

# Vector and matrix algebra

## Packages for this section

- This is (almost) all base R! We only need this for one thing later:

```
library(tidyverse)
```

# Vector addition

- Adding vectors:

```
u <- c(2, 3, 6, 5, 7)
v <- c(1, 8, 3, 2, 0)
u + v
```

```
[1] 3 11 9 7 7
```

- Elementwise addition. (Linear algebra: vector addition.)

## Adding a number to a vector

- Define a vector, then “add 2” to it:

```
u
```

```
[1] 2 3 6 5 7
```

```
k <- 2  
u + k
```

```
[1] 4 5 8 7 9
```

- adds 2 to *each* element of *u*.

# Scalar multiplication

As per linear algebra:

$k$

$[1] \ 2$

$u$

$[1] \ 2 \ 3 \ 6 \ 5 \ 7$

$k * u$

$[1] \ 4 \ 6 \ 12 \ 10 \ 14$

- Each element of vector multiplied by 2.

## “Vector multiplication”

What about this?

u

[1] 2 3 6 5 7

v

[1] 1 8 3 2 0

u \* v

[1] 2 24 18 10 0

Each element of u multiplied by *corresponding* element of v. Could be called elementwise multiplication.

(Don't confuse with “outer” or “vector” product from linear algebra, or indeed “inner” or “scalar” multiplication, for which the answer is a number.)

## Combining different-length vectors

- No error here (you get a warning). What happens?

```
u
```

```
[1] 2 3 6 5 7
```

```
w <- c(1, 2)
```

```
u + w
```

```
[1] 3 5 7 7 8
```

- Add 1 to first element of `u`, add 2 to second.
- Go back to beginning of `w` to find something to add: add 1 to 3rd element of `u`, 2 to 4th element, 1 to 5th.

## How R does this

- Keep re-using shorter vector until reach length of longer one.
- “Recycling”.
- If the longer vector's length not a multiple of the shorter vector's length, get a warning (probably not what you want).
- Same idea is used when multiplying a vector by a number: the number keeps getting recycled.



# Matrices

- Create matrix like this:

```
(A <- matrix(1:4, nrow = 2, ncol = 2))
```

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

- First: stuff to make matrix from, then how many rows and columns.
- R goes down columns by default. To go along rows instead:

```
(B <- matrix(5:8, nrow = 2, ncol = 2, byrow = TRUE))
```

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

- One of `nrow` and `ncol` enough, since R knows how many things in the matrix.

# Adding matrices

What happens if you add two matrices?

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A + B

	[,1]	[,2]
[1,]	6	9
[2,]	9	12

# Adding matrices

- Nothing surprising here. This is matrix addition as we and linear algebra know it.

# Multiplying matrices

- Now, what happens here?

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A \* B

	[,1]	[,2]
[1,]	5	18
[2,]	14	32

# Multiplying matrices?

- *Not* matrix multiplication (as per linear algebra).
- Elementwise multiplication. Also called *Hadamard product* of  $A$  and  $B$ .

# Legit matrix multiplication

Like this:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A %\*% B

	[,1]	[,2]
[1,]	26	30
[2,]	38	44

## Reading matrix from file

- The usual:

```
my_url <- "http://ritsokiguess.site/datafiles/m.txt"
M <- read_delim(my_url, " ", col_names = FALSE )
M
```

```
# A tibble: 3 x 2
```

```
      X1      X2
```

```
<dbl> <dbl>
```

```
1      10      9
```

```
2       8      7
```

```
3       6      5
```

```
class(M)
```

```
[1] "spec_tbl_df" "tbl_df"      "tbl"         "data.frame"
```

but...

- except that  $M$  is not an R matrix, and thus this doesn't work:

```
v <- c(1, 3)
M %*% v
```

Error in  $M \%*\% v$ : requires numeric/complex matrix/vector argument



# Making a genuine matrix

Do this first:

```
M <- as.matrix(M)
M
```

```
      X1 X2
[1,] 10  9
[2,]  8  7
[3,]  6  5
```

```
v
```

```
[1] 1 3
```

and then all is good:

```
M %*% v
```

```
      [,1]
[1,]    37
[2,]    29
```

## Linear algebra stuff

- To solve system of equations  $Ax = w$  for  $x$ :

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

w

[1] 1 2

```
solve(A, w)
```

[1] 1 0

# Matrix inverse

- To find the inverse of A:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

`solve(A)`

	[,1]	[,2]
[1,]	-2	1.5
[2,]	1	-0.5

# Checking

Matrix inverse:

```
A %% solve(A)
```

	[,1]	[,2]
[1,]	1	0
[2,]	0	1

System of equations:

```
A %% solve(A, w)
```

	[,1]
[1,]	1
[2,]	2

```
w
```

```
[1] 1 2
```

## Inner product

- Vectors in R are column vectors, so just do the matrix multiplication (`t()` is transpose):

```
a <- c(1, 2, 3)
b <- c(4, 5, 6)
t(a) %*% b
```

```
      [,1]
[1,]    32
```

- Note that the answer is actually a  $1 \times 1$  matrix.
- Or as the sum of the elementwise multiplication:

```
sum(a * b)
```

```
[1] 32
```

## Accessing parts of vector

- use square brackets and a number to get elements of a vector

```
b
```

```
[1] 4 5 6
```

```
b[2]
```

```
[1] 5
```

## Accessing parts of matrix

- use a row and column index to get an element of a matrix

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

A[2,1]

[1] 2

- leave the row or column index empty to get whole row or column, eg.

A[1,]

[1] 1 3

# Eigenvalues and eigenvectors 1/2

- For a matrix  $A$ , these are scalars  $\lambda$  and vectors  $v$  that solve

$$Av = \lambda v$$

- In R, `eigen` gets these:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

```
e <- eigen(A)
```



## Eigenvalues and eigenvectors 2/2

e

```
eigen() decomposition
```

```
$values
```

```
[1]  5.3722813 -0.3722813
```

```
$vectors
```

```
      [,1]      [,2]
```

```
[1,] -0.5657675 -0.9093767
```

```
[2,] -0.8245648  0.4159736
```

## Eigenvalues/vectors correct?

- $\lambda_1 v_1$ : (scalar) multiply first eigenvalue by first eigenvector (in column)

```
e$values[1] * e$vectors[,1]
```

```
[1] -3.039462 -4.429794
```

- $Av_1$ : (matrix) multiply matrix by first eigenvector (in column)

```
A %*% e$vectors[,1]
```

```
      [,1]
```

```
[1,] -3.039462
```

```
[2,] -4.429794
```

- These are (correctly) equal.
- The second one goes the same way.

# A statistical application of eigenvalues

- A negative correlation:

```
d <- tribble(
  ~x, ~y,
  10, 20,
  11, 18,
  12, 17,
  13, 14,
  14, 13
)
v <- cor(d)
v
```

	x	y
x	1.0000000	-0.9878783
y	-0.9878783	1.0000000

- `cor` gives the correlation matrix between each pair of variables (correlation between `x` and `y` is  $-0.988$ )

# Eigenanalysis of correlation matrix

```
eigen(v)
```

```
eigen() decomposition
```

```
$values
```

```
[1] 1.98787834 0.01212166
```

```
$vectors
```

```
          [,1]      [,2]  
[1,] -0.7071068 -0.7071068  
[2,]  0.7071068 -0.7071068
```

- first eigenvalue much bigger than second (second one near zero)
- two variables, but data nearly *one*-dimensional
- opposite signs in first eigenvector indicate that the one dimension is:
  - ▶ x small and y large at one end,
  - ▶ x large and y small at the other.