

# Analysis of variance revisited

# Analysis of variance

- Analysis of variance used with:
  - ▶ counted/measured response
  - ▶ categorical explanatory variable(s)
  - ▶ that is, data divided into groups, and see if response significantly different among groups
  - ▶ or, see whether knowing group membership helps to predict response.

## Two stages

- Typically two stages:
  - ▶  $F$ -test to detect *any* differences among/due to groups
  - ▶ if  $F$ -test significant, do *multiple comparisons* to see which groups significantly different from which.
- Need special multiple comparisons method because just doing (say) two-sample  $t$ -tests on each pair of groups gives too big a chance of finding “significant” differences by accident.

# Packages

These:

```
library(tidyverse)
library(broom)
library(car) # for Levene's test
```

## Example: Pain threshold and hair colour

- Do people with different hair colour have different abilities to deal with pain?
- Men and women of various ages divided into 4 groups by hair colour: light and dark blond, light and dark brown.
- Each subject given a pain sensitivity test resulting in pain threshold score: higher score is higher pain tolerance.
- 19 subjects altogether.

## The data

In `hairpain.txt` (some):

```
hair pain  
lightblond 62  
lightblond 60  
lightblond 71  
lightblond 55  
lightblond 48  
darkblond 63  
darkblond 57  
darkblond 52  
darkblond 41
```

## Summarizing the groups

```
my_url <- "http://ritsokiguess.site/datafiles/hairpain.txt"
hairpain <- read_delim(my_url, " ")
hairpain %>%
  group_by(hair) %>%
  summarize(
    n = n(),
    xbar = mean(pain),
    s = sd(pain)
  )
```

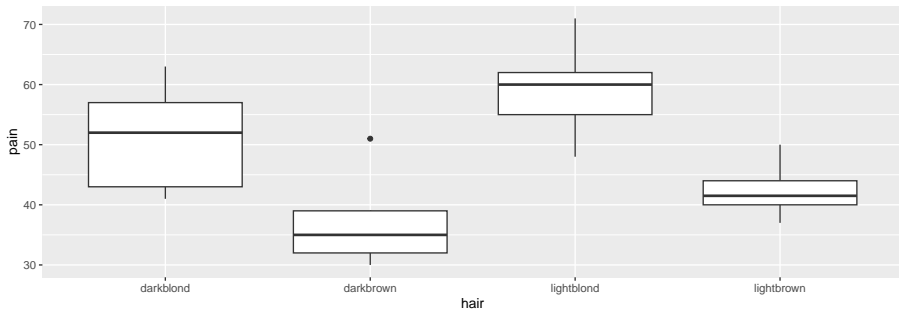
# A tibble: 4 x 4

	hair	n	xbar	s
	<chr>	<int>	<dbl>	<dbl>
1	darkblond	5	51.2	9.28
2	darkbrown	5	37.4	8.32
3	lightblond	5	59.2	8.53
4	lightbrown	4	42.5	5.45

Brown-haired people seem to have lower pain tolerance.

# Boxplot

```
ggplot(hairpain, aes(x = hair, y = pain)) + geom_boxplot()
```





# Assumptions

- Data should be:
  - ▶ normally distributed within each group
  - ▶ same spread for each group
- darkbrown group has upper outlier (suggests not normal)
- darkblond group has smaller IQR than other groups.
- But, groups *small*.
- Shrug shoulders and continue for moment.

# Testing equality of SDs

- via **Levene's test** in package `car`:

```
leveneTest(pain ~ hair, data = hairpain)
```

Levene's Test for Homogeneity of Variance (center = median)

	Df	F value	Pr(>F)
group	3	0.3927	0.76
	15		

- No evidence (at all) of difference among group SDs.
- Possibly because groups *small*.

# Analysis of variance

```
hairpain.1 <- aov(pain ~ hair, data = hairpain)
summary(hairpain.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
hair	3	1361	453.6	6.791	0.00411 **
Residuals	15	1002	66.8		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- P-value small: the mean pain tolerances for the four groups are *not* all the same.
- Which groups differ from which, and how?

## Multiple comparisons

- Which groups differ from which? Multiple comparisons method. Lots.
- Problem: by comparing all the groups with each other, doing many tests, have large chance to (possibly incorrectly) reject  $H_0$  : groups have equal means.
- 4 groups: 6 comparisons (1 vs 2, 1 vs 3, ..., 3 vs 4). 5 groups: 10 comparisons. Thus 6 (or 10) chances to make mistake.
- Get “familywise error rate” of 0.05 (whatever), no matter how many comparisons you’re doing.
- My favourite: Tukey, or “honestly significant differences”: how far apart might largest, smallest group means be (if actually no differences). Group means more different: significantly different.

# Tukey

- TukeyHSD:

```
TukeyHSD(hairpain.1)
```

Tukey multiple comparisons of means  
95% family-wise confidence level

```
Fit: aov(formula = pain ~ hair, data = hairpain)
```

```
$hair
```

	diff	lwr	upr	p adj
darkbrown-darkblond	-13.8	-28.696741	1.0967407	0.0740679
lightblond-darkblond	8.0	-6.896741	22.8967407	0.4355768
lightbrown-darkblond	-8.7	-24.500380	7.1003795	0.4147283
lightblond-darkbrown	21.8	6.903259	36.6967407	0.0037079
lightbrown-darkbrown	5.1	-10.700380	20.9003795	0.7893211
lightbrown-lightblond	-16.7	-32.500380	-0.8996205	0.0366467

## The old-fashioned way

- List group means in order
- Draw lines connecting groups that are *not* significantly different:

darkbrown	lightbrown	darkblond	lightblond
37.4	42.5	51.2	59.2
-----		-----	

- lightblond significantly higher than everything except darkblond (at  $\alpha = 0.05$ ).
- darkblond in middle ground: not significantly less than lightblond, not significantly greater than darkbrown and lightbrown.
- More data might resolve this.
- Looks as if blond-haired people do have higher pain tolerance, but not completely clear.

## Some other multiple-comparison methods

- Work any time you do  $k$  tests at once (not just ANOVA).
  - ▶ **Bonferroni**: multiply all P-values by  $k$ .
  - ▶ **Holm**: multiply smallest P-value by  $k$ , next-smallest by  $k - 1$ , etc.
  - ▶ **False discovery rate**: multiply smallest P-value by  $k/1$ , 2nd-smallest by  $k/2$ , ...,  $i$ -th smallest by  $k/i$ .
- Stop after non-rejection.

## Example

- P-values 0.005, 0.015, 0.03, 0.06 (4 tests all done at once) Use  $\alpha = 0.05$ .
- Bonferroni:
  - ▶ Multiply all P-values by 4 (4 tests).
  - ▶ Reject only 1st null.
- Holm:
  - ▶ Times smallest P-value by 4:  $0.005 * 4 = 0.020 < 0.05$ , reject.
  - ▶ Times next smallest by 3:  $0.015 * 3 = 0.045 < 0.05$ , reject.
  - ▶ Times next smallest by 2:  $0.03 * 2 = 0.06 > 0.05$ , do not reject. Stop.



## ...Continued

- With P-values 0.005, 0.015, 0.03, 0.06:
- False discovery rate:
  - ▶ Times smallest P-value by 4:  $0.005 * 4 = 0.02 < 0.05$ : reject.
  - ▶ Times second smallest by  $4/2$ :  $0.015 * 4/2 = 0.03 < 0.05$ , reject.
  - ▶ Times third smallest by  $4/3$ :  $0.03 * 4/3 = 0.04 < 0.05$ , reject.
  - ▶ Times fourth smallest by  $4/4$ :  $0.06 * 4/4 = 0.06 > 0.05$ , do not reject.  
Stop.

# pairwise.t.test

```
with(hairpain, pairwise.t.test(pain, hair, p.adj = "none"))
```

Pairwise comparisons using t tests with pooled SD

data: pain and hair

	darkblond	darkbrown	lightblond
darkbrown	0.01748	-	-
lightblond	0.14251	0.00075	-
lightbrown	0.13337	0.36695	0.00817

P value adjustment method: none

```
with(hairpain, pairwise.t.test(pain, hair, p.adj = "holm"))
```

Pairwise comparisons using t tests with pooled SD

data: pain and hair

	darkblond	darkbrown	lightblond
darkbrown	0.0699	-	-
lightblond	0.4001	0.0045	-
lightbrown	0.4001	0.4001	0.0408

P value adjustment method: holm

## pairwise.t.test part 2

```
with(hairpain, pairwise.t.test(pain, hair, p.adj = "fdr"))
```

Pairwise comparisons using t tests with pooled SD

data: pain and hair

	darkblond	darkbrown	lightblond
darkbrown	0.0350	-	-
lightblond	0.1710	0.0045	-
lightbrown	0.1710	0.3670	0.0245

P value adjustment method: fdr

```
with(hairpain, pairwise.t.test(pain, hair, p.adj = "bon"))
```

Pairwise comparisons using t tests with pooled SD

data: pain and hair

	darkblond	darkbrown	lightblond
darkbrown	0.1049	-	-
lightblond	0.8550	0.0045	-
lightbrown	0.8002	1.0000	0.0490

P value adjustment method: bonferroni

# Comments

- P-values all adjusted upwards from “none”.
- Required because 6 tests at once.
- Highest P-values for Bonferroni: most “conservative”.
- Prefer Tukey or FDR or Holm.
- Tukey only applies to ANOVA, not to other cases of multiple testing.

# Rats and vitamin B

- What is the effect of dietary vitamin B on the kidney?
- A number of rats were randomized to receive either a B-supplemented diet or a regular diet.
- Desired to control for initial size of rats, so classified into size classes lean and obese.
- After 20 weeks, rats' kidneys weighed.
- Variables:
  - ▶ Response: kidneyweight (grams).
  - ▶ Explanatory: diet, ratsize.
- Read in data:

```
my_url <- "http://ritsokiguess.site/datafiles/vitaminb.txt"  
vitaminb <- read_delim(my_url, " ")
```

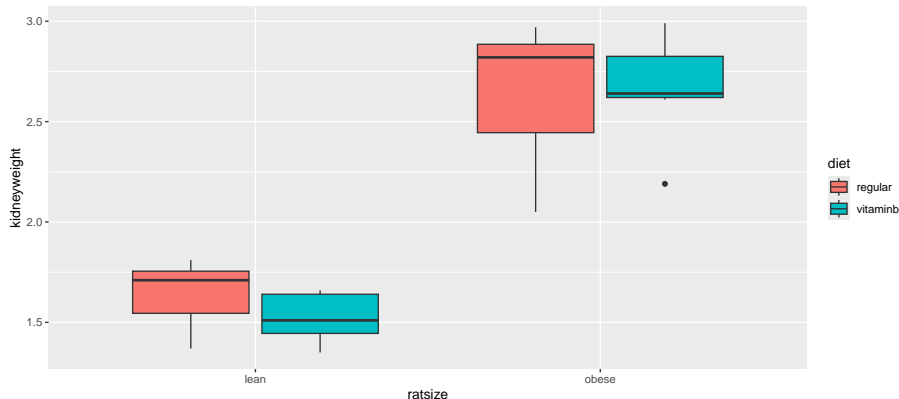
# The data

```
vitaminb
```

```
# A tibble: 28 x 3
  ratsize diet      kidneyweight
  <chr>    <chr>          <dbl>
1 lean    regular         1.62
2 lean    regular         1.8
3 lean    regular         1.71
4 lean    regular         1.81
5 lean    regular         1.47
6 lean    regular         1.37
7 lean    regular         1.71
8 lean    vitaminb        1.51
9 lean    vitaminb        1.65
10 lean    vitaminb        1.45
# i 18 more rows
```

# Grouped boxplot

```
ggplot(vitaminb, aes(  
  x = ratsize, y = kidneyweight,  
  fill = diet  
) + geom_boxplot())
```



# What's going on?

- Calculate group means:

```
summary <- vitaminb %>%  
  group_by(ratsize, diet) %>%  
  summarize(n = n(), mean = mean(kidneyweight))  
summary
```

# A tibble: 4 x 4

# Groups: ratsize [2]

	ratsize	diet	n	mean
	<chr>	<chr>	<int>	<dbl>
1	lean	regular	7	1.64
2	lean	vitaminb	7	1.53
3	obese	regular	7	2.64
4	obese	vitaminb	7	2.67

- Rat size: a large and consistent effect.
- Diet: small/no effect (compare same rat size, different diet).
- Effect of rat size *same* for each diet: no interaction.



# ANOVA with interaction

```
vitaminb.1 <- aov(kidneyweight ~ ratsize * diet,  
  data = vitaminb  
)  
summary(vitaminb.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
ratsize	1	8.068	8.068	141.179	1.53e-11 ***
diet	1	0.012	0.012	0.218	0.645
ratsize:diet	1	0.036	0.036	0.638	0.432
Residuals	24	1.372	0.057		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- Significance/nonsignificance as we expected.
- Note no significant interaction (can be removed).

# Interaction plot

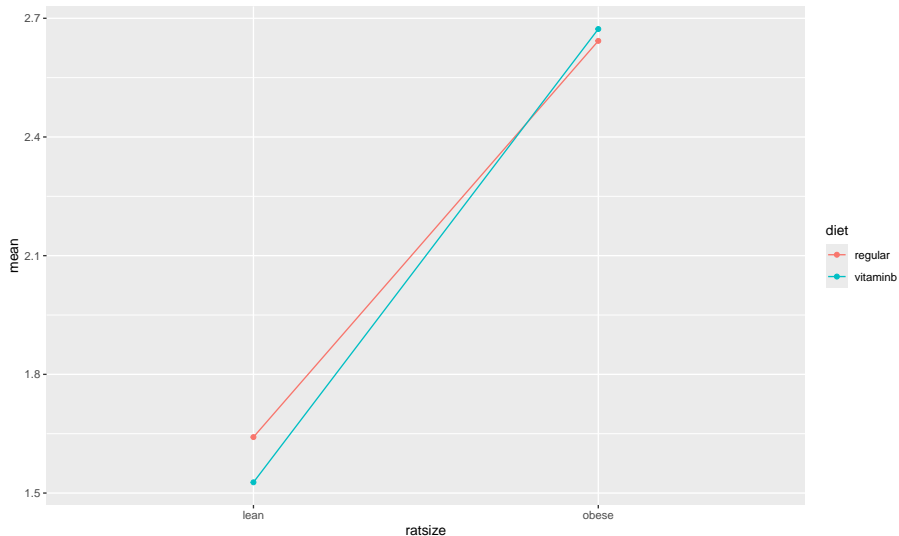
- Plot mean of response variable against one of the explanatory, using other one as groups. Start from summary:

```
g <- ggplot(summary, aes(  
  x = ratsize, y = mean,  
  colour = diet, group = diet  
)) +  
  geom_point() + geom_line()
```

- For this, have to give *both* group and colour.

# The interaction plot

g



## Take out interaction

```
vitaminb.2 <- update(vitaminb.1, . ~ . - ratsize:diet)
summary(vitaminb.2)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
ratsize	1	8.068	8.068	143.256	7.59e-12 ***
diet	1	0.012	0.012	0.221	0.643
Residuals	25	1.408	0.056		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- No Tukey for diet: not significant.
- No Tukey for ratsize: only two sizes, and already know that obese rats have larger kidneys than lean ones.
- Bottom line: diet has no effect on kidney size once you control for size of rat.

```
TukeyHSD(vitaminb.2)
```

## Assessing assumptions: residuals

- In two-way ANOVA, not many observations per treatment group.
- Difficult to check for normality / equal spreads.
- *But*, any regular ANOVA also a regression.
- Use regression residual ideas.
- In ANOVA, one fitted value per treatment group (based on means).
- Residual: observation minus fitted value.

## Previous ANOVA as regression

```
vitaminb.3 <- lm(kidneyweight ~ ratsize + diet, data = vitaminb)
summary(vitaminb.3)
```

Call:

```
lm(formula = kidneyweight ~ ratsize + diet, data = vitaminb)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.62893	-0.12625	0.04071	0.14607	0.35321

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.60536	0.07768	20.67	< 2e-16 ***
ratsizeobese	1.07357	0.08970	11.97	7.59e-12 ***
dietvitaminb	-0.04214	0.08970	-0.47	0.643

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2373 on 25 degrees of freedom

Multiple R-squared: 0.8516, Adjusted R-squared: 0.8397

# Reproduce ANOVA

```
drop1(vitaminb.3, test = "F")
```

Single term deletions

Model:

```
kidneyweight ~ ratsize + diet
```

	Df	Sum of Sq	RSS	AIC	F value	Pr(>F)
<none>		1.4079	-77.722			
ratsize	1	8.0679	9.4758	-26.337	143.2563	7.593e-12 ***
diet	1	0.0124	1.4204	-79.476	0.2207	0.6425

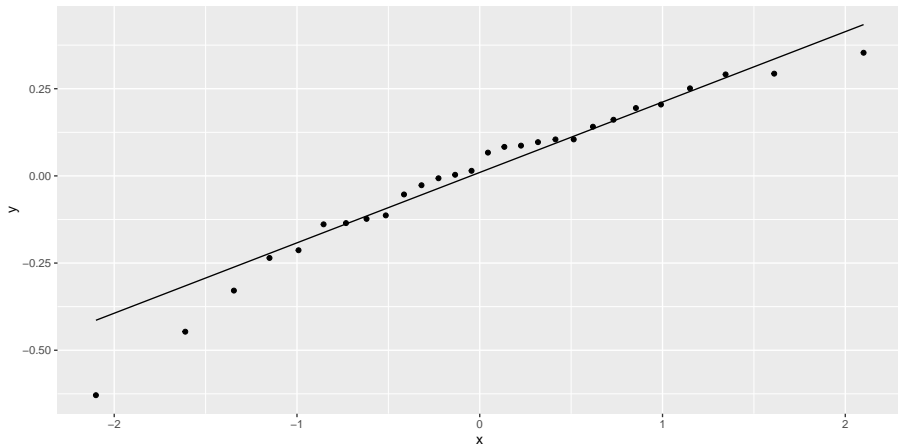
---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- ANOVA and regression drop1 output always the same.
- this time, ANOVA and regression summary output have same P-values, but only because categorical variables both have two levels.

# Are the residuals normal?

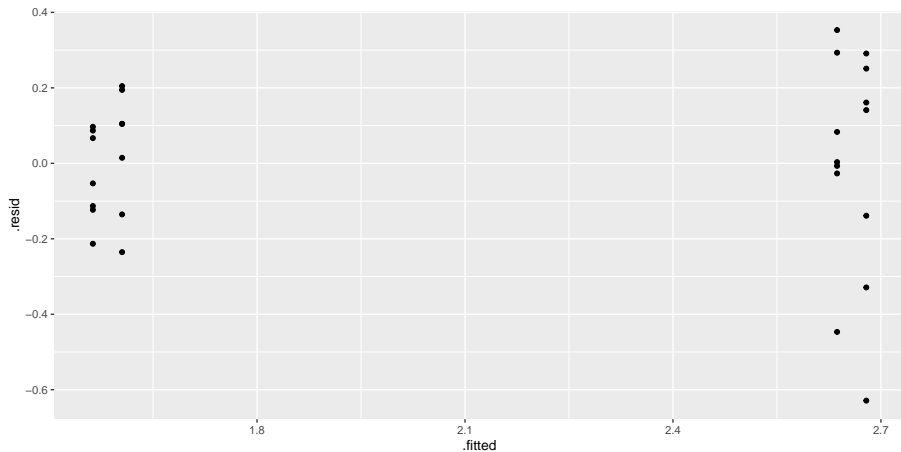
```
ggplot(vitaminb.3, aes(sample=.resid)) +  
  stat_qq() + stat_qq_line()
```





# Residuals against fitted

```
ggplot(vitaminb.3, aes(x=.fitted, y=.resid)) + geom_point()
```



## Comments

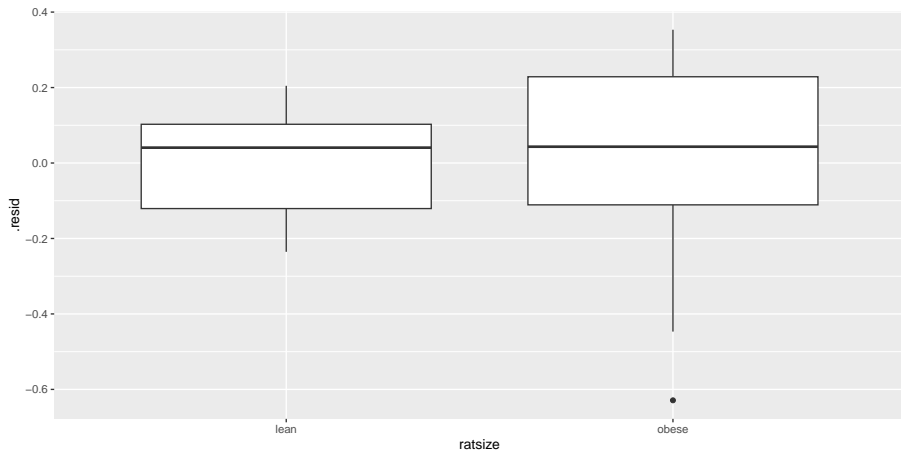
- 2 rat sizes, 2 diets: only  $2 \times 2 = 4$  different fitted values
- larger fitted values have greater spread (fan-out, transformation?)
- add residuals to data to plot residuals against size, diet (augment from broom):

```
vitaminb.3 %>% augment(vitaminb) -> vitaminb.3a
```

- explanatory rat size, diet categorical, so plot resid vs. them with *boxplots*.

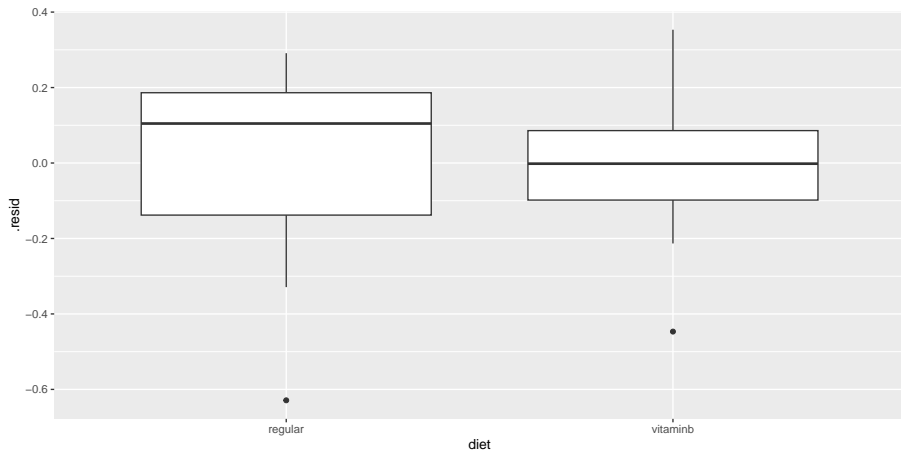
# Residuals vs rat size

```
ggplot(vitaminb.3a, aes(x = ratsize, y = .resid)) +  
  geom_boxplot()
```



# Residuals vs diet

```
ggplot(vitaminb.3a, aes(x = diet, y = .resid)) +  
  geom_boxplot()
```



# Comments

- there are low outliers on the plot against diet
- residuals for obese rats seem more spread out than for lean rats
- case for transformation of rat weights
- however, story from our analysis very clear:
  - ▶ rat size strongly significant
  - ▶ diet nowhere near significant
- and so expect transformation to make no difference to conclusions.

## The auto noise data

In 1973, the President of Texaco cited an automobile filter developed by Associated Octel Company as effective in reducing pollution. However, questions had been raised about the effects of filter silencing. He referred to the data included in the report (and below) as evidence that the silencing properties of the Octel filter were at least equal to those of standard silencers.

```
u <- "http://ritsokiguess.site/datafiles/autonoise.txt"
autonoise <- read_table(u)
```

# The data

```
autonoise
```

```
# A tibble: 36 x 4
  noise size  type  side
<dbl> <chr> <chr> <chr>
1   840 M      Std    R
2   770 L      Octel  L
3   820 M      Octel  R
4   775 L      Octel  R
5   825 M      Octel  L
6   840 M      Std    R
7   845 M      Std    L
8   825 M      Octel  L
9   815 M      Octel  L
10  845 M      Std    R
# i 26 more rows
```

# Making boxplot

- Make a boxplot, but have combinations of filter type and engine size.
- Use grouped boxplot again, thus:

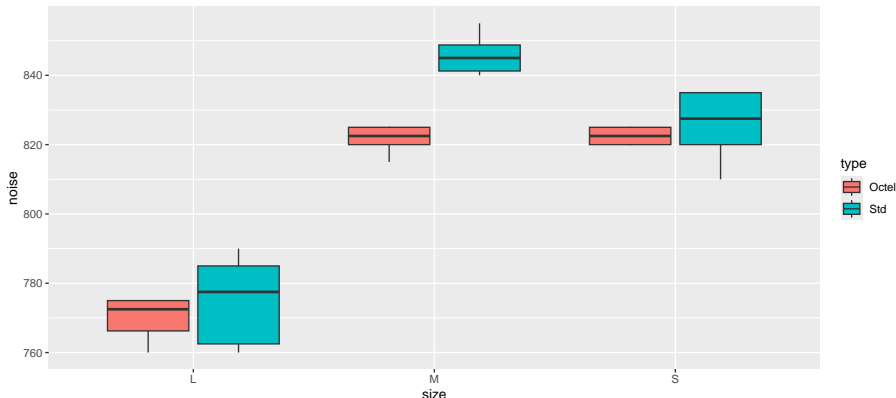
```
g <- autonoise %>%  
  ggplot(aes(x = size, y = noise, fill = type)) +  
  geom_boxplot()
```



# The boxplot

- See difference in engine noise between Octel and standard is larger for medium engine size than for large or small.
- Some evidence of differences in spreads (ignore for now):

gg



# ANOVA

```
autonoise.1 <- aov(noise ~ size * type, data = autonoise)
summary(autonoise.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
size	2	26051	13026	199.119	< 2e-16	***
type	1	1056	1056	16.146	0.000363	***
size:type	2	804	402	6.146	0.005792	**
Residuals	30	1962	65			

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- The interaction is significant, as we suspected from the boxplots.
- The within-group spreads don't look very equal, but only based on 6 obs each.

# Tukey: ouch!

```
autonoise.2 <- TukeyHSD(autonoise.1)
autonoise.2$`size:type`
```

	diff	lwr	upr	p adj
M:Octel-L:Octel	51.6666667	37.463511	65.869823	6.033496e-11
S:Octel-L:Octel	52.5000000	38.296844	66.703156	4.089762e-11
L:Std-L:Octel	5.0000000	-9.203156	19.203156	8.890358e-01
M:Std-L:Octel	75.8333333	61.630177	90.036489	4.962697e-14
S:Std-L:Octel	55.8333333	41.630177	70.036489	9.002910e-12
S:Octel-M:Octel	0.8333333	-13.369823	15.036489	9.999720e-01
L:Std-M:Octel	-46.6666667	-60.869823	-32.463511	6.766649e-10
M:Std-M:Octel	24.1666667	9.963511	38.369823	1.908995e-04
S:Std-M:Octel	4.1666667	-10.036489	18.369823	9.454142e-01
L:Std-S:Octel	-47.5000000	-61.703156	-33.296844	4.477636e-10
M:Std-S:Octel	23.3333333	9.130177	37.536489	3.129974e-04
S:Std-S:Octel	3.3333333	-10.869823	17.536489	9.787622e-01
M:Std-L:Std	70.8333333	56.630177	85.036489	6.583623e-14
S:Std-L:Std	50.8333333	36.630177	65.036489	8.937329e-11
S:Std-M:Std	-20.0000000	-34.203156	-5.796844	2.203265e-03

## Interaction plot

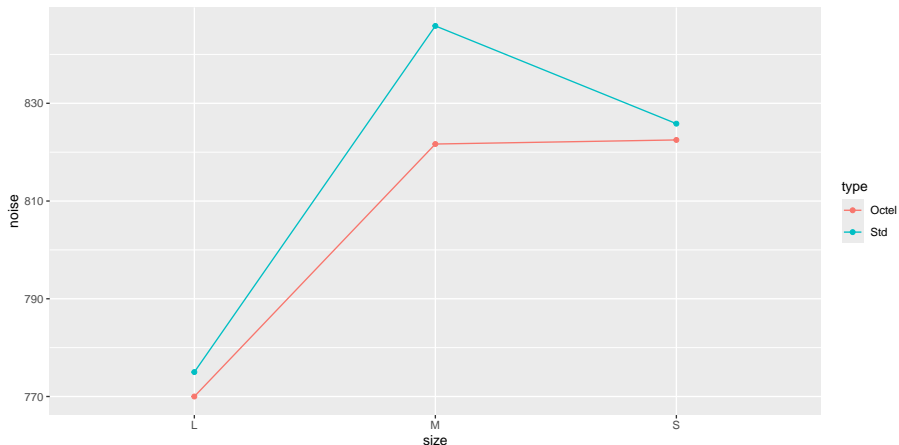
- This time, don't have summary of mean noise for each size-type combination.
- One way is to compute summaries (means) first, and feed into ggplot as in vitamin B example.
- Or, have ggplot compute them for us, thus:

```
g <- ggplot(autonoise, aes(  
  x = size, y = noise,  
  colour = type, group = type  
)) +  
  stat_summary(fun = mean, geom = "point") +  
  stat_summary(fun = mean, geom = "line")
```

## Interaction plot

The lines are definitely *not* parallel, showing that the effect of type is different for medium-sized engines than for others:

g



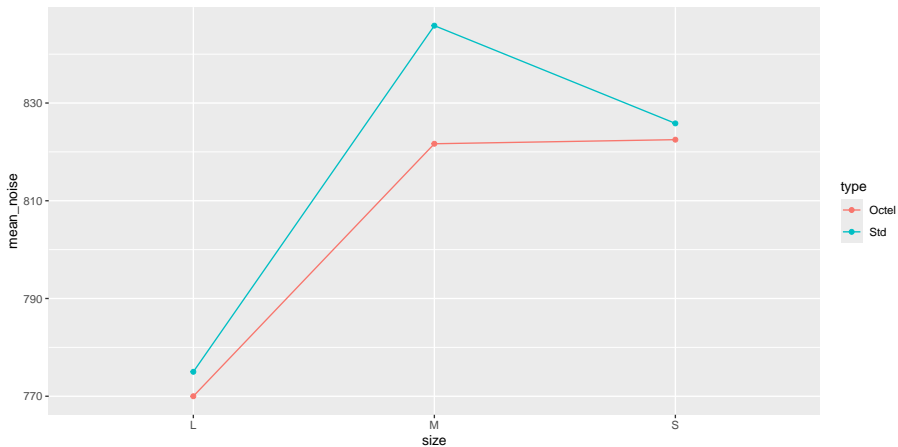
If you don't like that...

... then compute the means first:

```
autonoise %>%  
  group_by(size, type) %>%  
  summarize(mean_noise = mean(noise)) %>%  
  ggplot(aes(  
    x = size, y = mean_noise, group = type,  
    colour = type  
  )) + geom_point() + geom_line() -> g
```

# Interaction plot again

09



## Simple effects for auto noise example

- In auto noise example, weren't interested in all comparisons between car size and filter type combinations.
- Wanted to demonstrate (lack of) difference between filter types *for each engine type*.
- These are called **simple effects** of one variable (filter type) conditional on other variable (engine type).
- To do this, pull out just the data for small cars, compare noise for the two filter types. Then repeat for medium and large cars. (Three one-way ANOVAs.)



## Do it using dplyr tools

- Small cars:

```
autonoise %>%  
  filter(size == "S") %>%  
  aov(noise ~ type, data = .) %>%  
  summary()
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
type	1	33.3	33.33	0.548	0.476
Residuals	10	608.3	60.83		

- No filter difference for small cars.
- For Medium, change S to M and repeat.

## Simple effect of filter type for medium cars

```
autonoise %>%  
  filter(size == "M") %>%  
  aov(noise ~ type, data = .) %>%  
  summary()
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
type	1	1752.1	1752.1	68.93	8.49e-06 ***
Residuals	10	254.2	25.4		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

- There *is* an effect of filter type for medium cars. Look at means to investigate (over).

## Mean noise for each filter type

... for medium engine size:

```
autonoise %>%  
  filter(size == "M") %>%  
  group_by(type) %>%  
  summarize(m = mean(noise))
```

```
# A tibble: 2 x 2  
  type      m  
  <chr> <dbl>  
1 Octel  822.  
2 Std    846.
```

- Octel filters produce *less* noise for medium cars.

## Large cars

- Large cars:

```
autonoise %>%  
  filter(size == "L") %>%  
  aov(noise ~ type, data = .) %>%  
  summary()
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
type	1	75	75	0.682	0.428
Residuals	10	1100	110		

- No significant difference again.

## All at once, using split/apply/combine

The “split” part:

```
autonoise %>%  
  group_by(size) %>%  
  nest()
```

```
# A tibble: 3 x 2  
# Groups:   size [3]  
  size data  
  <chr> <list>  
1 M     <tibble [12 x 3]>  
2 L     <tibble [12 x 3]>  
3 S     <tibble [12 x 3]>
```

Now have *three* rows, with the data frame for each size encoded as *one element* of this data frame.

## Apply

- Write function to do aov on a data frame with columns `noise` and `type`, returning P-value:

```
aov_pval <- function(x) {  
  noise.1 <- aov(noise ~ type, data = x)  
  gg <- tidy(noise.1)  
  gg$p.value[1]  
}
```

- Test it:

```
autonoise %>%  
  filter(size == "L") %>%  
  aov_pval()
```

```
[1] 0.428221
```

- Check.

## Combine

- Apply this function to each of the nested data frames (one per engine size):

```
autonoise %>%  
  nest_by(size) %>%  
  rowwise() %>%  
  mutate(p_val = aov_pval(data)) %>%  
  select(-data)
```

```
# A tibble: 3 x 2  
# Rowwise:  
  size      p_val  
<chr>    <dbl>  
1 L      0.428  
2 M      0.00000849  
3 S      0.476
```

## Tidy up

- The data column was stepping-stone to getting answer. Don't need it any more:

```
autonoise %>%  
  nest_by(size) %>%  
  rowwise() %>%  
  mutate(p_val = aov_pval(data)) %>%  
  select(-data) -> simple_effects  
simple_effects
```

```
# A tibble: 3 x 2
```

```
# Rowwise:
```

	size	p_val
	<chr>	<dbl>
1	L	0.428
2	M	0.00000849
3	S	0.476



## Simultaneous tests

- When testing simple effects, doing several tests at once. (In this case, 3.) Should adjust P-values for this. Eg. Bonferroni:

```
simple_effects %>%  
  mutate(p_val_adj = p_val * 3)
```

```
# A tibble: 3 x 3
```

```
# Rowwise:
```

	size	p_val	p_val_adj
	<chr>	<dbl>	<dbl>
1	L	0.428	1.28
2	M	0.00000849	0.0000255
3	S	0.476	1.43

- No change in rejection decisions.
- Octel filters sig. better in terms of noise for medium cars, and not sig. different for other sizes.
- Octel filters never significantly worse than standard ones.

# Confidence intervals

- Perhaps better way of assessing simple effects: look at *confidence intervals* rather than tests.
- Gives us sense of accuracy of estimation, and thus whether non-significance might be lack of power: “absence of evidence is not evidence of absence”.
- Works here because *two* filter types, using *t*.test for each engine type.
- Want to show that the Octel filter is equivalent to or better than the standard filter, in terms of engine noise.

# Equivalence and noninferiority

- Known as “equivalence testing” in medical world. A good read: [link](#).  
Basic idea: decide on size of difference  $\delta$  that would be considered “equivalent”, and if CI entirely inside  $\pm\delta$ , have evidence in favour of equivalence.
- We really want to show that the Octel filters are “no worse” than the standard one: that is, equivalent *or better* than standard filters.
- Such a “noninferiority test” done by checking that upper limit of CI, new minus old, is *less* than  $\delta$ . (This requires careful thinking about (i) which way around the difference is and (ii) whether a higher or lower value is better.)

## CI for small cars

Same idea as for simple effect test:

```
autonoise %>%  
  filter(size == "S") %>%  
  t.test(noise ~ type, data = .) %>%  
  pluck("conf.int")
```

```
[1] -14.517462    7.850795
```

```
attr(,"conf.level")
```

```
[1] 0.95
```

## CI for medium cars

```
autonoise %>%  
  filter(size == "M") %>%  
  t.test(noise ~ type, data = .) %>%  
  pluck("conf.int")
```

```
[1] -30.75784 -17.57549  
attr(,"conf.level")  
[1] 0.95
```

## CI for large cars

```
autonoise %>%  
  filter(size == "L") %>%  
  t.test(noise ~ type, data = .) %>%  
  pluck("conf.int")
```

```
[1] -19.270673    9.270673  
attr(,"conf.level")  
[1] 0.95
```

## Or, all at once: split/apply/combine

```
ci_func <- function(x) {  
  tt <- t.test(noise ~ type, data = x)  
  tt$conf.int  
}  
  
autonoise %>% nest_by(size) %>%  
  rowwise() %>%  
  mutate(ci = list(ci_func(data))) %>%  
  unnest_wider(ci, names_sep = "_") -> cis
```

# Results

```
cis %>% select(size, starts_with("ci"))
```

```
# A tibble: 3 x 3
```

	size	ci_1	ci_2
	<chr>	<dbl>	<dbl>
1	L	-19.3	9.27
2	M	-30.8	-17.6
3	S	-14.5	7.85



# Procedure

- Function to get CI of difference in noise means for types of filter on input data frame
- Nest by size (mini-df data per size)
- Calculate CI for each thing in data: CI is two numbers long
- `unnest ci` column (wider) to see two numbers in each CI.

## CIs and noninferiority test

- Suppose we decide that a 20 dB difference would be considered equivalent. (I have no idea whether that is reasonable.)
- Intervals:

```
cis %>% select(-data)
```

```
# A tibble: 3 x 3  
  size    ci_1    ci_2  
  <chr> <dbl>   <dbl>  
1 L     -19.3    9.27  
2 M     -30.8  -17.6  
3 S     -14.5    7.85
```

# Comments

- In all cases, upper limit of CI is less than 20 dB. The Octel filters are “noninferior” to the standard ones.
- Caution: we did 3 procedures at once again. The true confidence level is not 95%. (Won't worry about that here.)

# Contrasts in ANOVA

- Sometimes, don't want to compare *all* groups, only *some* of them.
- Might be able to specify these comparisons ahead of time; other comparisons of no interest.
- Wasteful to do ANOVA and Tukey.

## Example: chainsaw kickback

- From [link](#).
- Forest manager concerned about safety of chainsaws issued to field crew. 4 models of chainsaws, measure “kickback” (degrees of deflection) for 5 of each:

A	B	C	D
-----			
42	28	57	29
17	50	45	29
24	44	48	22
39	32	41	34
43	61	54	30

- So far, standard 1-way ANOVA: what differences are there among models?

## chainsaw kickback (2)

- But: models A and D are designed to be used at home, while models B and C are industrial models.
- Suggests these comparisons of interest:
  - ▶ home vs. industrial
  - ▶ the two home models A vs. D
  - ▶ the two industrial models B vs. C.
- Don't need to compare *all* the pairs of models.

# What is a contrast?

- Contrast is a linear combination of group means.
- Notation:  $\mu_A$  for (population) mean of group  $A$ , and so on. In example:
  - ▶ compare two home models:  $H_0 : \mu_A - \mu_D = 0$ .
  - ▶ compare two industrial models:  $H_0 : \mu_B - \mu_C = 0$ .
  - ▶ compare average of two home models vs. average of two industrial models:  $H_0 : \frac{1}{2}(\mu_A + \mu_D) - \frac{1}{2}(\mu_B + \mu_C) = 0$  or  $H_0 : 0.5\mu_A - 0.5\mu_B - 0.5\mu_C + 0.5\mu_D = 0$ .
- Note that coefficients of contrasts add to 0, and right-hand side is 0.

## Contrasts in R

- Comparing two home models A and D ( $\mu_A - \mu_D = 0$ ):

```
c.home <- c(1, 0, 0, -1)
```

- Comparing two industrial models B and C ( $\mu_B - \mu_C = 0$ ):

```
c.industrial <- c(0, 1, -1, 0)
```

- Comparing home average vs. industrial average  
( $0.5\mu_A - 0.5\mu_B - 0.5\mu_C + 0.5\mu_D = 0$ ):

```
c.home.ind <- c(0.5, -0.5, -0.5, 0.5)
```



## Orthogonal contrasts

- What happens if we multiply the contrast coefficients one by one?

```
# c.home  
c.industrial
```

```
[1]  0  1 -1  0
```

```
c.home.ind
```

```
[1]  0.5 -0.5 -0.5  0.5
```

```
# c.home * c.industrial  
# c.home * c.home.ind  
c.industrial * c.home.ind
```

```
[1]  0.0 -0.5  0.5  0.0
```

- in each case, the results **add up to zero**. Such contrasts are called **orthogonal**.

## Orthogonal contrasts (2)

- Compare these:

```
c1 <- c(1, -1, 0)
c2 <- c(0, 1, -1)
sum(c1 * c2)
```

```
[1] -1
```

Not zero, so  $c1$  and  $c2$  are *not* orthogonal.

- Orthogonal contrasts are much easier to deal with.
- Can use non-orthogonal contrasts, but more trouble (beyond us).

## Read in data

```
url <- "http://ritsokiguess.site/datafiles/chainsaw.txt"
chain.wide <- read_table(url)
chain.wide
```

```
# A tibble: 5 x 4
```

	A	B	C	D
	<dbl>	<dbl>	<dbl>	<dbl>
1	42	28	57	29
2	17	50	45	29
3	24	44	48	22
4	39	32	41	34
5	43	61	54	30

# Tidying

Need all the kickbacks in *one* column:

```
chain.wide %>%  
  pivot_longer(A:D, names_to = "model",  
               names_ptypes = list(model = factor()),  
               values_to = "kickback") -> chain
```

## Starting the analysis (2)

The proper data frame:

```
chain
```

```
# A tibble: 20 x 2
```

```
  model kickback
```

```
<fct>
```

```
<dbl>
```

```
1 A      42
```

```
2 B      28
```

```
3 C      57
```

```
4 D      29
```

```
5 A      17
```

```
6 B      50
```

```
7 C      45
```

```
8 D      29
```

```
9 A      24
```

```
10 B     44
```

```
11 C     48
```

## Setting up contrasts

```
m <- cbind(c.home, c.industrial, c.home.ind)
m
```

	c.home	c.industrial	c.home.ind
[1,]	1	0	0.5
[2,]	0	1	-0.5
[3,]	0	-1	-0.5
[4,]	-1	0	0.5

```
contrasts(chain$model) <- m
```

## ANOVA as if regression

```
chain.1 <- lm(kickback ~ model, data = chain)
summary(chain.1)
```

Call:

```
lm(formula = kickback ~ model, data = chain)
```

Residuals:

Min	1Q	Median	3Q	Max
-16.00	-7.10	0.60	6.25	18.00

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	38.450	2.179	17.649	6.52e-12 ***
modelc.home	2.100	3.081	0.682	0.50524
modelc.industrial	-3.000	3.081	-0.974	0.34469
modelc.home.ind	-15.100	4.357	-3.466	0.00319 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.743 on 16 degrees of freedom

# Conclusions

```
tidy(chain.1) %>% select(term, p.value)
```

```
# A tibble: 4 x 2
```

	term	p.value
	<chr>	<dbl>
1	(Intercept)	6.52e-12
2	modelc.home	5.05e- 1
3	modelc.industrial	3.45e- 1
4	modelc.home.ind	3.19e- 3

- Two home models not sig. diff. (P-value 0.51)
- Two industrial models not sig. diff. (P-value 0.34)
- Home, industrial models *are* sig. diff. (P-value 0.0032).



## Means by model

- The means:

```
chain %>%  
  group_by(model) %>%  
  summarize(mean.kick = mean(kickback)) %>%  
  arrange(desc(mean.kick))
```

```
# A tibble: 4 x 2  
  model mean.kick  
  <fct>      <dbl>  
1 C          49  
2 B          43  
3 A          33  
4 D         28.8
```

- Home models A & D have less kickback than industrial ones B & C.
- Makes sense because industrial users should get training to cope with additional kickback.