

## Bootstrap for sampling distribution of sample mean

# Assessing assumptions

- ▶ Our  $t$ -tests assume normality of variable being tested
- ▶ but, Central Limit Theorem says that normality matters less if sample is “large”
- ▶ in practice “approximate normality” is enough, but how do we assess whether what we have is normal enough?
- ▶ so far, use histogram/boxplot and make a call, allowing for sample size.

# What actually has to be normal

- ▶ is: **sampling distribution of sample mean**
- ▶ the distribution of sample mean over *all possible samples*
- ▶ but we only have *one* sample!
- ▶ Idea: assume our sample is representative of the population, and draw samples from our sample (!), with replacement.
- ▶ This gives an idea of what different samples from the population might look like.
- ▶ Called *bootstrap*, after expression “to pull yourself up by your own bootstraps”.

# Packages

```
library(tidyverse)
```

## Blue Jays attendances

```
jays$attendance
```

```
[1] 48414 17264 15086 14433 21397 34743 44794 14184 15606  
[13] 21312 30430 42917 42419 29306 15062 16402 19014 21195  
[25] 17276
```

► A bootstrap sample:

```
s <- sample(jays$attendance, replace = TRUE)  
s
```

```
[1] 21195 34743 21312 44794 16402 19014 34743 21195 17264  
[13] 34743 19217 14433 15062 16402 15062 34743 15062 15086  
[25] 30430
```

► It is easier to see what is happening if we sort both the actual attendances and the bootstrap sample:

```
sort(jays$attendance)
```

## Getting mean of bootstrap sample

- ▶ A bootstrap sample is same size as original, but contains repeated values (eg. 15062) and missing ones (42917).
- ▶ We need the mean of our bootstrap sample:

```
mean(s)
```

```
[1] 23055.28
```

- ▶ This is a little different from the mean of our actual sample:

```
mean(jays$attendance)
```

```
[1] 25070.16
```

- ▶ Want a sense of how the sample mean might vary, if we were able to take repeated samples from our population.
- ▶ Idea: take lots of *bootstrap* samples, and see how *their* sample means vary.

## Setting up bootstrap sampling

- ▶ Begin by setting up a dataframe that contains a row for each bootstrap sample. I usually call this column `sim`. Do just 4 to get the idea:

```
tibble(sim = 1:4)
```

```
# A tibble: 4 x 1
```

```
  sim
```

```
<int>
```

|   |   |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |

## Drawing the bootstrap samples

- ▶ Then set up to work one row at a time, and draw a bootstrap sample of the attendances in each row:

```
tibble(sim = 1:4) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(jays$attendance, replace = TR
```

```
# A tibble: 4 x 2
```

```
# Rowwise:
```

```
      sim sample  
  <int> <list>  
1       1 <dbl [25]>  
2       2 <dbl [25]>  
3       3 <dbl [25]>  
4       4 <dbl [25]>
```

- ▶ Each row of our dataframe contains *all* of a bootstrap sample of 25 observations drawn with replacement from the attendances.



## Sample means

- Find the mean of each sample:

```
tibble(sim = 1:4) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(jays$attendance, replace = TRUE, size = 25)))  
  mutate(my_mean = mean(sample))
```

# A tibble: 4 x 3

# Rowwise:

|   | sim   | sample     | my_mean |
|---|-------|------------|---------|
|   | <int> | <list>     | <dbl>   |
| 1 | 1     | <dbl [25]> | 28472.  |
| 2 | 2     | <dbl [25]> | 28648.  |
| 3 | 3     | <dbl [25]> | 23329.  |
| 4 | 4     | <dbl [25]> | 24808.  |

- These are (four simulated values of) the bootstrapped sampling distribution of the sample mean.

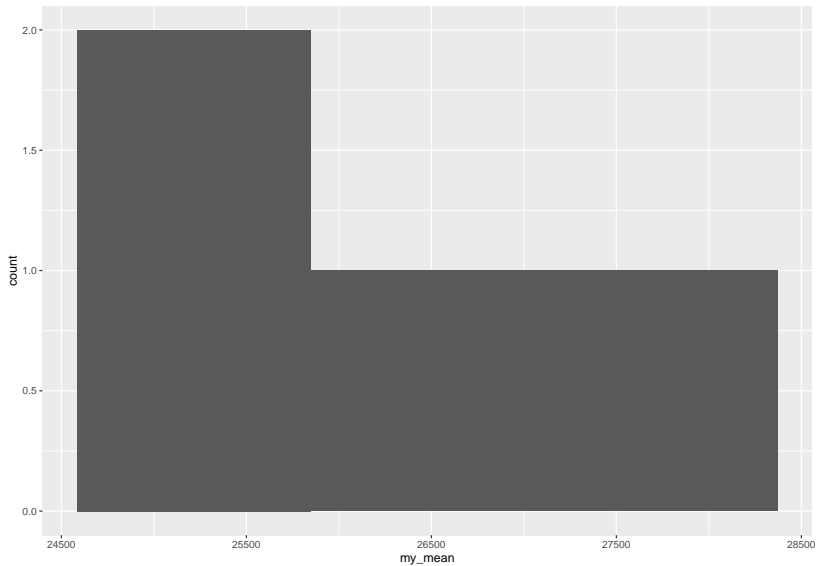
## Make a histogram of them

► rather pointless here, but to get the idea:

```
tibble(sim = 1:4) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(jays$attendance, replace = TRUE)))  
  mutate(my_mean = mean(sample)) %>%  
  ggplot(aes(x = my_mean)) + geom_histogram(bins = 3) -> g
```

# The (pointless) histogram

g



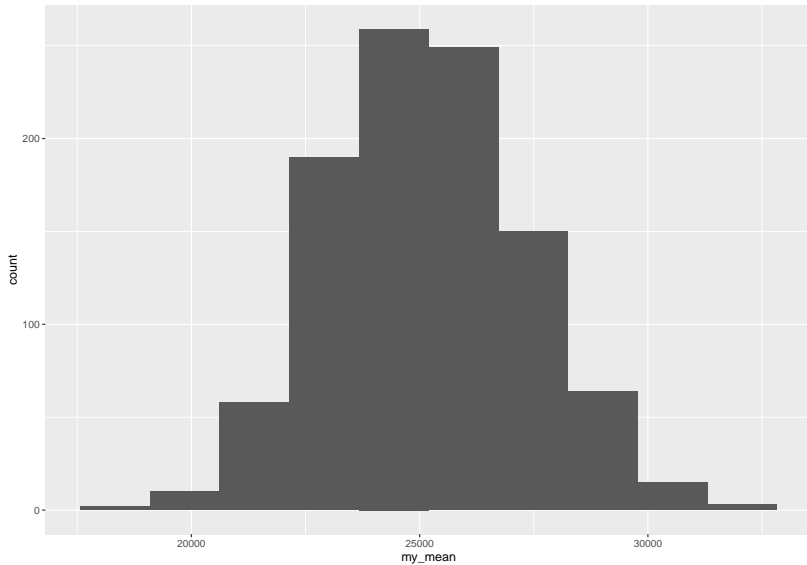
## Now do again with a decent number of bootstrap samples

- ▶ say 1000, and put a decent number of bins on the histogram also:

```
tibble(sim = 1:1000) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(jays$attendance, replace = TRUE, size = 100)))  
  mutate(my_mean = mean(sample)) %>%  
  ggplot(aes(x = my_mean)) + geom_histogram(bins = 10) -> g
```

# The (better) histogram

g



## Comments

- ▶ This is very close to normal
- ▶ The bootstrap says that the sampling distribution of the sample mean is close to normal, even though the distribution of the data is not
- ▶ A sample size of 25 is big enough to overcome the skewness that we saw
- ▶ This is the Central Limit Theorem in practice
- ▶ It is surprisingly powerful.
- ▶ Thus, the  $t$ -test is actually perfectly good here.

## Comments on the code 1/2

- You might have been wondering about this:

```
tibble(sim = 1:4) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(jays$attendance, replace = TR
```

```
# A tibble: 4 x 2
```

```
# Rowwise:
```

|   | sim   | sample     |
|---|-------|------------|
|   | <int> | <list>     |
| 1 | 1     | <dbl [25]> |
| 2 | 2     | <dbl [25]> |
| 3 | 3     | <dbl [25]> |
| 4 | 4     | <dbl [25]> |

## Comments on the code 2/2

- ▶ how did we squeeze all 25 sample values into one cell?
  - ▶ sample is a so-called “list-column” that can contain anything.
- ▶ why did we have to put `list()` around the `sample()`?
  - ▶ because `sample` produces a collection of numbers, not just a single one
  - ▶ the `list()` signals this: “make a list-column of samples”.



## Two samples

- ▶ Assumption: *both* samples are from a normal distribution.
- ▶ In this case, each sample should be “normal enough” given its sample size, since Central Limit Theorem will help.
- ▶ Use bootstrap on each group independently, as above.

## Kids learning to read

```
# A tibble: 44 x 2
  group score
  <chr> <dbl>
1 t      24
2 t      61
3 t      59
4 t      46
5 t      43
6 t      44
7 t      52
8 t      43
9 t      58
10 t     67
# i 34 more rows
```

```
ggplot(kids, aes(x=group, y=score)) + geom_boxplot()
```

## Getting just the control group

- Use filter to select rows where something is true:

```
kids %>% filter(group == "c") -> controls  
controls
```

```
# A tibble: 23 x 2
```

```
  group score
```

```
  <chr> <dbl>
```

```
1 c      42
```

```
2 c      33
```

```
3 c      46
```

```
4 c      37
```

```
5 c      43
```

```
6 c      41
```

```
7 c      10
```

```
8 c      42
```

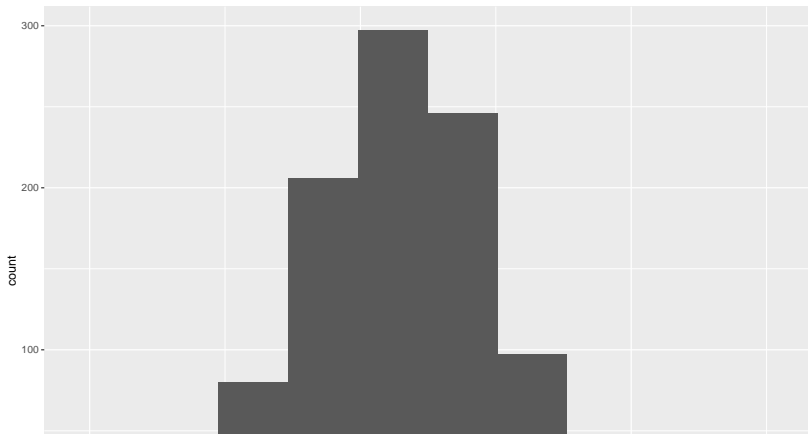
```
9 c      55
```

```
10 c      19
```

```
# i 13 more rows
```

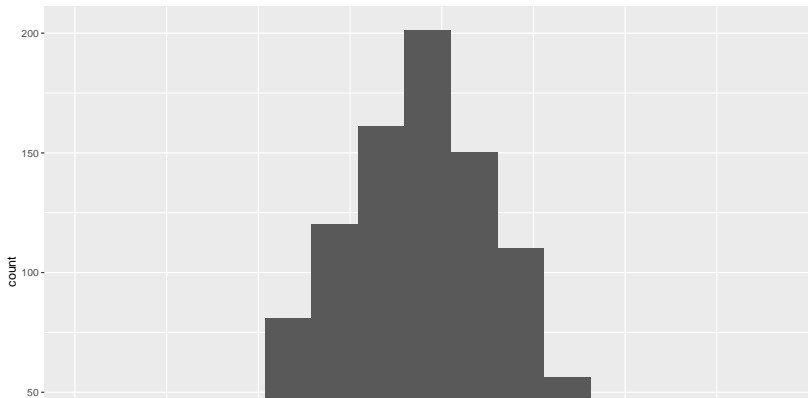
## Bootstrap these

```
tibble(sim = 1:1000) %>%  
  rowwise() %>%  
  mutate(sample = list(sample(controls$score, replace = TRUE, size = 10)))  
  mutate(my_mean = mean(sample)) %>%  
  ggplot(aes(x = my_mean)) + geom_histogram(bins = 10)
```



... and the treatment group:

```
kids %>% filter(group=="t") -> treats
tibble(sim = 1:1000) %>%
  rowwise() %>%
  mutate(sample = list(sample(treats$score, replace = TRUE)))
  mutate(my_mean = mean(sample)) %>%
  ggplot(aes(x = my_mean)) + geom_histogram(bins = 15)
```



## Comments

- ▶ sampling distributions of sample means both look pretty normal, though treatment group is a tiny bit left-skewed
- ▶ as we thought, no problems with our two-sample  $t$  at all.