

Logistic Regression

Logistic regression

- When response variable is measured/counted, regression can work well.
- But what if response is yes/no, lived/died, success/failure?
- Model *probability* of success.
- Probability must be between 0 and 1; need method that ensures this.
- *Logistic regression* does this. In R, is a *generalized linear model* with binomial “family”:

```
glm(y ~ x, family="binomial")
```

- Begin with simplest case.

Packages

```
library(MASS, exclude = "select")
library(tidyverse)
library(marginaleffects)
library(broom)
library(nnet)
# library(conflicted)
# conflict_prefer("select", "dplyr")
# conflict_prefer("filter", "dplyr")
# conflict_prefer("rename", "dplyr")
# conflict_prefer("summarize", "dplyr")
```

The rats, part 1

- Rats given dose of some poison; either live or die:

dose status

0 lived

1 died

2 lived

3 lived

4 died

5 died

Read in:

```
my_url <- "http://ritsokiguess.site/datafiles/rat.txt"
rats <- read_delim(my_url, " ")
rats
```

```
# A tibble: 6 x 2
```

```
  dose status
```

```
<dbl> <chr>
```

```
1      0 lived
```

```
2      1 died
```

```
3      2 lived
```

```
4      3 lived
```

```
5      4 died
```

```
6      5 died
```

This doesn't work

```
status.0 <- glm(status ~ dose, family = "binomial", data = rats)
```

Error in eval(family\$initialize): y values must be $0 \leq y \leq 1$

- Values of response variable (here status) must be either:
 - ▶ 1 = "success", 0 = "failure"
 - ▶ a factor (not text) with two levels.
- The error message doesn't say that the second is a possibility.

Basic logistic regression

- So, make response into a factor first:

```
rats2 <- rats %>% mutate(status = factor(status))  
rats2
```

```
# A tibble: 6 x 2
```

```
  dose status  
  <dbl> <fct>  
1     0 lived  
2     1 died  
3     2 lived  
4     3 lived  
5     4 died  
6     5 died
```

- then fit model:

```
status.1 <- glm(status ~ dose, family = "binomial",  
                 data = rats2)
```

Output

```
summary(status.1)
```

Call:

```
glm(formula = status ~ dose, family = "binomial", data = rats2)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.6841	1.7979	0.937	0.349
dose	-0.6736	0.6140	-1.097	0.273

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 8.3178 on 5 degrees of freedom
Residual deviance: 6.7728 on 4 degrees of freedom
AIC: 10.773

Interpreting the output

- Like (multiple) regression, get tests of significance of individual x 's
- Here not significant (only 6 observations).
- “Slope” for dose is negative, meaning that as dose increases, probability of event modelled (survival) decreases.

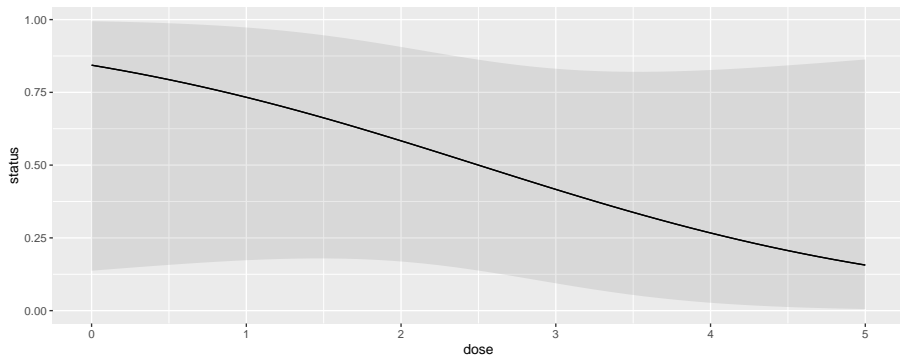
Output part 2: predicted survival probs

```
cbind(predictions(status.1)) %>%  
  select(dose, estimate, conf.low, conf.high)
```

	dose	estimate	conf.low	conf.high
1	0	0.8434490	0.137095792	0.9945564
2	1	0.7331122	0.173186479	0.9729896
3	2	0.5834187	0.168847561	0.9061463
4	3	0.4165813	0.093853682	0.8311524
5	4	0.2668878	0.027010413	0.8268135
6	5	0.1565510	0.005443589	0.8629042

On a graph

```
plot_predictions(status.1, condition = "dose")
```



The rats, more

- More realistic: more rats at each dose (say 10).
- Listing each rat on one line makes a big data file.
- Use format below: dose, number of survivals, number of deaths.

dose	lived	died
0	10	0
1	7	3
2	6	4
3	4	6
4	2	8
5	1	9

- 6 lines of data correspond to 60 actual rats.
- Saved in `rat2.txt`.

These data

```
my_url <- "http://ritsokiguess.site/datafiles/rat2.txt"
rat2 <- read_delim(my_url, " ")
rat2
```

```
# A tibble: 6 x 3
  dose lived died
<dbl> <dbl> <dbl>
1     0    10     0
2     1     7     3
3     2     6     4
4     3     4     6
5     4     2     8
6     5     1     9
```

Response matrix:

- Each row contains *multiple* observations.
- Create *two-column* response with `cbind`:
 - ▶ #survivals in first column,
 - ▶ #deaths in second.

```
with(rat2, cbind(lived, died))
```

	lived	died
[1,]	10	0
[2,]	7	3
[3,]	6	4
[4,]	4	6
[5,]	2	8
[6,]	1	9

Fit logistic regression

- constructing the response in the glm:

```
rat2.1 <- glm(cbind(lived, died) ~ dose,  
              family = "binomial", data = rat2)
```

Output

Significant effect of dose now:

```
summary(rat2.1)
```

Call:

```
glm(formula = cbind(lived, died) ~ dose, family = "binomial",  
     data = rat2)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	2.3619	0.6719	3.515	0.000439	***
dose	-0.9448	0.2351	-4.018	5.87e-05	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

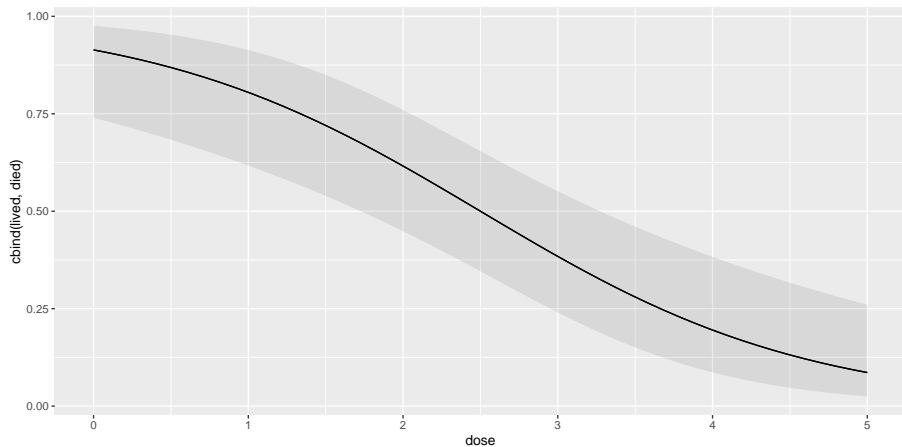
Predicted survival probs

```
new <- datagrid(model = rat2.1, dose = 0:5)
cbind(predictions(rat2.1, newdata = new)) %>%
  select(estimate, dose, conf.low, conf.high)
```

	estimate	dose	conf.low	conf.high
1	0.9138762	0	0.73983042	0.9753671
2	0.8048905	1	0.61695841	0.9135390
3	0.6159474	2	0.44876099	0.7595916
4	0.3840526	3	0.24040837	0.5512390
5	0.1951095	4	0.08646093	0.3830417
6	0.0861238	5	0.02463288	0.2601697

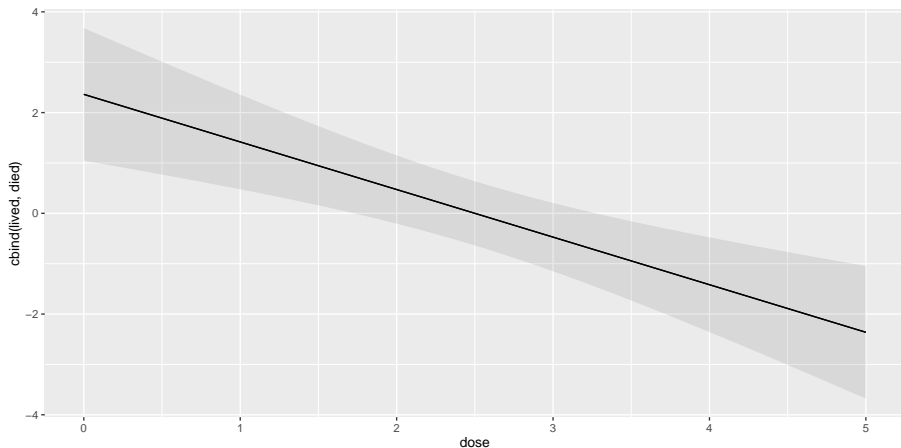
On a picture

```
plot_predictions(rat2.1, condition = "dose")
```



Dose and predicted log-odds

```
plot_predictions(rat2.1, condition = "dose", type = "link")
```



Comments

- Significant effect of dose.
- Effect of larger dose is to *decrease* survival probability (“slope” negative; also see in decreasing predictions.)
- Confidence intervals around prediction narrower (more data).

Multiple logistic regression

- With more than one x , works much like multiple regression.
- Example: study of patients with blood poisoning severe enough to warrant surgery. Relate survival to other potential risk factors.
- Variables, 1=present, 0=absent:
 - ▶ survival (death from sepsis=1), response
 - ▶ shock
 - ▶ malnutrition
 - ▶ alcoholism
 - ▶ age (as numerical variable)
 - ▶ bowel infarction
- See what relates to death.

Read in data

```
my_url <-  
  "http://ritsokiguess.site/datafiles/sepsis.txt"  
sepsis <- read_delim(my_url, " ")  
sepsis
```

```
# A tibble: 106 x 6
```

	death	shock	malnut	alcohol	age	bowelinf
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	0	0	0	0	56	0
2	0	0	0	0	80	0
3	0	0	0	0	61	0
4	0	0	0	0	26	0
5	0	0	0	0	53	0
6	1	0	1	0	87	0
7	0	0	0	0	21	0
8	1	0	0	1	69	0
9	0	0	0	0	57	0
10	0	0	1	0	76	0

```
# i 96 more rows
```

Make sure categoricals really are

```
sepsis %>%  
  mutate(across(-age, \(x) factor(x))) -> sepsis
```

The data (some)

```
sepsis
```

```
# A tibble: 106 x 6
```

	death	shock	malnut	alcohol	age	bowelinf
	<fct>	<fct>	<fct>	<fct>	<dbl>	<fct>
1	0	0	0	0	56	0
2	0	0	0	0	80	0
3	0	0	0	0	61	0
4	0	0	0	0	26	0
5	0	0	0	0	53	0
6	1	0	1	0	87	0
7	0	0	0	0	21	0
8	1	0	0	1	69	0
9	0	0	0	0	57	0
10	0	0	1	0	76	0

```
# i 96 more rows
```


Fit model

```
sepsis.1 <- glm(death ~ shock + malnut + alcohol + age +  
  bowelinf, family = "binomial", data = sepsis  
)
```

Output part 1

```
summary(sepsis.1)
```

Call:

```
glm(formula = death ~ shock + malnut + alcohol + age + bowelinf,  
     family = "binomial", data = sepsis)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-9.75391	2.54170	-3.838	0.000124	***
shock1	3.67387	1.16481	3.154	0.001610	**
malnut1	1.21658	0.72822	1.671	0.094798	.
alcohol1	3.35488	0.98210	3.416	0.000635	***
age	0.09215	0.03032	3.039	0.002374	**
bowelinf1	2.79759	1.16397	2.403	0.016240	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Or, with tidy (from broom)

```
tidy(sepsis.1)
```

```
# A tibble: 6 x 5
```

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	-9.75	2.54	-3.84	0.000124
2	shock1	3.67	1.16	3.15	0.00161
3	malnut1	1.22	0.728	1.67	0.0948
4	alcohol1	3.35	0.982	3.42	0.000635
5	age	0.0922	0.0303	3.04	0.00237
6	bowelinf1	2.80	1.16	2.40	0.0162

- All P-values fairly small
- but malnut not significant: remove.

Removing malnut

```
sepsis.2 <- update(sepsis.1, . ~ . - malnut)
summary(sepsis.2)
```

Call:

```
glm(formula = death ~ shock + alcohol + age + bowelinf, family = "b",
     data = sepsis)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-8.89459	2.31689	-3.839	0.000124	***
shock1	3.70119	1.10353	3.354	0.000797	***
alcohol1	3.18590	0.91725	3.473	0.000514	***
age	0.08983	0.02922	3.075	0.002106	**
bowelinf1	2.38647	1.07227	2.226	0.026039	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Comments

- Everything significant now.
- Most of the original x 's helped predict death. Only `malnut` seemed not to add anything.
- Removed `malnut` and tried again.
- Everything remaining is significant (though `bowelinf` actually became *less* significant).
- All coefficients are *positive*, so having any of the risk factors (or being older) *increases* risk of death.

Predictions from model without “malnut” 1/2

- A few (rows of original dataframe) chosen “at random”:

```
sepsis %>% slice(c(4, 1, 2, 11, 32)) -> new  
new
```

```
# A tibble: 5 x 6
```

	death	shock	malnut	alcohol	age	bowelinf
	<fct>	<fct>	<fct>	<fct>	<dbl>	<fct>
1	0	0	0	0	26	0
2	0	0	0	0	56	0
3	0	0	0	0	80	0
4	1	0	0	1	66	1
5	1	0	0	1	49	0

Predictions from model without “malnut” 2/2

```
cbind(predictions(sepsis.2, newdata = new)) %>%  
  select(estimate, conf.low, conf.high, shock:bowelinf)
```

	estimate	conf.low	conf.high	shock	malnut	alcohol	age	bowelinf
1	0.001415347	6.272642e-05	0.03103047	0	0	0	26	0
2	0.020552383	4.102504e-03	0.09656596	0	0	0	56	0
3	0.153416834	5.606838e-02	0.35603441	0	0	0	80	0
4	0.931290137	5.490986e-01	0.99341482	0	0	1	66	1
5	0.213000997	7.639063e-02	0.46967947	0	0	1	49	0

Comments

- Survival chances pretty good if no risk factors, though decreasing with age.
- Having more than one risk factor reduces survival chances dramatically.
- Usually good job of predicting survival; sometimes death predicted to survive.

Another way to assess effects

of age:

```
new <- datagrid(model = sepsis.2, age = seq(30, 70, 10))  
new
```

	shock	alcohol	bowelinf	age	rowid
1	0	0	0	30	1
2	0	0	0	40	2
3	0	0	0	50	3
4	0	0	0	60	4
5	0	0	0	70	5

Assessing age effect

```
cbind(predictions(sepsis.2, newdata = new)) %>%  
  select(estimate, shock:age)
```

	estimate	shock	alcohol	bowelinf	age
1	0.002026053	0	0	0	30
2	0.004960283	0	0	0	40
3	0.012092515	0	0	0	50
4	0.029179226	0	0	0	60
5	0.068729752	0	0	0	70

Assessing shock effect

```
new <- datagrid(shock = c(0, 1), model = sepsis.2)
new
```

	alcohol	age	bowelinf	shock	rowid
1	0	51.28302	0	0	1
2	0	51.28302	0	1	2

```
cbind(predictions(sepsis.2, newdata = new)) %>%
  select(estimate, alcohol:shock)
```

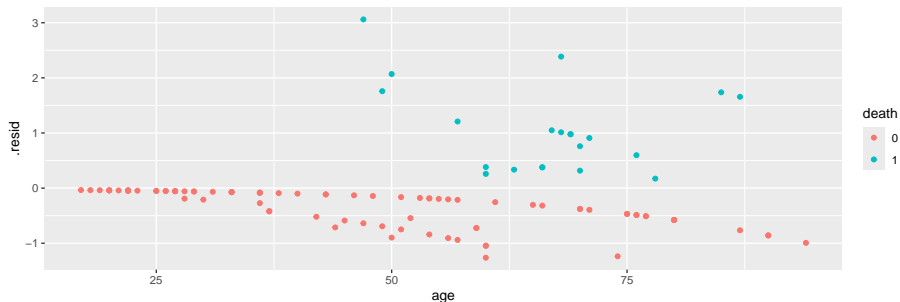
	estimate	alcohol	age	bowelinf	shock
1	0.01354973	0	51.28302	0	0
2	0.35742607	0	51.28302	0	1

Assessing proportionality of odds for age

- An assumption we made is that log-odds of survival depends linearly on age.
- Hard to get your head around, but basic idea is that survival chances go continuously up (or down) with age, instead of (for example) going up and then down.
- In this case, seems reasonable, but should check:

Residuals vs. age

```
sepsis.2 %>% augment(sepsis) %>%  
  ggplot(aes(x = age, y = .resid, colour = death)) +  
  geom_point()
```



Comments

- No apparent problems overall.
- Confusing “line” across: no risk factors, survived.

Probability and odds

For probability p , odds is $p/(1 - p)$:

Prob	Odds	Log-odds	Words
0.5	$0.5 / 0.5 = 1.00$	0.00	even money
0.1	$0.1 / 0.9 = 0.11$	-2.20	9 to 1
0.4	$0.4 / 0.6 = 0.67$	-0.41	1.5 to 1
0.8	$0.8 / 0.2 = 4.00$	1.39	4 to 1 on

- Gamblers use odds: if you win at 9 to 1 odds, get original stake back plus 9 times the stake.
- Probability has to be between 0 and 1
- Odds between 0 and infinity
- *Log-odds* can be anything: any log-odds corresponds to valid probability.
- Thus, predict *log-odds of probability* from explanatory variable(s), rather than probability itself.

Odds ratio

- Suppose 90 of 100 men drank wine last week, but only 20 of 100 women.
- Prob of man drinking wine $90/100 = 0.9$, woman $20/100 = 0.2$.
- Odds of man drinking wine $0.9/0.1 = 9$, woman $0.2/0.8 = 0.25$.
- Ratio of odds is $9/0.25 = 36$.
- Way of quantifying difference between men and women: “odds of drinking wine 36 times larger for males than females”.

Sepsis data again

- Recall prediction of probability of death from risk factors:

```
sepsis
```

```
# A tibble: 106 x 6
```

	death	shock	malnut	alcohol	age	bowelinf
	<fct>	<fct>	<fct>	<fct>	<dbl>	<fct>
1	0	0	0	0	56	0
2	0	0	0	0	80	0
3	0	0	0	0	61	0
4	0	0	0	0	26	0
5	0	0	0	0	53	0
6	1	0	1	0	87	0
7	0	0	0	0	21	0
8	1	0	0	1	69	0
9	0	0	0	0	57	0
10	0	0	1	0	76	0

```
# i 96 more rows
```

Multiplying the odds

- Can interpret slopes by taking “exp” of them. We ignore intercept.

```
sepsis.2.tidy %>%  
  mutate(exp_coeff=exp(estimate)) %>%  
  select(term, exp_coeff)
```

```
# A tibble: 5 x 2  
  term      exp_coeff  
  <chr>      <dbl>  
1 (Intercept) 0.000137  
2 shock1      40.5  
3 alcohol1    24.2  
4 age         1.09  
5 bowelinf1   10.9
```

Interpretation

```
# A tibble: 5 x 2
  term      exp_coeff
<chr>      <dbl>
1 (Intercept) 0.000137
2 shock1      40.5
3 alcohol1    24.2
4 age         1.09
5 bowelinf1   10.9
```

- These say “how much do you *multiply* odds of death by for increase of 1 in corresponding risk factor?” Or, what is odds ratio for that factor being 1 (present) vs. 0 (absent)?
- Eg. being alcoholic vs. not increases odds of death by 24 times
- One year older multiplies odds by about 1.1 times. Over 40 years, about $1.09^{40} = 31$ times.

Odds ratio and relative risk

- **Relative risk** is ratio of probabilities.
- Above: 90 of 100 men (0.9) drank wine, 20 of 100 women (0.2).
- Relative risk $0.9/0.2=4.5$. (odds ratio was 36).
- When probabilities small, relative risk and odds ratio similar.
- Eg. prob of man having disease 0.02, woman 0.01.
- Relative risk $0.02/0.01 = 2$.

Odds ratio vs. relative risk

- Odds for men and for women:

```
(od1 <- 0.02 / 0.98) # men
```

```
[1] 0.02040816
```

```
(od2 <- 0.01 / 0.99) # women
```

```
[1] 0.01010101
```

- Odds ratio

```
od1 / od2
```

```
[1] 2.020408
```

- Very close to relative risk of 2.

More than 2 response categories

- With 2 response categories, model the probability of one, and prob of other is one minus that. So doesn't matter which category you model.
- With more than 2 categories, have to think more carefully about the categories: are they
- *ordered*: you can put them in a natural order (like low, medium, high)
- *nominal*: ordering the categories doesn't make sense (like red, green, blue).
- R handles both kinds of response; learn how.

Ordinal response: the miners

- Model probability of being in given category *or lower*.
- Example: coal-miners often suffer disease pneumoconiosis. Likelihood of disease believed to be greater among miners who have worked longer.
- Severity of disease measured on categorical scale: none, moderate, severe.

Miners data

- Data are frequencies:

Exposure	None	Moderate	Severe
5.8	98	0	0
15.0	51	2	1
21.5	34	6	3
27.5	35	5	8
33.5	32	10	9
39.5	23	7	8
46.0	12	6	10
51.5	4	2	5

Reading the data

Data in aligned columns with more than one space between, so:

```
my_url <- "http://ritsokiguess.site/datafiles/miners-tab.txt"  
freqs <- read_table(my_url)
```

The data

```
freqs
```

```
# A tibble: 8 x 4
```

	Exposure	None	Moderate	Severe
	<dbl>	<dbl>	<dbl>	<dbl>
1	5.8	98	0	0
2	15	51	2	1
3	21.5	34	6	3
4	27.5	35	5	8
5	33.5	32	10	9
6	39.5	23	7	8
7	46	12	6	10
8	51.5	4	2	5

Tidying

```
freqs %>%  
  pivot_longer(-Exposure, names_to = "Severity", values_to = "  
  mutate(Severity = fct_inorder(Severity)) -> miners
```

Result

```
miners
```

```
# A tibble: 24 x 3
  Exposure Severity  Freq
    <dbl> <fct>    <dbl>
1     5.8 None      98
2     5.8 Moderate    0
3     5.8 Severe     0
4    15  None     51
5    15  Moderate    2
6    15  Severe     1
7   21.5 None     34
8   21.5 Moderate    6
9   21.5 Severe     3
10   27.5 None     35
# i 14 more rows
```

```
levels(miners$Severity)
```

Plot proportions against exposure 1/2

```
miners %>%  
  group_by(Exposure) %>%  
  mutate(proportion = Freq / sum(Freq)) -> prop  
prop
```

A tibble: 24 x 4

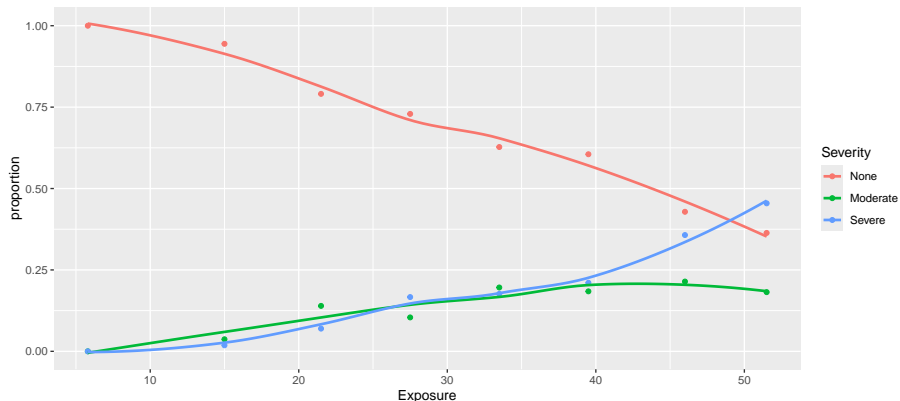
Groups: Exposure [8]

	Exposure	Severity	Freq	proportion
	<dbl>	<fct>	<dbl>	<dbl>
1	5.8	None	98	1
2	5.8	Moderate	0	0
3	5.8	Severe	0	0
4	15	None	51	0.944
5	15	Moderate	2	0.0370
6	15	Severe	1	0.0185
7	21.5	None	34	0.791
8	21.5	Moderate	6	0.140
9	21.5	Severe	3	0.0698
10	27.5	None	35	0.729

#> # A tibble: 14 x 4

Plot proportions against exposure 2/2

```
ggplot(prop, aes(x = Exposure, y = proportion,  
                 colour = Severity)) +  
  geom_point() + geom_smooth(se = F)
```



Reminder of data setup

```
miners
```

```
# A tibble: 24 x 3
  Exposure Severity  Freq
  <dbl>   <fct>    <dbl>
1     5.8   None      98
2     5.8 Moderate    0
3     5.8 Severe     0
4    15    None     51
5    15  Moderate    2
6    15  Severe     1
7   21.5   None    34
8   21.5 Moderate    6
9   21.5 Severe     3
10  27.5   None    35
# i 14 more rows
```

Fitting ordered logistic model

Use function `polr` from package `MASS`. Like `glm`.

```
sev.1 <- polr(Severity ~ Exposure,  
  weights = Freq,  
  data = miners  
)
```


Output: not very illuminating

```
sev.1 <- polr(Severity ~ Exposure,  
  weights = Freq,  
  data = miners,  
)
```

```
summary(sev.1)
```

Call:

```
polr(formula = Severity ~ Exposure, data = miners, weights = Freq)
```

Coefficients:

	Value	Std. Error	t value
Exposure	0.0959	0.01194	8.034

Intercepts:

	Value	Std. Error	t value
None Moderate	3.9558	0.4097	9.6558
Moderate Severe	4.8690	0.4411	11.0383

Residual Deviance: 416.9188

AIC: 422.9188

Does exposure have an effect?

Fit model without Exposure, and compare using anova. Note 1 for model with just intercept:

```
sev.0 <- polr(Severity ~ 1, weights = Freq, data = miners)
anova(sev.0, sev.1)
```

Likelihood ratio tests of ordinal regression models

Response: Severity

	Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
1	1	369	505.1621				
2	Exposure	368	416.9188	1 vs 2	1	88.24324	0

Exposure definitely has effect on severity of disease.

Another way

- What (if anything) can we drop from model with exposure?

```
drop1(sev.1, test = "Chisq")
```

Single term deletions

Model:

Severity ~ Exposure

	Df	AIC	LRT	Pr(>Chi)
<none>		422.92		
Exposure	1	509.16	88.243	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

- Nothing. Exposure definitely has effect.

Predicted probabilities 1/2

```
freqs %>% select(Exposure) -> new  
new
```

```
# A tibble: 8 x 1
```

```
  Exposure
```

```
    <dbl>
```

```
1      5.8
```

```
2     15
```

```
3    21.5
```

```
4    27.5
```

```
5    33.5
```

```
6    39.5
```

```
7     46
```

```
8    51.5
```

Predicted probabilities 2/2

```
cbind(predictions(sev.1, newdata = new)) %>%  
  select(group, estimate, Exposure) %>%  
  pivot_wider(names_from = group, values_from = estimate)
```

A tibble: 8 x 4

	Exposure	None	Moderate	Severe
	<dbl>	<dbl>	<dbl>	<dbl>
1	5.8	0.968	0.0191	0.0132
2	15	0.925	0.0433	0.0314
3	21.5	0.869	0.0739	0.0569
4	27.5	0.789	0.114	0.0969
5	33.5	0.678	0.162	0.160
6	39.5	0.542	0.205	0.253
7	46	0.388	0.224	0.388
8	51.5	0.272	0.210	0.517

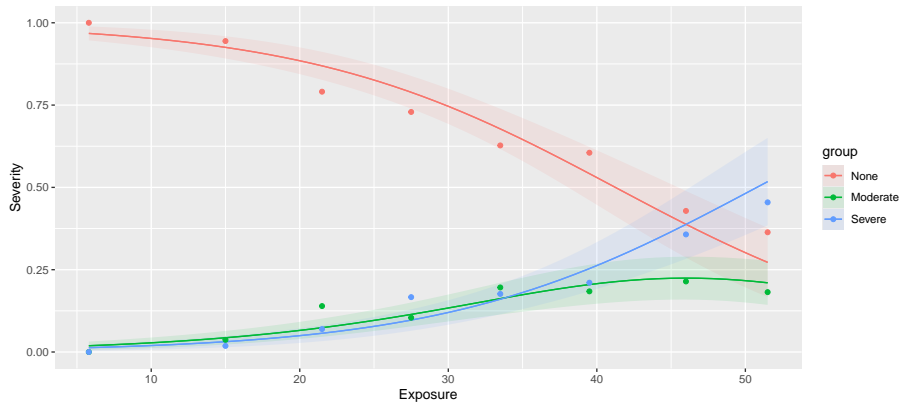
Plot of predicted probabilities

- Wider for looking at, longer for graph:

```
plot_predictions(model = sev.1,  
                 condition = c("Exposure", "group"),  
                 type = "probs") +  
  geom_point(data = prop, aes(x = Exposure, y = proportion,  
                             colour = Severity)) -> ggg
```

The graph

ggg



Comments

- Model appears to match data well enough.
- As exposure goes up, prob of None goes down, Severe goes up (sharply for high exposure).
- So more exposure means worse disease.

Unordered responses

- With unordered (nominal) responses, can use *generalized logit*.
- Example: 735 people, record age and sex (male 0, female 1), which of 3 brands of some product preferred.
- Data in `mlogit.csv` separated by commas (so `read_csv` will work):

```
my_url <- "http://ritsokiguess.site/datafiles/mlogit.csv"  
brandpref <- read_csv(my_url)
```

The data (some)

```
brandpref
```

```
# A tibble: 735 x 3
  brand    sex  age
  <dbl> <dbl> <dbl>
1     1     0   24
2     1     0   26
3     1     0   26
4     1     1   27
5     1     1   27
6     3     1   27
7     1     0   27
8     1     0   27
9     1     1   27
10    1     0   27
# i 725 more rows
```

Bashing into shape

- sex and brand not meaningful as numbers, so turn into factors:

```
brandpref %>%  
  mutate(sex = ifelse(sex == 1, "female", "male"),  
         sex = factor(sex),  
         brand = factor(brand)  
  ) -> brandpref
```

```
brandpref
```

```
# A tibble: 735 x 3
```

	brand	sex	age
	<fct>	<fct>	<dbl>
1	1	male	24
2	1	male	26
3	1	male	26
4	1	female	27
5	1	female	27
6	3	female	27
7	1	male	27

Fitting model

- We use `multinom` from package `nnet`. Works like `polr`.

```
library(nnet)
# levels(brandpref$sex)

brands.1 <- multinom(brand ~ age + sex, data = brandpref)

# weights:  12 (6 variable)
initial  value 807.480032
iter   10 value 702.990572
final   value 702.970704
converged
```

- `summary` output not helpful.

Can we drop anything?

- Unfortunately drop1 seems not to work:

```
drop1(brands.1, test = "Chisq", trace = 0)
```

```
trying - age
```

```
Error in if (trace) {: argument is not interpretable as logical}
```

- So, fall back on fitting model without what you want to test, and comparing using anova.

Do age/sex help predict brand? 1/3

Fit models without each of age and sex:

```
brands.2 <- multinom(brand ~ age, data = brandpref)
```

```
# weights:  9 (4 variable)
initial  value 807.480032
iter   10 value 706.796323
iter   10 value 706.796322
final   value 706.796322
converged
```

```
brands.3 <- multinom(brand ~ sex, data = brandpref)
```

```
# weights:  9 (4 variable)
initial  value 807.480032
final   value 791.861266
converged
```

Do age/sex help predict brand? 2/3

```
anova(brands.2, brands.1)
```

Likelihood ratio tests of Multinomial Models

Response: brand

	Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
1	age	1466	1413.593				
2	age + sex	1464	1405.941	1 vs 2	2	7.651236	0.02180496

Do age/sex help predict brand? 3/3

```
anova(brands.3, brands.1)
```

Likelihood ratio tests of Multinomial Models

Response: brand

	Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
1	sex	1466	1583.723				
2	age + sex	1464	1405.941	1 vs 2	2	177.7811	0

Comments

- age definitely significant (second anova)
- sex significant also (first anova), though P-value less dramatic
- Keep both.
- Expect to see a large effect of age, and a smaller one of sex.

Another way to build model

- Start from model with everything and run step:

```
step(brands.1, trace = 0)
```

```
trying - age
```

```
trying - sex
```

Call:

```
multinom(formula = brand ~ age + sex)
```

Coefficients:

	(Intercept)	age	sexmale
2	-11.25127	0.3682202	-0.5237736
3	-22.25571	0.6859149	-0.4658215

Residual Deviance: 1405.941

AIC: 1417.941

- Final model contains both age and sex so neither could be removed.

Making predictions

Find age 5-number summary, and the two sexes:

```
summary(brandpref)
```

brand	sex	age
1:207	female:466	Min. :24.0
2:307	male :269	1st Qu.:32.0
3:221		Median :32.0
		Mean :32.9
		3rd Qu.:34.0
		Max. :38.0

Space the ages out a bit for prediction (see over).

Combinations

```
new <- datagrid(age = seq(24, 40, 4), # cover age range  
               sex = c("female", "male"), model = brands.1)  
new
```

	age	sex	rowid
1	24	female	1
2	24	male	2
3	28	female	3
4	28	male	4
5	32	female	5
6	32	male	6
7	36	female	7
8	36	male	8
9	40	female	9
10	40	male	10

The predictions

```
cbind(predictions(brands.1, newdata = new)) %>%  
  select(group, estimate, age, sex) %>%  
  pivot_wider(names_from = group, values_from = estimate)
```

A tibble: 10 x 5

	age	sex	`1`	`2`	`3`
	<dbl>	<fct>	<dbl>	<dbl>	<dbl>
1	24	female	0.915	0.0819	0.00279
2	24	male	0.948	0.0502	0.00181
3	28	female	0.696	0.271	0.0329
4	28	male	0.793	0.183	0.0236
5	32	female	0.291	0.495	0.214
6	32	male	0.405	0.408	0.187
7	36	female	0.0503	0.374	0.576
8	36	male	0.0795	0.350	0.571
9	40	female	0.00473	0.153	0.842
10	40	male	0.00759	0.146	0.847

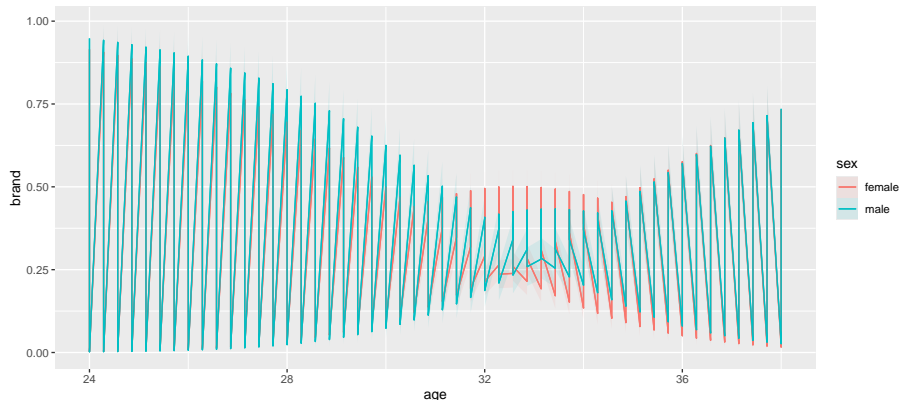
Comments

- Young males prefer brand 1, but older males prefer brand 3.
- Females similar, but like brand 1 less and brand 2 more.
- A clear brand effect, but the sex effect is less clear.

Making a plot

- I thought `plot_predictions` doesn't work as we want, but I was (sort of) wrong about that:

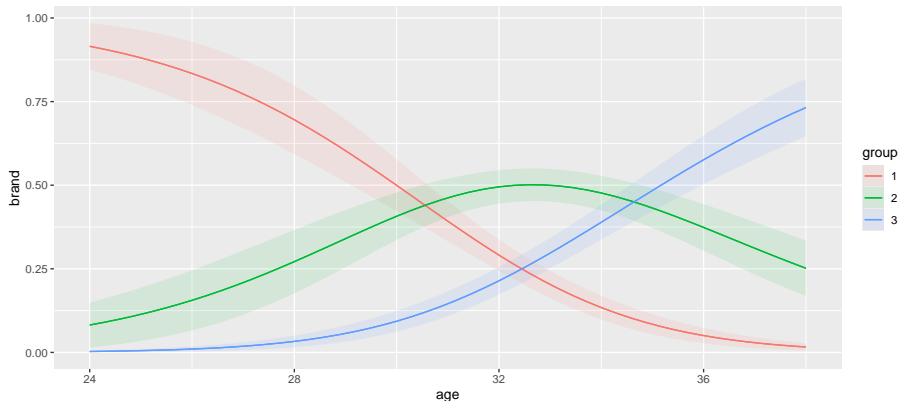
```
plot_predictions(brands.1, condition = c("age", "sex"),  
  type = "probs")
```



Making it go

- We have to include group in the condition:

```
plot_predictions(brands.1, condition = c("age", "group"))
```



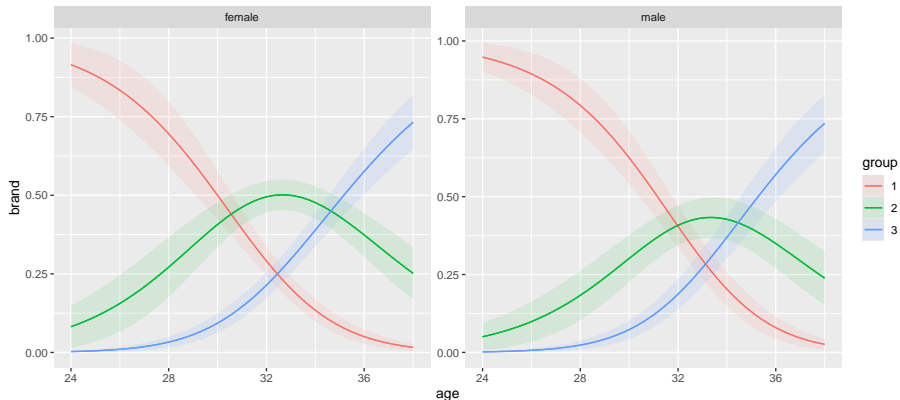
Comments

- This picks the most common sex in the data (females).
- See younger females prefer brand 1, older ones preferring brand 3.

For each sex

If we add the other variable to the *end*, we get facets for sex:

```
plot_predictions(brands.1, condition = c("age", "group", "sex"))
```



Not actually much difference between males and females.

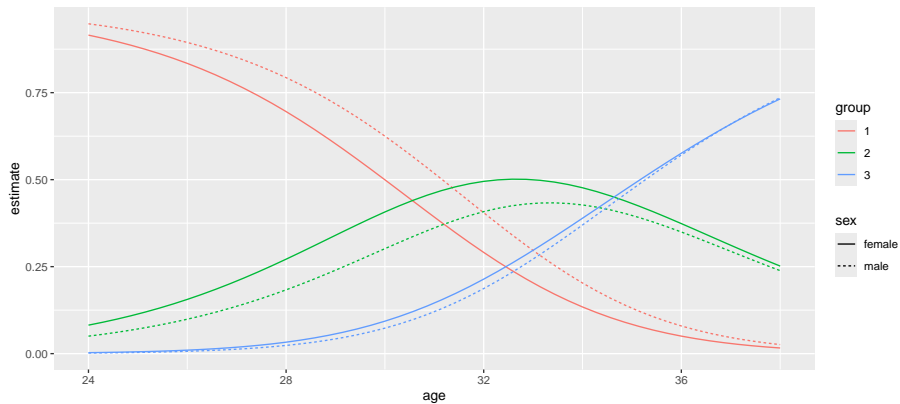
A better graph

- but the male-female difference *was* significant. How?
- *don't* actually plot the graph, then plot the right things:

```
plot_predictions(brands.1,  
                  condition = c("age", "brand", "sex"),  
                  type = "probs", draw = FALSE) %>%  
  ggplot(aes(x = age, y = estimate, colour = group,  
             linetype = sex)) +  
  geom_line() -> g
```

The graph

09



Digesting the plot

- Brand vs. age: younger people (of both genders) prefer brand 1, but older people (of both genders) prefer brand 3. (Explains significant age effect.)
- Brand vs. sex: females (solid) like brand 1 less than males (dashed), like brand 2 more (for all ages).
- Not much brand difference between genders (solid and dashed lines of same colours close), but enough to be significant.
- Model didn't include interaction, so modelled effect of gender on brand same for each age, modelled effect of age same for each gender. (See also later.)

Alternative data format

Summarize all people of same brand preference, same sex, same age on one line of data file with frequency on end:

```
1 0 24 1
1 0 26 2
1 0 27 4
1 0 28 4
1 0 29 7
1 0 30 3
...
```

Whole data set in 65 lines not 735! But how?

Getting alternative data format

```
brandpref %>%  
  group_by(age, sex, brand) %>%  
  summarize(Freq = n()) %>%  
  ungroup() -> b  
b
```

A tibble: 65 x 4

	age	sex	brand	Freq
	<dbl>	<fct>	<fct>	<int>
1	24	male	1	1
2	26	male	1	2
3	27	female	1	4
4	27	female	3	1
5	27	male	1	4
6	28	female	1	6
7	28	female	2	2
8	28	female	3	1

Fitting models, almost the same

- Just have to remember weights to incorporate frequencies.
- Otherwise multinom assumes you have just 1 obs on each line!
- Again turn (numerical) sex and brand into factors:

```
b %>%  
  mutate(sex = factor(sex)) %>%  
  mutate(brand = factor(brand)) -> bf  
b.1 <- multinom(brand ~ age + sex, data = bf, weights = Freq)  
b.2 <- multinom(brand ~ age, data = bf, weights = Freq)
```


P-value for sex identical

```
anova(b.2, b.1)
```

Likelihood ratio tests of Multinomial Models

Response: brand

	Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
1	age	126	1413.593				
2	age + sex	124	1405.941	1 vs 2	2	7.651236	0.02180496

Same P-value as before, so we haven't changed anything important.

Trying interaction between age and sex

```
brands.4 <- update(brands.1, . ~ . + age:sex)
```

```
anova(brands.1, brands.4)
```

Likelihood ratio tests of Multinomial Models

Response: brand

	Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
1	age + sex	1464	1405.941				
2	age + sex + age:sex	1462	1405.142	1 vs 2	2	0.7996223	0.6704466

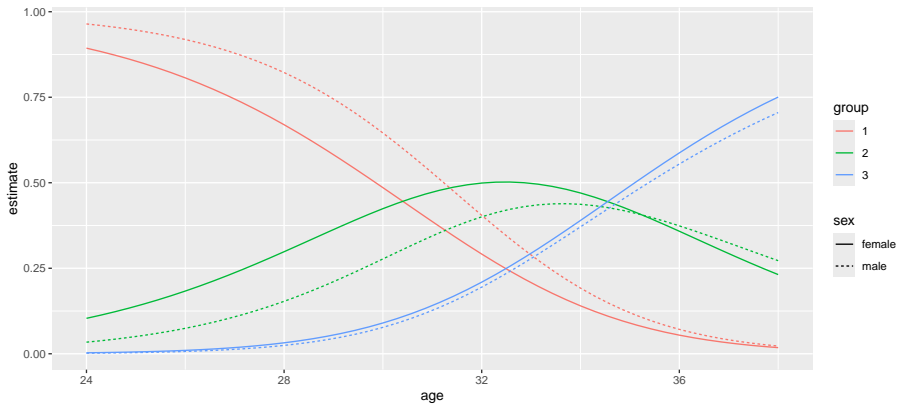
- No evidence that effect of age on brand preference differs for the two genders.

Make graph again

```
plot_predictions(brands.4,  
                  condition = c("age", "brand", "sex"),  
                  type = "probs", draw = FALSE) %>%  
  ggplot(aes(x = age, y = estimate, colour = group,  
             linetype = sex)) +  
  geom_line() -> g4
```

Not much difference in the graph

g4



Compare model without interaction

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