Learning Goal: Calculate and interpret a standardized score and use the Standard Normal probability density curve to estimate probabilities.

Introduction

In this activity we continue to estimate probabilities using normal curves but with a bit of a historical twist that will help us prepare for what will come later in the course.

We will learn to *standardize* a data value so that we can compare values from distributions with different means and standard deviations. Standardizing is a process that puts different variables on the same scale for easier comparison.

Standardized scores

In statistics a standardized score measures the distance between a value and the mean using standard deviation as a yard stick. In other words, a standardized score answers the question: how many standard deviations is this value from the mean?

Here is the formula for standardizing a value. The standardized score is called a Z-score.

$$Z = \frac{value - mean}{standard\ deviation}$$

Check your understanding:

1) Suppose that you took a test and scored an 85 out of 100 possible points. Suppose also that your class had a mean of 75 with a standard deviation of 10. Your friend took a test and scored a 16 out of 20 possible points. The mean for her class was 10 with a standard deviation of 5.

   Who performed better? To answer this question, we cannot compare your score of 85 to her score of 16 because the scales for the two tests differ: 100 possible points vs. 20 possible points. In addition, the mean and standard deviation are different for the two classes.

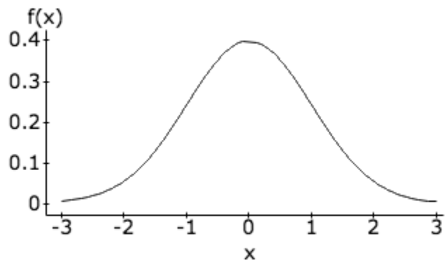
   a) Who scored a higher percentage? A percentage scales both scores to a score out of 100 so that we can more easily compare the scores. Find each percentage to answer the question.

   b) Who scored higher relative to their classmates? A Z-score scales both scores to a common measurement of "number of standard deviations from the class mean" so that we can compare the scores with standard deviation as the yardstick. Find each Z-score to answer the question.

The Standard Normal Curve

Before the advent of the sophisticated technology that is available today, statisticians used standardized values (Z-scores) to estimate probabilities.

The probability density curve for Z-scores is a normal curve with a mean of 0 and a standard deviation of 1. This curve is called the Standard Normal Curve.



Without the benefit of applets and statistical programs like StatCrunch, statisticians used tables to find probabilities.

Obviously, they could not produce a table for a normal curve with every possible mean and standard deviation, so they standardized data-values and used a single table associated with the Standard Normal Curve. For a given Z-score, they could look up the associated probability for values greater that Z-score.

Check your understanding:

2) Use the Empirical Rule and the Standard Normal Curve to answer the following:

   a) $P(-1 \leq Z \leq 1) =$

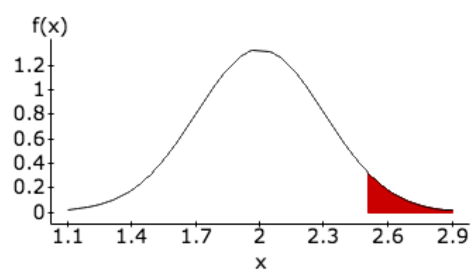
   b) $P(Z \geq 1) =$

   c) $P(-2 \leq Z \leq 2) =$

   d) $P(Z \leq -2) =$

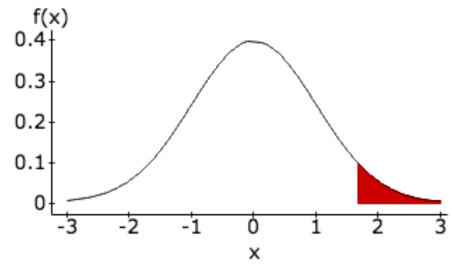
3) Suppose that the distribution of chicken egg weights is normal with a mean of 2 oz. and standard deviation of 0.3 oz. The minimum weight of a jumbo egg is 2.5 oz.

   a) What is the Z-score for the smallest jumbo egg? What does the Z-score tell us? Show the Z-score on the probability distribution of egg weights.



b) What is the probability that a randomly selected egg is classified as a jumbo egg?

Use StatCrunch and the Standard Normal Curve to estimate the probability. Fill in the missing values and label the probability in the image.

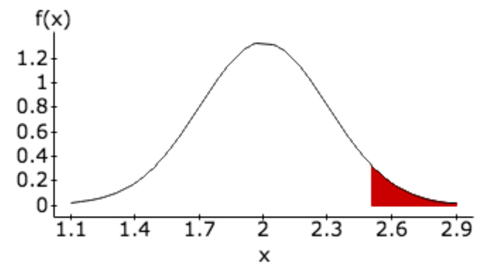


(In StatCrunch, choose Open StatCrunch, Stat, Calculator, Normal. Note that the default settings give the Standard Normal Curve.)

Mean: ☐ Std. Dev.: ☐ Here X represents ____________

P (X ☐) = ☐

c) Now find the probability again using the same StatCrunch Normal Calculator, but this time enter the mean and standard deviation for the probability distribution of egg weights as we have done before. Fill in the missing values and label the probability in the image.



Mean: ☐ Std. Dev.: ☐

Here X represents ____________

P (X ☐) = ☐

d) Theoretically, both methods should give the same probability, but the answers you found may differ a bit. What adjustment could you make to rectify this?

When do you use a Z-score and the Standard Normal Curve instead of a normal probability distribution based on a given mean and standard deviation?

The answer is that you can use either method.

Why do we teach the Z-score and the Standard Normal Curve when we really don't need it to find a probability?

The answer is that we will use Z-scores later in the course. In addition, when we get into statistical inference methods, we will always be calculating a test statistic (a Z-score or some other metric) and using a density curve based on those scores to find probabilities. Standardized scores are our first introduction to a test statistic and the Standard Normal Curve is the first example of a density curve based on a test statistic. Later, we will use probability distributions for different test statistics that are not normal curves.

Group work

4) Weights of one-year-old boys are normally distributed with a mean of 22.8 pounds and a standard deviation of 2.15 pounds. (Source: About.com). Children whose weights are more than two standards deviations from the mean are unusually large or small.

   a) What is the probability that a randomly selected one-year-old boy will be unusually small based on the definition above? Show your work.

   b) Ann's one-year-old son weighs 18 pounds. What is his Z-score? Is he unusually small relative to other one-year-old boys?

   c) Why is Ann's son's Z-score negative?

   d) What is the probability that a randomly selected one-year-old boy weighs less than Ann's son? Document your use of StatCrunch to find the answer.