Assumptions

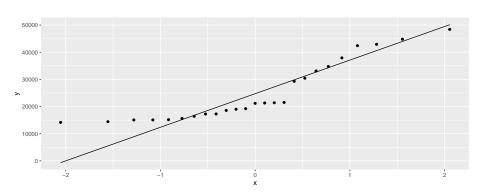
Assumptions

- The t procedures we have seen so far come with assumption of normally-distributed data
- but how much does that normality matter?
- Central Limit Theorem says that sampling distribution of sample mean is "approximately normal" if sample size is "large".
- Hence same applies to difference of two sample means.
- How to use this in practice? Draw a picture and make a call about whether sample size large enough.

Assumptions 2 / 11

Blue Jays attendances

```
ggplot(jays, aes(sample = attendance)) +
stat_qq() + stat_qq_line()
```



Assumptions 3 / 11

Comments

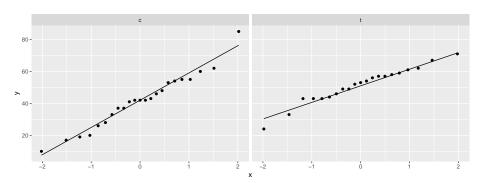
- Distribution of attendances somewhat skewed to the right (because of the short lower tail and the sort-of curve)
- \bullet Sample size n=25 is reasonably large in Central Limit Theorem terms
- Use of t may be OK here despite skewed shape.

Assumptions 4 / 11

Learning to read

• Make normal quantile plots, one for each sample:

```
ggplot(kids, aes(sample = score)) +
  stat_qq() + stat_qq_line() +
  facet_wrap(~ group)
```



Assumptions 5 / 11

Comments

- with sample sizes over 20 in each group, these are easily normal enough to use a t-test.
- the (sampling distribution of the) difference between two sample means tends to have a more normal distribution than either sample mean individually, so that two-sample t tends to be better than you'd guess.

Assumptions 6/11

Pain relief

• With matched pairs, assumption is of normality of *differences*, so work those out first:

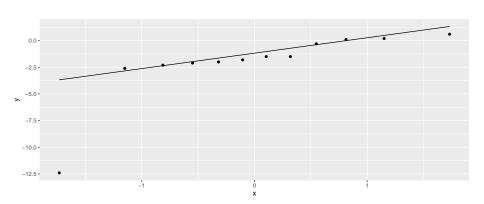
```
pain %>% mutate(diff = druga - drugb) -> pain
pain
```

```
A tibble: 12 \times 4
 subject druga drugb diff
   <dbl> <dbl> <dbl> <dbl>
           2
                 3.5 - 1.5
         3.6 \quad 5.7 \quad -2.1
3
       3 2.6 2.9 -0.300
       4 2.6 2.4 0.200
5
       5 7.3 9.9 -2.6
6
       6 3.4 3.3 0.100
          14.9 16.7 -1.80
8
       8
           6.6
                 6
                      0.600
           2.3
                 3.8 - 1.5
```

Assumptions 7 / 11

Normality of differences

ggplot(pain,aes(sample=diff)) + stat_qq() + stat_qq_line()



Assumptions 8 / 11

Comments

- This is very non-normal (the low outlier)
- The sample size of n=12 is not large
- We should have concerns about our matched pairs *t*-test.

Assumptions 9 / 11

Doing things properly

- The right way to use a t procedure:
 - draw a graph of our data (one of the standard graphs, or normal quantile plot)
 - ▶ use the graph to assess sufficient normality given the sample size
 - ▶ for a two-sample test, assess equality of spreads (boxplot easier for this)
 - ▶ if necessary, express our doubts about the *t* procedure (for now), or do a better test (later).

Assumptions 10 / 11

Looking ahead

- Looking at a normal quantile plot and assessing it with the sample size seems rather arbitrary. Can we do better? (Yes: using the bootstrap, later.)
- What to do if the t procedure is not to be trusted? Use a different test (later):
 - ▶ one sample: sign test
 - two samples: Mood's median test
 - matched pairs: sign test on differences.
- If you have heard about the signed rank or rank sum tests: they come with extra assumptions that are usually not satisfied if normality fails.

Assumptions 11 / 11