Multivariate analysis of variance (MANOVA)

Multivariate analysis of variance

- Standard ANOVA has just one response variable.
- What if you have more than one response?
- Try an ANOVA on each response separately.
- But might miss some kinds of interesting dependence between the responses that distinguish the groups.

Packages

```
library(car)
library(tidyverse)
```

Small example

- Measure yield and seed weight of plants grown under 2 conditions: low and high amounts of fertilizer.
- Data (fertilizer, yield, seed weight):

```
url <- "http://www.utsc.utoronto.ca/~butler/d29/manova1.txt"
hilo <- read_delim(url, " ")
```

```
##
## -- Column specification ------------------------------
## cols(
##   fertilizer = col_character(),
##   yield = col_double(),
##   weight = col_double()
## )
```

- 2 responses, yield and seed weight.

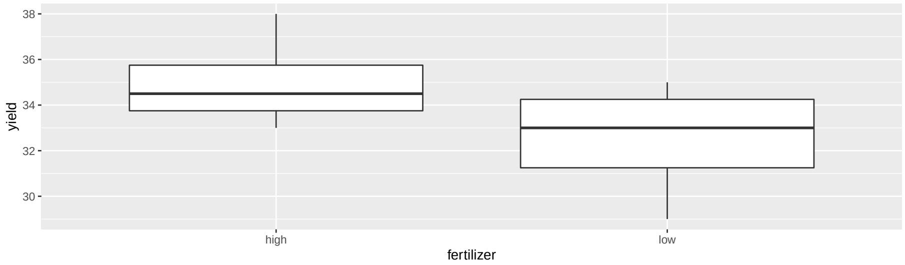
The data

```
hilo
```

fertilizer	yield	weight
low	34	10
low	29	14
low	35	11
low	32	13
high	33	14
high	38	12
high	34	13
high	35	14

Boxplot for yield for each fertilizer group

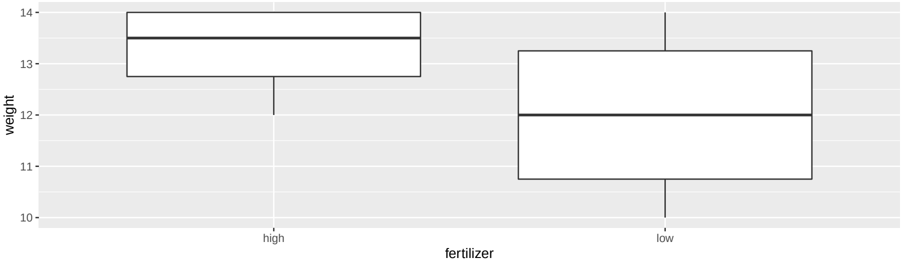
```r
ggplot(hilo, aes(x = fertilizer, y = yield)) + geom_boxplot()
```



Yields overlap for fertilizer groups.

Boxplot for weight for each fertilizer group

```
ggplot(hilo, aes(x = fertilizer, y = weight)) + geom_boxplot()
```



Weights overlap for fertilizer groups.

ANOVAs for yield and weight

```
hilo.y <- aov(yield ~ fertilizer, data = hilo)
summary(hilo.y)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## fertilizer   1   12.5  12.500   2.143  0.194
## Residuals    6   35.0   5.833
```

```
hilo.w <- aov(weight ~ fertilizer, data = hilo)
summary(hilo.w)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## fertilizer   1  3.125   3.125   1.471  0.271
## Residuals    6 12.750   2.125
```

Neither response depends significantly on fertilizer. But…

Plotting both responses at once

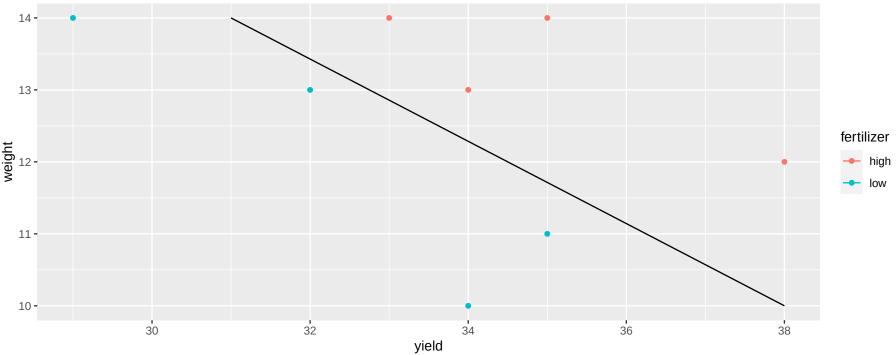
- Have two response variables (not more), so can plot the response variables against *each other*, labelling points by which fertilizer group they're from.
- First, create data frame with points $(31, 14)$ and $(38, 10)$ (why? Later):

```
d <- tribble(
  ~line_x, ~line_y,
  31, 14,
  38, 10
)
```

- Then plot data as points, and add line through points in `d`:

```
g <- ggplot(hilo, aes(x = yield, y = weight,
                      colour = fertilizer)) + geom_point() +
  geom_line(data = d,
```

The plot



Comments

- Graph construction:
  - Joining points in `d` by line.
  - `geom_line` inherits `colour` from `aes` in `ggplot`.
  - Data frame `d` has no `fertilizer` (previous `colour`), so have to unset.
- Results:
  - High-fertilizer plants have both yield and weight high.
  - True even though no sig difference in yield or weight individually.
  - Drew line separating highs from lows on plot.

MANOVA finds multivariate differences

- Is difference found by diagonal line significant? MANOVA finds out.

```
response <- with(hilo, cbind(yield, weight))
hilo.1 <- manova(response ~ fertilizer, data = hilo)
summary(hilo.1)
```

```
##             Df  Pillai approx F num Df den Df
## fertilizer  1 0.80154   10.097      2      5
## Residuals   6
##              Pr(>F)
## fertilizer 0.01755 *
## Residuals
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Yes! Difference between groups is *diagonally*, not just up/down (weight) or left-right (yield). The *yield-weight combination* matters.

Strategy

- Create new response variable by gluing together columns of responses, using `cbind`.
- Use `manova` with new response, looks like `lm` otherwise.
- With more than 2 responses, cannot draw graph. What then?
- If MANOVA test significant, cannot use Tukey. What then?
- Use *discriminant analysis* (of which more later).

Another way to do MANOVA

Install (once) and load package `car`:

```
library(car)
```

Another way...

```
hilo.2.lm <- lm(response ~ fertilizer, data = hilo)
hilo.2 <- Manova(hilo.2.lm)
hilo.2
```

```
##
## Type II MANOVA Tests: Pillai test statistic
##            Df test stat approx F num Df den Df
## fertilizer  1   0.80154   10.097      2      5
##             Pr(>F)
## fertilizer 0.01755 *
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Same result as small-m manova.
- Manova will also do *repeated measures*, coming up later.

Another example: peanuts

- Three different varieties of peanuts (mysteriously, 5, 6 and 8) planted in two different locations.
- Three response variables: `y`, `smk` and `w`.

```
u <- "http://www.utsc.utoronto.ca/~butler/d29/peanuts.txt"
peanuts.orig <- read_delim(u, " ")
```

```
##
## -- Column specification ----------------------------
## cols(
##   obs = col_double(),
##   location = col_double(),
##   variety = col_double(),
##   y = col_double(),
##   smk = col_double(),
##   w = col_double()
## )
```

The data

```
peanuts.orig
```

obs	location	variety	y	smk	w
1	1	5	195.3	153.1	51.4
2	1	5	194.3	167.7	53.7
3	2	5	189.7	139.5	55.5
4	2	5	180.4	121.1	44.4
5	1	6	203.0	156.8	49.8
6	1	6	195.9	166.0	45.8
7	2	6	202.7	166.1	60.4
8	2	6	197.6	161.8	54.1
9	1	8	193.5	164.5	57.8
10	1	8	187.0	165.1	58.6
11	2	8	201.5	166.8	65.0
12	2	8	200.0	173.8	67.2

Setup for analysis

```
peanuts <- peanuts.orig %>%
  mutate(
    location = factor(location),
    variety = factor(variety)
  )
response <- with(peanuts, cbind(y, smk, w))
head(response)
```

```
##          y   smk    w
## [1,] 195.3 153.1 51.4
## [2,] 194.3 167.7 53.7
## [3,] 189.7 139.5 55.5
## [4,] 180.4 121.1 44.4
## [5,] 203.0 156.8 49.8
## [6,] 195.9 166.0 45.8
```

Analysis (using Manova)

```
peanuts.1 <- lm(response ~ location * variety, data = peanuts)
peanuts.2 <- Manova(peanuts.1)
peanuts.2
```

```
##
## Type II MANOVA Tests: Pillai test statistic
##                   Df test stat approx F num Df
## location           1   0.89348  11.1843      3
## variety            2   1.70911   9.7924      6
## location:variety   2   1.29086   3.0339      6
##                   den Df   Pr(>F)
## location               4 0.020502 *
## variety               10 0.001056 **
## location:variety      10 0.058708 .
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comments

- Interaction not quite significant, but main effects are.
- Combined response variable `(y,smk,w)` definitely depends on location and on variety
- Weak dependence of `(y,smk,w)` on the location-variety *combination.*
- Understanding that dependence beyond our scope right now.