Learning Goal: Use the properties of the normal density curve to estimate probabilities using the Empirical Rule and standardized scores.

Introduction

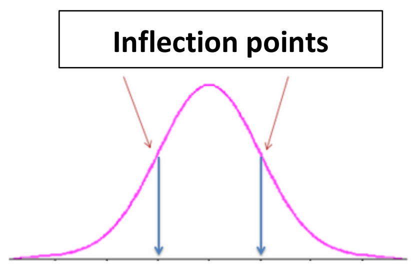
In this activity we continue estimating probabilities using normal curves, but we will also investigate general properties of the normal curve.

General properties of normal curves:

<u>Shape:</u> Every normal curve has a bell-shape.
<u>Center</u>: Because of the symmetry in the curve, the mean is equal to the median; both are associated with the peak and divide the area under the curve in half.
<u>Spread</u>: As before, the standard deviation is roughly the average distance of values from the mean. It is associated with the inflection point on the curve where the curve transitions from concave down to concave up or vice versa.

For a density curves, we use Greek letters to represent the mean (μ) and the standard deviation (σ).

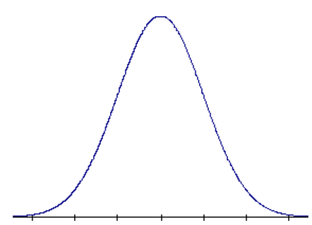
(Why the special notation? Because we are working with a mathematical model, not real data or relative frequencies from data.)



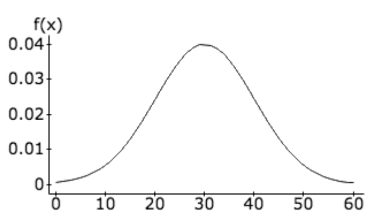
On the horizontal axis, label the mean as μ, the point that is one standard deviation below the mean as μ−σ, and the point that is one standard deviation above the mean as μ+σ.

Check your understanding:

1) For the normal curve at the right, mark a scale on the horizontal axis so that the mean is 6 and the standard deviation is 2.



2) Estimate the mean and standard deviation of the normal curve.



μ =

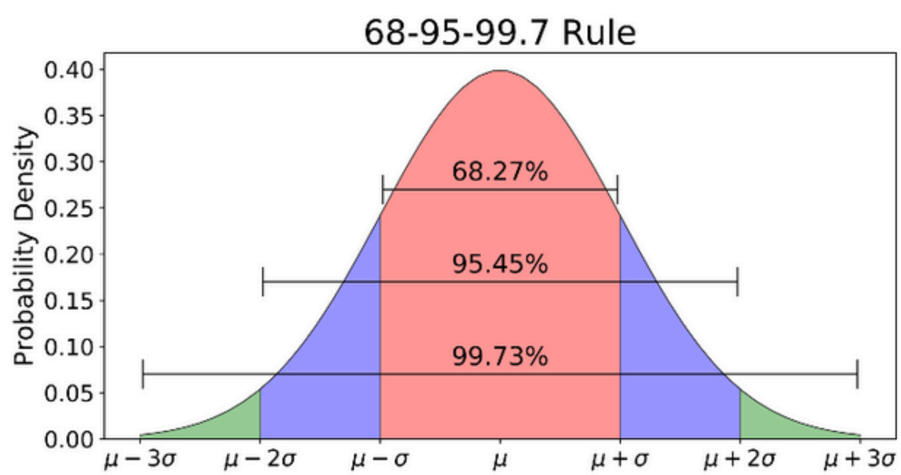
σ =

The Empirical Rule

For every normal curve, the probability that a value lies with one, two or three standard deviations from the mean is predictable and does not depend on the value of the mean or standard deviation. Specifically, the probability that a value …

- Is within one standard deviation of the mean is 68% (this is the area between $\mu - \sigma$ and $\mu + \sigma$)
- Is within two standard deviations of the mean is 95% (area between $\mu - 2\sigma$ and $\mu + 2\sigma$)
- Is within three standard deviations of the mean is 99.7% (area between $\mu - 3\sigma$ and $\mu + 3\sigma$)

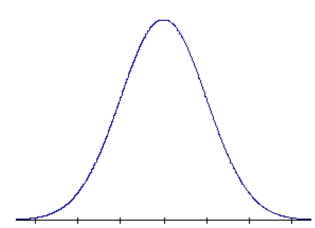
This special property is called the Empirical Rule or the 68-95-99.7 Rule.



Check your understanding:

3) Weights of 1-year old boys have a normal distribution; therefore, a normal curve is a good mathematical model for the probability distribution. The mean is 22.8 lbs and the standard deviation is 2.2 lbs.

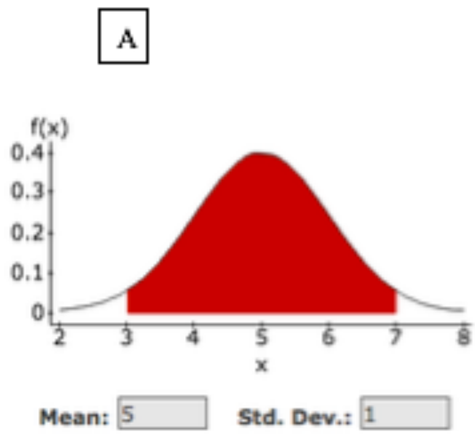
   a) Label the horizontal axis of the normal curve to fit this scenario.

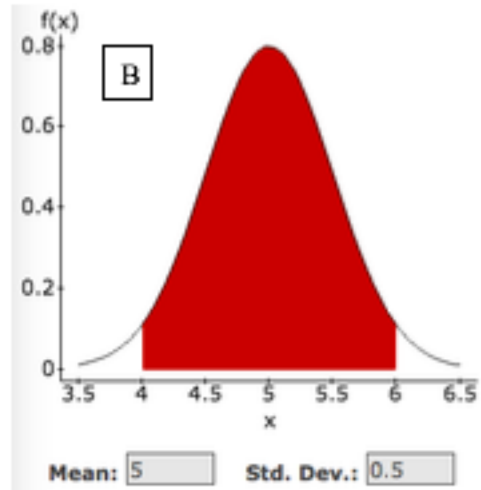


   b) What is the probability that a randomly selected 1-year old boy weighs between 20.6 lbs and 25 lbs?

   c) What is the probability that a randomly selected 1-year boy weighs more than 25 lbs?

4) Which probability represented by the shaded area is larger? Why do you think so?





Group work:

5) Birth weights have a normal distribution and so the probability distribution is normal in shape. The mean is 120 ounces and the standard deviation is 20 ounces.

   a) What is the probability that a randomly selected infant weighs less than 100 ounces? Show your work so that someone else can follow your thinking.

   b) Doctors define "normal" birth weight as weights that are within two standard deviations of the mean. (Here "normal" is the everyday use of that word, not the statistical use.)

      - What is the probability that a randomly selected baby will have a "normal" birthweight?

      - What is the range for "normal" birth weights by this definition?

   c) Doctors define "low birth weight" as a birthweight more than 2 standard deviations below the mean. What is the probability that a randomly selected baby has a low birth weight by this definition? Show your work.