Factor Analysis

Principal components and factor analysis

- Principal components:
  - Purely mathematical.
  - Find eigenvalues, eigenvectors of correlation matrix.
  - No testing whether observed components reproducible, or even probability model behind it.
- Factor analysis:
  - some way towards fixing this (get test of appropriateness)
  - In factor analysis, each variable modelled as: "common factor" (eg. verbal ability) and "specific factor" (left over).
  - Choose the common factors to "best" reproduce pattern seen in correlation matrix.
  - Iterative procedure, different answer from principal components.

Packages

```
library(lavaan) # for confirmatory, later
library(ggbiplot)
library(tidyverse)
```

Example

- 145 children given 5 tests, called PARA, SENT, WORD, ADD and DOTS. 3 linguistic tasks (paragraph comprehension, sentence completion and word meaning), 2 mathematical ones (addition and counting dots).
- Correlation matrix of scores on the tests:

```
para 1     0.722 0.714 0.203 0.095
sent 0.722 1     0.685 0.246 0.181
word 0.714 0.685 1     0.170 0.113
add  0.203 0.246 0.170 1     0.585
dots 0.095 0.181 0.113 0.585 1
```

- Is there small number of underlying "constructs" (unobservable) that explains this pattern of correlations?

To start: principal components

Using correlation matrix. Read that first:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/rex2.txt"
kids <- read_delim(my_url, " ")
kids
```

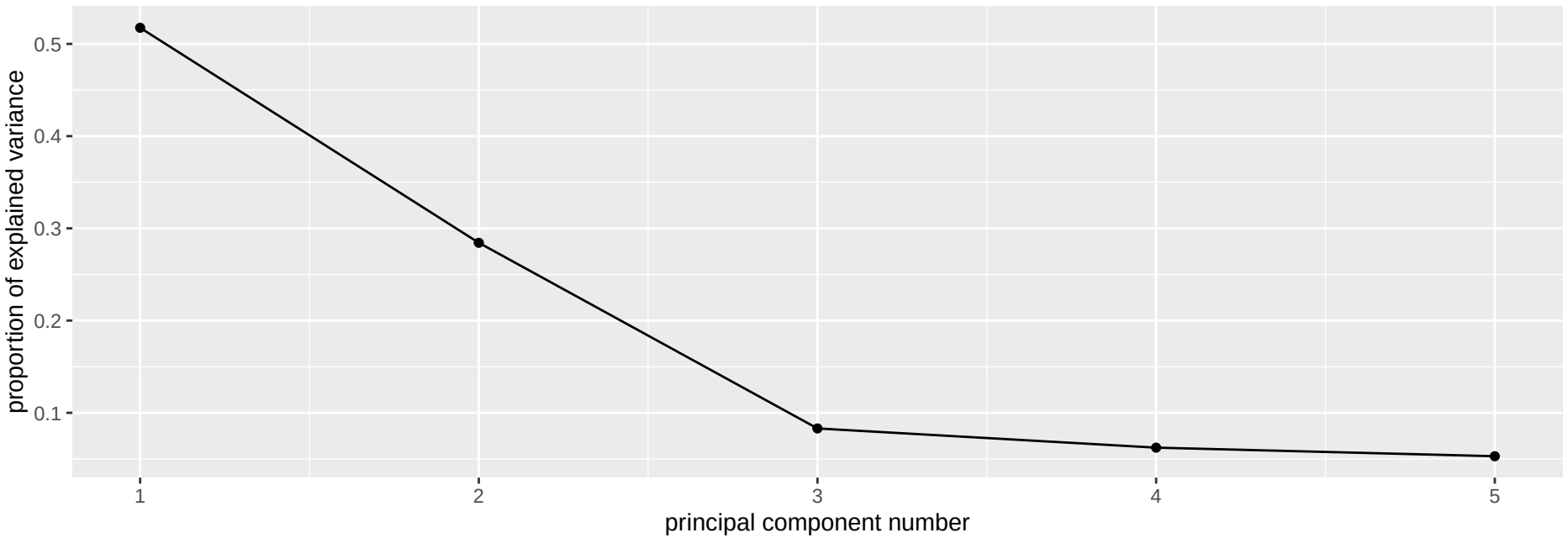
test	para	sent	word	add	dots
para	1.000	0.722	0.714	0.203	0.095
sent	0.722	1.000	0.685	0.246	0.181
word	0.714	0.685	1.000	0.170	0.113
add	0.203	0.246	0.170	1.000	0.585
dots	0.095	0.181	0.113	0.585	1.000

Principal components on correlation matrix

```
kids %>%
  select_if(is.numeric) %>%
  as.matrix() %>%
  princomp(covmat = .) -> kids.pc
```

Scree plot

```
ggscreeplot(kids.pc)
```



Principal component results

- Need 2 components. Loadings:

```
kids.pc$loadings
```

```
##
## Loadings:
##       Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## para   0.534  0.245  0.114          0.795
## sent   0.542  0.164         0.660 -0.489
## word   0.523  0.247 -0.144 -0.738 -0.316
## add    0.297 -0.627  0.707
## dots   0.241 -0.678 -0.680         0.143
##
##                Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## SS loadings       1.0    1.0    1.0    1.0    1.0
## Proportion Var    0.2    0.2    0.2    0.2    0.2
## Cumulative Var    0.2    0.4    0.6    0.8    1.0
```

Comments

- First component has a bit of everything, though especially the first three tests.
- Second component rather more clearly `add` and `dots`.
- No scores, plots since no actual data.

Factor analysis

- Specify number of factors first, get solution with exactly that many factors.
- Includes hypothesis test, need to specify how many children wrote the tests.
- Works from correlation matrix via `covmat` or actual data, like `princomp`.
- Introduces extra feature, *rotation*, to make interpretation of loadings (factor-variable relation) easier.

Factor analysis for the kids data

- Create "covariance list" to include number of children who wrote the tests.
- Feed this into `factanal`, specifying how many factors (2).

```
km <- kids %>%
  select_if(is.numeric) %>%
  as.matrix()
km2 <- list(cov = km, n.obs = 145)
kids.f2 <- factanal(factors = 2, covmat = km2)
```

Uniquenesses

```
kids.f2$uniquenesses
```

```
##      para      sent      word       add      dots
## 0.2424457 0.2997349 0.3272312 0.5743568 0.1554076
```

- Uniquenesses say how "unique" a variable is (size of specific factor). Small uniqueness means that the variable is summarized by a factor (good).
- Very large uniquenesses are bad; add's uniqueness is largest but not large enough to be worried about.
- Also see "communality" for this idea, where *large* is good and *small* is bad.

Loadings

```
kids.f2$loadings
```

```
##
## Loadings:
##      Factor1 Factor2
## [1,] 0.867
## [2,] 0.820   0.166
## [3,] 0.816
## [4,] 0.167   0.631
## [5,]         0.918
##
##                Factor1 Factor2
## SS loadings      2.119   1.282
## Proportion Var   0.424   0.256
## Cumulative Var   0.424   0.680
```

- Loadings show how each factor depends on variables. Blanks indicate "small", less than 0.1.

Comments

- Factor 1 clearly the "linguistic" tasks, factor 2 clearly the "mathematical" ones.
- Two factors together explain 68% of variability (like regression R-squared).
- Which variables belong to which factor is *much* clearer than with principal components.

Are 2 factors enough?

```
kids.f2$STATISTIC
```

```
## objective
## 0.5810578
```

```
kids.f2$dof
```

```
## [1] 1
```

```
kids.f2$PVAL
```

```
## objective
##  0.445898
```

P-value not small, so 2 factors OK.

1 factor

```
kids.f1 <- factanal(factors = 1, covmat = km2)
kids.f1$STATISTIC
```

```
## objective
##  58.16534
```

```
kids.f1$dof
```

```
## [1] 5
```

```
kids.f1$PVAL
```

```
##    objective
## 2.907856e-11
```

1 factor rejected (P-value small). Definitely need more than 1.

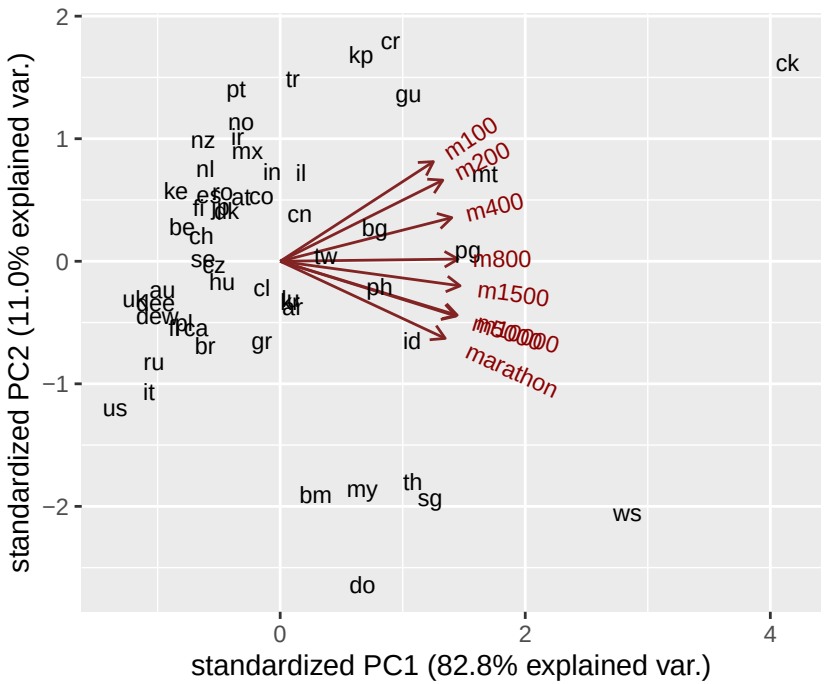
Track running records revisited

Read the data, run principal components, get biplot:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/men_track_f
track <- read_table(my_url)
track %>% select_if(is.numeric) -> track_num
track.pc <- princomp(track_num, cor = T)
g2 <- ggbiplot(track.pc, labels = track$country)
```

The biplot

g2



Benefit of rotation

- 100m and marathon arrows almost perpendicular, but components don't match anything much:
- sprinting: bottom left and top right
- distance running: top left and bottom right.
- Can we arrange things so that components (factors) correspond to something meaningful?

Track records by factor analysis

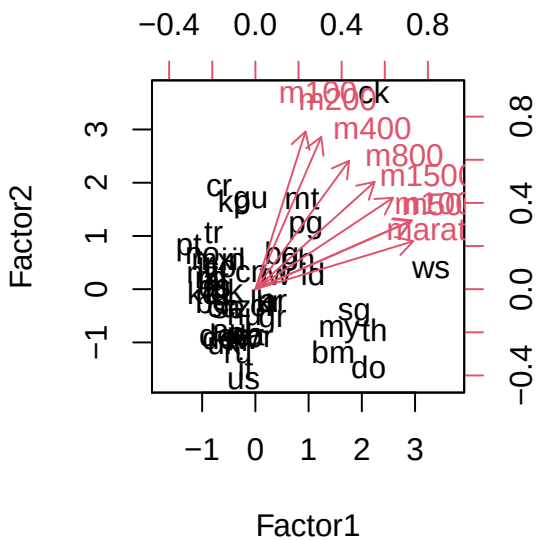
Obtain factor scores (have actual data):

```
track %>%
  select_if(is.numeric) %>%
  factanal(2, scores = "r") -> track.f
```

Track data biplot

Not so nice-looking:

```
biplot(track.f$scores, track.f$loadings,
  xlabs = track$country
)
```



Comments

- This time 100m “up” (factor 2), marathon “right” (factor 1).
- Countries most negative on factor 2 good at sprinting.
- Countries most negative on factor 1 good at distance running.

Rotated factor loadings

```
track.f$loadings
```

```
##
## Loadings:
##          Factor1 Factor2
## m100     0.291   0.914
## m200     0.382   0.882
## m400     0.543   0.744
## m800     0.691   0.622
## m1500    0.799   0.530
## m5000    0.901   0.394
## m10000   0.907   0.399
## marathon 0.915   0.278
##
##                Factor1 Factor2
## SS loadings      4.112   3.225
## Proportion Var   0.514   0.403
## Cumulative Var   0.514   0.917
```

Which countries are good at sprinting or distance running?

Make a data frame with the countries and scores in:

```
scores <- data.frame(
  country = track$country,
  track.f$scores
)
scores %>% slice(1:6)
```

country	Factor1	Factor2
ar	0.3363378	-0.2651512
au	-0.4939579	-0.8121335
at	-0.7419991	0.1764151
be	-0.7960275	-0.2388525
bm	1.4654159	-1.1704466
br	0.0778016	-0.8871291

The best sprinting countries

Most negative on factor 2:

```
scores %>%
  arrange(Factor2) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Country, Factor1, Factor2) %>%
  slice(1:10)
```

Country	Factor1	Factor2
United States of America	-0.2194270	-1.7251036
Italy	-0.1843670	-1.4990521
Dominican Republic	2.1290655	-1.4666402
Russian Federation	-0.3247311	-1.2236590
Bermuda	1.4654159	-1.1704466
United Kingdom	-0.5896906	-1.0139983
France	-0.2530185	-0.9519162
West Germany	-0.4674888	-0.9079005
Canada	-0.1369016	-0.8920777
Brazil	0.0778016	-0.8871291

The best distance-running countries

Most negative on factor 1:

```
scores %>%
  arrange(Factor1) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Country, Factor1, Factor2) %>%
  slice(1:10)
```

Country	Factor1	Factor2
Portugal	-1.2509805	0.7836689
Norway	-0.9920727	0.6229956
New Zealand	-0.9813348	0.2660349
Kenya	-0.9749696	-0.0709948
Iran, Islamic Republic of	-0.9231505	0.5027121
Netherlands	-0.9078661	0.2394820
Romania	-0.8178386	0.1855500
Mexico	-0.8096291	0.5144676
Finland	-0.8094725	-0.0570522
Belgium	-0.7960275	-0.2388525

A bigger example: BEM sex role inventory

- 369 women asked to rate themselves on 60 traits, like "self-reliant" or "shy".
- Rating 1 "never or almost never true of me" to 7 "always or almost always true of me' '.
- 60 personality traits is a lot. Can we find a smaller number of factors that capture aspects of personality?
- The whole BEM sex role inventory on next page.

The whole inventory

1. self reliant
2. yielding
3. helpful
4. defends own beliefs
5. cheerful
6. moody
7. independent
8. shy
9. conscientious
10. athletic
11. affectionate
12. theatrical
13. assertive
14. flatterable
15. happy
16. strong personality
17. loyal
18. unpredictable
19. forceful
20. feminine
21. reliable
22. analytical
23. sympathetic
24. jealous
25. leadership ability
26. sensitive to other's needs
27. truthful
28. willing to take risks
29. understanding
30. secretive
31. makes decisions easily
32. compassionate
33. sincere
34. self-sufficient
35. eager to soothe hurt feelings
36. conceited
37. dominant
38. soft spoken
39. likable
40. masculine
41. warm
42. solemn
43. willing to take a stand
44. tender
45. friendly
46. aggressive
47. gullible
48. inefficient
49. acts as a leader
50. childlike
51. adaptable
52. individualistic
53. does not use harsh language
54. unsystematic
55. competitive
56. loves children
57. tactful
58. ambitious
59. gentle
60. conventional

Some of the data

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/factor.txt"
bem <- read_tsv(my_url)
bem
```

subno	helpful	reliant	defbel	yielding	cheerful	indpt	athlet	shy	assert	strpers	forceful	affect	flatter	loyal	analyt	feminine	sympathy	mood	sensidty	understand	compass	leader	absoot
1	7	7	5	5	7	7	7	1	7	7	2	7	4	7	1	2	6	3	7	7	7	6	7
2	5	6	6	6	2	3	3	3	4	1	3	5	4	5	6	5	5	4	5	5	4	4	4
3	7	6	4	4	5	5	2	3	4	4	3	5	2	7	5	6	5	2	4	6	6	4	7
4	6	6	7	4	6	6	3	4	4	3	3	5	3	7	6	6	5	2	6	6	6	2	6
5	6	6	7	4	7	7	7	2	7	7	5	7	4	7	7	4	7	3	7	6	7	7	7
7	5	6	7	4	6	6	2	4	4	2	2	5	3	7	7	7	6	4	5	5	5	3	5
8	6	4	6	6	6	3	1	3	3	4	1	7	7	6	4	4	7	3	6	6	6	1	6
9	7	6	7	5	6	7	5	2	5	6	6	7	3	7	7	6	6	4	6	6	6	6	5
10	7	6	6	4	4	5	2	2	5	7	6	6	6	6	7	6	4	4	6	6	5	5	5
11	7	4	7	4	7	5	2	1	5	6	4	7	7	7	7	5	5	1	5	6	6	4	6
13	7	7	7	4	6	7	1	3	6	6	4	6	5	7	7	5	7	6	6	7	7	5	6
14	7	7	5	5	7	1	3	5	6	7	1	7	7	7	3	5	7	5	7	7	7	2	7
15	7	7	7	7	7	7	7	1	7	7	7	7	5	7	7	7	7	1	7	7	7	7	7
23	5	6	7	5	6	6	1	1	6	6	7	7	7	7	7	6	6	4	6	6	5	4	4
25	6	6	7	4	4	7	7	1	5	5	7	6	1	7	4	4	7	5	4	7	5	4	5
26	6	6	5	5	7	6	2	2	5	5	4	4	5	7	7	4	6	2	6	6	5	6	6

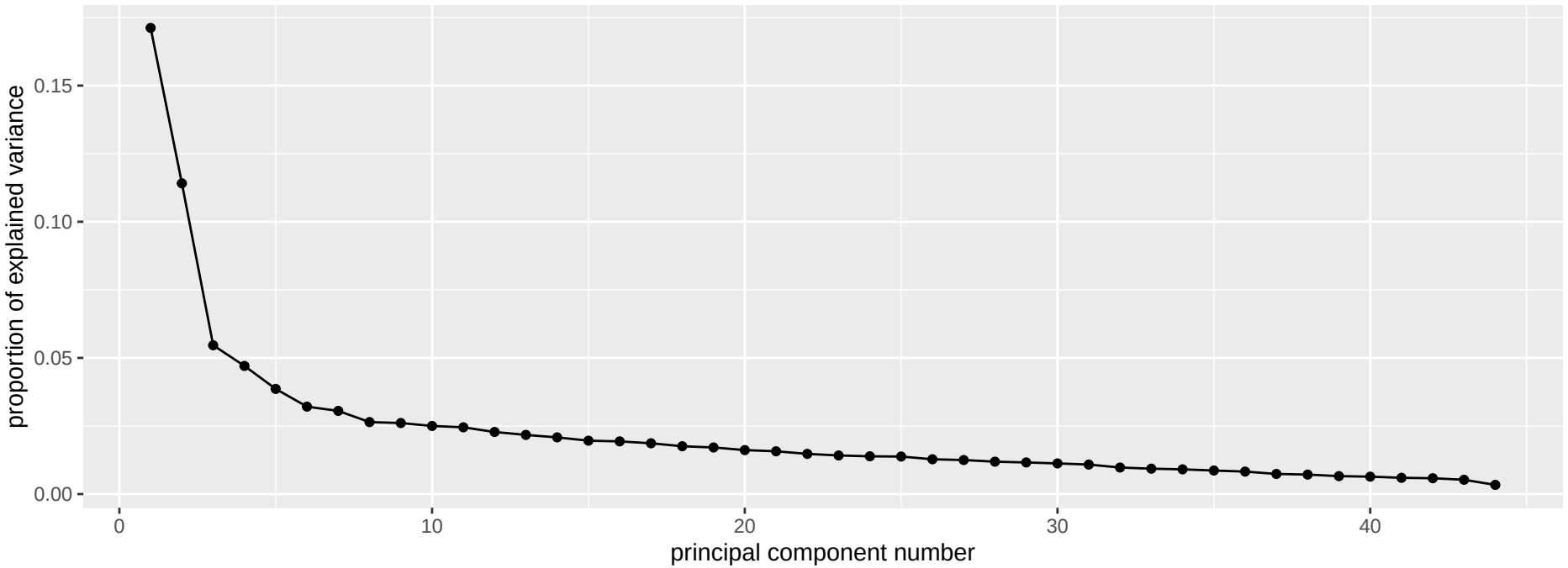
Principal components first

...to decide on number of factors:

```
bem.pc <- bem %>%
  select(-subno) %>%
  princomp(cor = T)
```

The scree plot

```
(g <- ggscreeplot(bem.pc))
```

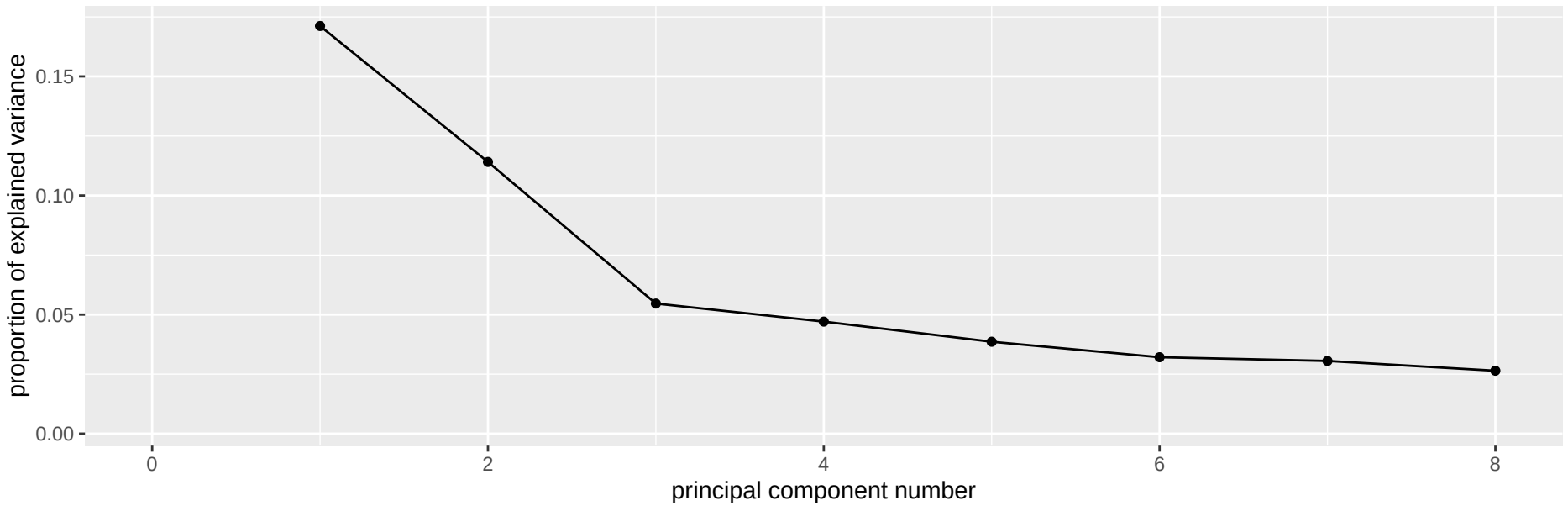


- No obvious elbow.

Zoom in to search for elbow

Possible elbows at 3 (2 factors) and 6 (5):

```
g + scale_x_continuous(limits = c(0, 8))
```



but is 2 really good?

```
summary(bem.pc)
```

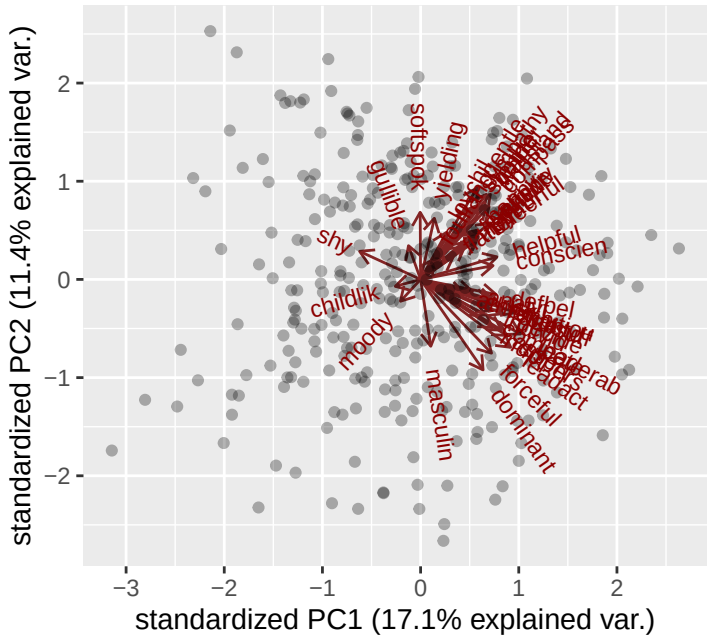
```
## Importance of components:
##                           Comp.1    Comp.2
## Standard deviation     2.7444993 2.2405789
## Proportion of Variance 0.1711881 0.1140953
## Cumulative Proportion  0.1711881 0.2852834
##                           Comp.3     Comp.4
## Standard deviation     1.55049106 1.43886350
## Proportion of Variance 0.05463688 0.04705291
## Cumulative Proportion  0.33992029 0.38697320
##                           Comp.5     Comp.6
## Standard deviation     1.30318840 1.18837867
## Proportion of Variance 0.03859773 0.03209645
## Cumulative Proportion  0.42557093 0.45766738
##                           Comp.7     Comp.8
## Standard deviation     1.15919129 1.07838912
## Proportion of Variance 0.03053919 0.02643007
## Cumulative Proportion  0.48820657 0.51463664
##                           Comp.9    Comp.10
## Standard deviation     1.07120568 1.04901318
## Proportion of Variance 0.02607913 0.02500974
## Cumulative Proportion  0.54071577 0.56572551
##                          Comp.11    Comp.12
```

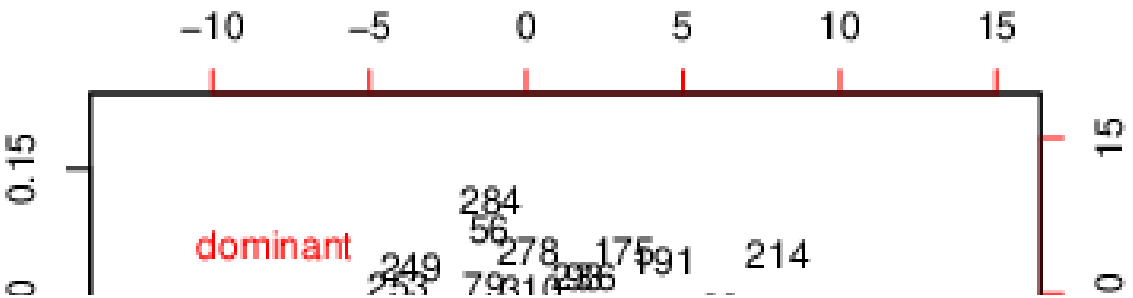
Comments

- Want overall fraction of variance explained ("cumulative proportion'') to be reasonably high.
- 2 factors, 28.5%. Terrible!
- Even 56% (10 factors) not that good!
- Have to live with that.

Biplot

```
ggbiplot(bem.pc, alpha = 0.3)
```





Comments

- Ignore individuals for now.
- Most variables point to 10 o'clock or 7 o'clock.
- Suggests factor analysis with rotation will get interpretable factors (rotate to 6 o'clock and 9 o'clock, for example).
- Try for 2-factor solution (rough interpretation, will be bad):

```
bem.2 <- bem %>%
  select(-subno) %>%
  factanal(factors = 2)
```

- Show output in pieces (just print `bem.2` to see all of it).

Uniquenesses, sorted

```
sort(bem.2$uniquenesses)
```

```
##  leaderab   leadact      warm    tender  dominant    gentle
## 0.4091894 0.4166153 0.4764762 0.4928919 0.4942909 0.5064551
##  forceful   strpers   compass     stand  undstand    assert
## 0.5631857 0.5679398 0.5937073 0.6024001 0.6194392 0.6329347
##    soothe    affect    decide  selfsuff  sympathy     indpt
## 0.6596103 0.6616625 0.6938578 0.7210246 0.7231450 0.7282742
##   helpful    defbel      risk   reliant   individ   compete
## 0.7598223 0.7748448 0.7789761 0.7808058 0.7941998 0.7942910
##  conscien     happy  sensitiv     loyal  ambitiou       shy
## 0.7974820 0.8008966 0.8018851 0.8035264 0.8101599 0.8239496
##  softspok  cheerful  masculin  yielding  feminine  truthful
## 0.8339058 0.8394916 0.8453368 0.8688473 0.8829927 0.8889983
##   lovchil    analyt    athlet   flatter  gullible     moody
## 0.8924392 0.8968744 0.9229702 0.9409500 0.9583435 0.9730607
##  childlik  foullang
## 0.9800360 0.9821662
```

Comments

- Mostly high or very high (bad).
- Some smaller, eg.: Leadership ability (0.409), Acts like leader (0.417), Warm (0.476), Tender (0.493).
- Smaller uniquenesses captured by one of our two factors.
- Larger uniquenesses are not: need more factors to capture them.

Factor loadings, some

```
bem.2$loadings
```

```
##
## Loadings:
##          Factor1 Factor2
## helpful   0.314   0.376
## reliant   0.453   0.117
## defbel    0.434   0.193
## yielding -0.131   0.338
## cheerful  0.152   0.371
## indpt     0.521
## athlet    0.267
## shy      -0.414
## assert    0.605
## strpers   0.657
## forceful  0.649  -0.126
## affect    0.178   0.554
## flatter           0.223
## loyal     0.151   0.417
## analyt    0.295   0.127
## feminine  0.113   0.323
## sympathy          0.526
## moody            -0.162
## sensitiv  0.135   0.424
```

Making a data frame

There are too many to read easily, so make a data frame. A bit tricky:

```
loadings <- as.data.frame(unclass(bem.2$loadings)) %>%
  mutate(trait = rownames(bem.2$loadings))
loadings %>% slice(1:12)
```

Factor1	Factor2	trait
0.3137466	0.3764849	helpful
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
-0.1309965	0.3376293	yielding
0.1523718	0.3705305	cheerful
0.5212403	0.0058703	indpt
0.2670788	0.0755429	athlet
-0.4144579	-0.0653728	shy
0.6049588	0.0330048	assert
0.6569855	0.0207776	strpers
0.6487190	-0.1264058	forceful
0.1778911	0.5537994	affect

Pick out the big ones on factor 1

Arbitrarily defining > 0.4 or < -0.4 as "big":

```
loadings %>% filter(abs(Factor1) > 0.4)
```

Factor1	Factor2	trait
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
0.5212403	0.0058703	indpt
-0.4144579	-0.0653728	shy
0.6049588	0.0330048	assert
0.6569855	0.0207776	strpers
0.6487190	-0.1264058	forceful
0.7654924	0.0695136	leaderab
0.4416176	0.1612384	risk
0.5416796	0.1128080	decide
0.5109964	0.1336268	selfsuff
0.6676490	-0.2448558	dominant
0.6066864	0.1718489	stand
0.7627129	-0.0406672	leadact
0.4448064	0.0891461	individ
0.4504188	0.0532073	compete
0.4136498	0.1368696	ambitiou

Factor 2, the big ones

```
loadings %>% filter(abs(Factor2) > 0.4)
```

Factor1	Factor2	trait
0.1778911	0.5537994	affect
0.1512127	0.4166622	loyal
0.0230146	0.5256654	sympathy
0.1347697	0.4242037	sensitiv
0.0911130	0.6101294	undstand
0.1135064	0.6272223	compass
0.0606175	0.5802714	soothe
0.1189301	0.4300698	happy
0.0795698	0.7191610	warm
0.0511381	0.7102763	tender
-0.0187322	0.7022768	gentle

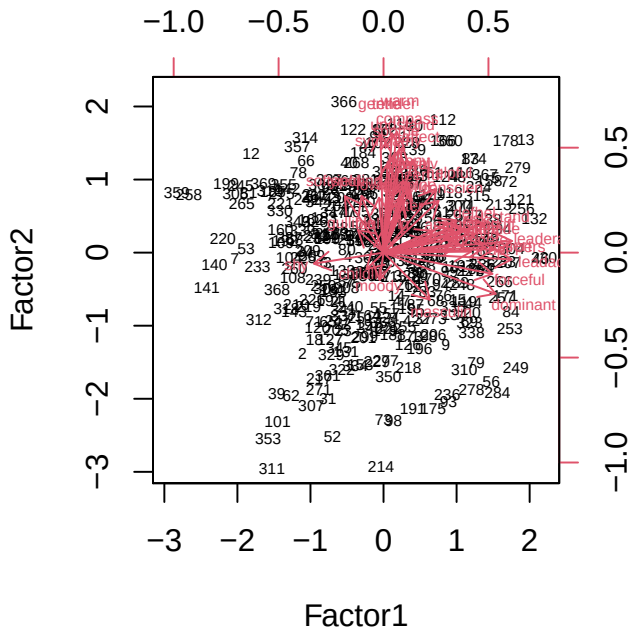
Plotting the two factors

- A bi-plot, this time with the variables reduced in size. Looking for unusual individuals.
- Have to run `factanal` again to get factor scores for plotting.

```
bem %>% select(-subno) %>%
  factanal(factors = 2, scores = "r") -> bem.2a
biplot(bem.2a$scores, bem.2a$loadings, cex = c(0.5, 0.5))
```

- Numbers on plot are row numbers of `bem` data frame.

The (awful) biplot



Comments

- Variables mostly up ("feminine") and right ("masculine"), accomplished by rotation.
- Some unusual individuals: 311, 214 (low on factor 2), 366 (high on factor 2), 359, 258 (low on factor 1), 230 (high on factor 1).

Individual 366

```
bem %>% slice(366) %>% glimpse()
```

```
## Rows: 1
## Columns: 45
## $ subno    <dbl> 755
## $ helpful  <dbl> 7
## $ reliant  <dbl> 7
## $ defbel   <dbl> 5
## $ yielding <dbl> 7
## $ cheerful <dbl> 7
## $ indpt    <dbl> 7
## $ athlet   <dbl> 7
## $ shy      <dbl> 2
## $ assert   <dbl> 1
## $ strpers  <dbl> 3
## $ forceful <dbl> 1
## $ affect   <dbl> 7
## $ flatter  <dbl> 9
## $ loyal    <dbl> 7
## $ analyt   <dbl> 7
## $ feminine <dbl> 7
## $ sympathy <dbl> 7
## $ moody    <dbl> 1
## $ sensitiv <dbl> 7
## $ undstand <dbl> 7
## $ compass  <dbl> 6
## $ leaderab <dbl> 3
## $ soothe   <dbl> 7
## $ risk     <dbl> 7
## $ decide   <dbl> 7
## $ selfsuff <dbl> 7
## $ conscien <dbl> 7
```

Comments

- Individual 366 high on factor 2, but hard to see which traits should have high scores (unless we remember).
- Idea: *tidy* original data frame to make easier to look things up.

Tidying original data

```
bem %>%
  mutate(row = row_number()) %>%
  pivot_longer(c(-subno, -row), names_to="trait",
               values_to="score") -> bem_tidy
bem_tidy
```

subno	row	trait	score
1	1	helpful	7
1	1	reliant	7
1	1	defbel	5
1	1	yielding	5
1	1	cheerful	7
1	1	indpt	7
1	1	athlet	7
1	1	shy	1
1	1	assert	7
1	1	strpers	7
1	1	forceful	2
1	1	affect	7
1	1	flatter	4
1	1	loyal	7
1	1	analyt	1
1	1	feminine	2

Recall data frame of loadings

```
loadings %>% slice(1:10)
```

Factor1	Factor2	trait
0.3137466	0.3764849	helpful
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
-0.1309965	0.3376293	yielding
0.1523718	0.3705305	cheerful
0.5212403	0.0058703	indpt
0.2670788	0.0755429	athlet
-0.4144579	-0.0653728	shy
0.6049588	0.0330048	assert
0.6569855	0.0207776	strpers

Want to add the factor scores for each trait to our tidy data frame `bem_tidy`. This is a left-join (over), matching on the column `trait` that is in both data frames (thus, the default):

Looking up loadings

```
bem_tidy %>% left_join(loadings) -> bem_tidy
```

```
## Joining, by = "trait"
```

```
bem_tidy %>% sample_n(12)
```

subno	row	trait	score	Factor1	Factor2
583	342	childlik	3	-0.1014384	-0.0983365
324	190	strpers	7	0.6569855	0.0207776
418	237	happy	7	0.1189301	0.4300698
203	115	risk	4	0.4416176	0.1612384
314	183	reliant	6	0.4532904	0.1171406
483	273	sensitiv	7	0.1347697	0.4242037
398	224	conscien	7	0.3277630	0.3083647
515	298	childlik	2	-0.1014384	-0.0983365
357	208	sympathy	6	0.0230146	0.5256654
454	256	softspok	5	-0.2303283	0.3362171
242	137	shy	2	-0.4144579	-0.0653728
150	98	gentle	4	-0.0187322	0.7022768

Individual 366, high on Factor 2

So now pick out the rows of the tidy data frame that belong to individual 366 (`row`=366) and for which the `Factor2` score exceeds 0.4 in absolute value (our "big" from before):

```
bem_tidy %>% filter(row == 366, abs(Factor2) > 0.4)
```

subno	row	trait	score	Factor1	Factor2
755	366	affect	7	0.1778911	0.5537994
755	366	loyal	7	0.1512127	0.4166622
755	366	sympathy	7	0.0230146	0.5256654
755	366	sensitiv	7	0.1347697	0.4242037
755	366	undstand	7	0.0911130	0.6101294
755	366	compass	6	0.1135064	0.6272223
755	366	soothe	7	0.0606175	0.5802714
755	366	happy	7	0.1189301	0.4300698
755	366	warm	7	0.0795698	0.7191610
755	366	tender	7	0.0511381	0.7102763
755	366	gentle	7	-0.0187322	0.7022768

As expected, high scorer on these.

Several individuals

Rows 311 and 214 were *low* on Factor 2, so their scores should be low. Can we do them all at once?

```
bem_tidy %>% filter(
  row %in% c(366, 311, 214),
  abs(Factor2) > 0.4
)
```

subno	row	trait	score	Factor1	Factor2
369	214	affect	1	0.1778911	0.5537994
369	214	loyal	7	0.1512127	0.4166622
369	214	sympathy	4	0.0230146	0.5256654
369	214	sensitiv	7	0.1347697	0.4242037
369	214	undstand	5	0.0911130	0.6101294
369	214	compass	5	0.1135064	0.6272223
369	214	soothe	3	0.0606175	0.5802714
369	214	happy	4	0.1189301	0.4300698
369	214	warm	1	0.0795698	0.7191610
369	214	tender	3	0.0511381	0.7102763
369	214	gentle	2	-0.0187322	0.7022768

Individual by column

Un-`tidy`, that is, `spread`:

```
bem_tidy %>%
  filter(
    row %in% c(366, 311, 214),
    abs(Factor2) > 0.4
  ) %>%
  select(-subno, -Factor1, -Factor2) %>%
  pivot_wider(names_from=row, values_from=score)
```

trait	214	311	366
affect	1	5	7
loyal	7	4	7
sympathy	4	4	7
sensitiv	7	4	7
undstand	5	3	7
compass	5	4	6
soothe	3	4	7
happy	4	3	7
warm	1	3	7
tender	3	4	7
gentle	2	3	7

366 high, 311 middling, 214 (sometimes) low.

Individuals 230, 258, 359

These were high, low, low on factor 1. Adapt code:

```
bem_tidy %>%
  filter(row %in% c(359, 258, 230), abs(Factor1) > 0.4) %>%
  select(-subno, -Factor1, -Factor2) %>%
  pivot_wider(names_from=row, values_from=score)
```

trait	230	258	359
reliant	7	4	1
defbel	7	1	1
indpt	7	7	1
shy	2	7	5
assert	7	3	1
strpers	7	1	3
forceful	7	1	1
leaderab	7	1	1
risk	7	5	7
decide	7	1	2
selfsuff	7	4	1
dominant	7	1	1
stand	7	1	6
leadact	7	1	1
individ	7	3	3
compete	6	2	1
ambitiou	7	2	4

Is 2 factors enough?

Suspect not:

```
bem.2$PVAL
```

```
##     objective
## 1.458183e-150
```

2 factors resoundingly rejected. Need more. Have to go all the way to 15 factors to not reject:

```
bem.15 <- bem %>%
  select(-subno) %>%
  factanal(factors = 15)
bem.15$PVAL
```

```
## objective
##  0.132617
```

Even then, only just over 50% of variability explained.

Get 15-factor loadings

into a data frame, as before:

```
loadings <- as.data.frame(unclass(bem.15$loadings)) %>%
  mutate(trait = rownames(bem.15$loadings))
```

then show the highest few loadings on each factor.

Factor 1 (of 15)

```
loadings %>%
  arrange(desc(abs(Factor1))) %>%
  select(Factor1, trait) %>%
  slice(1:10)
```

Factor1	trait
0.8127595	compass
0.6756043	undstand
0.6611293	sympathy
0.6408327	sensitiv
0.5971006	soothe
0.3481290	warm
0.2797159	gentle
0.2788627	tender
0.2501505	helpful
0.2340594	conscien

Compassionate, understanding, sympathetic, soothing: thoughtful of others

Factor 2

```
loadings %>%
  arrange(desc(abs(Factor2))) %>%
  select(Factor2, trait) %>%
  slice(1:10)
```

Factor2	trait
0.7615492	strpers
0.7160312	forceful
0.6981500	assert
0.5041921	dominant
0.3929344	leaderab
0.3669560	stand
0.3507080	leadact
-0.3131682	softspok
-0.2866862	shy
0.2602525	analyt

Strong personality, forceful, assertive, dominant: getting ahead.

Factor 3

```
loadings %>%
  arrange(desc(abs(Factor3))) %>%
  select(Factor3, trait) %>%
  slice(1:10)
```

Factor3	trait
0.6697542	reliant
0.6475496	selfsuff
0.6204018	indpt
0.3899607	helpful
-0.3393605	gullible
0.3333813	individ
0.3319003	decide
0.3294806	conscien
0.2877396	leaderab
0.2804170	defbel

Self-reliant, self-sufficient, independent: going it alone.

Factor 4

```
loadings %>%
  arrange(desc(abs(Factor4))) %>%
  select(Factor4, trait) %>%
  slice(1:10)
```

Factor4	trait
0.6956206	gentle
0.6920303	tender
0.5992467	warm
0.4465546	affect
0.3942568	softspok
0.2779793	lovchil
0.2444249	undstand
0.2442119	happy
0.2125905	loyal
0.2022861	soothe

Gentle, tender, warm (affectionate): caring for others.

Factor 5

```
loadings %>%
  arrange(desc(abs(Factor5))) %>%
  select(Factor5, trait) %>%
  slice(1:10)
```

Factor5	trait
0.6956846	compete
0.6743459	ambitiou
0.3453425	risk
0.3423456	individ
0.2808623	athlet
0.2695570	leaderab
0.2449656	decide
0.2064415	dominant
0.1928159	leadact
0.1854989	strpers

Ambitious, competitive (with a bit of risk-taking and individualism): Being the best

Factor 6

```
loadings %>%
  arrange(desc(abs(Factor6))) %>%
  select(Factor6, trait) %>%
  slice(1:10)
```

Factor6	trait
0.8675651	leadact
0.6078869	leaderab
0.3378645	dominant
0.2014835	forceful
-0.1915632	shy
0.1789256	risk
0.1703440	masculin
0.1639190	decide
0.1594585	compete
0.1466037	athlet

Acts like a leader, leadership ability (with a bit of Dominant): Taking charge

Factor 7

```
loadings %>%
  arrange(desc(abs(Factor7))) %>%
  select(Factor7, trait) %>%
  slice(1:10)
```

Factor7	trait
0.6698996	happy
0.6667105	cheerful
-0.5219125	moody
0.2191425	athlet
0.2126626	warm
0.1719953	gentle
-0.1640302	masculin
0.1601472	reliant
0.1472926	yielding
0.1410481	lovchil

Acts like a leader, leadership ability (with a bit of Dominant): Taking charge

Factor 8

```
loadings %>%
  arrange(desc(abs(Factor8))) %>%
  select(Factor8, trait) %>%
  slice(1:10)
```

Factor8	trait
0.6296764	affect
0.5158355	flatter
-0.2512066	softspok
0.2214623	warm
0.1878549	tender
0.1846225	strpers
-0.1804838	shy
0.1801992	compete
0.1658105	loyal
0.1548617	helpful

Affectionate, flattering: Making others feel good.

Factor 9

```
loadings %>%
  arrange(desc(abs(Factor9))) %>%
  select(Factor9, trait) %>%
  slice(1:10)
```

Factor9	trait
0.8633171	stand
0.3403294	defbel
0.2446971	individ
0.1941110	risk
-0.1715481	shy
0.1710978	decide
0.1197126	assert
0.1157729	conscien
0.1120308	analyt
-0.1115140	gullible

Taking a stand.

Factor 10

```
loadings %>%
  arrange(desc(abs(Factor10))) %>%
  select(Factor10, trait) %>%
  slice(1:10)
```

Factor10	trait
0.8075127	feminine
-0.2637851	masculin
0.2450718	softspok
0.2317560	conscien
0.2019203	selfsuff
0.1758423	yielding
0.1412707	gentle
0.1128203	flatter
0.1093453	decide
-0.0940798	lovchil

Feminine. (A little bit of not-masculine!)

Factor 11

```
loadings %>%
  arrange(desc(abs(Factor11))) %>%
  select(Factor11, trait) %>%
  slice(1:10)
```

Factor11	trait
0.9162259	loyal
0.1894908	affect
0.1588386	truthful
0.1246453	helpful
0.1044066	analyt
0.1007679	tender
0.0972046	lovchil
0.0963522	gullible
0.0935062	cheerful
0.0820760	conscien

Loyal.

Factor 12

```
loadings %>%
  arrange(desc(abs(Factor12))) %>%
  select(Factor12, trait) %>%
  slice(1:10)
```

Factor12	trait
0.6106933	childlik
-0.2845004	selfsuff
-0.2786751	conscien
0.2588843	moody
0.2013245	shy
-0.1669301	decide
0.1542031	masculin
0.1455526	dominant
0.1379163	compass
-0.1297408	leaderab

Childlike. (With a bit of moody, shy, not-self-sufficient, not-conscientious.)

Factor 13

```
loadings %>%
  arrange(desc(abs(Factor13))) %>%
  select(Factor13, trait) %>%
  slice(1:10)
```

Factor13	trait
0.5729242	truthful
-0.2776490	gullible
0.2631046	happy
0.1885152	warm
-0.1671924	shy
0.1646031	loyal
-0.1438127	yielding
-0.1302900	assert
0.1137074	defbel
-0.1105583	lovchil

Truthful. (With a bit of happy and not-gullible.)

Factor 14

```
loadings %>%
  arrange(desc(abs(Factor14))) %>%
  select(Factor14, trait) %>%
  slice(1:10)
```

Factor14	trait
0.4429926	decide
0.2369714	selfsuff
0.1945034	forceful
-0.1862756	softspok
0.1604175	risk
-0.1484606	strpers
0.1461972	dominant
0.1279456	happy
0.1154479	compass
0.1054078	masculin

Decisive. (With a bit of self-sufficient and not-soft-spoken.)

Factor 15

```
loadings %>%
  arrange(desc(abs(Factor15))) %>%
  select(Factor15, trait) %>%
  slice(1:10)
```

Factor15	trait
-0.3244092	compass
0.2471884	athlet
0.2292980	sensitiv
0.1986878	risk
-0.1638296	affect
0.1632164	moody
-0.1118135	individ
0.1100678	warm
0.1047347	cheerful
0.1012342	reliant

Not-compassionate, athletic, sensitive: A mixed bag. ("Cares about self"?)

Anything left out? Uniquenesses

```
enframe(bem.15$uniquenesses, name="quality", value="uniq") %>%
  arrange(desc(uniq)) %>%
  slice(1:10)
```

quality	uniq
foullang	0.9136126
lovchil	0.8242992
analyt	0.8120934
yielding	0.7911748
masculin	0.7228739
athlet	0.7217327
shy	0.7033071
gullible	0.7000779
flatter	0.6625008
helpful	0.6516863

Uses foul language especially, also loves children and analytical. So could use even more factors.

Section 1

Confirmatory factor analysis}

Confirmatory factor analysis

- Exploratory: what do data suggest as hidden underlying factors (in terms of variables observed)?
- Confirmatory: have *theory* about how underlying factors depend on observed variables; test whether theory supported by data:
- does theory provide *some* explanation (better than nothing)
- can we do better?
- Also can compare two theories about factors: is more complicated one significantly better than simpler one?

Children and tests again

- Previously had this correlation matrix of test scores (based on 145 children):

```
km
```

```
##       para  sent  word   add  dots
## [1,] 1.000 0.722 0.714 0.203 0.095
## [2,] 0.722 1.000 0.685 0.246 0.181
## [3,] 0.714 0.685 1.000 0.170 0.113
## [4,] 0.203 0.246 0.170 1.000 0.585
## [5,] 0.095 0.181 0.113 0.585 1.000
```

- Will use package `lavaan` for confirmatory analysis.
- Can use actual data or correlation matrix.
- Latter (a bit) more work, as we see.

Two or three steps

- Make sure correlation matrix (if needed) is handy.
- Specify factor model (from theory)
- Fit factor model: does it fit acceptably?

Terminology

- Thing you cannot observe called **latent variable**.
- Thing you *can* observe called **manifest variable**.
- Model predicts latent variables from manifest variables.
  - asserts a relationship between latent and manifest.
- We need to invent names for the latent variables.

Specifying a factor model

- Model with one factor including all the tests:

```
test.model.1 <- "ability=~para+sent+word+add+dots"
```

- and a model that we really believe, that there are two factors, a verbal and a mathematical:

```
test.model.2 <- "verbal=~para+sent+word
                 math=~add+dots"
```

- Note the format: really all one line between single quotes, but putting it on several lines makes the layout clearer.
- Also note special notation =~ for "this latent variable depends on these observed variables' '.

Fitting a 1-factor model

- Need to specify model, correlation matrix, n like this:

```
fit1 <- cfa(test.model.1,
  sample.cov = km,
  sample.nobs = 145
)
```

- Has summary, or briefer version like this:

```
fit1
```

```
## lavaan 0.6-7 ended normally after 16 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         10
##
##   Number of observations                           145
##
## Model Test User Model:
##
##   Test statistic                                59.886
```

Two-factor model

```
fit2 <- cfa(test.model.2, sample.cov = km, sample.nobs = 145)
fit2
```

```
## lavaan 0.6-7 ended normally after 25 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         11
##
##   Number of observations                           145
##
## Model Test User Model:
##
##   Test statistic                                 2.951
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.566
```

- This fits OK: 2-factor model supported by the data.
- 1-factor model did not fit. We really need 2 factors.
- Same conclusion as from `factanal` earlier.

Comparing models

- Use `anova` as if this were a regression:

```
anova(fit1, fit2)
```

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
fit2	4	1776.673	1809.417	2.950949	NA	NA	NA
fit1	5	1831.608	1861.375	59.886210	56.93526	1	0

- 2-factor model fits significantly better than 1-factor.
- No surprise!

Track and field data, yet again

- `cfa` works easier on actual data, such as the running records:

```
track %>% print(n = 6)
```

```
## # A tibble: 55 x 9
##    m100  m200  m400  m800 m1500 m5000 m10000 marathon
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>    <dbl>
## 1  10.4  20.8  46.8  1.81  3.7   14.0   29.4     138.
## 2  10.3  20.1  44.8  1.74  3.57  13.3   27.7     128.
## 3  10.4  20.8  46.8  1.79  3.6   13.3   27.7     136.
## 4  10.3  20.7  45.0  1.73  3.6   13.2   27.4     130.
## 5  10.3  20.6  45.9  1.8   3.75  14.7   30.6     147.
## 6  10.2  20.4  45.2  1.73  3.66  13.6   28.6     133.
## # ... with 49 more rows, and 1 more variable: country <chr>
```

- Specify factor model. Factors seemed to be "sprinting" (up to 800m) and "distance running" (beyond):

```
track.model <- "sprint=~m100+m200+m400+m800
                distance=~m1500+m5000+m10000+marathon"
```

Fit and examine the model

- Fit the model. The observed variables are on different scales, so we should standardize them first via `std.ov`:

```
track.1 <- track %>%
  select(-country) %>%
  cfa(track.model, data = ., std.ov = T)
track.1
```

```
## lavaan 0.6-7 ended normally after 59 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         17
##
##   Number of observations                            55
##
## Model Test User Model:
##
##   Test statistic                                87.608
##   Degrees of freedom                                19
##   P-value (Chi-square)                           0.000
```

- This fits badly. Can we do better?

Factor model 2

- Define factor model:

```
track.model.2 <- "sprint=~m100+m200+m400
                  middle=~m800+m1500
                  distance=~m5000+m10000+marathon"
```

- Fit:

```
track %>%
  select(-country) %>%
  cfa(track.model.2, data = ., std.ov = T) -> track.2
```

Examine

```
track.2
```

```
## lavaan 0.6-7 ended normally after 72 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         19
##
##   Number of observations                            55
##
## Model Test User Model:
##
##   Test statistic                                40.089
##   Degrees of freedom                                17
##   P-value (Chi-square)                           0.001
```

- Fits marginally better, though still badly.

Comparing the two models

- Second model doesn't fit well, but is it better than first?

```
anova(track.1, track.2)
```

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
track.2	17	535.4894	573.6288	40.08919	NA	NA	NA
track.1	19	579.0083	613.1329	87.60804	47.51885	2	0

- Oh yes, a lot better.