Learning Goals:

- Describe the sampling distribution of sample means.
- Describe the effect of sample size on the variability of sample means.

Introduction:

Up to this point we have tested hypotheses and found confidence intervals in order to draw conclusions about a population proportion. Proportions summarize categorical data. We will now focus on quantitative data. We summarize quantitative data with means.

In this Unit we will use quantitative data to test hypotheses and find confidence intervals in order to draw conclusions about a population mean or a difference in population means.

1) Which of the following research questions involves quantitative data?

    a) What proportion of community college students has a student loan?
    b) What is the average loan amount owed by community college students?
    c) On average do college students borrow more than $5,000 before they graduate?
    d) Do more than 80% of college graduates have credit card debt?
    e) Is the average interest rate on a student loan greater than 8%?
    f) Do the majority of community college students work on average 20 hours or more each week?

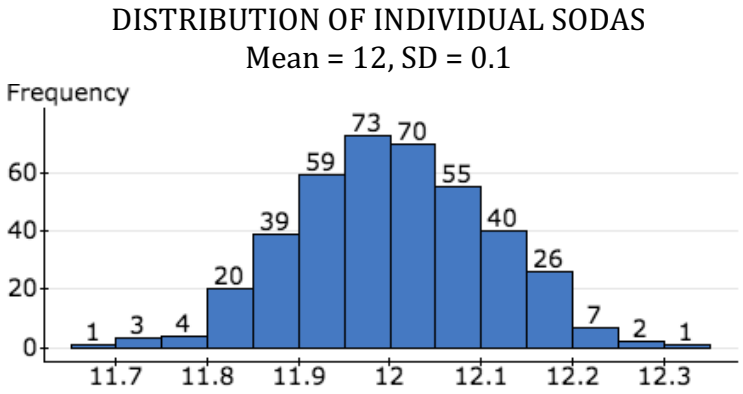
Example:

In a quality control process at a soda bottling plant, 400 bottles, filled during a 10-minute period, are pulled from the assembly line. These bottles will represent our population of soda bottles. The bottles are labeled with a volume of 12 fluid ounces, but there is variability in the amount each bottle actually contains.

2) The histogram shows the distribution of the volumes for the population of 400 bottles. The mean is 12fl.oz. and the standard deviation is 0.1fl.oz.

    a) Select a random soda from the list of 400 sodas.

b) Draw an arrow to show the bin that your soda lies in. Is the volume in your soda unusual?



c) Is it unusual for a bottle of soda to contain 11.9fl.oz.? What makes you think so?

d) What is the probability that a bottle of soda contains less than 11.9fl.oz.?

e) Do you think that the quality control engineer will be happy with this distribution? Why or why not?

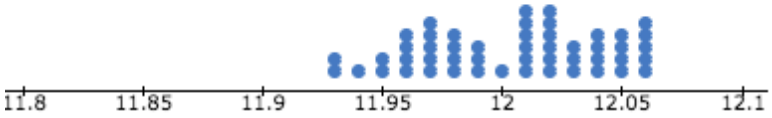
3) Now, let's think about a 6-pack of soda. If a 6-pack is randomly selected from this population of 400 sodas, is it unusual for the mean amount of soda to be 11.9floz in the 6-pack?

To answer this question, we need to examine the variability in the distribution of sample means.

a) Collect your own random sample of 6 sodas and determine the mean volume for your 6-pack.

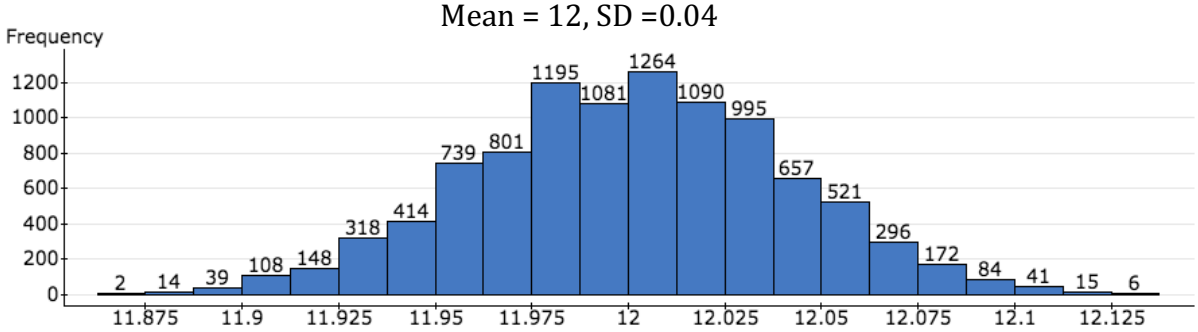
b) Here is a dot plot of the mean volumes of 50 randomly selected 6-packs of soda. Plot your mean volume in the dot plot.

DISTRIBUTION OF SAMPLE MEANS



c) Here is a histogram of the mean volumes of 10,000 6-packs of soda. The mean of the sample means is 12floz. The standard deviation of the sample means is 0.04floz.

DISTRIBUTION OF SAMPLE MEANS
Mean = 12, SD =0.04



Draw an arrow to show the bin where your sample mean lies.
Is the mean volume of your 6-pack unusual? How do you know?

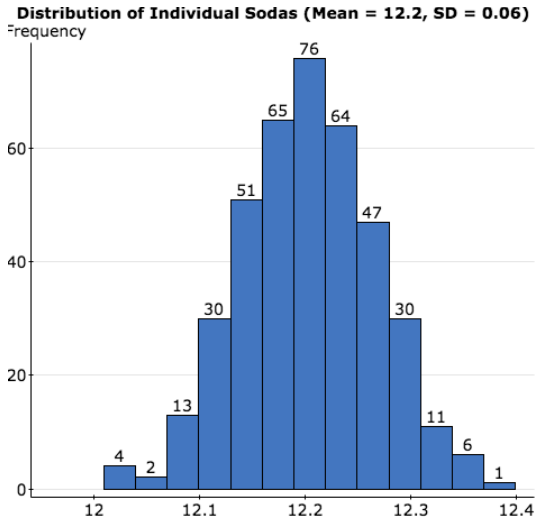
d) Is it unusual for the mean amount of soda to be 11.9floz in the 6-pack? How do you know?

e) What is the probability that the mean amount of soda in a 6-pack is 11.9floz or less?

4) Which is more unusual? Why?

- A 12-pack of soda with a mean volume of 12.2 fl.oz,
- A 6-pack of soda with a mean volume of 12.2 fl.oz, or
- A single soda with a volume of 12.2 fl.oz?

5) The quality control engineer resets the bottle-filling machine and selects another 400 bottles during a 10-minute run. He produces the following histogram. Are the new settings better or worse? Why do you think so?



Group work:

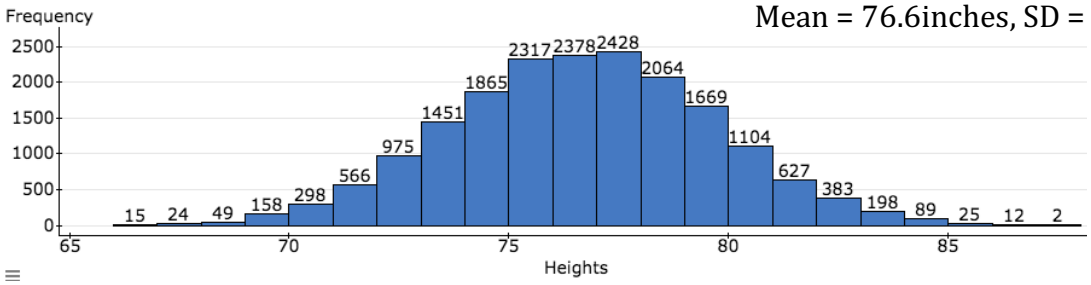
1) There are 18,697 men who play NCAA (National Collegiate Athletic Association) basketball. Their mean height is 76.6 inches with a 3-inch standard deviation.

   Which do you think is more likely? Randomly selecting one NCAA male basketball player who is 79 inches (6'7") tall or randomly selecting a team of 5 players that is 79 inches tall? Why do you think so?

2) A randomly selected NCAA male basketball player is 79.2inches tall.

a) Locate the bin that contains this player in the histogram of individual heights.

DISTRIBUTION OF INDIVIDUALS
Mean = 76.6inches, SD = 3.0inches



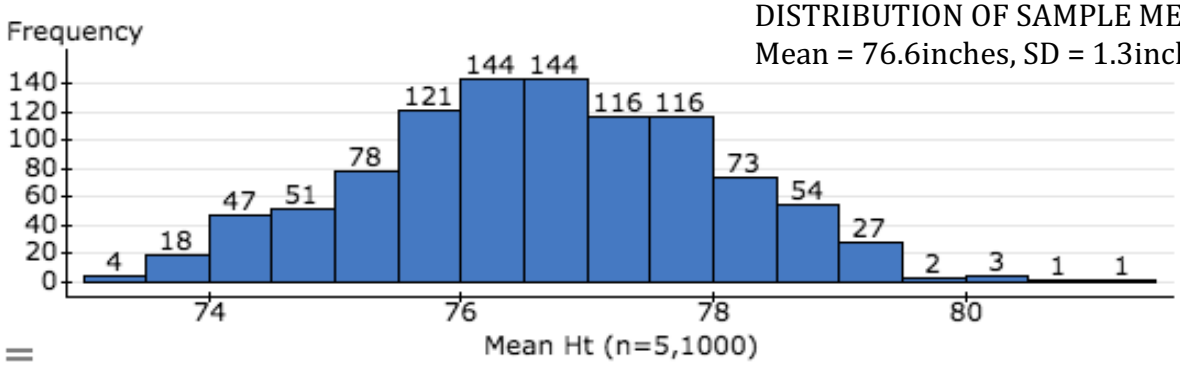
b) Give an interval of typical heights for these players.

c) What is the probability that a randomly selected player is at least 79inches tall?

3) We randomly select a team of 5 players. The team members have the following heights (in inches): 80.1, 84.6, 78.9, 77.5, 75.2.

a) Locate the bin that contains this team's mean height in the histogram of 1,000 sample means.

DISTRIBUTION OF SAMPLE MEANS
Mean = 76.6inches, SD = 1.3inches



b) Give an interval of typical mean heights for randomly selected teams of 5 players.

c) What is the probability that a randomly selected team of 5 players has a mean height of at least 79inches?

4) If we randomly chose a team of 15 players is it more likely or less likely that the mean height of a team is 79inches or more? Why do you think so?

5) Generalize your observations from this activity by completing the following sentences

- If a population has a mean of μ, then the mean of a distribution of sample means will be (circle one: equal to, less than, greater than) μ.
- If a population has a standard deviation of σ, then the standard deviation of a distribution of sample means will be (circle one: equal to, less than, greater than) σ.
- If we increase the sample size, the standard deviation of the distribution of sample means will (circle one: stay the same, decrease, increase).
- If the distribution of a variable (such as height) in the population has a bell-shape, then what can we say about the shape of the distribution of sample means?

Volume (fluid ounces) for 400 bottles of soda from a normal distribution with mean of 12 fl.oz. and standard deviation of 0.1 fl.oz.

Bottle	Volume (fl.oz.)								
1	12.09	41	12.01	81	12.04	121	12.13	161	11.96
2	12.03	42	11.91	82	12.02	122	11.96	162	11.87
3	12.02	43	12.31	83	12	123	11.91	163	11.98
4	11.84	44	11.98	84	11.95	124	12.17	164	11.99
5	11.95	45	11.97	85	12	125	12	165	11.99
6	12.01	46	12.14	86	11.93	126	12.02	166	12.19
7	12.1	47	12.02	87	11.95	127	12.02	167	12.17
8	11.94	48	11.74	88	12.11	128	11.95	168	12.06
9	12.09	49	12.03	89	12	129	11.97	169	12.05
10	12.1	50	11.95	90	11.87	130	11.94	170	12.04
11	11.98	51	11.93	91	12.04	131	12.13	171	11.97
12	12.3	52	11.99	92	12.03	132	11.81	172	12.12
13	11.96	53	11.76	93	12.04	133	11.92	173	11.86
14	11.79	54	12.14	94	11.91	134	11.92	174	11.94
15	12.03	55	12.01	95	12.09	135	12.05	175	11.97
16	11.85	56	12.09	96	11.97	136	12.03	176	11.88
17	12.32	57	12.06	97	11.81	137	11.88	177	12.25
18	11.99	58	12.1	98	11.89	138	12.1	178	11.98
19	11.89	59	11.94	99	11.97	139	12.09	179	12.08
20	11.95	60	11.84	100	11.95	140	11.98	180	12
21	12.01	61	12.03	101	12.22	141	11.98	181	11.84
22	12.16	62	12.03	102	11.94	142	11.82	182	11.99
23	12.03	63	12.04	103	12.11	143	11.91	183	11.94
24	12	64	12	104	12.07	144	11.96	184	12.17
25	12.06	65	11.82	105	11.9	145	12.07	185	12.02
26	11.98	66	12.01	106	12.09	146	12.06	186	12.1
27	12.2	67	11.99	107	11.88	147	12.08	187	12.08
28	12.17	68	12.18	108	11.79	148	12.01	188	12
29	11.97	69	11.98	109	11.95	149	12.02	189	11.77
30	12	70	12	110	12.11	150	12.08	190	12.03
31	12.02	71	11.96	111	12.14	151	12.26	191	11.77
32	12.21	72	12.08	112	12.17	152	12.22	192	11.89
33	11.94	73	12.05	113	11.91	153	12.11	193	12.15
34	12.04	74	12.1	114	12.23	154	12.11	194	11.99
35	12.05	75	12.11	115	11.98	155	12.18	195	12.03
36	11.98	76	11.97	116	12	156	11.98	196	11.95
37	12.04	77	11.87	117	12.1	157	11.98	197	11.97
38	12.05	78	12.1	118	11.88	158	12.03	198	12.11
39	12.07	79	12.01	119	11.95	159	12.06	199	11.99
40	12	80	11.88	120	12.02	160	11.95	200	11.78

Bottle	Volume (fl.oz.)								
201	12.06	241	11.98	281	12.11	321	11.82	361	12.03
202	12.02	242	12.02	282	12.16	322	11.85	362	11.76
203	12.15	243	12.11	283	12.2	323	11.93	363	11.98
204	11.84	244	12.05	284	11.88	324	12.09	364	11.98
205	11.93	245	11.92	285	11.85	325	12.02	365	12.01
206	11.82	246	11.93	286	12.02	326	12.02	366	11.93
207	12	247	12.04	287	12.05	327	12	367	11.94
208	12.02	248	11.96	288	11.95	328	11.99	368	12.01
209	12.12	249	11.98	289	11.96	329	11.81	369	12.04
210	11.92	250	12.08	290	12.14	330	11.84	370	12.17
211	12.09	251	12.06	291	12.13	331	12.15	371	11.98
212	12.06	252	11.96	292	11.99	332	12	372	12.03
213	12.08	253	11.85	293	12.14	333	11.82	373	11.94
214	12.07	254	11.88	294	11.94	334	12.04	374	12.11
215	12.05	255	11.96	295	11.95	335	11.82	375	12.03
216	12.18	256	12.01	296	11.85	336	11.89	376	11.9
217	11.83	257	12.19	297	11.99	337	12.11	377	12.01
218	12.01	258	11.92	298	11.99	338	12.12	378	12.22
219	12.08	259	11.97	299	11.99	339	12.06	379	11.93
220	12.09	260	11.93	300	12.01	340	12.01	380	12.04
221	11.89	261	11.99	301	12.05	341	11.87	381	12.04
222	12.04	262	11.86	302	12.08	342	12.11	382	11.95
223	11.93	263	11.97	303	11.91	343	11.99	383	11.94
224	11.98	264	11.87	304	11.91	344	12.08	384	11.91
225	12.04	265	12.22	305	12.08	345	12.28	385	11.95
226	12.16	266	12.17	306	11.99	346	11.97	386	11.98
227	11.98	267	11.98	307	12.03	347	12.2	387	11.96
228	11.89	268	12.07	308	12.1	348	12.2	388	12.04
229	11.98	269	12.13	309	12.08	349	12.01	389	12.11
230	12.02	270	12.03	310	11.92	350	11.98	390	11.79
231	11.99	271	12.05	311	12.14	351	12.07	391	12.01
232	11.98	272	12.03	312	12.08	352	11.97	392	12.24
233	12.12	273	12.06	313	11.93	353	11.92	393	12.03
234	12.28	274	12.05	314	12.07	354	11.94	394	12.08
235	11.85	275	11.99	315	11.87	355	11.88	395	12.1
236	11.94	276	11.87	316	12.04	356	12.06	396	11.92
237	12.02	277	12.16	317	12	357	12.02	397	11.84
238	12.05	278	12.22	318	11.88	358	12.18	398	12.03
239	11.94	279	12	319	12.02	359	12.06	399	11.9
240	12.16	280	11.92	320	11.95	360	12.11	400	11.95