Additional definitions:

The **sample space S** of a random phenomenon such as flipping a coin or rolling dice is the set of all possible outcomes. For example, if we flip one coin twice, one possible outcome is we get a heads on the first flip and a tails on the second flip or HT. Another possible outcome is that we get a tails on the first flip and a heads on the second flip or TH. So the sample space of flipping one coin twice is

S = {HH, HT, TH, TT}.

An **event** is an outcome or a set of outcomes of a random phenomenon. That is, an event is a subset of the sample space. For example, the event that we get exactly one heads when we flip two coins in order is

E = {HT, TH}.

And the probability of an event E is

$$P(E) = \frac{\#\ of\ outcomes\ in\ E}{\#\ of\ outcomes\ in\ S} = \frac{2}{4} = \frac{1}{2}$$

Discussion 6: Common gambling games involve rolling two dice. The sample space is the set of all possible outcomes for the two dice. Here are three common ways to find the sample space: picture, table, or tree.

Picture:

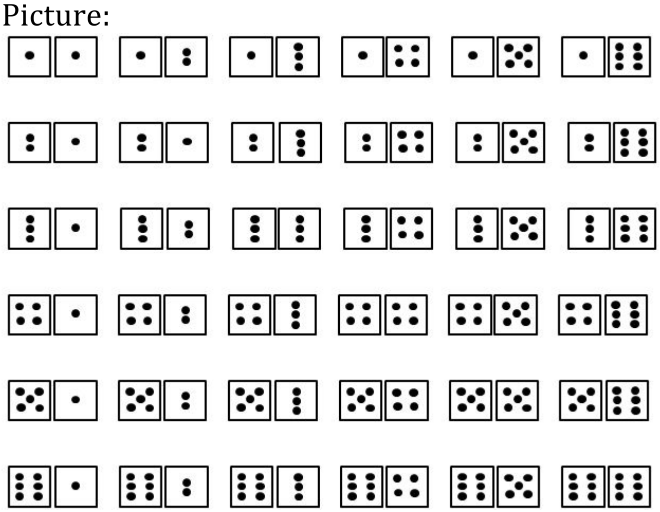


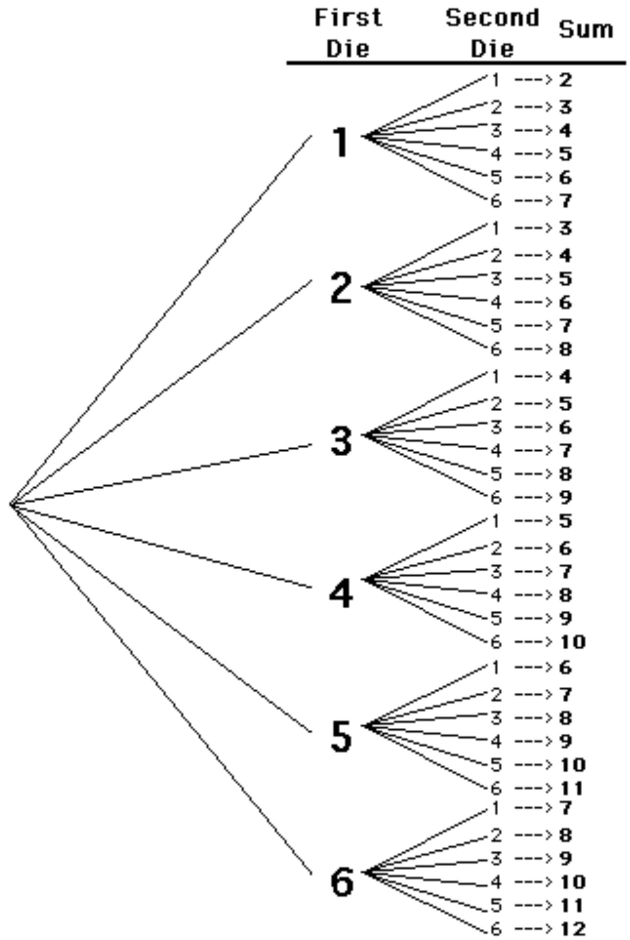
Table:

Dice #2 \ Dice #1	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

a. Using the preferred diagram for the sample space, complete the following probability distribution table:

X = sum of dice	P(X) = probability
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Tree:



b. Are the probabilities in the table above theoretical or empirical probabilities? Explain your reasoning.

c. Many gambling games are based upon the rolls that occur most frequently. For example, both craps and roulette pay out based on frequency. Which face totals have the highest frequency?

d. To calculate the average (mean) roll for two dice, we have to take into account that some face totals occur more often than other face totals. This is known as a weighted average (weighted mean). To calculate the weighted average (weighted mean) for two dice, we multiply each face by its probability. Finish the calculation to find the weighted average:

$$2*\frac{1}{36}+3*\frac{2}{36}+4*\frac{3}{36}+5*\frac{4}{36}+6*\frac{5}{36}+7*\frac{6}{36}+8*\frac{5}{36}+9*\frac{4}{36}+10*\frac{3}{36}+11*\frac{2}{36}+12*\frac{1}{36}=$$

e. Are you surprised by the result? The weighted average (weighted mean) is also known as the expected value. Why does this make sense?

f. Similar to the weighted mean calculation, the standard deviation calculation also needs to include the probability weight. Start the standard deviation calculation by completing the table:

X	P(X)	$(X-7)$	$(X-7)^2$	$(X-7)^2 * P(X)$
2	$\frac{1}{36}$	$2-7=-5$	$\mathbf{(-5)^2=25}$	$25*\frac{1}{36}=\frac{25}{36}=0.6944$
3	$\frac{2}{36}$			
4	$\frac{}{36}$			
5	$\frac{}{36}$			
6	$\frac{}{36}$			
7	$\frac{}{36}$			
8	$\frac{}{36}$			
9	$\frac{}{36}$			
10	$\frac{}{36}$			
11	$\frac{}{36}$			
12	$\frac{}{36}$			

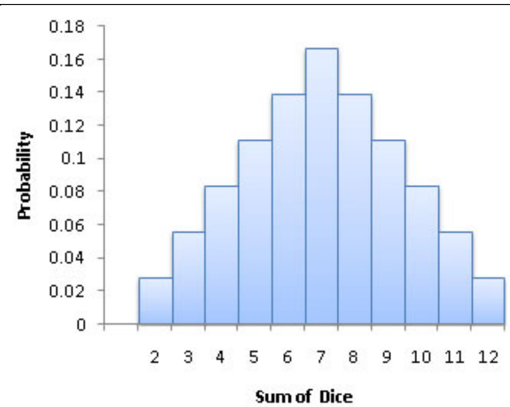
g. Continue the standard deviation calculation by summing up the values in the $(X-7)^2 * P(X)$ column.

Finish the standard deviation calculation by taking the square root of the sum above.

We have just worked through the formula $\sqrt{\sum(x-\mu_x)^2 * P(x)}$ where μ_x represents the mean of the variable x. In our calculations, the weighted mean was 7.

In the dice example, the total of the two faces of the dice is a *discrete random variable*.

Since the dice face total only takes on whole numbers 2 through 12, the probability distribution is a histogram with bars for each discrete value.



For *continuous random variables*, the probability distribution can be approximated by a smooth curve called a *probability density curve*.

Recall that these smooth curves are mathematical models. We use a mathematical model to describe a probability distribution so that we can use technology and the equation of this model to estimate probabilities.