

# Basic statistical inference

## Packages for this section

```
library(tidyverse)
```

# Inference for means (from STAB57)

Three kinds of inference for means of normally-distributed data:

- **One-sample  $t$ :** a single sample from a population, estimate that population's mean
- **Two-sample  $t$ :** one sample from each of 2 populations, estimate difference in population means
- **Matched pairs  $t$ :** two paired measurements on same (or matched) individuals, estimate population mean difference

Two forms of inference for a population parameter:

- **Confidence interval:** “what is the population parameter?”
- **Hypothesis test:** “could the population parameter be equal to this value?”

## Examples:

- Blue jays attendances (one-sample)
- Kids learning to read (two-sample)
- Pain relief (matched pairs)

# Confidence interval

- You have a sample from some population
- Imagine repeated sampling from that population
- Procedure that gives an interval containing the true parameter in 95% (or 90% or 99%) of all possible samples

# Hypothesis test

- Null hypothesis gives value for population parameter
- Alternative hypothesis says how you are trying to prove the null hypothesis wrong (not equal, greater, less).
- Test statistic measures “distance” between data and null hypothesis
- P-value gives probability of observing test statistic *as extreme or more extreme*, **if the null hypothesis is true**.
- Reject null hypothesis if P-value small enough (eg smaller than 0.05).

## Why 0.05? This man.



- analysis of variance
- Fisher information
- Linear discriminant analysis
- Fisher's  $z$ -transformation
- Fisher-Yates shuffle
- Behrens-Fisher problem

Sir Ronald A. Fisher, 1890–1962.

## Why 0.05? (2)

- From The Arrangement of Field Experiments (1926):

the line at about the level at which we can say: "Either there is something in the treatment, or a coincidence has occurred such as does not occur more than once in twenty trials." This level, which we may call the 5 per cent. point, would be indicated, though very roughly, by the greatest chance deviation observed in twenty successive trials. To

- and

If one in twenty does not seem high enough odds, we may, if we prefer it, draw the line at one in fifty (the 2 per cent. point), or one in a hundred (the 1 per cent. point). Personally, the writer prefers to set a low standard of significance at the 5 per cent. point, and ignore entirely all results which fail to reach this level. A scientific fact should be regarded as experimentally established only if a properly designed experiment rarely fails to give this level of significance. The very high



## $\alpha$ and errors

- Hypothesis test ends with decision:
  - ▶ reject null hypothesis
  - ▶ do not reject null hypothesis.
- but decision may be wrong:

	Decision	
Truth	Do not reject	reject null
Null true	Correct	Type I error
Null false	Type II error	Correct

- Either type of error is bad, but for now focus on controlling Type I error: write  $\alpha = P(\text{type I error})$ , and devise test so that  $\alpha$  small, typically 0.05.
- That is, **if null hypothesis true**, have only small chance to reject it (which would be a mistake).
- Worry about type II errors later (when we consider power of test).

# One sample: the Blue Jays attendances

- The Toronto Blue Jays' average home attendance in part of 2015 season was 25,070 (up to May 27 2015, from [baseball-reference.com](http://baseball-reference.com)).
- Does that mean the attendance at every game was exactly 25,070?  
Certainly not. Actual attendance depends on many things, eg.:
  - ▶ how well the Jays are playing
  - ▶ the opposition
  - ▶ day of week
  - ▶ weather
  - ▶ random chance

# Reading the attendances

...as a .csv file:

```
my_url <- "http://ritsokiguess.site/datafiles/jays15-home.csv"
jays <- read_csv(my_url)
jays
```

# A tibble: 25 x 21

	row	game	date	box	team	venue	opp	result	runs	Oppruns	innings
	<dbl>	<dbl>	<chr>	<chr>	<chr>	<lg1>	<chr>	<chr>	<dbl>	<dbl>	<dbl>
1	82	7	Monda~	boxs~	TOR	NA	TBR	L	1	2	NA
2	83	8	Tuesd~	boxs~	TOR	NA	TBR	L	2	3	NA
3	84	9	Wedne~	boxs~	TOR	NA	TBR	W	12	7	NA
4	85	10	Thurs~	boxs~	TOR	NA	TBR	L	2	4	NA
5	86	11	Frida~	boxs~	TOR	NA	ATL	L	7	8	NA
6	87	12	Satur~	boxs~	TOR	NA	ATL	W-wo	6	5	10
7	88	13	Sunda~	boxs~	TOR	NA	ATL	L	2	5	NA
8	89	14	Tuesd~	boxs~	TOR	NA	BAL	W	13	6	NA
9	90	15	Wedne~	boxs~	TOR	NA	BAL	W	4	2	NA
10	91	16	Thurs~	boxs~	TOR	NA	BAL	W	7	6	NA

# i 15 more rows

# i 9 more variables: position <dbl>, gb <chr>, winner <chr>, loser <chr>,

# save <chr>, game\_time <time>, Daynight <chr>, attendance <dbl>

## Another way

- This is a “big” data set: only 25 observations, but a lot of *variables*.
- To see the first few values in all the variables, can also use `glimpse`:

```
glimpse(jays)
```

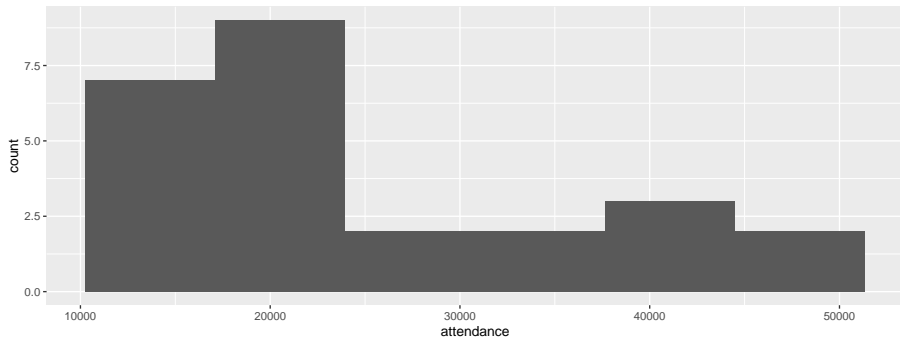
```
Rows: 25
```

```
Columns: 21
```

```
$ row      <dbl> 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96~
$ game     <dbl> 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 27, 28, 29, 30, 31, 3~
$ date     <chr> "Monday, Apr 13", "Tuesday, Apr 14", "Wednesday, Apr 15", ~
$ box      <chr> "boxscore", "boxscore", "boxscore", "boxscore", "boxscore"~
$ team     <chr> "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "TOR", "T~
$ venue    <lgl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA~
$ opp      <chr> "TBR", "TBR", "TBR", "TBR", "ATL", "ATL", "ATL", "BAL", "B~
$ result   <chr> "L", "L", "W", "L", "L", "W-wo", "L", "W", "W", "W", "W", ~
$ runs     <dbl> 1, 2, 12, 2, 7, 6, 2, 13, 4, 7, 3, 3, 5, 7, 7, 3, 10, 2, 3~
$ Oppruns  <dbl> 2, 3, 7, 4, 8, 5, 5, 6, 2, 6, 1, 6, 1, 0, 1, 6, 6, 3, 4, 4~
$ innings  <dbl> NA, NA, NA, NA, NA, NA, 10, NA, NA, NA, NA, NA, NA, NA, NA, NA~
$ wl       <chr> "4-3", "4-4", "5-4", "5-5", "5-6", "6-6", "6-7", "7-7", "8~
$ position <dbl> 2, 3, 2, 4, 4, 3, 4, 2, 2, 1, 4, 5, 3, 3, 3, 3, 5, 5, 5, 5~
$ gb       <chr> "1", "2", "1", "1.5", "2.5", "1.5", "1.5", "2", "1", "Tied~
$ winner   <chr> "Odorizzi", "Geltz", "Buehrle", "Archer", "Martin", "Cecil~
$ loser    <chr> "Dickey", "Castro", "Ramirez", "Sanchez", "Cecil", "Marimo~
$ save     <chr> "Boxberger", "Jepsen", NA, "Boxberger", "Grilli", NA, "Gri~
```

# Attendance histogram

```
ggplot(jays, aes(x = attendance)) + geom_histogram(bins = 6)
```



# Comments

- Attendances have substantial variability, ranging from just over 10,000 to around 50,000.
- Distribution somewhat skewed to right (but no outliers).
- These are a sample of “all possible games” (or maybe “all possible games played in April and May”). What can we say about mean attendance in all possible games based on this evidence?

## CI for mean attendance

- `t.test` function does CI and test. Look at CI first:

```
t.test(jays$attendance)
```

### One Sample t-test

```
data:  jays$attendance
t = 11.389, df = 24, p-value = 3.661e-11
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 20526.82 29613.50
sample estimates:
mean of x
 25070.16
```

- From 20,500 to 29,600.

## Or, 90% CI

- by including a value for `conf.level`:

```
t.test(jays$attendance, conf.level = 0.90)
```

### One Sample t-test

```
data: jays$attendance
```

```
t = 11.389, df = 24, p-value = 3.661e-11
```

```
alternative hypothesis: true mean is not equal to 0
```

```
90 percent confidence interval:
```

```
21303.93 28836.39
```

```
sample estimates:
```

```
mean of x
```

```
25070.16
```

- From 21,300 to 28,800. (Shorter, as it should be.)



## Comments

- Need to say “column attendance within data frame jays” using \$.
- 95% CI from about 20,000 to about 30,000.
- Not estimating mean attendance well at all!
- Generally want confidence interval to be shorter, which happens if:
  - ▶ SD smaller
  - ▶ sample size bigger
  - ▶ confidence level smaller
- Last one is a cheat, really, since reducing confidence level increases chance that interval won't contain pop. mean at all!

## Another way to access data frame columns

```
with(jays, t.test(attendance))
```

### One Sample t-test

```
data:  attendance
t = 11.389, df = 24, p-value = 3.661e-11
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 20526.82 29613.50
sample estimates:
mean of x
 25070.16
```

## Hypothesis testing for Blue Jays attendances

- Previous year's mean attendance was 29,327, so test to see whether the mean is different from that in any way (two-sided test):

```
t.test(jays$attendance, mu = 29327)
```

One Sample t-test

data: jays\$attendance

t = -1.9338, df = 24, p-value = 0.06502

alternative hypothesis: true mean is not equal to 29327

95 percent confidence interval:

20526.82 29613.50

sample estimates:

mean of x

25070.16

- See test statistic  $-1.93$ , P-value 0.065.

## Another example: learning to read

- You devised new method for teaching children to read.
- Guess it will be more effective than current methods.
- To support this guess, collect data.
- Want to generalize to “all children in Canada”.
- So take random sample of all children in Canada.
- Or, argue that sample you actually have is “typical” of all children in Canada.
- Randomization (1): whether or not a child in sample or not has nothing to do with anything else about that child.
- Randomization (2): randomly choose whether each child gets new reading method (t) or standard one (c).

## Reading in data

- File at <http://ritsokiguess.site/datafiles/drp.txt>.
- Proper reading-in function is `read_delim` (check file to see)
- Read in thus:

```
my_url <- "http://ritsokiguess.site/datafiles/drp.txt"
kids <- read_delim(my_url," ")
```

# The data (some)

```
kids
```

```
# A tibble: 44 x 2
```

```
  group score
```

```
<chr> <dbl>
```

```
1 t      24
```

```
2 t      61
```

```
3 t      59
```

```
4 t      46
```

```
5 t      43
```

```
6 t      44
```

```
7 t      52
```

```
8 t      43
```

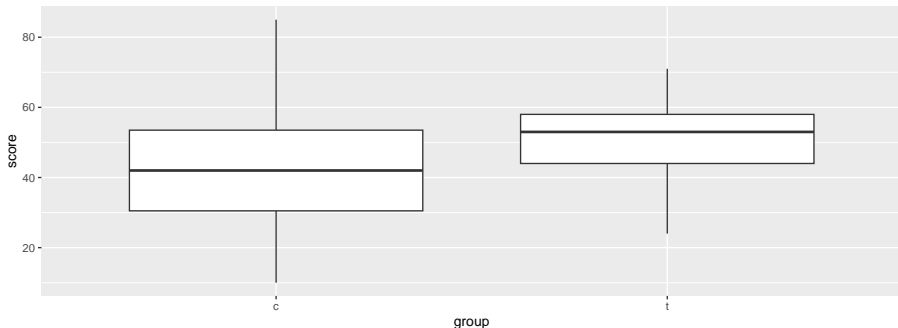
```
9 t      58
```

```
10 t     67
```

```
# i 34 more rows
```

# Boxplots

```
ggplot(kids, aes(x = group, y = score)) + geom_boxplot()
```



## Two kinds of two-sample t-test

- Do the two groups have same spread (SD, variance)?
  - ▶ If yes (shaky assumption here), can use pooled t-test.
  - ▶ If not, use Welch-Satterthwaite t-test (safe).
- Pooled test derived in STAB57 (easier to derive, but assumes equal variances).
- Welch-Satterthwaite does not assume equality of variances.
- Assess (approx) equality of spreads using boxplot.



# The (Welch-Satterthwaite) t-test

- c (control) before t (treatment) alphabetically, so proper alternative is “less”.
- R does Welch-Satterthwaite test by default
- Answer to “does the new reading program really help?”
- (in a moment) how to get R to do pooled test?

# Welch-Satterthwaite

```
t.test(score ~ group, data = kids, alternative = "less")
```

Welch Two Sample t-test

data: score by group

t = -2.3109, df = 37.855, p-value = 0.01319

alternative hypothesis: true difference in means between group

95 percent confidence interval:

-Inf -2.691293

sample estimates:

mean in group c mean in group t

41.52174

51.47619

# The pooled t-test

```
t.test(score ~ group, data = kids,  
       alternative = "less", var.equal = TRUE)
```

## Two Sample t-test

data: score by group

t = -2.2666, df = 42, p-value = 0.01431

alternative hypothesis: true difference in means between group

95 percent confidence interval:

-Inf -2.567497

sample estimates:

mean in group c mean in group t

41.52174

51.47619

## Two-sided test; CI

- To do 2-sided test, leave out alternative:

```
t.test(score ~ group, data = kids)
```

### Welch Two Sample t-test

data: score by group

t = -2.3109, df = 37.855, p-value = 0.02638

alternative hypothesis: true difference in means between group

95 percent confidence interval:

-18.67588 -1.23302

sample estimates:

mean in group c mean in group t

41.52174

51.47619

## Comments:

- P-values for pooled and Welch-Satterthwaite tests very similar (even though the pooled test seemed inferior): 0.013 vs. 0.014.
- Two-sided test also gives CI: new reading program increases average scores by somewhere between about 1 and 19 points.
- Confidence intervals inherently two-sided, so do 2-sided test to get them.

# Pain relief

Some data:

subject	druga	drugb
1	2.0	3.5
2	3.6	5.7
3	2.6	2.9
4	2.6	2.4
5	7.3	9.9
6	3.4	3.3
7	14.9	16.7
8	6.6	6.0
9	2.3	3.8
10	2.0	4.0
11	6.8	9.1
12	8.5	20.9

## Matched pairs data

- Data are comparison of 2 drugs for effectiveness at reducing pain.
  - ▶ 12 subjects (cases) were arthritis sufferers
  - ▶ Response is #hours of pain relief from each drug.
- In reading example, each child tried only one reading method.
- But here, each subject tried out both drugs, giving us two measurements.
  - ▶ Possible because, if you wait long enough, one drug has no influence over effect of other.
  - ▶ Advantage: focused comparison of drugs. Compare one drug with another on same person, removes a lot of variability due to differences between people.
  - ▶ Matched pairs, requires different analysis.
- Design: randomly choose 6 of 12 subjects to get drug A first, other 6 get drug B first.

## Paired t test: reading the data

Values aligned in columns:

```
my_url <- "http://ritsokiguess.site/datafiles/analgesic.txt"  
pain <- read_table(my_url)
```



# The data

```
pain
```

```
# A tibble: 12 x 3
```

	subject	druga	drugb
	<dbl>	<dbl>	<dbl>
1	1	2	3.5
2	2	3.6	5.7
3	3	2.6	2.9
4	4	2.6	2.4
5	5	7.3	9.9
6	6	3.4	3.3
7	7	14.9	16.7
8	8	6.6	6
9	9	2.3	3.8
10	10	2	4
11	11	6.8	9.1
12	12	8.5	20.9

## Paired $t$ -test

```
with(pain, t.test(druga, drugb, paired = T))
```

### Paired $t$ -test

data: druga and drugb

$t = -2.1677$ ,  $df = 11$ ,  $p\text{-value} = 0.05299$

alternative hypothesis: true mean difference is not equal to 0

95 percent confidence interval:

-4.29941513 0.03274847

sample estimates:

mean difference

-2.133333

- $P$ -value is 0.053.
- Not quite evidence of difference between drugs.

## t-testing the differences

- Likewise, you can calculate the differences yourself and do a 1-sample t-test on them.
- First calculate a column of differences:

```
(pain %>% mutate(diff=druga-drugb) -> pain)
```

```
# A tibble: 12 x 4
  subject druga drugb    diff
  <dbl> <dbl> <dbl> <dbl>
1       1     2   3.5  -1.5
2       2     3   5.7  -2.1
3       3     2   2.9 -0.300
4       4     2   2.4  0.200
5       5     7   9.9  -2.6
6       6     3   3.3  0.100
7       7    14.9 16.7  -1.80
8       8     6   6    0.600
9       9     2   3.8  -1.5
10      10     2    4    -2
11      11     6   9.1  -2.3
12      12     8  20.9 -12.4
```

## t-test on the differences

- then throw them into `t.test`, testing that the mean is zero, with same result as before:

```
with(pain, t.test(diff, mu=0))
```

### One Sample t-test

```
data: diff
```

```
t = -2.1677, df = 11, p-value = 0.05299
```

```
alternative hypothesis: true mean is not equal to 0
```

```
95 percent confidence interval:
```

```
-4.29941513  0.03274847
```

```
sample estimates:
```

```
mean of x
```

```
-2.133333
```