

## Figures

```
library(tidyverse)
library(MASS, exclude = "select")
library(marginaleffects)
library(broom)
library(car)
library(survival)
```

Figure 1: Packages

	Year	Seed	Final4	Izzo
1	2003	7	0	1
2	2007	10	0	0
3	1992	16	0	0
4	2002	3	0	0
5	1986	6	0	0
6	1994	7	0	0
7	2003	1	0	0
8	1985	3	0	0
9	1987	3	0	0
10	1993	8	0	0
11	2004	7	0	0
12	1990	13	0	0
13	1992	15	0	0
14	2003	12	0	0
15	1998	9	0	0
16	1999	9	0	0
17	2001	5	0	0
18	2007	15	0	0
19	2000	8	0	0
20	1986	4	0	0

Figure 2: Izzo data (20 randomly chosen rows)

```
izzo.1 <- glm(Final4 ~ Seed + Izzo, data = FinalFourIzzo, family = "binomial")
summary(izzo.1)
```

Call:

```
glm(formula = Final4 ~ Seed + Izzo, family = "binomial", data = FinalFourIzzo)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	0.02776	0.19967	0.139	0.88942
Seed	-0.58809	0.05909	-9.953	< 2e-16 ***
Izzo	2.32441	0.73971	3.142	0.00168 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 778.06 on 1663 degrees of freedom  
Residual deviance: 519.56 on 1661 degrees of freedom  
AIC: 525.56

Number of Fisher Scoring iterations: 8

Figure 3: Izzo data logistic regression

```
new <- datagrid(model = izzo.1, Seed = c(1, 5, 9, 13), Izzo = c(0, 1))
cbind(predictions(izzo.1, new)) %>%
  select(Seed, Izzo, estimate, conf.low, conf.high)
```

	Seed	Izzo	estimate	conf.low	conf.high
1	1	0	0.3634724918	0.2963916353	0.436320736
2	1	1	0.8537203338	0.5735880449	0.962008512
3	5	0	0.0515307373	0.0370854692	0.071186657
4	5	1	0.3570333767	0.1174754651	0.698471533
5	9	0	0.0051427167	0.0024183140	0.010902808
6	9	1	0.0501821692	0.0112579158	0.196888042
7	13	0	0.0004915945	0.0001470338	0.001642276
8	13	1	0.0050017290	0.0008634866	0.028408518

Figure 4: Izzo data predictions

```
plot_predictions(izzo.1, condition = c("Seed", "Izzo"))
```

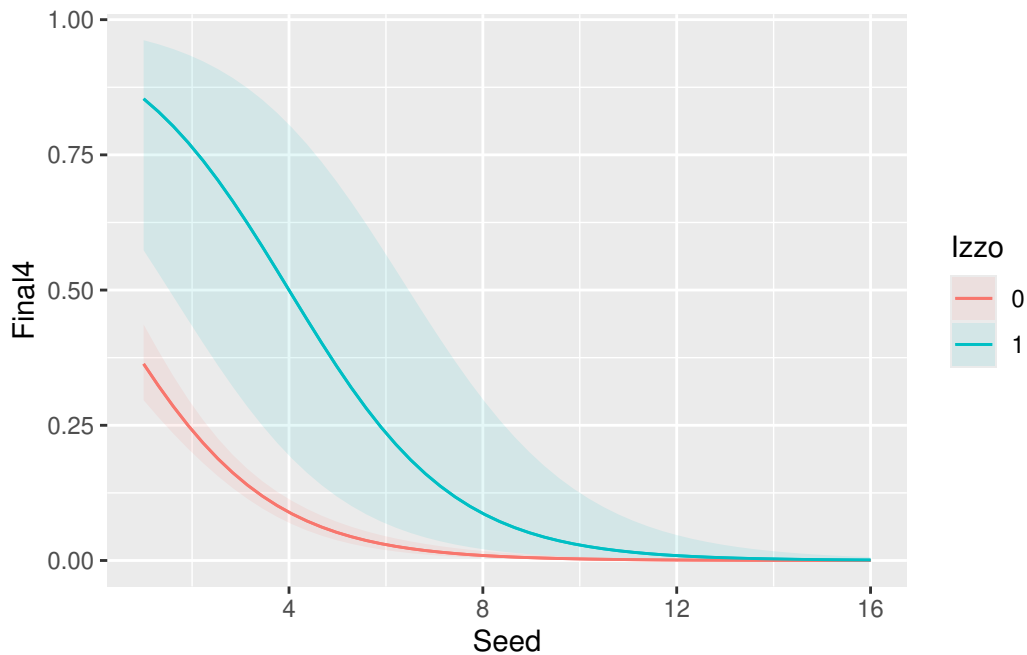


Figure 5: Graph of predictions for Izzo data

	Task	Report	Time
1	Visual	Verbal	5.74
2	Visual	Visual	28.15
3	Visual	Verbal	6.68
4	Visual	Visual	15.85
5	Verbal	Visual	8.44
6	Verbal	Visual	13.16
7	Visual	Verbal	5.88
8	Verbal	Verbal	11.37
9	Verbal	Verbal	18.28
10	Verbal	Visual	14.69

Figure 6: Brain side data (10 randomly chosen rows)

```
ggplot(VisualVerbal, aes(x = Task, y = Time, fill = Report)) + geom_boxplot()
```

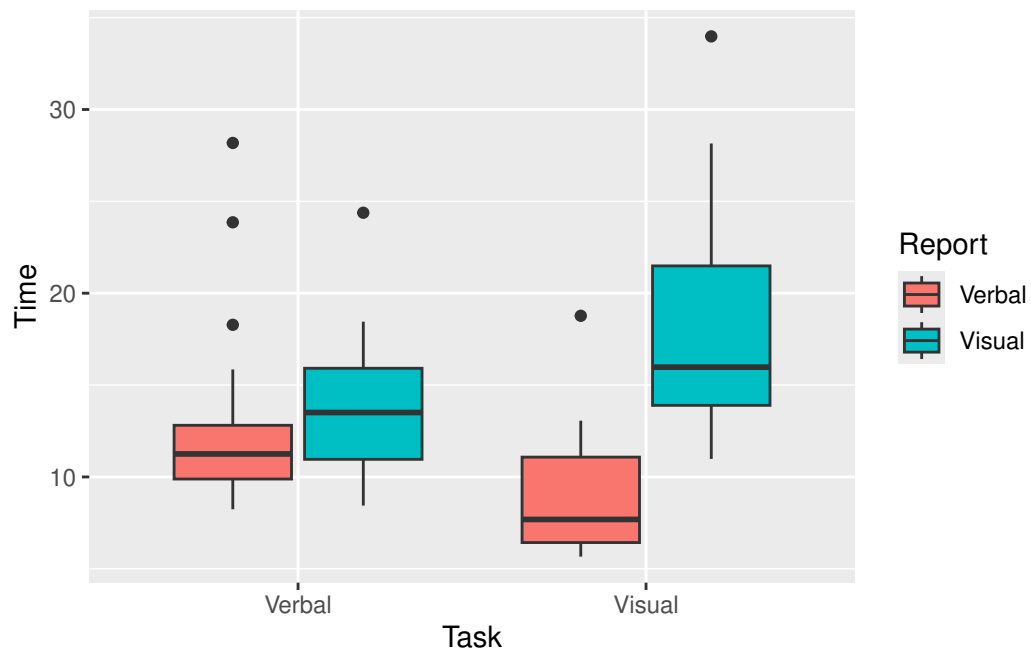


Figure 7: Brain side boxplots

```
vis.1 <- aov(log(Time) ~ Task * Report, data = VisualVerbal)
summary(vis.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Task	1	0.033	0.0335	0.33	0.567
Report	1	3.136	3.1360	30.92	3.83e-07 ***
Task:Report	1	1.963	1.9630	19.36	3.49e-05 ***
Residuals	76	7.708	0.1014		

---  
 Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Figure 8: Brain side ANOVA

```
VisualVerbal %>% filter(Task == "Verbal") -> verbals
verbals.1 <- aov(log(Time) ~ Report, data = verbals)
summary(verbals.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Report	1	0.068	0.06839	0.722	0.401
Residuals	38	3.599	0.09470		

Figure 9: Brain side further analysis part 1

```
VisualVerbal %>% filter(Task == "Visual") -> visuals
visuals.1 <- aov(log(Time) ~ Report, data = visuals)
summary(visuals.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Report	1	5.031	5.031	46.52	4.3e-08 ***
Residuals	38	4.109	0.108		

---  
 Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Figure 10: Brain side further analysis part 2

---

	Loc	Var	Y1	Y2
1	UF	M	81.0	80.7
2	UF	S	105.4	82.3
3	UF	V	119.7	80.4
4	UF	T	109.7	87.2
5	UF	P	98.3	84.2
6	W	M	146.6	100.4
7	W	S	142.0	115.5
8	W	V	150.7	112.2
9	W	T	191.5	147.7
10	W	P	145.7	108.1
11	M	M	82.3	103.1
12	M	S	77.3	105.1
13	M	V	78.4	116.5
14	M	T	131.3	139.9
15	M	P	89.6	129.6
16	C	M	119.8	98.9
17	C	S	121.4	61.9
18	C	V	124.0	96.2
19	C	T	140.8	125.5
20	C	P	124.8	75.7
21	GR	M	98.9	66.4
22	GR	S	89.0	49.9
23	GR	V	69.1	96.7
24	GR	T	89.3	61.9
25	GR	P	104.1	80.3
26	D	M	86.9	67.7
27	D	S	77.1	66.7
28	D	V	78.9	67.4
29	D	T	101.8	91.8
30	D	P	96.0	94.1

Figure 11: Barley yield data (all)

```
immer %>% select(starts_with("Y")) %>% as.matrix() -> y
immer.1 <- manova(y ~ Var + Loc, data = immer)
summary(immer.1)
```

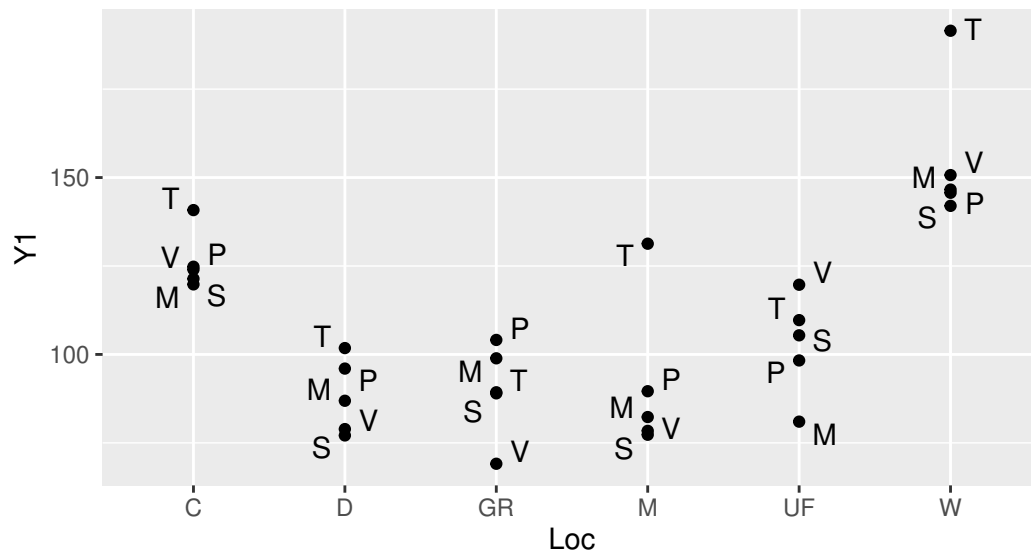
	Df	Pillai	approx F	num Df	den Df	Pr(>F)
Var	4	0.64205	2.364	8	40	0.03469 *
Loc	5	1.50658	12.213	10	40	2.543e-09 ***
Residuals	20					

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Figure 12: Barley yield MANOVA

```
ggplot(immer, aes(x = Loc, y = Y1, label = Var)) + geom_point() +  
  geom_text_repel()
```



```
ggplot(immer, aes(x = Loc, y = Y2, label = Var)) + geom_point() +  
  geom_text_repel()
```

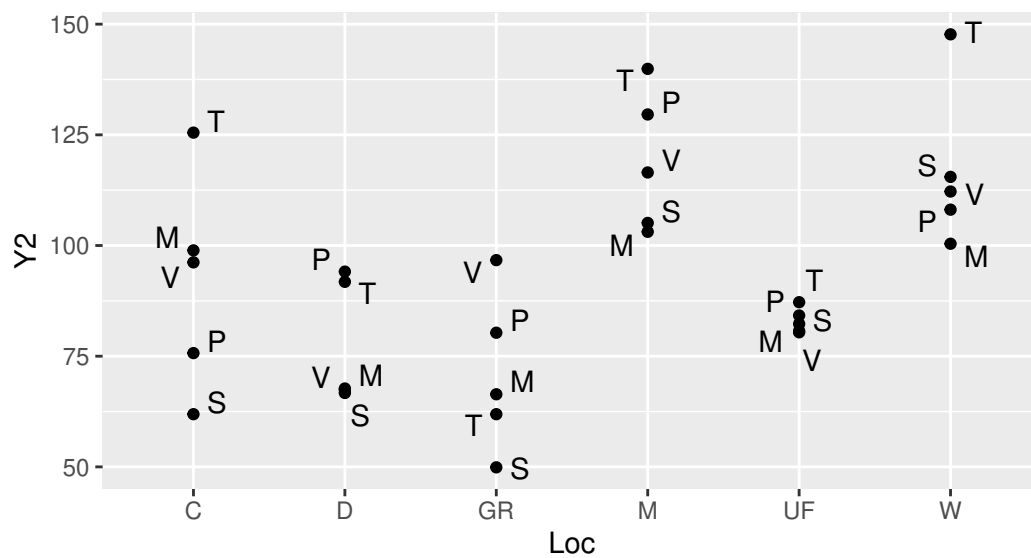


Figure 13: Barley yield plots



---

	id	grip.type	replicate	UBP
1	c_1	classic	1	168.2084
2	c_1	classic	2	161.4141
3	c_1	classic	3	163.2345
4	c_2	classic	1	155.9429
5	c_2	classic	2	168.5388
6	c_2	classic	3	166.3163
7	c_3	classic	1	162.6191
8	c_3	classic	2	157.8030
9	c_3	classic	3	171.6529
10	c_4	classic	1	165.1400
11	c_4	classic	2	164.9525
12	c_4	classic	3	158.2008
13	m_1	modern	1	160.0739
14	m_1	modern	2	161.2383
15	m_1	modern	3	166.7635
16	m_2	modern	1	161.8334
17	m_2	modern	2	162.7900
18	m_2	modern	3	157.5793
19	m_3	modern	1	165.2248
20	m_3	modern	2	162.7804
21	m_3	modern	3	159.7632
22	m_4	modern	1	160.3049
23	m_4	modern	2	168.5381
24	m_4	modern	3	164.4688
25	i_1	integrated	1	166.7134
26	i_1	integrated	2	173.0319
27	i_1	integrated	3	173.2537
28	i_2	integrated	1	165.4825
29	i_2	integrated	2	166.0498
30	i_2	integrated	3	170.5794
31	i_3	integrated	1	174.8182
32	i_3	integrated	2	166.8222
33	i_3	integrated	3	165.2776
34	i_4	integrated	1	174.8661
35	i_4	integrated	2	173.0058
36	i_4	integrated	3	165.1532

Figure 14: Ski grip data (all)

```
grip %>%  
  group_by(grip.type, replicate) %>%  
  summarize(mean_ubp = mean(UBP)) %>%  
  ggplot(aes(x = replicate, y = mean_ubp,  
             colour = grip.type, group = grip.type)) +  
    geom_point() + geom_line()
```

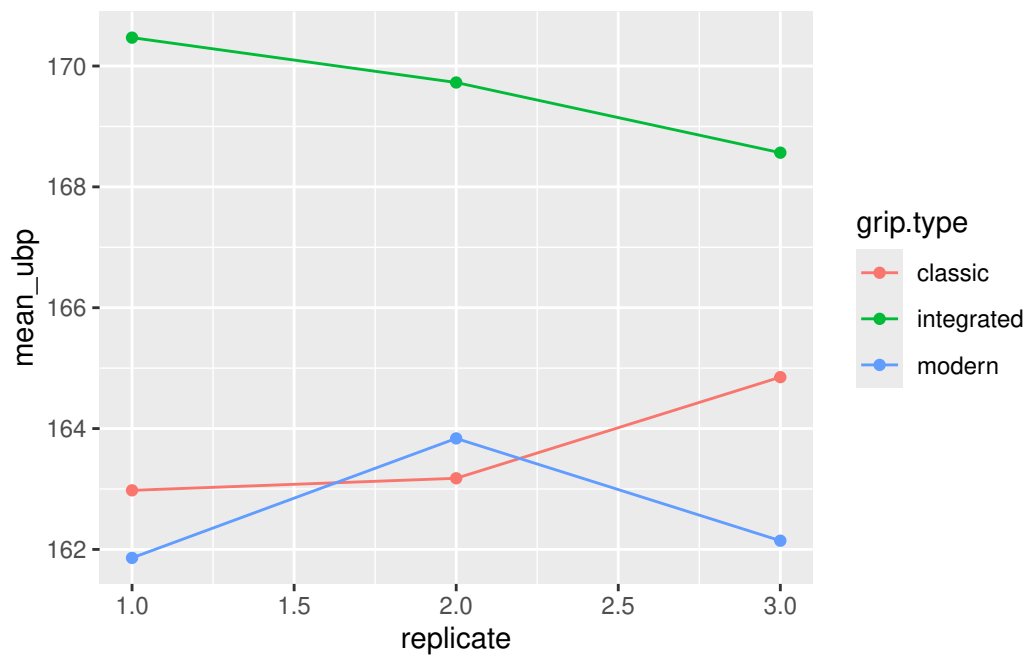


Figure 15: Ski grip interaction plot

```
ggplot(grip, aes(x = replicate, y = UBP, colour = grip.type, group = id)) +
  geom_point() + geom_line()
```

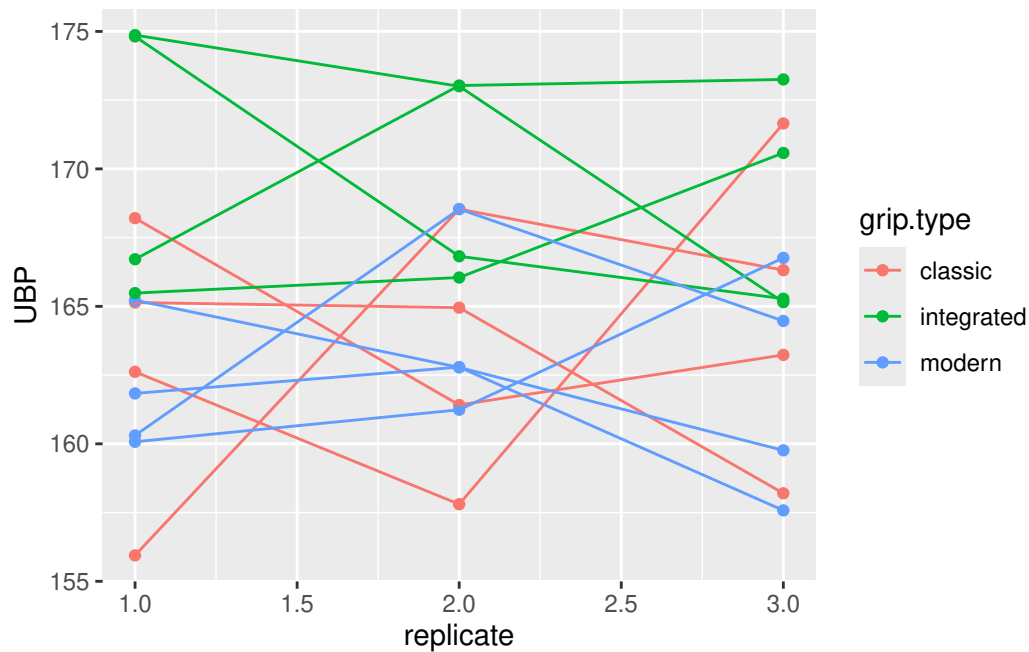


Figure 16: Ski grip spaghetti plot

```
grip %>% pivot_wider(names_from = replicate, values_from = UBP) -> grip_wide
```

Figure 17: Ski grip code

```
grip_wide %>%
  select(`1`:`3`) %>%
  as.matrix() -> y
grip.1a <- lm(y ~ grip.type, data = grip_wide)
times <- colnames(y)
times.df <- data.frame(times = factor(times))
grip.1 <- Manova(grip.1a, idata = times.df, idesign = ~ times)
```

Figure 18: Ski grip analysis code

## Univariate Type II Repeated-Measures ANOVA Assuming Sphericity

	Sum Sq	num Df	Error SS	den Df	F value	Pr(>F)
(Intercept)	983547	1	52.50	9	1.6861e+05	< 2.2e-16 ***
grip.type	339	2	52.50	9	2.9073e+01	0.0001182 ***
times	2	2	458.88	18	3.0600e-02	0.9698719
grip.type:times	23	4	458.88	18	2.2970e-01	0.9181327

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

## Mauchly Tests for Sphericity

	Test statistic	p-value
times	0.9088	0.68214
grip.type:times	0.9088	0.68214

Greenhouse-Geisser and Huynh-Feldt Corrections  
for Departure from Sphericity

	GG eps	Pr(>F[GG])
times	0.91642	0.9616
grip.type:times	0.91642	0.9057

	HF eps	Pr(>F[HF])
times	1.139118	0.9698719
grip.type:times	1.139118	0.9181327

Figure 19: Ski grip analysis output

Player	position	FGPct	FG3Pct	FTPct	Rebounds	Steals	Fouls
Rudy Gay	forward	0.455	0.359	0.858	5.867647	1.0441176	2.308823
Jamal Crawford	guard	0.396	0.327	0.901	1.937500	0.9218750	1.687500
J.J. Redick	guard	0.477	0.437	0.901	2.141026	0.5000000	1.717949
Robin Lopez	centre	0.535	0.000	0.772	6.677966	0.2711864	2.067797
Greivis Vasquez	guard	0.408	0.379	0.758	2.634146	0.5609756	2.158537
Brandon Jennings	guard	0.401	0.360	0.839	2.536585	1.0731707	1.560976
Zach Randolph	forward	0.487	0.350	0.765	10.521127	0.9718310	2.464789
Tony Allen	guard	0.495	0.345	0.627	4.444444	2.0476190	2.634921
Al Jefferson	centre	0.481	0.400	0.655	8.430769	0.7230769	2.138462
Jeff Teague	guard	0.460	0.343	0.862	2.520548	1.7123288	1.904110
Derrick Rose	guard	0.405	0.280	0.813	3.156863	0.7058824	1.235294
Andre Drummond	centre	0.514	0.000	0.389	13.463415	0.8902439	3.475610
Luol Deng	forward	0.469	0.355	0.761	5.222222	0.9027778	1.513889
Jeff Green	forward	0.430	0.332	0.833	4.205128	0.6794872	1.884615
Norris Cole	guard	0.412	0.313	0.716	2.120000	0.7600000	1.653333

Figure 20: NBA 2015 data (15 randomly selected rows)

```

y <- with(nba, cbind(FGPct, FG3Pct, FTPct, Rebounds, Steals, Fouls))
nba.2 <- manova(y ~ position, data = nba)
summary(nba.2)

```

```

              Df  Pillai approx F num Df den Df      Pr(>F)
position      2 0.72575   15.852    12    334 < 2.2e-16 ***
Residuals 171
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Figure 21: NBA 2015 MANOVA

```
nba.1 <- lda(position ~ FGPct + FG3Pct + FTPct + Rebounds + Steals +  
              Fouls, data = nba)  
nba.1
```

Call:

```
lda(position ~ FGPct + FG3Pct + FTPct + Rebounds + Steals + Fouls,  
     data = nba)
```

Prior probabilities of groups:

	centre	forward	guard
	0.1379310	0.3563218	0.5057471

Group means:

	FGPct	FG3Pct	FTPct	Rebounds	Steals	Fouls
centre	0.5214167	0.2160417	0.6998750	8.642295	0.6699533	2.614796
forward	0.4553065	0.3293065	0.7515161	5.941030	0.9615395	2.228547
guard	0.4277500	0.3467841	0.7935682	3.518978	1.1223494	2.085363

Coefficients of linear discriminants:

	LD1	LD2
FGPct	-6.7149807	17.4822905
FG3Pct	1.1377356	-5.7597904
FTPct	0.3022703	3.2938611
Rebounds	-0.4652831	-0.4747002
Steals	1.4300714	-0.1587456
Fouls	-0.1288587	0.9834488

Proportion of trace:

	LD1	LD2
	0.9537	0.0463

Figure 22: NBA 2015 discriminant analysis

```
p <- predict(nba.1)
d <- cbind(nba, p)
with(d, table(obs = position, pred = class))
```

obs	pred		
	centre	forward	guard
centre	18	6	0
forward	9	31	22
guard	1	4	83

Figure 23: NBA 2015 further discriminant analysis

```
ggplot(d, aes(x = x.LD1, y = x.LD2, colour = position)) + geom_point()
```

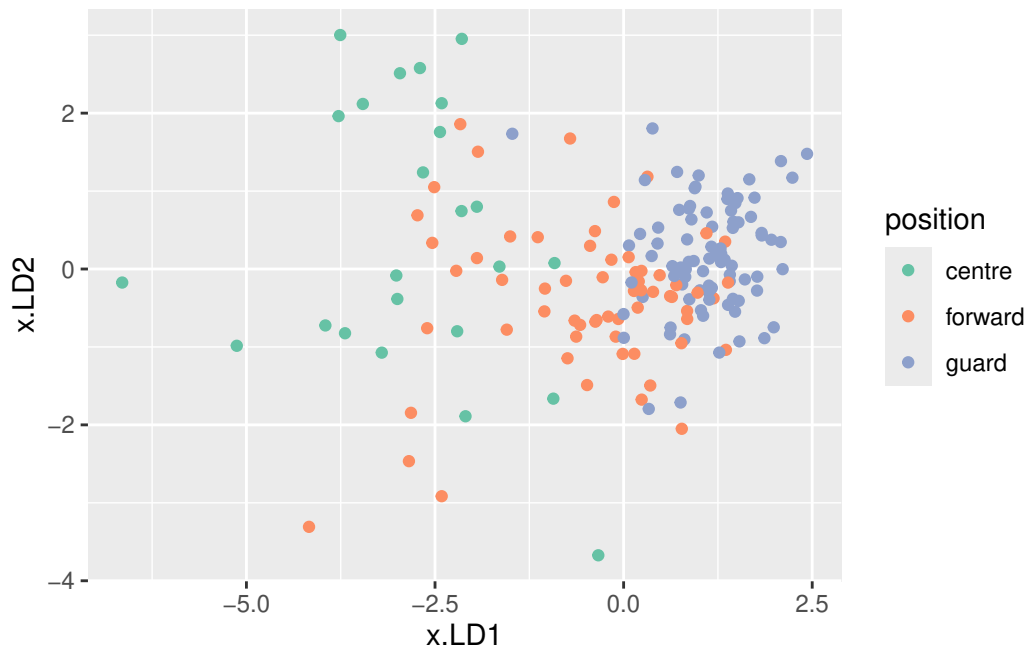


Figure 24: NBA 2015 discriminant analysis plot

	Player	position	class	p.centre	p.forward	p.guard
25	Jose Calderon	guard	guard	0.000	0.171	0.829
173	Thaddeus Young	forward	guard	0.001	0.329	0.670
103	Kawhi Leonard	forward	guard	0.000	0.349	0.651
57	Danilo Gallinari	forward	guard	0.000	0.275	0.724
105	Jeremy Lin	guard	guard	0.000	0.061	0.939
13	Nicolas Batum	forward	forward	0.001	0.647	0.353
167	Andrew Wiggins	guard	guard	0.001	0.354	0.645
140	Iman Shumpert	guard	guard	0.000	0.133	0.867
168	Deron Williams	guard	guard	0.000	0.169	0.831
141	Marcus Smart	guard	guard	0.000	0.064	0.936

Note: The columns with names starting with **p.** originally started with **posterior**. The column **p.centre**, for example, was originally called **posterior.centre**. I changed this so that the table would fit on the page.

Figure 25: NBA 2015 posterior probabilities (selected)

	Sl_No	L500	L1000	L2000	L4000	R500	R1000	R2000	R4000
1	47	5	0	10	70	-5	5	15	40
2	14	5	15	5	60	5	5	0	50
3	55	15	20	10	60	20	20	0	25
4	66	-10	0	5	60	-10	-5	0	65
5	71	0	10	40	60	-5	0	25	50
6	75	0	-10	0	60	15	0	5	50
7	28	-5	-5	-5	55	-5	5	10	70
8	50	-5	0	10	55	-10	0	5	50
9	67	5	10	40	55	0	5	30	40
10	98	10	10	15	55	0	0	5	75
11	18	5	0	0	50	10	10	5	65
12	27	0	0	5	50	5	0	5	40
13	73	0	5	45	50	0	10	15	50
14	34	-10	-10	-10	45	-10	-10	5	45
15	35	-5	10	20	45	-5	10	35	60

Figure 26: Hearing data (15 selected rows)



```
hearing %>% select(-Sl_No) -> hearing0
hearing.1 <- princomp(hearing0, cor = TRUE)
hearing.1
```

Call:

```
princomp(x = hearing0, cor = TRUE)
```

Standard deviations:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8
	1.9821719	1.2721328	0.9875853	0.6832146	0.5831723	0.5620420	0.4473378	0.3930313

8 variables and 100 observations.

```
hearing.1$loadings
```

Loadings:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8
L500	0.401	0.317	0.158	0.328		0.446	0.329	0.546
L1000	0.421	0.225		0.482	-0.379			-0.623
L2000	0.366	-0.239	-0.470	0.282	0.439		-0.526	0.186
L4000	0.281	-0.474	0.430	0.161	0.350	-0.417	0.427	
R500	0.343	0.386	0.259	-0.488	0.498	0.195	-0.159	-0.343
R1000	0.411	0.232		-0.372	-0.351	-0.614		0.361
R2000	0.312	-0.317	-0.563	-0.391	-0.111	0.265	0.478	-0.147
R4000	0.254	-0.514	0.426	-0.159	-0.396	0.366	-0.414	

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8
SS loadings	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Proportion Var	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
Cumulative Var	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000

Figure 27: Hearing data principal components

```
ggscreeplot(hearing.1)
```

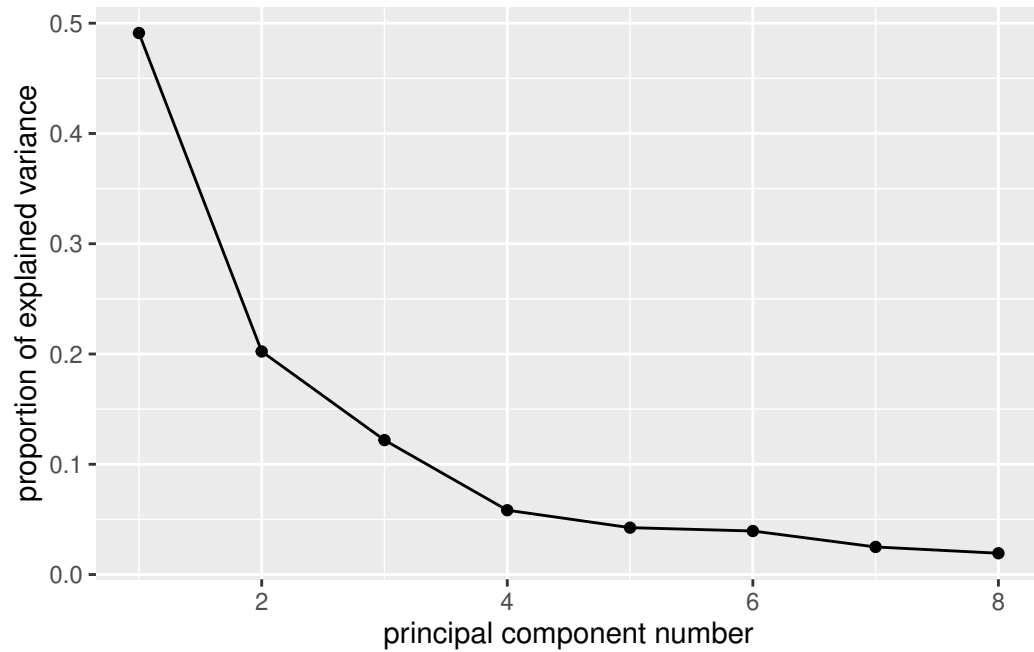


Figure 28: Hearing data screeplot



```
# A tibble: 20 x 4
  age      left_side right_side     n
<fct>   <chr>      <chr>   <int>
1 (15,30] no        no      1913
2 (15,30] no        yes       41
3 (15,30] yes       no      149
4 (15,30] yes       yes       63
5 (30,40] no        no     2226
6 (30,40] no        yes       48
7 (30,40] yes       no      190
8 (30,40] yes       yes       84
9 (40,50] no        no     2262
10 (40,50] no       yes       40
11 (40,50] yes      no      148
12 (40,50] yes      yes       70
13 (50,70] no       no     1974
14 (50,70] no       yes       31
15 (50,70] yes      no      113
16 (50,70] yes      yes       60
17 (70,90] no       no     671
18 (70,90] no       yes       10
19 (70,90] yes      no       55
20 (70,90] yes      yes       38
```

Figure 31: Chest pain data

```
chest.1 <- glm(n ~ age * left_side * right_side, data = chest, family = "poisson")
drop1(chest.1, test = "Chisq")
```

Single term deletions

Model:

```
n ~ age * left_side * right_side
```

	Df	Deviance	AIC	LRT	Pr(>Chi)
<none>		0.0000	175.45		
age:left_side:right_side	4	6.6877	174.13	6.6877	0.1533

```
chest.2 <- update(chest.1, . ~ . - age:left_side:right_side)
drop1(chest.2, test = "Chisq")
```

Single term deletions

Model:

```
n ~ age + left_side + right_side + age:left_side + age:right_side +
    left_side:right_side
```

	Df	Deviance	AIC	LRT	Pr(>Chi)
<none>		6.69	174.13		
age:left_side	4	19.52	178.97	12.83	0.01211 *
age:right_side	4	7.71	167.15	1.02	0.90674
left_side:right_side	1	983.32	1148.76	976.63	< 2e-16 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
chest.3 <- update(chest.2, . ~ . - age:right_side)
drop1(chest.3, test = "Chisq")
```

Single term deletions

Model:

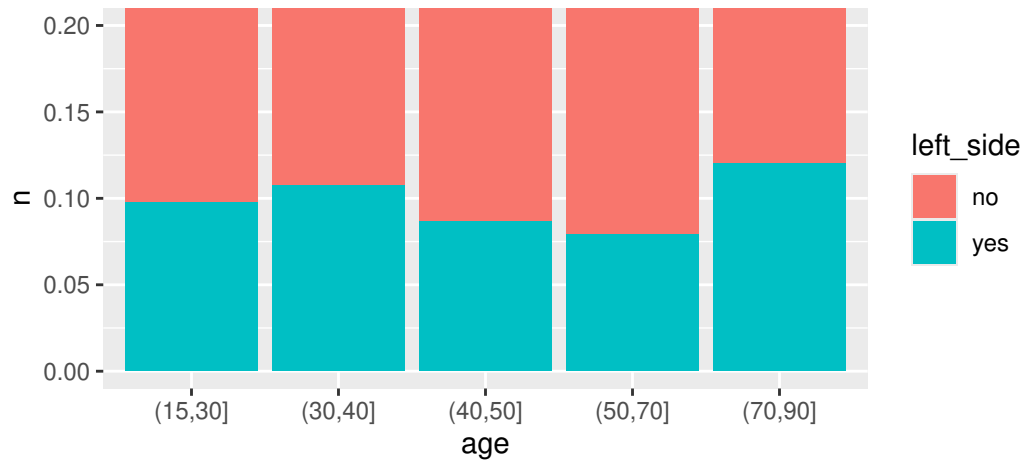
```
n ~ age + left_side + right_side + age:left_side + left_side:right_side
```

	Df	Deviance	AIC	LRT	Pr(>Chi)
<none>		7.71	167.15		
age:left_side	4	26.33	177.78	18.63	0.0009308 ***
left_side:right_side	1	990.13	1147.58	982.42	< 2.2e-16 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Figure 32: Chest pain model fitting

```
ggplot(chest, aes(x = age, y = n, fill = left_side)) +  
  geom_col(position = "fill") +  
  coord_cartesian(ylim = c(0, 0.2))
```



Note: the `coord_cartesian` is used to truncate the  $y$ -scale, as on Worksheet 12.

Figure 33: Chest pain graph 1

```
ggplot(chest, aes(x = left_side, y = n, fill = right_side)) +  
  geom_col(position = "fill")
```

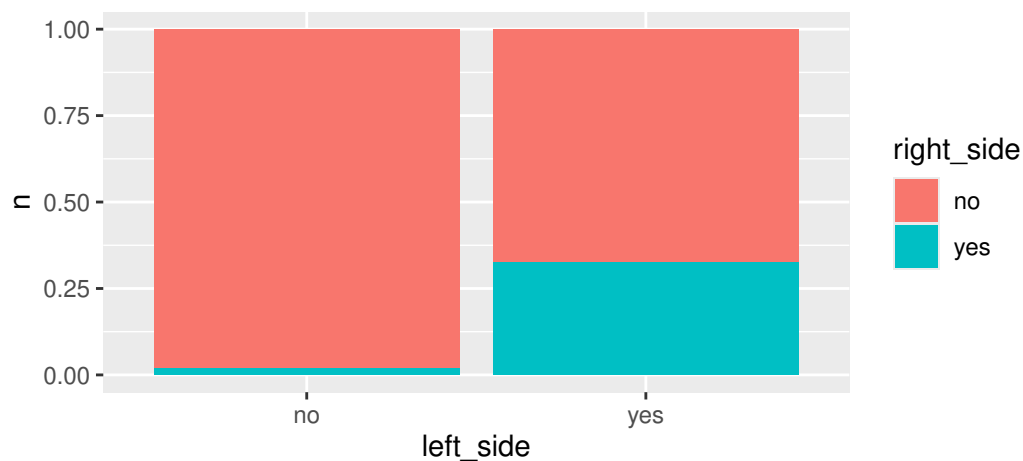


Figure 34: Chest pain graph 2