## STAD29 / STA 1007 assignment 4

## Due Tuesday Feb 4 at 11:59pm on Quercus

Hand in the indicated questions. In preparation for the questions you hand in, it is worth your while to work through (or at least read through) the other questions as well.

Hand in your work on Quercus. If you did STAC32 last fall, it's the same procedure. A reminder is here: https://www.utsc.utoronto.ca/~butler/c32/quercus1.nb.html

You are reminded that work handed in with your name on it must be entirely your own work. It is as if you have signed your name under it. If it was done wholly or partly by someone else, you have committed an academic offence, and you can expect to be asked to explain yourself. The same applies if you allow someone else to copy your work. The grader will be watching out for assignments that look suspiciously similar to each other (or to my solutions). Besides which, if you do not do your own assignments, you will do badly on the exams, because the struggle to figure things out for yourself is an important part of the learning process.

You will need this first:

library(tidyverse)

Hand in questions 2 and 4.

- 1. Work through, or look at the problems in Chapter 20 of PASIAS that relate to nominal responses: 20.4, 20.5, 20.9, 20.11. (The other questions in Chapter 20 are about the organization of some of the data sets used in the chapter; feel free to go through those too.)
- 2. A survey called High School and Beyond was given to a large number of American high school seniors (grade 12) by the National Center of Education Statistics. The data set at http://ritsokiguess.site/STAD29/hsb.txt is a random sample of 200 of those students.

The variables collected are:

- gender: student's gender, female or male.
- ses: Socio-economic status of student's family (low, middle, or high)
- schtyp: School type, public or private.
- prog: Student's program, general, academic, or vocational.
- read: Score on standardized reading test.
- write: Score on standardized writing test.
- math: Score on standardized math test.
- science: Score on standardized science test.
- socst: Score on standardized social studies test.

Some of these variables are quantitative and some are categorical.

You will recognize this as one of the data sets from PASIAS. This time, we take a different angle: we try to predict which program a student went into based on the values of the other variables.

(a) (2 marks) Read in and display (some of) the data.

```
Solution: The usual thing:
my_url <- "http://ritsokiguess.site/STAD29/hsb.txt"</pre>
ml <- read delim(my url," ")</pre>
## Parsed with column specification:
##
   cols(
##
     row, = col_double(),
##
    id = col_double(),
##
    female = col_character(),
##
    ses = col_character(),
    schtyp = col_character(),
##
##
    prog = col character(),
    read = col_double(),
##
    write = col double(),
##
##
    math = col_double(),
    science = col_double(),
##
##
    socst = col double(),
##
   honors = col character(),
    awards = col_double(),
##
##
    cid = col_double()
## )
ml
##
   # A tibble: 200 x 14
##
       `row,`
                 id female ses
                                   schtyp prog
                                                  read write
                                                               math science socst honors
##
        <dbl> <dbl> <chr>
                            <chr> <chr>
                                          <chr> <dbl> <dbl>
                                                              <dbl>
                                                                       <dbl> <dbl> <chr>
                                                                          29
##
    1
            1
                 45 female low
                                   public voca~
                                                    34
                                                           35
                                                                                 26 not
                                                           33
##
    2
            2
                108 male
                            midd~ public gene~
                                                    34
                                                                  41
                                                                          36
                                                                                 36 not
##
    3
            3
                 15 male
                            high public voca~
                                                    39
                                                           39
                                                                  44
                                                                          26
                                                                                 42 not
                                                                                         e~
    4
            4
                                                    37
                                                           37
##
                 67 male
                            low
                                   public voca~
                                                                  42
                                                                          33
                                                                                 32 not
##
                153 male
                            midd~ public voca~
                                                    39
                                                                                 51 not
                                                                                         e~
##
            6
                 51 female high public gene~
                                                    42
                                                           36
                                                                          31
                                                                                 39 not
    6
                                                                  42
                                                                                        e~
##
    7
            7
                164 male
                            midd~ public voca~
                                                    31
                                                           36
                                                                  46
                                                                          39
                                                                                 46 not
                                                                                         e.~
                                                    50
                133 male
##
    8
            8
                            midd~ public voca~
                                                           31
                                                                  40
                                                                          34
                                                                                 31 not
##
            9
                  2 female midd~ public voca~
                                                    39
                                                           41
                                                                  33
                                                                          42
                                                                                 41 not
           10
                 53 male
                            midd~ public voca~
                                                    34
                                                           37
                                                                  46
                                                                          39
## 10
                                                                                 31 not
## # ... with 190 more rows, and 2 more variables: awards <dbl>, cid <dbl>
My data frame is called ml. Yours probably isn't. It looks as if I have the right thing: 200 rows
and 13 columns plus a row number.
```

(b) (2 marks) Use multinom to fit a model predicting program from everything else that makes sense as an explanatory variable. Exclude everything after socst. No need to display the output.

```
Solution: Load package nnet first, since that's where multinom lives. You might need to install it first, but don't include the installation in what you hand in:

library(nnet)
ml.1 <- multinom(prog~female+ses+schtyp+read+write+math+science+socst, data=ml)
## # weights: 33 (20 variable)
## initial value 219.722458
## iter 10 value 172.925326
## iter 20 value 156.065379
## final value 155.776076
```

```
## converged
I think those are all the variables it makes sense to include. I said to exclude everything after
socst, and row and id are identifiers.
Extra: summary of one of these models looks like this:
summary(ml.1)
## Call:
## multinom(formula = prog ~ female + ses + schtyp + read + write +
       math + science + socst, data = ml)
##
##
##
   Coefficients:
##
            (Intercept) femalemale
                                        seslow sesmiddle schtyppublic
               3.161817 -0.1400384 0.91165962 0.6737407
                                                             0.6021655 -0.04355380
##
   general
                                                             1.9403196 -0.03402511
##
   vocation
               7.327585 -0.3526068 0.01878735 1.1740961
##
                   write
                                math
                                         science
##
   general -0.02672221 -0.09961354 0.10264775 -0.02550275
   vocation -0.03292013 -0.11559404 0.06001292 -0.07677874
##
##
## Std. Errors:
##
            (Intercept) femalemale
                                       seslow sesmiddle schtyppublic
               1.766449 0.4485411 0.5852751 0.4990211
                                                            0.5549095 0.03049076
## general
               0.8203955 0.03398077
## vocation
##
                 write
                              math
                                      science
## general 0.03295135 0.03423444 0.03132909 0.02646662
   vocation 0.03491891 0.03821745 0.03227780 0.02893924
##
## Residual Deviance: 311.5522
## AIC: 351.5522
There is one row for each response category apart from the baseline academic, and one column
for each explanatory variable (plus the intercept). The numbers are not helpful (yet), but
they're log-odds as for a regular logistic regression.
```

(c) (3 marks) There are a lot of explanatory variables. To see whether any of them can be removed, we can use step on one of these models. Run step with direction="backward" on your model from the previous part, saving the result and then displaying the summary of it. Which explanatory variables seem to be still in the model? Hint: step produces a lot of output; the input trace controls how much output there is. See if you can minimize the amount of output.

**Solution:** This. If your model in the previous part was different from mine, your answer here might be a little different from mine, but the point here is the procedure. The default value of trace is 1 (you can tell by reading the help for trace), with a higher value indicating more output, so let's try 0:

```
output, so let's try 0:
ml.2 <- step(ml.1, direction="backward", trace=0)
## trying - female
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst</pre>
```

```
## # weights: 30 (18 variable)
## initial value 219.722458
## iter 10 value 172.662548
## iter 20 value 156.063823
## final value 156.032828
## converged
## trying - ses
## trying - schtyp
## trying - read
## trying - write
## trying - math
## trying - science
## trying - socst
## # weights: 27 (16 variable)
## initial value 219.722458
## iter 10 value 176.827677
## iter 20 value 156.410686
## final value 156.406678
## converged
## trying - ses
## trying - schtyp
## trying - read
## trying - math
## trying - science
## trying - socst
## # weights: 24 (14 variable)
## initial value 219.722458
## iter 10 value 171.169761
## iter 20 value 157.775586
## final value 157.775540
## converged
## trying - ses
## trying - schtyp
## trying - math
## trying - science
## trying - socst
I think that's pretty minimal. The implication here is that it tried to remove ses, schtyp,
math, science and socst and failed to do so, so these are the ones that are left:
summary(m1.2)
## Call:
## multinom(formula = prog ~ ses + schtyp + math + science + socst,
##
       data = ml)
##
## Coefficients:
##
            (Intercept)
                             seslow sesmiddle schtyppublic
                                                                  math
                                                                           science
## general
              2.587029 0.87607389 0.6978995 0.6468812 -0.1212242 0.08209791
               6.687272 -0.01569301 1.2065000
                                                  1.9955504 -0.1369641 0.03941237
## vocation
                  socst
## general -0.04441228
## vocation -0.09363417
##
```

```
## Std. Errors:
##
            (Intercept)
                            seslow sesmiddle schtyppublic
                                                                  math
                                                                           science
               1.686492 0.5758781 0.4930330
                                                  0.545598 0.03213345 0.02787694
##
  general
##
   vocation
               1.945363 0.6690861 0.5571202
                                                  0.812881 0.03591701 0.02864929
##
                  socst
            0.02344856
##
   general
##
   vocation 0.02586717
##
## Residual Deviance: 315.5511
## AIC: 343.5511
```

This is indeed the case. Careful that ses is the name of the categorical variable, and the values in seslow and sesmiddle are the estimates for two of its *levels* (the other one high being the baseline). The call line at the top of the output also shows the model that was fitted, which has the same five things in it.

(d) (3 marks) The summary output is very hard to understand. Let's set up to do some predictions. First, for each of the explanatory variables in your final model from step, find out all its levels if it is categorical, and find its (first and third) quartiles if it is quantitative. Do this how you like. You don't need to be clever.

```
Solution: The lazy way to do this is to look at the summary of the entire data frame:
summary(ml)
##
                              id
                                              female
                                                                     ses
                                          Length:200
##
            :
               1.00
                       Min.
                                  1.00
                                                                Length:200
    Min.
                               :
##
    1st Qu.: 50.75
                       1st Qu.: 50.75
                                          Class : character
                                                                Class : character
##
    Median :100.50
                       Median :100.50
                                          Mode
                                                 :character
                                                                Mode : character
##
    Mean
            :100.50
                       Mean
                               :100.50
                       3rd Qu.:150.25
##
    3rd Qu.:150.25
##
    Max.
            :200.00
                       Max.
                               :200.00
##
       schtyp
                                                     read
                                                                      write
                              prog
    Length: 200
                                                                 Min.
##
                                                       :28.00
                                                                         :31.00
                         Length:200
                                               Min.
##
    Class : character
                          Class : character
                                               1st Qu.:44.00
                                                                 1st Qu.:45.75
##
           :character
                         Mode
                                :character
                                               Median :50.00
                                                                 Median :54.00
##
                                               Mean
                                                       :52.23
                                                                 Mean
                                                                         :52.77
##
                                               3rd Qu.:60.00
                                                                 3rd Qu.:60.00
##
                                               Max.
                                                       :76.00
                                                                 Max.
                                                                         :67.00
##
          math
                          science
                                             socst
                                                              honors
##
            :33.00
                              :26.00
                                        Min.
                                                :26.00
                                                          Length: 200
    Min.
                      Min.
    1st Qu.:45.00
                      1st Qu.:44.00
                                        1st Qu.:46.00
##
                                                          Class : character
##
    Median :52.00
                      Median :53.00
                                        Median :52.00
                                                          Mode
                                                                 :character
##
    Mean
            :52.65
                              :51.85
                                                :52.41
                      Mean
                                        Mean
##
    3rd Qu.:59.00
                      3rd Qu.:58.00
                                        3rd Qu.:61.00
##
    Max.
            :75.00
                      Max.
                              :74.00
                                        Max.
                                                :71.00
##
        awards
                           cid
                             : 1.00
##
    Min.
            :0.00
                     Min.
##
    1st Qu.:0.00
                     1st Qu.: 5.00
    Median:1.00
                     Median :10.50
##
##
    Mean
            :1.67
                     Mean
                             :10.43
##
    3rd Qu.:2.00
                     3rd Qu.:15.00
##
    Max.
            :7.00
                     Max.
                             :20.00
This will give you the quartiles for all the quantitative variables. To deal with the categorical
```

```
variables ses and schtyp, you can count up how many times each one appears:
ml %>% count(ses)
## # A tibble: 3 x 2
##
     ses
                 n
##
     <chr> <int>
## 1 high
                58
## 2 low
                47
## 3 middle
ml %>% count(schtyp)
## # A tibble: 2 x 2
##
     schtyp
                  n
##
     <chr>
              <int>
## 1 private
                 32
## 2 public
                168
Or get the distinct ones:
ml %>% distinct(ses)
## # A tibble: 3 x 1
##
     ses
##
     <chr>
## 1 low
## 2 middle
## 3 high
ml %>% distinct(schtyp)
## # A tibble: 2 x 1
##
     schtyp
##
     <chr>
## 1 public
## 2 private
Or turn it into a factor (it's currently text) and get its levels:
levels(factor(ml$ses))
## [1] "high"
                 "low"
                           "middle"
levels(factor(ml$schtyp))
## [1] "private" "public"
Going back to the quantitative variables, to get the Q1 and Q3 of everything you want, perhaps
the most obvious way is to construct a great big summarize:
ml %>% summarize(
            mathq1=quantile(math, 0.25),
           mathq3=quantile(math, 0.75),
            scienceq1=quantile(science, 0.25),
            scienceq3=quantile(science, 0.75),
            socstq1=quantile(socst, 0.25),
            socstq3=quantile(socst, 0.75)
        )
## # A tibble: 1 x 6
     mathq1 mathq3 scienceq1 scienceq3 socstq1 socstq3
##
      <dbl>
              <dbl>
                         <dbl>
                                    <dbl>
                                             <dbl>
                                                     <dbl>
## 1
                                       58
                                                46
Or, you can use summarize to get Q1 and Q3 of everything quantitative, and then pick out
what you want. With more than one summary statistic, you have to put them in a list. These
```

```
variants of summarize use the squiggle and dot thing like map does: "for each variable that is
numeric, work out the first and third quartiles of it":
ml %>% summarize_if(is.numeric, list(q1=~quantile(., 0.25), q3=~quantile(., 0.75)))
   # A tibble: 1 x 18
##
     `row,_q1` id_q1 read_q1 write_q1 math_q1 science_q1 socst_q1 awards_q1 cid_q1
##
          <dbl> <dbl>
                         <dbl>
                                   <dbl>
                                            <dbl>
                                                        <dbl>
                                                                  <dbl>
                                                                            <dbl>
                                                                                   <db1>
## 1
           50.8 50.8
                            44
                                    45.8
                                               45
                                                           44
                                                                     46
                                                                                 \cap
## # ... with 9 more variables: `row,_q3` <dbl>, id_q3 <dbl>, read_q3 <dbl>,
       write_q3 <dbl>, math_q3 <dbl>, science_q3 <dbl>, socst_q3 <dbl>,
       awards_q3 <dbl>, cid_q3 <dbl>
## #
There are more columns than will display in my output, but you'll be able to scroll across to
get the ones you want.
Or select the columns first, and then summarize all of them:
ml %>% select(math, science, socst) %>%
    summarize_all(list(q1=~quantile(., 0.25), q3=~quantile(., 0.75)))
## # A tibble: 1 x 6
##
     math_q1 science_q1 socst_q1 math_q3 science_q3 socst_q3
##
        <dbl>
                   <dbl>
                             <dbl>
                                      <dbl>
                                                  <dbl>
                                                            <dbl>
## 1
                       44
                                 46
                                         59
                                                     58
Find a way that works. ses has levels low, middle, high; schtyp has levels private and
public; the quartiles of math are 45 and 59; of science are 44 and 58, and of socst are 46
and 61.
```

(e) (3 marks) Make a data frame that includes all combinations of values you obtained in the previous part. (I have five variables, two of which are categorical, and one of those has three levels, so I have 48 rows.)

```
Solution: crossing, as ever for these. I like to set up the pairs or trios of values for each
variable first, using plural names. But if you can do it all in one go without getting confused,
go for it:
sess <- c("low", "middle", "high")</pre>
schtyps <- c("private", "public")</pre>
sciences \leftarrow c(44, 58)
socsts < -c(46, 61)
maths <-c(45, 59)
new <- crossing(ses=sess, schtyp=schtyps, science=sciences, socst=socsts, math=maths)
new
## # A tibble: 48 x 5
##
      ses
             schtyp science socst
                                      math
##
      <chr> <chr>
                        <dbl> <dbl> <dbl>
##
    1 high private
                           44
                                  46
                                         45
##
    2 high private
                           44
                                  46
                                         59
##
    3 high
            private
                           44
                                  61
                                         45
##
    4 high
             private
                           44
                                  61
                                         59
##
    5 high
                           58
                                  46
                                         45
            private
    6 high
            private
                           58
                                  46
                                         59
                           58
                                         45
##
    7 high
            private
                                  61
                           58
    8 high
             private
                                  61
                                         59
   9 high public
                            44
                                  46
                                         45
## 10 high public
                           44
                                  46
```

```
## # ... with 38 more rows
48 rows.
```

(f) (3 marks) Obtain predictions, using your model that came from step and the data frame of values to predict for that you just created. Be sure to obtain predicted *probabilities* for each response category. Display (some of) your predictions side by side with the values they are predictions for. Save your final data frame.

```
Solution: Now that we've done the setup, that is this:
p <- predict(ml.2, new, type="probs")</pre>
preds <- cbind(new, p)</pre>
preds
##
          ses schtyp science socst math
                                            academic
                                                         general
                                                                     vocation
## 1
                            44
                                  46
                                       45 0.7134056 0.19467533 0.091919090
        high private
                                  46
##
        high private
                            44
                                       59 0.9355143 0.04676994 0.017715808
##
   .3
        high private
                            44
                                  61
                                       45 0.8533882 0.11961922 0.026992544
##
   4
        high private
                            44
                                  61
                                       59 0.9705639 0.02492415 0.004511933
## 5
                            58
                                  46
        high private
                                       45 0.4796188 0.41308175 0.107299437
## 6
        high private
                            58
                                  46
                                       59 0.8398621 0.13252254 0.027615351
                            58
                                       45 0.6678579 0.29546346 0.036678643
##
   7
        high private
                                  61
## 8
        high private
                            58
                                  61
                                       59 0.9181696 0.07441913 0.007411284
                                       45 0.4050374 0.21106025 0.383902398
##
   9
        high public
                            44
                                  46
   10
        high
              public
                                  46
                                       59 0.8098660 0.07731549 0.112818486
##
                                       45 0.6665147 0.17840254 0.155082736
##
   11
        high
              public
                            44
                                  61
        high
                                       59 0.9231602 0.04526998 0.031569821
##
   12
              public
                            44
                                  61
        high
##
   13
              public
                            58
                                  46
                                       45 0.2330791 0.38333639 0.383584523
## 14
        high
              public
                            58
                                  46
                                       59 0.6480070 0.19525333 0.156739636
              public
##
   15
        high
                            58
                                  61
                                       45 0.4446798 0.37566783 0.179652424
                            58
##
   16
        high
              public
                                  61
                                       59 0.8236198 0.12747511 0.048905067
   17
                            44
                                  46
                                       45 0.5611194 0.36770859 0.071171992
##
         low private
##
   18
         low private
                            44
                                  46
                                       59 0.8781945 0.10543411 0.016371402
##
   19
         low private
                            44
                                  61
                                       45 0.7311286 0.24610598 0.022765414
##
   20
                            44
                                  61
                                       59 0.9378700 0.05783796 0.004292060
         low private
##
   21
         low private
                            58
                                  46
                                        45 0.3040867 0.62894290 0.066970449
##
   22
                                       59 0.7085685 0.26849592 0.022935544
         low private
                            58
                                  46
##
   23
         low private
                            58
                                  61
                                       45 0.4724829 0.50197244 0.025544648
##
   24
                            58
                                       59 0.8315402 0.16185233 0.006607518
         low private
                                  61
##
   25
         low
              public
                            44
                                  46
                                       45 0.3140279 0.39296476 0.293007372
##
   26
              public
                            44
                                  46
                                       59 0.7318525 0.16778415 0.100363373
         low
##
   27
               public
                            44
                                        45 0.5342341 0.34339734 0.122368578
         low
                                  61
   28
                            44
                                       59 0.8684870 0.10227527 0.029237686
##
         low
               public
                                  61
   29
                                       45 0.1522144 0.60118276 0.246602854
##
         low
              public
                            58
                                  46
##
   30
         low
               public
                            58
                                  46
                                       59 0.5097610 0.36885813 0.121380889
##
   31
         low
              public
                            58
                                  61
                                       45 0.2918452 0.59208398 0.116070791
   32
                            58
##
         low
              public
                                  61
                                       59 0.6992339 0.25989333 0.040872777
   33 middle private
                            44
                                  46
                                       45 0.5053223 0.27710018 0.217577504
      middle private
                            44
                                  46
                                       59 0.8592933 0.08632801 0.054378635
##
##
   35 middle private
                            44
                                  61
                                       45 0.7207859 0.20302733 0.076186800
                                       59 0.9370842 0.04835806 0.014557722
   36 middle private
                            44
                                  61
## 37 middle private
                            58
                                  46
                                        45 0.2874917 0.49757548 0.214932851
## 38 middle private
                            58
                                  46
                                       59 0.7007880 0.22220936 0.077002627
```

```
## 39 middle private
                          58
                                 61
                                      45 0.4824966 0.42895116 0.088552275
## 40 middle private
                          58
                                 61
                                      59 0.8404424 0.13688742 0.022670171
                          44
## 41 middle public
                                 46
                                      45 0.1917717 0.20081201 0.607416331
## 42 middle
             public
                          44
                                 46
                                      59 0.6033659 0.11575188 0.280882177
## 43 middle
              public
                          44
                                 61
                                      45 0.4318847 0.23230203 0.335813256
## 44 middle
              public
                          44
                                 61
                                      59 0.8245222 0.08125115 0.094226640
## 45 middle
             public
                          58
                                 46
                                      45 0.1019926 0.33708496 0.560922421
## 46 middle
             public
                          58
                                 46
                                      59 0.4142840 0.25084807 0.334867934
                                      45 0.2470510 0.41940845 0.333540553
## 47 middle
              public
                          58
                                 61
## 48 middle public
                          58
                                 61
                                      59 0.6624924 0.20605039 0.131457174
```

Mine displays all 48 rows. Yours will probably display the first 10, with a Next button to click on to see more.

I saved my final output to use again in a moment. (That's why I asked you to save yours.)

(g) (3 marks) Assess the effect of ses on the probabilities of a student being in each of the three programs by displaying appropriate rows of your saved data frame of predictions (using slice or filter). Which programs do students of each socioeconomic status mostly end up in?

**Solution:** Find three rows of your data frame of predictions that have (i) the *same* values for schtyp, science, socst, math and (ii) *different* values for ses. This is assessing what happens when you change ses but leave everything else the same.

I looked through my data frame and saw that rows 1, 17, 33 would do this:

```
preds %>% slice(1, 17, 33)
##
        ses schtyp science socst math academic
                                                    general
                                     45 0.7134056 0.1946753 0.09191909
       high private
                          44
## 2
        low private
                          44
                                46
                                     45 0.5611194 0.3677086 0.07117199
## 3 middle private
                          44
                                46
                                     45 0.5053223 0.2771002 0.21757750
```

The precise rows depend on the order you used in the crossing earlier. Or you could use filter to pick out the rows where you want the values to stay the same (schtyp, science, socst, math). These are actually different probabilities (just fine) but the comparison is the same:

```
preds %>% filter(
              schtyp=="public",
              science==44,
              socst==46.
              math==45
        ses schtyp science socst math academic
##
                                                   general vocation
## 1
       high public
                        44
                                    45 0.4050374 0.2110602 0.3839024
                               46
## 2
        low public
                        44
                               46
                                    45 0.3140279 0.3929648 0.2930074
## 3 middle public
                        44
                                    45 0.1917717 0.2008120 0.6074163
```

Use either value for each of those; it doesn't matter.

So, what are those predictions telling us? High-SES students are most likely to go into the academic program, while middle-SES students are most likely to go into the vocational program.

What you say about low-SES students will depend on precisely which values you pick for the other variables, but those seem most likely to end up in the general program, at least for my values.

Comment, if you want to add any: the high-SES students may mainly have in mind to go to university later, so it would make sense that most of them would go into the academic program.

Maybe most of the middle-SES students reckon they're *not* going to university, so they would tend to go into a program that would more immediately help them get a job.

(h) (3 marks) By looking at an appropriate choice of rows from your data frame of predictions, assess the effect of an increase in math score on a student's choice of program.

**Solution:** Now pick two rows from your data frame of predictions that differ on the value of  $\mathtt{math}$  but are the same on everything else. In mine, rows 1 and 2 will do that:

Or, use filter to grab *any* rows of math, but pick a value of the others to be consistent with, for example:

```
preds %>% filter(
          ses=="middle",
          schtyp=="public",
          science==58,
          socst == 46
          )
                                                    general vocation
##
        ses schtvp science socst math academic
## 1 middle public
                         58
                               46
                                    45 0.1019926 0.3370850 0.5609224
## 2 middle public
                         58
                               46
                                    59 0.4142840 0.2508481 0.3348679
```

This gives a different pair of rows, but the picture should be the same whichever ones you pick.<sup>2</sup> The first time, I gave **socst** a value that I didn't predict for at all, and wondered why the result had no rows!

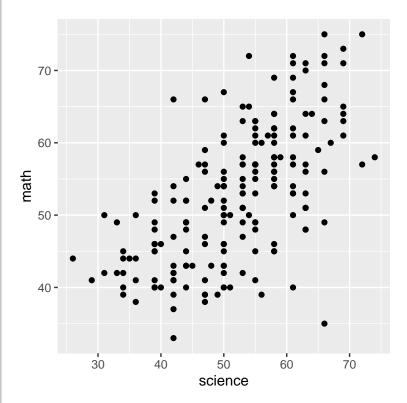
The effect, then: whichever values you look at, a student with a higher math score is much more likely to go into the academic program, and much less likely to go into either of the others.

Extra: the effect seems to be biggest for math. I tried the others too. Science: preds %>% slice(1,5)

```
## ses schtyp science socst math academic general vocation
## 1 high private 44 46 45 0.7134056 0.1946753 0.09191909
## 2 high private 58 46 45 0.4796188 0.4130818 0.10729944
```

This effect looks backwards to me, but remember it's the effect of a change in science all else remaining equal, which may not be the case; a student with a high science score may tend to have a high math score as well:

```
ggplot(ml, aes(x=science, y=math)) + geom_point()
```



At least kinda. So if you know about math, you kinda know about science too.

The effect of social studies is like the one for math:

though evidently a student who scores high on social studies will tend to take *different* academic courses than one who scores high on math.

- 3. Work through, or at least read through, chapter 21 of PASIAS.
- 4. A small clinical trial is run to compare two combination treatments in patients with advanced gastric cancer. Twenty participants with stage IV gastric cancer who consent to participate in the trial are randomly assigned to receive chemotherapy before surgery or chemotherapy after surgery. The primary outcome is death and participants are followed for up to 48 months (4 years) following enrollment into the trial. (The foregoing is directly taken from the website where I got the data; it's not how I would have written it, but I think it's good for you to see how this kind of thing is written by others.)

The data are shown in http://ritsokiguess.site/STAD29/chemo.csv. There are three columns: whether each patient had chemotherapy before or after surgery, how many months they were observed for, and whether or not they were observed to have died.

(a) (2 marks) Read in and display (some of) the data.

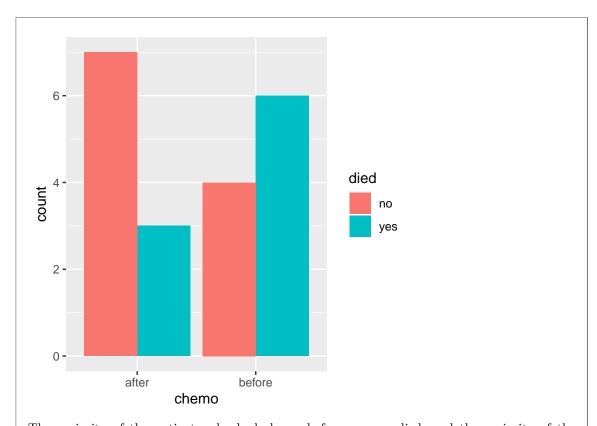
```
Solution: Nothing new here:
my_url <- "http://ritsokiguess.site/STAD29/chemo.csv"
trial <- read_csv(my_url)</pre>
```

```
## Parsed with column specification:
## cols(
## chemo = col_character(),
## months = col_double(),
## died = col_character()
## )
trial
## # A tibble: 20 x 3
     chemo months died
##
##
     <chr> <dbl> <chr>
##
  1 before
             8 yes
##
  2 before 12 yes
   3 before 26 yes
             14 yes
   4 before
##
##
   5 before
                21 yes
##
  6 before
              27 yes
   7 before
                8 no
## 8 before
                32 no
## 9 before
                20 no
## 10 before
                40 no
## 11 after
              33 yes
              28 yes
41 yes
## 12 after
## 13 after
## 14 after
                48 no
## 15 after
                48 no
                25 no
## 16 after
## 17 after
                37 no
## 18 after
                48 no
## 19 after
                25 no
## 20 after
                43 no
```

I got all 20 rows, but you will probably see only the first ten, and you can scroll down to see that there were also patients who had chemotherapy after surgery.

Extra: if you ignore the months column, you could make a graph of chemo and died; these are both categorical so a grouped bar chart would be the thing:

```
ggplot(trial, aes(x=chemo, fill=died)) + geom_bar(position="dodge")
```



The majority of the patients who had chemo before surgery died, and the majority of the patients who had chemo after surgery were still alive when last observed. This suggests that having chemo after surgery is better, but it fails to consider how long the patients were observed for. We will consider this in the modelling we do shortly, but for now consider the possibility that the patients who had chemo after surgery might not have been observed for very long, and that would make this treatment look better than it actually was.

(b) (2 marks) In the context of *this* data set, what would a censored observation look like? Give an example of one from the data set.

**Solution:** A censored observation is one where the event of interest (what the description calls the "primary endpoint"), that is, death, was never observed to occur. For example, patient 7 was only observed for 8 weeks and was never observed to die. (Or patients 8, 9 or 10.)

Extra: patients can be censored for a variety of reasons: maybe they decide to withdraw from the study, or maybe they stopped coming to doctor's appointments (I think, though, this study was in a hospital). Or they moved house or hospital. Or, maybe they died of something else and not of the gastric cancer directly.

(c) (3 marks) Create a response variable suitable for a Cox proportional-hazards regression. Do this outside the data frame, and display at least some of its values. How are the censored observations distinguished?

```
Solution: At some point you'll need this:
library(survival)
and then (I used my customary name y for a response):
y <- with(trial, Surv(months, died=="yes"))</pre>
```

```
У
##
    [1]
             12 26 14 21 27
                                      8+ 32+ 20+ 40+ 33 28 41 48+ 48+ 25+ 37+ 48+ 25+
## [20] 43+
The censored observations are displayed with a plus.
Extra: though y looks like an ordinary vector and you would expect to be able to create it with
mutate, it actually has two columns, and it just displays by default with special formatting.
This is how it really looks:
print.default(y)
##
          time status
##
    [1,]
             8
                      1
##
    [2,]
            12
                      1
    [3,]
##
            26
                      1
##
    [4,]
            14
                      1
##
    [5,]
            21
                      1
##
    [6,]
            27
                      1
##
    [7,]
             8
                      0
##
    [8,]
            32
                      0
    [9,]
                      0
##
            20
## [10,]
            40
                      0
##
   [11,]
            33
                      1
##
   [12,]
            28
                      1
   [13,]
            41
                      1
   [14,]
            48
                      0
##
## [15,]
            48
                      0
                      0
## [16,]
            25
## [17,]
            37
                      0
## [18,]
            48
                      0
## [19,]
            25
                      0
## [20,]
                      0
            43
## attr(,"type")
## [1] "right"
## attr(,"class")
## [1] "Surv"
The censoring information is obtained from the status column in there.
```

(d) (3 marks) Fit a Cox proportional-hazards model to predict survival time from the treatment chemo. Display the results.

care of the censoring:
trial.1 <- coxph(y~chemo, data=trial)
summary(trial.1)
## Call:
## coxph(formula = y ~ chemo, data = trial)
##
## n= 20, number of events= 9
##
## coef exp(coef) se(coef) z Pr(>|z|)

0.8474 2.321

0.0203 \*

Solution: This is very like a regression, only the computation behind the scenes properly takes

7.1515

chemobefore 1.9673

##

## ---

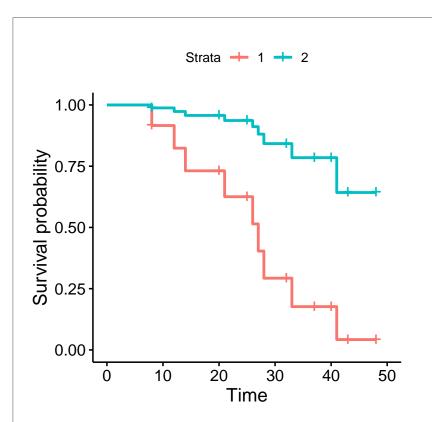
```
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
##
               exp(coef) exp(-coef) lower .95 upper .95
##
  chemobefore
                  7.151
                             0.1398
                                        1.358
                                                  37.65
##
## Concordance= 0.748 (se = 0.047)
## Likelihood ratio test= 6.46 on 1 df,
                                           p=0.01
## Wald test
                        = 5.39 on 1 df,
                                          p=0.02
## Score (logrank) test = 6.92 on 1 df,
                                          p=0.009
```

(e) (2 marks) Is there any evidence that one of the treatments is better than the other? If so, which treatment is better? Explain briefly. (Hint: you will get this backwards unless you are careful.)

Solution: The P-value of 0.0203 on the chemobefore line says that there is a significant difference in survival between having chemo before surgery and having it after (the baseline). To see which way around it goes, look at the estimate (coefficient) 1.9673 for chemobefore. This is positive, and larger than the 0 for the baseline chemoafter. A more positive coefficient goes with a higher hazard of event: that is, the event (death) is more likely to happen sooner for before than for after. Thus, having chemotherapy after surgery is better for survival. Extra: this is easy to get backwards, but remember that a higher hazard of death is bad. (Usually the event of interest is something bad like death, but not always; I had an example once where the event of interest was passing a driving test, and in that case a higher hazard of event was a good thing in that people were more likely to pass the driving test sooner.)

(f) (3 marks) Make a suitable plot that shows which treatment is better, and explain briefly why it shows that.

```
Solution: This is ggsurvplot. That goes like this:
library(survminer)
## Loading required package: ggpubr
## Loading required package: magrittr
##
## Attaching package: 'magrittr'
## The following object is masked from 'package:purrr':
##
##
      set names
## The following object is masked from 'package:tidyr':
##
##
      extract
trial.new <- tribble(</pre>
    ~chemo.
    "before",
    "after"
)
s1 <- do.call(survfit, list(formula=trial.1, newdata=trial.new, data=trial))
ggsurvplot(s1, conf.int = F)
```



The first step is to create a new data frame with values to predict for (survfit is like predict). Our one explanatory variable is chemo and its values are before and after (either order is good). Then that rather arcane thing with do.call in it (copy the example from lecture notes and modify), and finally the plot. The strata in the plot are in the same order as in trial.new. By way of comment: the best survival goes with the upper right survival curve, because that is associated with surviving longest. In mine, that is stratum 2, which is after. (If you had after first in your data frame, it'll be your stratum 1, and your red and blue survival curves will be switched around from mine.)

In short, if the event is undesirable, the upper right survival curve goes with avoiding it for as long as possible, and is thus best.

## Notes

<sup>&</sup>lt;sup>1</sup>Oddly, step works but drop1 doesn't. I have yet to figure out why that is.

<sup>&</sup>lt;sup>2</sup>The principle with filter is that you omit the explanatory variable whose effect you want to see, and supply values, any ones, for the others.