence interval to estimate the proportion of the population of U.S. adults in 2003 that had a high level of confidence in the U.S. military (answered "great deal" or "quite a lot" of confidence).

   b) Use a 90% confidence interval to estimate the proportion of the population of U.S. adults in 2012 that had a high level of confidence in the U.S. military.

   c) Based on your confidence intervals, could the proportion of U.S. adults with a high level of confidence in the U.S. military be the same in 2003 and as in 2012? Why or why not?

   d) What does a statistician mean by "90% confident?"

8) According to the same survey, "The church or organized religion has lost twenty-four percentage points with 44% of those surveyed reporting a great deal or

quite a lot of confidence in the church in June 2012 compared to 68% in May 1975."

The following StatCrunch print-outs show the 90% and 99% confidence intervals for estimating the proportion of U.S. adults with a high level of confidence in churches or organized religion in 2012.

One sample proportion confidence interval:
p : Proportion of successes
Method: Standard-Wald

90% confidence interval results:

Proportion	Count	Total	Sample Prop.	Std. Err.	L. Limit	U. Limit
p	198	450	0.44	0.023399905	0.40151058	0.47848942

One sample proportion confidence interval:
p : Proportion of successes
Method: Standard-Wald

99% confidence interval results:

Proportion	Count	Total	Sample Prop.	Std. Err.	L. Limit	U. Limit
p	198	450	0.44	0.023399905	0.37972584	0.50027416

a) Write the 90% and 99% confidence intervals from the StatCrunch print-outs.

b) Which interval has the largest margin of error? How do you know?

c) Which interval is the most likely to actually contain the proportion of U.S. adults with a high level of confidence in churches or organized religion in 2012? How do you know?

d) Which interval do you think is best for estimating the proportion of U.S. adults with a high level of confidence in churches or organized religion in 2012? Why?

e) What could Gallup do to decrease the margin of error in the 99% confidence interval?