

Case study: windmill

The windmill data

- ▶ Engineer: does amount of electricity generated by windmill depend on how strongly wind blowing?
- ▶ Measurements of wind speed and DC current generated at various times.
- ▶ Assume the “various times” to be randomly selected — aim to generalize to “this windmill at all times”.
- ▶ Research questions:
 - ▶ Relationship between wind speed and current generated?
 - ▶ If so, what kind of relationship?
 - ▶ Can we model relationship to do predictions?

Packages for this section

```
library(tidyverse)  
library(broom)
```

Reading in the data

```
my_url <-  
  "http://ritsokiguess.site/datafiles/windmill.csv"  
windmill <- read_csv(my_url)  
windmill
```

```
# A tibble: 25 x 2
```

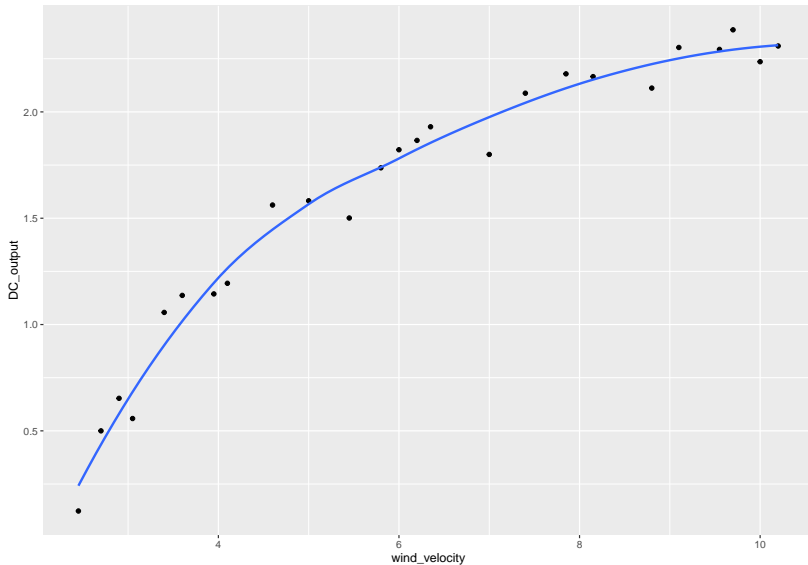
	wind_velocity	DC_output
	<dbl>	<dbl>
1	5	1.58
2	6	1.82
3	3.4	1.06
4	2.7	0.5
5	10	2.24
6	9.7	2.39
7	9.55	2.29
8	3.05	0.558
9	8.15	2.17
10	6.2	1.87

Strategy

- ▶ Two quantitative variables, looking for relationship: regression methods.
- ▶ Start with picture (scatterplot).
- ▶ Fit models and do model checking, fixing up things as necessary.
- ▶ Scatterplot:
 - ▶ 2 variables, DC_output and wind_velocity.
 - ▶ First is output/response, other is input/explanatory.
 - ▶ Put DC_output on vertical scale.
- ▶ Add trend, but don't want to assume linear:

```
ggplot(windmill, aes(y = DC_output, x = wind_velocity)) +  
  geom_point() + geom_smooth()
```

Scatterplot



Comments

- ▶ Definitely a relationship: as wind velocity increases, so does DC output. (As you'd expect.)
- ▶ Is relationship linear? To help judge, `geom_smooth` smooths scatterplot trend. (Trend called “loess”, “Locally weighted least squares” which downweights outliers. Not constrained to be straight.)
- ▶ Trend more or less linear for while, then curves downwards (levelling off?). Straight line not so good here.

Fit a straight line (and see what happens)

```
DC.1 <- lm(DC_output ~ wind_velocity, data = windmill)
summary(DC.1)
```

Call:

```
lm(formula = DC_output ~ wind_velocity, data = windmill)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.59869	-0.14099	0.06059	0.17262	0.32184

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.13088	0.12599	1.039	0.31
wind_velocity	0.24115	0.01905	12.659	7.55e-12 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Another way of looking at the output

- ▶ The standard output tends to go off the bottom of the page rather easily. Package broom has these:

```
glance(DC.1)
```

```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
    <dbl>      <dbl> <dbl>    <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl>
1   0.874        0.869 0.236    160. 7.55e-12     1   1.66  2.68  6.33
# i 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

showing that the R-squared is 87%, and

```
tidy(DC.1)
```

```
# A tibble: 2 x 5
  term          estimate std.error statistic p.value
  <chr>        <dbl>    <dbl>    <dbl>    <dbl>
1 (Intercept)  0.131    0.126     1.04 3.10e- 1
2 wind_velocity 0.241    0.0190    12.7 7.55e-12
```

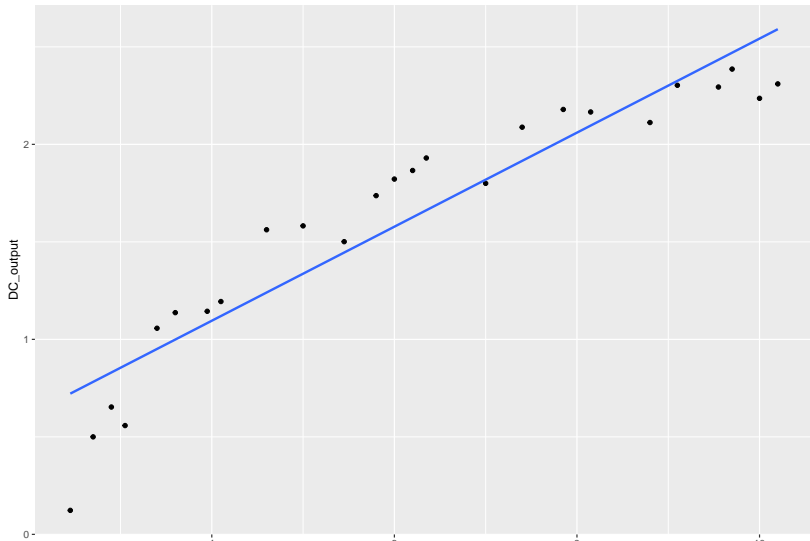
showing the intercept and slope and their significance.

Comments

- ▶ Strategy: `lm` actually fits the regression. Store results in a variable. Then look at the results, eg. via `summary` or `glance/tidy`.
- ▶ My strategy for model names: base on response variable (or data frame name) and a number. Allows me to fit several models to same data and keep track of which is which.
- ▶ Results actually pretty good: `wind.velocity` strongly significant, R-squared (87%) high.
- ▶ How to check whether regression is appropriate? Look at the residuals, observed minus predicted, plotted against fitted (predicted).
- ▶ Plot using the regression object as “data frame” (in a couple of slides).

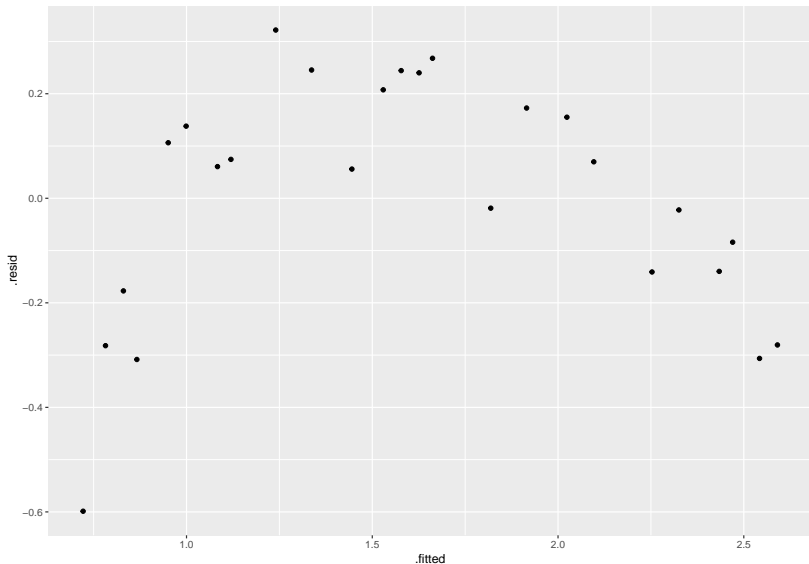
Scatterplot, but with line

```
ggplot(windmill, aes(y = DC_output, x = wind_velocity)) +  
  geom_point() + geom_smooth(method="lm", se = FALSE)
```



Plot of residuals against fitted values

```
ggplot(DC.1, aes(y = .resid, x = .fitted)) + geom_point()
```

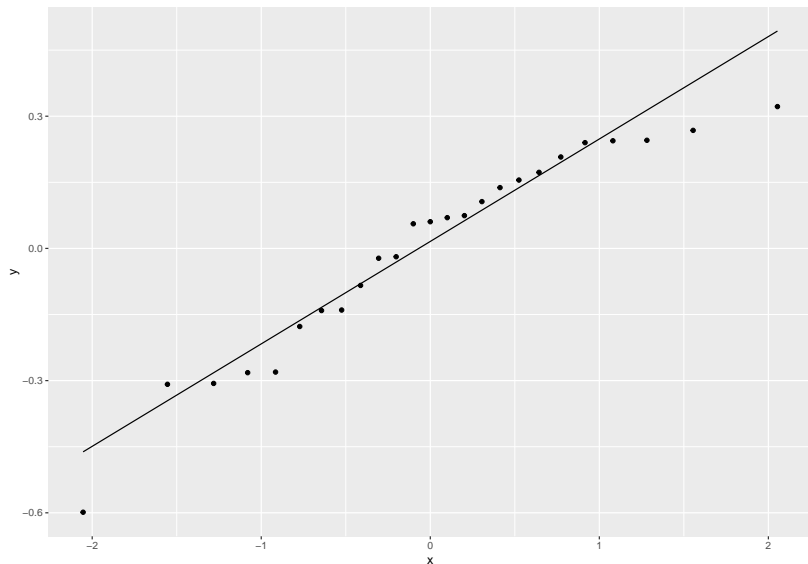


Comments on residual plot

- ▶ Residual plot should be a random scatter of points.
- ▶ Should be no pattern “left over” after fitting the regression.
- ▶ Smooth trend should be more or less straight across at 0.
- ▶ Here, have a curved trend on residual plot.
- ▶ This means original relationship must have been a curve (as we saw on original scatterplot).
- ▶ Possible ways to fit a curve:
 - ▶ Add a squared term in explanatory variable.
 - ▶ Transform response variable (doesn't work well here).
 - ▶ See what science tells you about mathematical form of relationship, and try to apply.

normal quantile plot of residuals

```
ggplot(DC.1, aes(sample = .resid)) + stat_qq() + stat_qq_line()
```



Parabolas and fitting parabola model

- ▶ A parabola has equation

$$y = ax^2 + bx + c$$

with coefficients a, b, c . About the simplest function that is not a straight line.

- ▶ Fit one using `lm` by adding x^2 to right side of model formula with `+`:

```
DC.2 <- lm(DC_output ~ wind_velocity + I(wind_velocity^2),  
  data = windmill  
)
```

- ▶ The `I()` necessary because \wedge in model formula otherwise means something different (to do with interactions in ANOVA).
- ▶ Call it *parabola model*.

Parabola model output

```
summary(DC.2)
```

Call:

```
lm(formula = DC_output ~ wind_velocity + I(wind_velocity^2),  
    data = windmill)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.26347	-0.02537	0.01264	0.03908	0.19903

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-1.155898	0.174650	-6.618	1.18e-06	**
wind_velocity	0.722936	0.061425	11.769	5.77e-11	**
I(wind_velocity^2)	-0.038121	0.004797	-7.947	6.59e-08	**

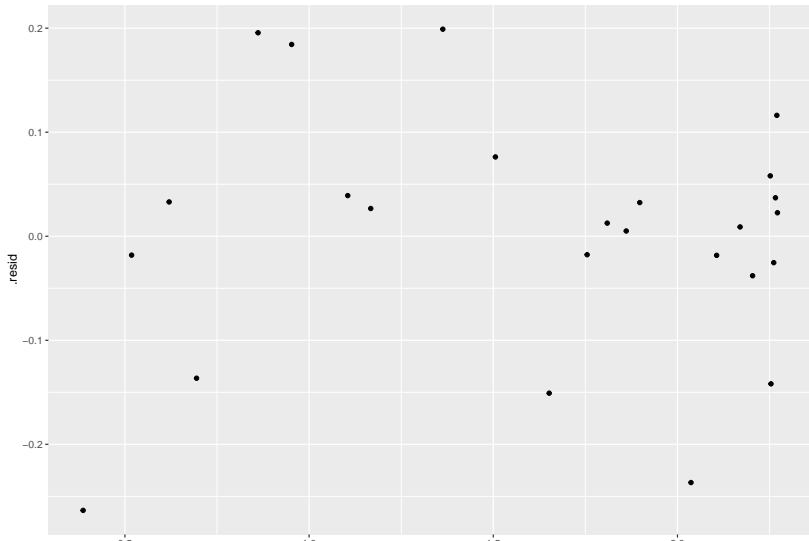
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Comments on output

- ▶ R-squared has gone up a lot, from 87% (line) to 97% (parabola).
- ▶ Coefficient of squared term strongly significant (P-value 6.59×10^{-8}).
- ▶ Adding squared term has definitely improved fit of model.
- ▶ Parabola model better than linear one.
- ▶ But...need to check residuals again.

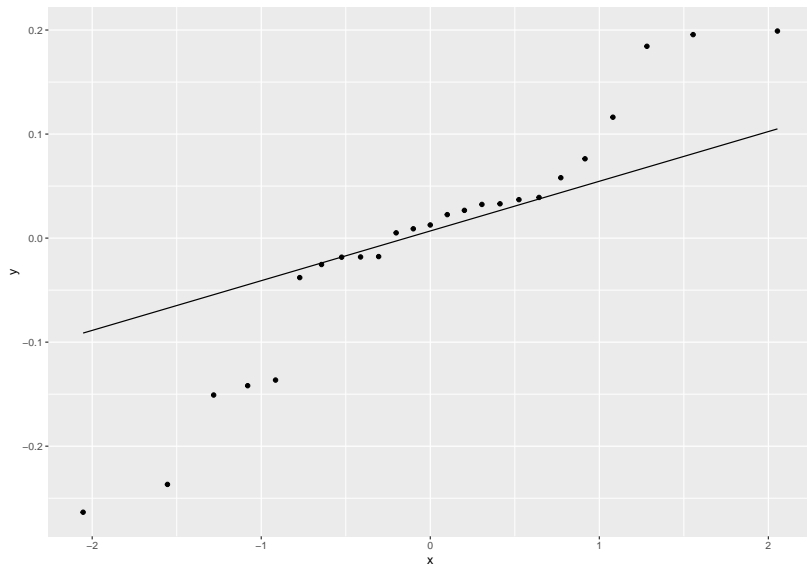
Residual plot from parabola model

```
ggplot(DC.2, aes(y = .resid, x = .fitted)) +  
  geom_point()
```



normal quantile plot of residuals

```
ggplot(DC.2, aes(sample = .resid)) + stat_qq() + stat_qq_l
```



Make scatterplot with fitted line and curve

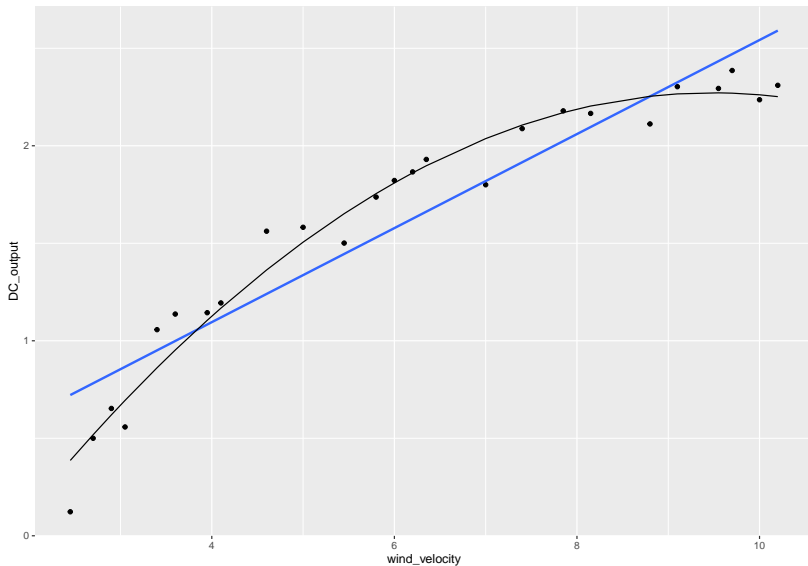
- ▶ Residual plot basically random. Good.
- ▶ Scatterplot with fitted line and curve like this:

```
ggplot(windmill, aes(y = DC_output, x = wind_velocity)) +  
  geom_point() + geom_smooth(method = "lm", se = F) +  
  geom_line(data = DC.2, aes(y = .fitted))
```

Comments

- ▶ This plots:
 - ▶ scatterplot (`geom_point`);
 - ▶ straight line (via tweak to `geom_smooth`, which draws best-fitting line);
 - ▶ fitted curve, using the predicted `DC_output` values, joined by lines (with points not shown).
- ▶ Trick in the `geom_line` is use the predictions as the y-points to join by lines (from `DC.2`), instead of the original data points. Without the data and `aes` in the `geom_line`, original data points would be joined by lines.

Scatterplot with fitted line and curve



Curve clearly fits better than line.

Another approach to a curve

- ▶ There is a problem with parabolas, which we'll see later.
- ▶ Ask engineer, "what should happen as wind velocity increases?":
 - ▶ Upper limit on electricity generated, but otherwise, the larger the wind velocity, the more electricity generated.
- ▶ Mathematically, *asymptote*. Straight lines and parabolas don't have them, but eg. $y = 1/x$ does: as x gets bigger, y approaches zero without reaching it.
- ▶ What happens to $y = a + b(1/x)$ as x gets large?
 - ▶ y gets closer and closer to a : that is, a is asymptote.
- ▶ Fit this, call it asymptote model.
- ▶ Fitting the model here because we have math to justify it.
 - ▶ Alternative, $y = a + be^{-x}$, approaches asymptote faster.

How to fit asymptote model?

- ▶ Define new explanatory variable to be $1/x$, and predict y from it.
- ▶ x is velocity, distance over time.
- ▶ So $1/x$ is time over distance. In walking world, if you walk 5 km/h, take 12 minutes to walk 1 km, called your pace. So $1/\text{wind_velocity}$ we call `wind_pace`.
- ▶ Make a scatterplot first to check for straightness (next page).

```
windmill %>% mutate(wind_pace = 1 / wind_velocity) -> windr  
ggplot(windmill, aes(y = DC_output, x = wind_pace)) +  
  geom_point() + geom_smooth(se = F)
```


and run regression like this:

```
DC.3 <- lm(DC_output ~ wind_pace, data = windmill)
summary(DC.3)
```

Call:

```
lm(formula = DC_output ~ wind_pace, data = windmill)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.20547	-0.04940	0.01100	0.08352	0.12204

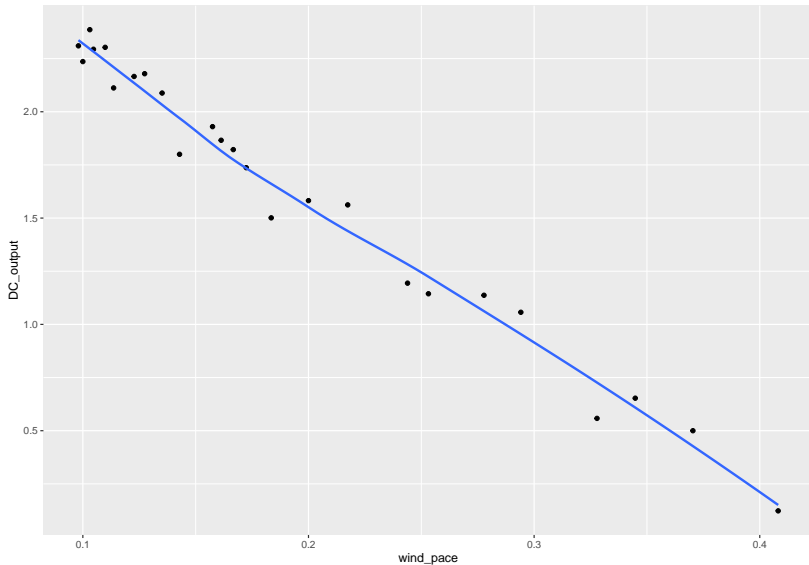
Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	2.9789	0.0449	66.34	<2e-16	***
wind_pace	-6.9345	0.2064	-33.59	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Scatterplot for wind_pace

Pretty straight. Blue actually smooth curve not line:



Regression output

```
glance(DC.3)
```

```
# A tibble: 1 x 12
```

	r.squared	adj.r.squared	sigma	statistic	p.value	df
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	0.980	0.979	0.0942	1128.	4.74e-21	1

```
# i 3 more variables: deviance <dbl>, df.residual <int>, no
```

```
tidy(DC.3)
```

```
# A tibble: 2 x 5
```

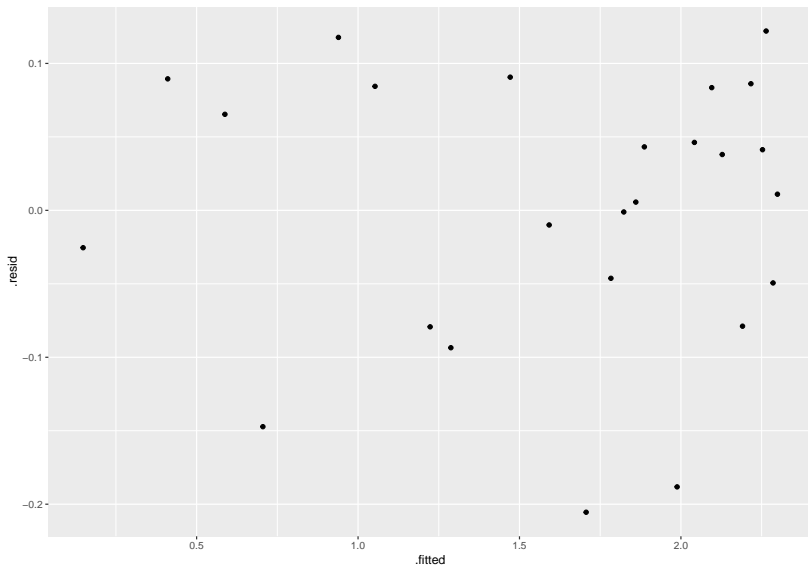
	term	estimate	std.error	statistic	p.value
	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	(Intercept)	2.98	0.0449	66.3	8.92e-28
2	wind_pace	-6.93	0.206	-33.6	4.74e-21

Comments

- ▶ R-squared, 98%, even higher than for parabola model (97%).
- ▶ Simpler model, only one explanatory variable (`wind.pace`) vs. 2 for parabola model (`wind.velocity` and its square).
- ▶ `wind.pace` (unsurprisingly) strongly significant.
- ▶ Looks good, but check residual plot (over).

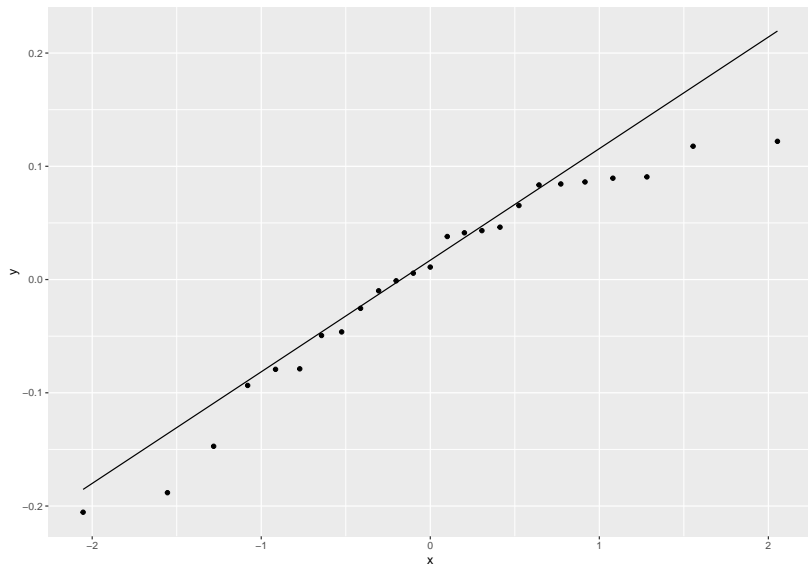
Residual plot for asymptote model

```
ggplot(DC.3, aes(y = .resid, x = .fitted)) + geom_point()
```



normal quantile plot of residuals

```
ggplot(DC.3, aes(sample = .resid)) + stat_qq() + stat_qq_line()
```



Plotting trends on scatterplot

- ▶ Residual plot not bad. But residuals go up to 0.10 and down to -0.20 , suggesting possible skewness (not normal). I think it's not perfect, but OK overall.
- ▶ Next: plot scatterplot with all three fitted lines/curves on it (for comparison), with legend saying which is which.
- ▶ First make data frame containing what we need, taken from the right places:

```
w2 <- tibble(  
  wind_velocity = windmill$wind_velocity,  
  DC_output = windmill$DC_output,  
  linear = fitted(DC.1),  
  parabola = fitted(DC.2),  
  asymptote = fitted(DC.3)  
)
```

What's in w2

w2

```
# A tibble: 25 x 5
```

	wind_velocity	DC_output	linear	parabola	asymptote
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	5	1.58	1.34	1.51	1.59
2	6	1.82	1.58	1.81	1.82
3	3.4	1.06	0.951	0.861	0.939
4	2.7	0.5	0.782	0.518	0.411
5	10	2.24	2.54	2.26	2.29
6	9.7	2.39	2.47	2.27	2.26
7	9.55	2.29	2.43	2.27	2.25
8	3.05	0.558	0.866	0.694	0.705
9	8.15	2.17	2.10	2.20	2.13
10	6.2	1.87	1.63	1.86	1.86

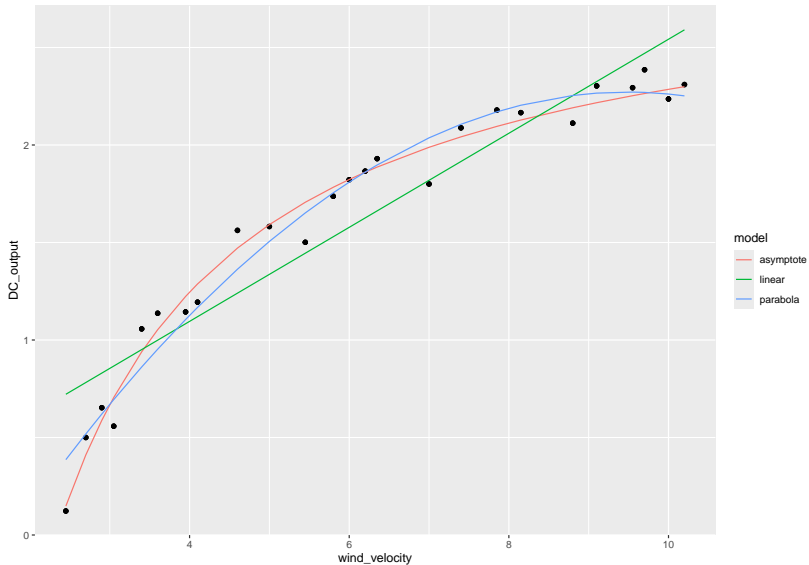
```
# i 15 more rows
```


Making the plot

- ▶ `ggplot` likes to have one column of x 's to plot, and one column of y 's, with another column for distinguishing things.
- ▶ But we have three columns of fitted values, that need to be combined into one.
- ▶ `pivot_longer`, then plot:

```
w2 %>%  
  pivot_longer(linear:asymptote, names_to="model",  
               values_to="fit") %>%  
  ggplot(aes(x = wind_velocity, y = DC_output)) +  
  geom_point() +  
  geom_line(aes(y = fit, colour = model))
```

Scatterplot with fitted curves



Comments

- ▶ Predictions from curves are very similar.
- ▶ Predictions from asymptote model as good, and from simpler model (one x not two), so prefer those.
- ▶ Go back to asymptote model summary.

Asymptote model summary

```
tidy(DC.3)
```

```
# A tibble: 2 x 5
```

	term	estimate	std.error	statistic	p.value
	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	(Intercept)	2.98	0.0449	66.3	8.92e-28
2	wind_pace	-6.93	0.206	-33.6	4.74e-21

Comments

- ▶ Intercept in this model about 3.
- ▶ Intercept of asymptote model is the asymptote (upper limit of DC.output).
- ▶ Not close to asymptote yet.
- ▶ Therefore, from this model, wind could get stronger and would generate appreciably more electricity.
- ▶ This is extrapolation! Would like more data from times when wind.velocity higher.
- ▶ Slope -7 . Why negative?
 - ▶ As wind.velocity increases, wind.pace goes down, and DC.output goes up. Check.
- ▶ Actual slope number hard to interpret.

Checking back in with research questions

- ▶ Is there a relationship between wind speed and current generated?
 - ▶ Yes.
- ▶ If so, what kind of relationship is it?
 - ▶ One with an asymptote.
- ▶ Can we model the relationship, in such a way that we can do predictions?
 - ▶ Yes, see model DC.3 and plot of fitted curve.
- ▶ Good. Job done.

Job done, kinda

- ▶ Just because the parabola model and asymptote model agree over the range of the data, doesn't necessarily mean they agree everywhere.
- ▶ Extend range of wind.velocity to 1 to 16 (steps of 0.5), and predict DC.output according to the two models:

```
wv <- seq(1, 16, 0.5)
```

```
wv
```

```
[1] 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
[14] 7.5 8.0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 12.0 12.5
[27] 14.0 14.5 15.0 15.5 16.0
```

- ▶ R has `predict`, which requires what to predict for, as data frame. The data frame has to contain values, with matching names, for all explanatory variables in `regression(s)`.

Setting up data frame to predict from

- ▶ Linear model had just `wind_velocity`.
- ▶ Parabola model had that as well (squared one will be calculated)
- ▶ Asymptote model had just `wind_pace` (reciprocal of velocity).
- ▶ So create data frame called `wv_new` with those in:

```
wv_new <- tibble(wind_velocity = wv, wind_pace = 1 / wv)
```


wv_new

wv_new

A tibble: 31 x 2

	wind_velocity	wind_pace
	<dbl>	<dbl>
1	1	1
2	1.5	0.667
3	2	0.5
4	2.5	0.4
5	3	0.333
6	3.5	0.286
7	4	0.25
8	4.5	0.222
9	5	0.2
10	5.5	0.182

i 21 more rows

Doing predictions, one for each model

- ▶ Use same names as before:

```
linear <- predict(DC.1, wv_new)
parabola <- predict(DC.2, wv_new)
asymptote <- predict(DC.3, wv_new)
```

- ▶ Put it all into a data frame for plotting, along with original data:

```
my_fits <- tibble(
  wind_velocity = wv_new$wind_velocity,
  linear, parabola, asymptote
)
```

my_fits

```
my_fits
```

```
# A tibble: 31 x 4
```

	wind_velocity	linear	parabola	asymptote
	<dbl>	<dbl>	<dbl>	<dbl>
1	1	0.372	-0.471	-3.96
2	1.5	0.493	-0.157	-1.64
3	2	0.613	0.137	-0.488
4	2.5	0.734	0.413	0.205
5	3	0.854	0.670	0.667
6	3.5	0.975	0.907	0.998
7	4	1.10	1.13	1.25
8	4.5	1.22	1.33	1.44
9	5	1.34	1.51	1.59
10	5.5	1.46	1.67	1.72

```
# i 21 more rows
```

Making a plot 1/2

- ▶ To make a plot, we use the same trick as last time to get all three predictions on a plot with a legend (saving result to add to later):

```
my_fits %>%  
  pivot_longer(  
    linear:asymptote,  
    names_to="model",  
    values_to="fit"  
  ) %>%  
  ggplot(aes(  
    y = fit, x = wind_velocity,  
    colour = model  
  )) + geom_line() -> g  
g
```



Making a plot 2/2

- The observed wind velocities were in this range:

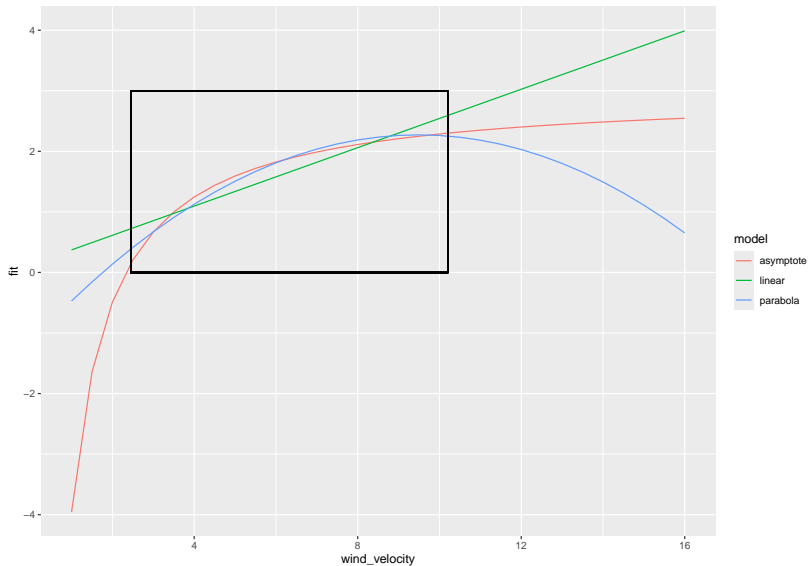
```
(vels <- range(windmill$wind_velocity))
```

```
[1] 2.45 10.20
```

- DC.output between 0 and 3 from asymptote model. Add rectangle to graph around where the data were:

```
g + geom_rect(  
  xmin = vels[1], xmax = vels[2], ymin = 0, ymax = 3,  
  alpha = 0, colour = "black"  
)
```

The plot



Comments (1)

- ▶ Over range of data, two models agree with each other well.
- ▶ Outside range of data, they disagree violently!
- ▶ For larger `wind.velocity`, asymptote model behaves reasonably, parabola model does not.
- ▶ What happens as `wind.velocity` goes to zero? Should find `DC.output` goes to zero as well. Does it?

Comments (2)

- For parabola model:

```
tidy(DC.2)
```

```
# A tibble: 3 x 5
```

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	-1.16	0.175	-6.62	1.18e- 6
2	wind_velocity	0.723	0.0614	11.8	5.77e-11
3	I(wind_velocity^2)	-0.0381	0.00480	-7.95	6.59e- 8

- Nope, goes to -1.16 (intercept), actually significantly different from zero.

Comments (3): asymptote model

```
tidy(DC.3)
```

```
# A tibble: 2 x 5
```

	term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>	p.value <dbl>
1	(Intercept)	2.98	0.0449	66.3	8.92e-28
2	wind_pace	-6.93	0.206	-33.6	4.74e-21

- ▶ As `wind.velocity` heads to 0, `wind.pace` heads to $+\infty$, so `DC.output` heads to $-\infty$!
- ▶ Also need more data for small `wind.velocity` to understand relationship. (Is there a lower asymptote?)
- ▶ Best we can do now is to predict `DC.output` to be zero for small `wind.velocity`.
- ▶ Assumes a “threshold” wind velocity below which no electricity generated at all.

Summary

- ▶ Often, in data analysis, there is no completely satisfactory conclusion, as here.
- ▶ Have to settle for model that works OK, with restrictions.
- ▶ Always something else you can try.
- ▶ At some point you have to say “I stop.”