Cluster Analysis

- One side-effect of discriminant analysis: could draw picture of data (if 1st 2 canonical variables told most of story) and see which individuals “close” to each other.
- Discriminant analysis requires knowledge of groups.
- Without knowledge of groups, use *cluster analysis*: see which individuals close, which groups suggested by data.
- Idea: see how individuals group into “clusters” of nearby individuals.
- Base on “dissimilarities” between individuals.
- Or base on standard deviations and correlations between variables (assesses dissimilarity behind scenes).

One to ten in 11 languages

	English	Norwegian	Danish	Dutch	German
1	one	en	en	een	eins
2	two	to	to	twee	zwei
3	three	tre	tre	drie	drei
4	four	fire	fire	vier	vier
5	five	fem	fem	vijf	funf
6	six	seks	seks	zes	sechs
7	seven	sju	syv	zeven	sieben
8	eight	atte	otte	acht	acht
9	nine	ni	ni	negen	neun
10	ten	ti	ti	tien	zehn

One to ten

	French	Spanish	Italian	Polish	Hungarian	Finnish
1	un	uno	uno	jeden	egy	yksi
2	deux	dos	due	dwa	ketto	kaksi
3	trois	tres	tre	trzy	harom	kolme
4	quatre	cuatro	quattro	cztery	negy	nelja
5	cinq	cinco	cinque	piec	ot	viisi
6	six	seis	sei	szesc	hat	kuusi
7	sept	siete	sette	siedem	het	seitseman
8	huit	ocho	otto	osiem	nyolc	kahdeksan
9	neuf	nueve	nove	dziewiec	kilenc	yhdeksan
10	dix	diez	dieci	dziesiec	tiz	kymmenen

Dissimilarities and languages example

- Can define dissimilarities how you like (whatever makes sense in application).
- Sometimes defining “similarity” makes more sense; can turn this into dissimilarity by subtracting from some maximum.
- Example: numbers 1–10 in various European languages. Define similarity between two languages by counting how often the same number has a name starting with the same letter (and dissimilarity by how often number has names starting with different letter).
- Crude (doesn’t even look at most of the words), but see how effective.

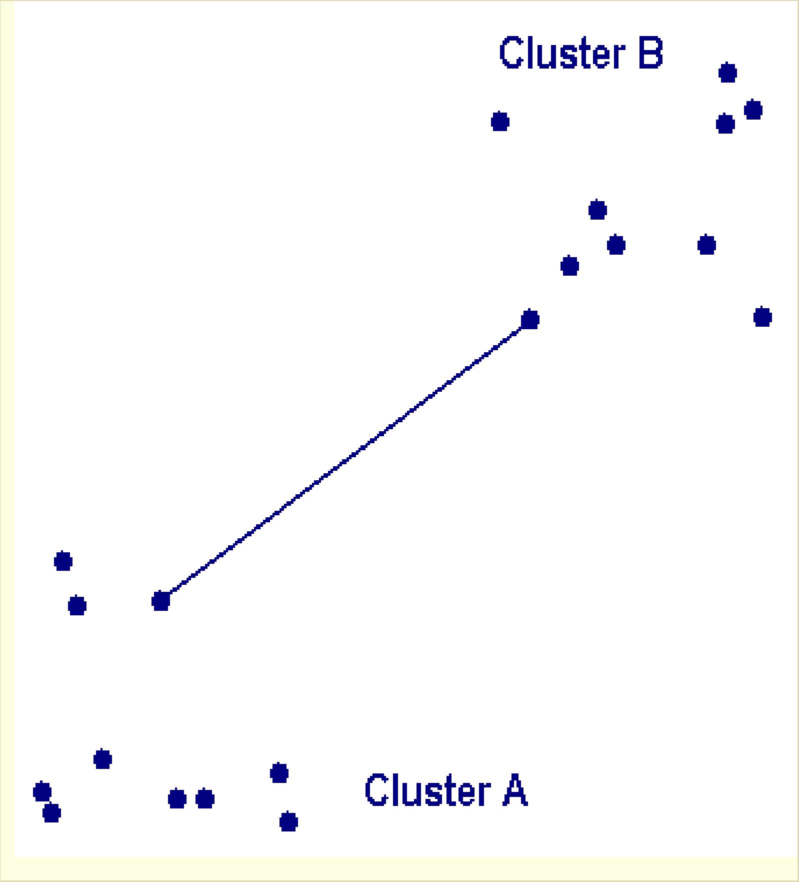
Two kinds of cluster analysis

- Looking at process of forming clusters (of similar languages): PROC CLUSTER, hierarchical cluster analysis.
  - Start with each individual in cluster by itself.
  - Join “closest” clusters one by one until all individuals in one cluster.
  - Rule to join clusters: single-linkage, complete linkage, Ward’s method, etc.
- Know how many clusters: which division into that many clusters is “best” for individuals? PROC FASTCLUS, K-means clustering.

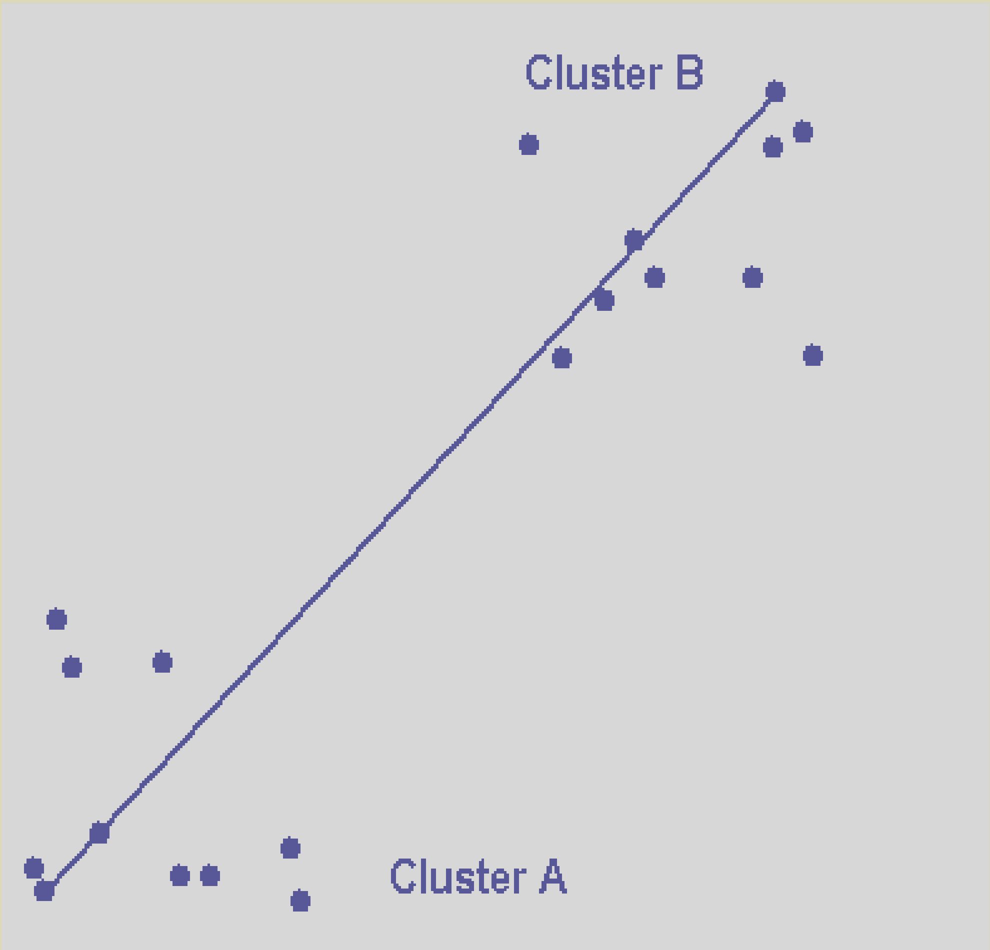
Hierarchical cluster analysis: joining rules

Join the two clusters that are “closest”, but how to define?

Single-linkage (from `http://www.resample.com`)



Complete linkage



Also average linkage (obvious?)

Ward’s method example

- Easiest to illustrate how Ward’s method works by example.
- Data (one variable): 1, 2, 3, 7, 8, 9, 11, 12, 13. Suppose currently have 3 clusters 1,2,3; 7,8,9; 11,12,13. Measure dissimilarity by absolute difference (throw away minus sign).
- Which 2 of these 3 clusters to join together?
- Single-linkage distances: $7 - 3 = 4$, $11 - 3 = 8$, $11 - 9 = 2$; join 2nd and 3rd.
- Complete-linkage distances: $9 - 1 = 8$, $13 - 1 = 12$, $13 - 7 = 6$; also join 2nd and 3rd.

. . . continued

- Suppose join 1st 2 clusters. Joined cluster has mean $(1+2+3+7+8+9)/6=5$; new sum of squared distances from mean $(1-5)^2+(2-5)^2+(3-5)^2+(7-5)^2+(8-5)^2+(9-5)^2=58$.
- Join 1st and 3rd (obviously bad idea): mean now 7, sum of squared distances $(1-7)^2+(2-7)^2+(3-7)^2+(11-7)^2+(12-7)^2+(13-7)^2=154$.
- Join 2nd and 3rd: mean now 10, sum of squared distances $(7-10)^2+(8-10)^2+(9-10)^2+(11-10)^2+(12-10)^2+(13-10)^2=28$.
- Smallest of these three sums is 28, so join 1st and 3rd clusters.
- Much computation, especially early with many clusters. But we don't care!

Ward’s method in general

- Work out sum of squared distances/dissimilarities from each observation to centre of its current cluster. Like error SS in ANOVA. Call it ESS.
- At start, each point in own cluster, so ESS 0.
- At each stage, join the two clusters that make resulting ESS smallest.
- Favours joining small clusters.
- Like linkage methods, joins “similar” clusters.

Dissimilarity data in SAS

Dissimilarities for language data (first line for reference, not in data file):

```
  en no dk nl de fr es it pl hu sf
en 0  2  2  7  6  6  6  6  7  9  9
no 2  0  1  5  4  6  6  6  7  8  9
dk 2  1  0  6  5  6  5  5  6  8  9
nl 7  5  6  0  5  9  9  9 10  8  9
de 6  4  5  5  0  7  7  7  8  9  9
fr 6  6  6  9  7  0  2  1  5 10  9
es 6  6  5  9  7  2  0  1  3 10  9
it 6  6  5  9  7  1  1  0  4 10  8
pl 7  7  6 10  8  5  3  4  0 10  9
hu 9  8  8  8  9 10 10 10 10  0  8
sf 9  9  9  9  9  9  9  8  9  8  0
```

SAS has special `type=distance` for data like these:

```
data lang(type=distance);
infile "one-ten.dat";
input lang $ en no dk nl de fr es it pl hu sf;
```

Variable `lang` has names of languages; variable names given on `input` line must match.

Doing a hierarchical cluster analysis

- Here, interested in clustering *process* more than final result, so hierarchical analysis appropriate: PROC CLUSTER.
- Choose single-linkage method for combining clusters (that is, combine clusters whose closest members are closest).
- Draw clustering “tree” from output data set. Trees by default vertical and (try to) use fancy graphics.

```
proc cluster method=single outtree=tree;
  id lang;

proc tree horizontal lineprinter;
  id lang;
```

Output: cluster history

The CLUSTER Procedure
Single Linkage Cluster Analysis

Mean Distance Between Observations 6.672727

Cluster History

NCL	--Clusters Joined---		FREQ	Norm Min Dist	Tie
10	no	dk	2	0.1499	T
9	fr	it	2	0.1499	T
8	CL9	es	3	0.1499	
7	en	CL10	3	0.2997	
6	CL8	pl	4	0.4496	
5	CL7	de	4	0.5995	
4	CL5	nl	5	0.7493	T
3	CL4	CL6	9	0.7493	
2	CL3	hu	10	1.1989	T
1	CL2	sf	11	1.1989	

Summary of clustering history

- Join Norwegian and Danish.
- Join French and Italian.
- Join Spanish to the French-Italian cluster.
- Join English to the Norwegian-Danish cluster.
- Then: German and Dutch joined to Germanic languages cluster, Polish to Romance language cluster (!)
- Then join these two clusters together, and join Hungarian and Finnish to them.

Output from PROC TREE (read from *right*)

```
                        The TREE Procedure
                  Single Linkage Cluster Analysis

                    Minimum Distance Between Clusters

      1.2 1.1   1   0.9  0.8  0.7  0.6  0.5  0.4  0.3  0.2  0.1   0
      +----+----+----+----+----+----+----+----+----+----+----+----+
N  en XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX...............
a     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
m  no XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
e     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   dk XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
o     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
f  de XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX..............................
      XXXXXXXXXXXXXXXXXXXXXXXX
O  nl XXXXXXXXXXXXXXXXXXXXXXXX.....................................
b     XXXXXXXXXXXXXXXXXXXXXXXX
s  fr XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
e     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
r  it XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
v     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
a  es XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
t     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
i  pl XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX......................
o     X
n  hu X............................................................
      X
   sf X............................................................
```

Checking our intuition about languages

- Have a Germanic cluster (English, Norwegian, Danish, German, Dutch)
- Have a Romance cluster (French, Italian, Spanish, maybe Polish)
- Have two odd languages (Hungarian, Finnish).
- Corresponds to linguistics/geography pretty well (for such a crude measure).
- Maybe Dutch joins Germanic cluster late. Dutch number words much like German, but often happen not to start with same letter.
- Clustering method: single linkage may join languages that happen to have words starting with same letter, but not otherwise similar. Ward's method joins clusters that are more "alike". Change "method=" on PROC DISCRIM line.

Tree from Ward’s method

```
                         The TREE Procedure
               Ward’s Minimum Variance Cluster Analysis

                           Semi-Partial R-Squared

      0.4   0.35     0.3    0.25     0.2    0.15     0.1    0.05      0
      +-------+-------+-------+-------+-------+-------+-------+-------+
   en        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX..
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   no        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   dk        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   nl        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX........
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
l  de        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX........
a            XXXXXXXXXXXX
n  hu        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX....................
g            XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   sf        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX....................
             X
   fr        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   it        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   es        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   pl        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX........
```

Comparing single-linkage and Ward

- In Ward, Dutch and German get joined earlier (before joining to Germanic cluster).
- Also Hungarian and Finnish get combined earlier.
- Consider which clustering method makes sense for data like these.

Another example

Birth, death and infant mortality rates for 96 countries
(variables not dissimilarities):

```
   24.7  5.7  30.8 Albania               12.5 11.9  14.4 Bulgaria
   13.4 11.7  11.3 Czechoslovakia        12   12.4   7.6 Former_E._Germany
   11.6 13.4  14.8 Hungary               14.3 10.2    16 Poland
   13.6 10.7  26.9 Romania                 14    9  20.2 Yugoslavia
   17.7   10    23 USSR                  15.2  9.5  13.1 Byelorussia_SSR
   13.4 11.6    13 Ukrainian_SSR         20.7  8.4  25.7 Argentina
   46.6   18   111 Bolivia               28.6  7.9    63 Brazil
   23.4  5.8  17.1 Chile                 27.4  6.1    40 Columbia
   32.9  7.4    63 Ecuador               28.3  7.3    56 Guyana
...
```

- Want to find groups of similar countries (and how many groups, which countries in each group).
- Tree would be unwieldy with 96 countries.
- More automatic way of finding number of clusters?
- Two countries per line: how to read into SAS?

SAS code and issues

```
data birthrate;
  infile "birthrate.dat";
  input birth death infant country $ @@;

proc cluster method=average ccc standard;
  id country;
```

- In DATA step, @@ means “continue reading on same line”.
- Using average linkage.
- “CCC” is “cubic clustering criterion”, helps us decide how many clusters.
- “standard” means to use standardized data (scaled to have mean 0 and SD 1) so each variable truly comparable.

Clustering history (a little)

96 lines, just show some:

Cluster History

NCL	--Clusters Joined---		FREQ	SPRSQ	RSQ	ERSQ	CCC	Norm RMS Dist	Tie
96	Austria	Canada	2	0.0000	1.00	.	.	0.0165	
95	Czechosl	Ukrainia	2	0.0000	1.00	.	.	0.0175	
...									
20	CL82	CL34	6	0.0016	.967	.	.	0.2664	
19	CL32	CL38	7	0.0018	.965	.952	4.10	0.2709	
18	Bolivia	CL29	6	0.0011	.964	.949	4.53	0.2794	
17	CL21	Oman	6	0.0014	.963	.945	4.87	0.3191	
16	CL23	CL26	16	0.0059	.957	.942	3.84	0.3225	
...									
8	CL12	CL74	24	0.0067	.907	.887	2.16	0.4773	
7	Mexico	Korea	2	0.0026	.904	.873	3.27	0.5037	
6	Afghanis	CL13	8	0.0045	.900	.854	4.47	0.5328	
5	CL15	CL10	45	0.0517	.848	.827	1.57	0.5697	
4	CL9	CL8	42	0.1001	.748	.788	-2.3	0.7742	
3	CL5	CL4	87	0.3980	.350	.723	-12	1.0708	
2	CL3	CL7	89	0.0385	.311	.593	-6.8	1.1662	
1	CL2	CL6	97	0.3114	.000	.000	0.00	1.5693	

Look for large values of CCC compared to neighbours, here 17 clusters or 6. We'll try 6.

The 6 best clusters

- Only purpose for running previous analysis was to get good number of clusters.
- 6 clusters obtained by “chopping the tree” may not be best division of countries into 6 clusters.
- Do better by deciding on 6 (or however many) clusters first, *then* trying for best division of countries into 6 clusters.
- This is where K-means clustering comes in. Choose best division of individuals (countries) into K (6) clusters so that sum of squared distances from individuals to cluster averages made smallest (over all possible divisions into K clusters).
- Use PROC FASTCLUS (which does not have “standard” option so have to standardize first).

Code

```
proc standard mean=0 std=1;

proc fastclus maxclusters=6 out=clust;
  id country;

proc sort data=clust;
  by cluster;

proc print data=clust;
  by cluster;
```

Sort data by cluster and print sorted data.

Cluster means and SDs

Cluster Means

Cluster	birth	death	infant
1	-0.435769031	-1.143859869	-0.728110805
2	1.204946595	0.697233337	1.016509747
3	1.301924159	2.117634622	1.866220472
4	-0.219972241	2.111657686	-0.454443499
5	-1.173710389	-0.185637473	-0.953436985
6	0.416099253	-0.516998811	0.264875362

Cluster Standard Deviations

Cluster	birth	death	infant
1	0.3560992452	0.3384785179	0.2086886380
2	0.2838078359	0.3886873578	0.4595354494
3	0.2072519523	0.4982442191	0.4178547653
4	0.2870875322	0.7759545638	0.2767385711
5	0.1523496837	0.3449633244	0.1225870222
6	0.3884813426	0.2398267650	0.4102515861

Cluster membership

------------------------- Cluster=1 ------------------------------------

Obs	birth	death	infant	country	DISTANCE
1	-0.33439	-1.10513	-0.52402	Albania	0.17862
2	-0.62967	-0.52417	-0.63491	Argentin	0.53740
3	-0.43036	-1.08361	-0.82189	Chile	0.18142
4	-0.13508	-1.01906	-0.32399	Columbia	0.42671
5	-0.12770	-1.38485	-0.68709	Venezuel	0.46416
6	-0.06126	-1.51395	-0.84581	Bahrain	0.63514
7	-0.51156	-0.97603	-0.98279	Israel	0.34370
8	-0.17937	-1.85822	-0.85451	Kuwait	0.89881
9	-0.47465	-1.51395	-0.62838	United_A	0.51244
10	-0.59276	-0.88996	-0.49793	China	0.27987
11	-1.29403	-1.27727	-1.06106	Hong_Kon	1.01806
12	0.17496	-1.12665	-0.67187	Malaysia	0.58493
13	-0.84374	-1.21271	-1.03062	Singapor	0.61480
14	-0.58538	-0.99754	-0.77189	Sri_Lank	0.21867
15	-0.51156	-0.67479	-0.58490	Thailand	0.36033

Cluster 2

------------------------- Cluster=2 -------------------------

Obs	birth	death	infant	country	DISTANCE
16	0.97958	0.14285	1.15669	Iran	0.53619
17	0.95744	1.00353	1.39368	Banglade	0.73436
18	0.89838	1.24022	1.63285	Cambodia	1.07380
19	0.76551	0.85291	1.58936	Nepal	0.88866
20	1.42249	0.16437	0.26306	Botswana	0.75211
21	1.24533	0.80988	0.39352	Congo	0.53986
22	0.75074	1.28325	1.04580	Gabon	0.86684
23	1.11984	0.48713	0.76314	Ghana	0.12230
24	1.31177	0.09982	0.37178	Kenya	0.67632
25	1.09031	0.27196	1.74156	Namibia	0.92807
26	1.42249	1.02505	1.08928	Nigeria	0.58894
27	1.13460	1.06808	1.15451	Sudan	0.60039
28	1.29700	0.35802	1.37194	Swazilan	0.56207
29	1.69562	1.02505	1.04580	Uganda	0.73647
30	1.57013	0.68078	1.11103	Tanzania	0.49353
31	1.20842	0.72381	0.61095	Zaire	0.30705
32	1.61442	0.61623	0.54572	Zambia	0.54965

Cluster 3 and 4

--------------------------- Cluster=3 --

Obs	birth	death	infant	country	DISTANCE
33	1.28224	1.54146	1.21974	Bolivia	0.97448
34	0.82456	1.69208	2.75477	Afghanis	1.06314
35	1.32653	2.01483	1.78505	Angola	0.23936
36	1.42988	2.12242	1.78505	Ethiopia	0.21566
37	1.34129	2.27303	1.91550	Gambia	0.09648
38	1.40773	3.04765	1.63285	Malawi	0.91953
39	1.16413	1.64904	1.87202	Mozambiq	0.56353
40	1.40035	2.70337	2.15467	Sierra_L	0.56234
41	1.54060	2.01483	1.67633	Somalia	0.39955

--------------------------- Cluster=4 --

Obs	birth	death	infant	country	DISTANCE
42	-0.01697	2.66034	-0.25876	Mexico	0.61689
43	-0.42297	1.56297	-0.65013	Korea	0.61689

Cluster 5

Obs	birth	death	infant	country	DISTANCE
44	-1.23498	0.22892	-0.88060	Bulgaria	0.33826
45	-1.16854	0.18589	-0.94800	Czechosl	0.28590
46	-1.27189	0.33651	-1.02845	Former_E	0.45403
47	-1.30142	0.55168	-0.87190	Hungary	0.66583
48	-1.10211	-0.13687	-0.84581	Poland	0.12899
49	-1.15378	-0.02928	-0.60882	Romania	0.34023
50	-1.12425	-0.39507	-0.75449	Yugoslav	0.35379
51	-0.85112	-0.17990	-0.69361	USSR	0.42007
52	-1.03567	-0.28748	-0.90886	Byelorus	0.23981
53	-1.16854	0.16437	-0.91104	Ukrainia	0.26595
54	-0.82898	-0.26597	-0.71753	Uruguay	0.44895
55	-1.27189	-0.05080	-1.02193	Belgium	0.13118
56	-1.18331	-0.15838	-1.06759	Finland	0.13997
57	-1.24236	0.22892	-1.03062	Denmark	0.34609
58	-1.15378	-0.30900	-1.03280	France	0.23045
59	-1.31618	0.07830	-1.03280	Germany	0.24157
60	-1.41214	-0.35204	-0.95452	Greece	0.34233
61	-1.04305	-0.37355	-1.03062	Ireland	0.31979
62	-1.44167	-0.37355	-1.00236	Italy	0.38293
63	-1.18331	-0.48114	-1.03932	Netherla	0.39409
64	-1.10211	-0.02928	-1.02410	Norway	0.13492
65	-1.27927	-0.28748	-0.90886	Portugal	0.21422
66	-1.36785	-0.56721	-1.01758	Spain	0.50927
67	-1.08734	0.05679	-1.07193	Sweden	0.22485
68	-1.23498	-0.28748	-1.03932	Switzerl	0.21892
69	-1.15378	0.14285	-1.01105	U.K.	0.25402
70	-1.05781	-0.73934	-1.01975	Austria	0.65632
71	-1.42691	-0.88996	-1.09585	Japan	0.84206
72	-1.08734	-0.76086	-1.03715	Canada	0.67483
73	-0.92494	-0.58872	-0.99584	U.S.A.	0.55498

Cluster 6

Obs	birth	death	infant	country	DISTANCE
74	-0.04650	-0.63176	0.17609	Brazil	0.47528
75	0.27092	-0.73934	0.17609	Ecuador	0.38853
76	-0.06864	-0.76086	0.02389	Guyana	0.63871
77	0.41118	-0.91148	-0.28050	Paraguay	0.86208
78	0.27092	-0.54569	1.19582	Peru	0.75143
79	0.98696	-0.65327	0.30655	Iraq	0.72179
80	0.71383	-0.95451	-0.23702	Jordan	0.93690
81	0.18234	-0.45962	-0.15005	Lebanon	0.61194
82	1.20842	-0.65327	-0.32399	Oman	1.20413
83	0.95005	-0.69631	0.35003	Saudi_Ar	0.69244
84	-0.00221	-0.52417	0.45875	Turkey	0.31280
85	0.09376	-0.13687	0.78489	India	0.51477
86	-0.04650	-0.30900	0.43700	Indonesi	0.38480
87	0.50714	-0.43810	0.28481	Mongolia	0.26224
88	0.07899	-0.58872	1.14799	Pakistan	0.74443
89	0.29307	-0.67479	-0.21527	Philippi	0.69706
90	0.18972	-0.28748	0.19784	Vietnam	0.32897
91	0.46285	-0.54569	0.41526	Algeria	0.18049
92	0.70645	-0.28748	-0.11961	Egypt	0.71885
93	1.09031	-0.30900	0.58920	Libya	0.81294
94	0.46285	-0.22293	0.58920	Morocco	0.32133
95	0.21187	-0.20142	0.37178	South_Af	0.29044
96	0.13805	-0.76086	-0.06308	Tunisia	0.61470
97	0.92053	-0.11535	0.24132	Zimbabwe	0.73814

Summary of clusters

- Cluster 3 has highest means on all variables; describe as “very poor” countries.
- Cluster 2 also higher than average on all, but not as high as Cluster 3:”poor” but not “very poor”.
- Cluster 4 has high death rate but low birth rates and infant mortality rates: “would-be western”.
- Cluster 6 has slightly above-average birth and infant mortality rates, and lower-than-average death rate: “third world”.
- Cluster 1 has lower-than-average everything, and especially low death rate: “becoming western”.
- Cluster 5 also is low on everything, and especially low on birth rate: “western world”.
- New variable “distance” shows how far a country is from its cluster average. Small value means “typical of its cluster”; large implies “does not fit any cluster very well”. Eg. Afghanistan vs. cluster 3.

Using PROC DISCRIM on clusters

- Summary on previous page took some working out.
- Idea: use output clusters as "grouping" variable for PROC DISCRIM with "can" option: get canonical variables that might shed some light on how clusters differ.
- Code below. Add onto end of previous (uses output data set with cluster membership in it):

```
proc discrim can out=zz;
  class cluster;
  var birth death infant;

proc sort;
  by cluster;

proc print;
  var country birth death infant can1 can2 can3;
  by cluster;
```

Output from discriminant analysis

The DISCRIM Procedure

Observations	97	DF Total	96
Variables	3	DF Within Classes	91
Classes	6	DF Between Classes	5

Eigenvalues of Inv(E)*H
= CanRsq/(1-CanRsq)

	Eigenvalue	Difference	Proportion	Cumulative
1	25.2031	20.7304	0.8449	0.8449
2	4.4727	4.3181	0.1499	0.9948
3	0.1546		0.0052	1.0000

Test of H0: The canonical correlations in the
current row and all that follow are zero

	Likelihood Ratio	Approximate F Value	Num DF	Den DF	Pr > F
1	0.00603979	88.02	15	246.09	<.0001
2	0.15826107	34.06	8	180	<.0001
3	0.86611339	4.69	3	91	0.0043

The canonical variables

3 canonical variables possible, all significant (though eigenvalue for last very small).

Raw Canonical Coefficients

Variable	Can1	Can2	Can3
birth	2.706578482	-1.095888137	-1.935532844
death	0.683696755	2.777557832	-0.585485143
infant	2.017039026	-0.834166707	2.334460951

- Can1 positive where birth rate and infant mortality rate both high, negative where both low.
- Can2 positive where death rate high, negative where low.
- Can3 positive where infant mortality rate high compared to birth rate, negative where low.

The clusters

Class Means on Canonical Variables

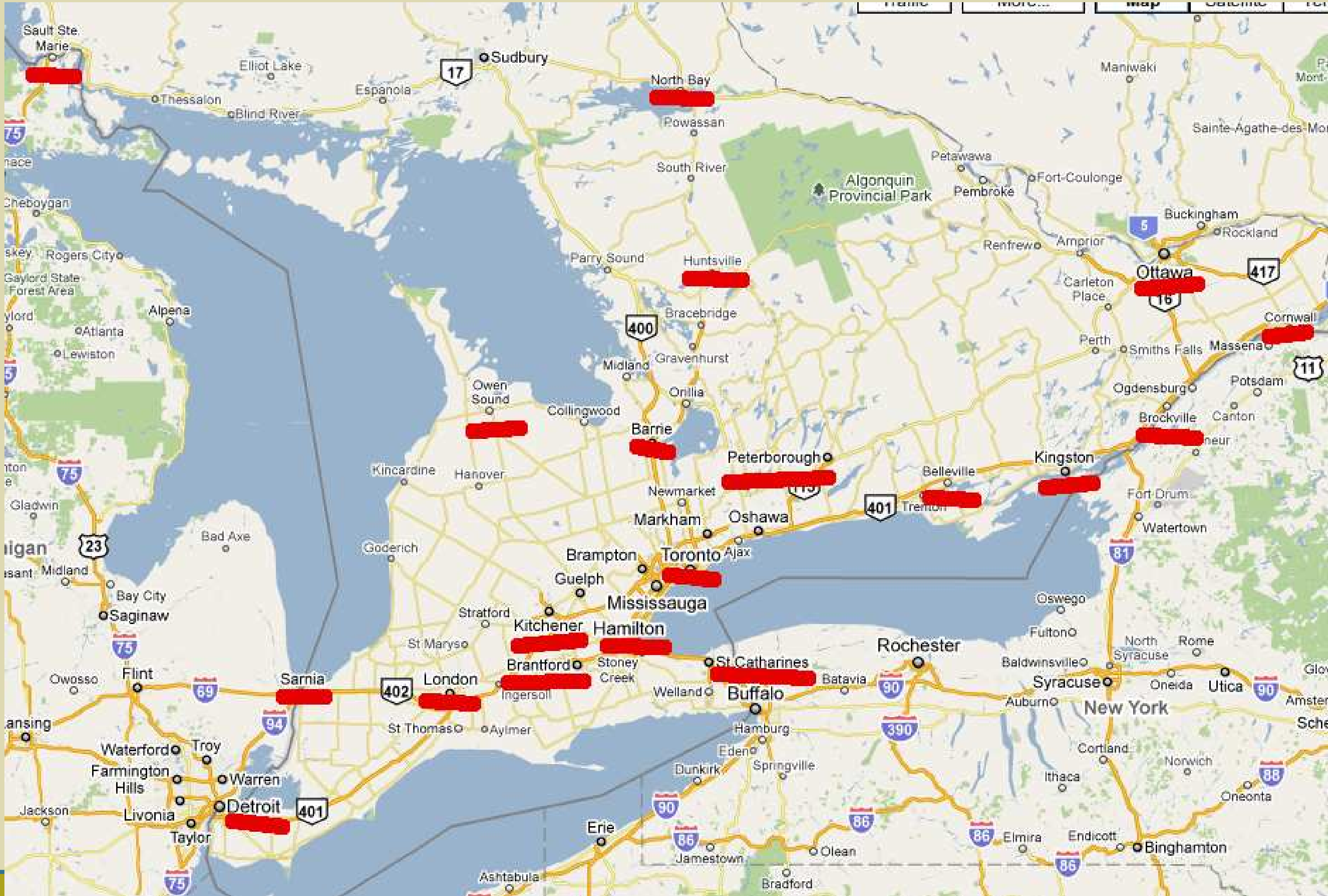
CLUSTER	Can1	Can2	Can3
1	-3.430124271	-2.092217034	-0.186588012
2	5.788318527	-0.231819353	-0.367431160
3	8.735819354	2.898350402	0.596858239
4	-0.068268900	6.485397951	-1.871461306
5	-5.226778630	1.565961865	0.154681580
6	1.306998818	-2.112942540	0.115662540

1. Low on everything (Chile).
2. High birth and infant mortality, average death rate (Ghana).
3. High (or very high) on everything (Gambia).
4. High death rate, high birth rate compared to infant mortality rate (Mexico).
5. Very low birth and infant mortality, highish death rate (Canada).
6. High birth and infant mortality but low death rate (Algeria).

Final example: a hockey league

- An Ontario hockey league has teams in 21 cities. How can we arrange those teams into 4 geographical divisions?
- Distance data in spreadsheet.
- Take out spaces in team names.
- Save as “text/csv”, and use text editor to remove all double-quotes.
- Open new file on Mathlab.
- Copy lines with team names and distances to clipboard, paste into Mathlab file.
- PROC FASTCLUS doesn’t work on distance data, so go back to PROC CLUSTER.

A map


Sault Ste. Marie
Elliot Lake
Thessalon
Blind River
Espanola
Sudbury
North Bay
Powassan
South River
Algonquin Provincial Park
Petawawa
Pembroke
Fort-Coulonge
Maniwaki
Sainte-Agathe-des-Mon
Buckingham
Rockland
Renfrew
Arnprior
Ottawa
Carleton Place
Cornwall
Massena
Perth
Smiths Falls
Potsdam
Ogdensburg
Brockville
Canton
Kingston
Fort Drum
Watertown
Belleville
Trenton
Peterborough
Parry Sound
Huntsville
Bracebridge
Gravenhurst
Midland
Orillia
Owen Sound
Collingwood
Barrie
Newmarket
Markham
Oshawa
Ajax
Toronto
Brampton
Guelph
Mississauga
Kitchener
Hamilton
Stratford
St Marys
Brantford
Stoney Creek
St Catharines
Welland
Buffalo
Batavia
Rochester
Oswego
Fulton
Baldwinsville
North Syracuse
Rome
Syracuse
Oneida
Utica
Auburn
New York
Cortland
Norwich
Ithaca
Oneonta
Endicott
Binghamton
Elmira
Olean
Bradford
Jamestown
Springville
Eden
Hamburg
Dunkirk
Erie
Ashtabula
Kincardine
Hanover
Goderich
London
Ingersoll
St Thomas
Aylmer
Sarnia
Cheboygan
Rogers City
Gaylord State Forest Area
Alpena
Atlanta
Lewiston
Gladwin
Bad Axe
Midland
Bay City
Saginaw
Owosso
Flint
Lansing
Waterford
Troy
Farmington Hills
Warren
Detroit
Livonia
Taylor
Jackson

My code

```
options linesize=75;

data dist(type=distance);
  infile "ontario-road-distances.dat" delimiter=",";
  input team $ Barrie Belleville Brantford Brockville
    Cornwall Hamilton Huntsville Kingston Kitchener
    London NiagaraFalls NorthBay Ottawa OwenSound
    Peterborough Sarnia SaultSteMarie StCatharines
    ThunderBay Toronto Windsor;

proc cluster method=ward outtree=tree;
  id team;

proc tree horizontal lineprinter;
  id team;
```

Use same team names in same order as data file. Hope tree output gives some idea of which teams to place in which divisions.

Clustering history

Cluster History

NCL	--Clusters Joined---		FREQ	Norm RMS Dist	Tie
20	NiagaraF	StCathar	2	0.0339	
19	Brantfor	Hamilton	2	0.0678	T
18	CL19	Kitchene	3	0.0864	
17	Bellevil	Kingston	2	0.1271	
16	CL18	Toronto	4	0.1489	
15	Brockvil	Cornwall	2	0.161	
14	London	Sarnia	2	0.1695	
13	CL16	CL20	6	0.1742	
12	CL15	Ottawa	3	0.1782	
11	Barrie	OwenSoun	2	0.2034	
10	Huntsvil	NorthBay	2	0.2203	
9	CL17	Peterbor	3	0.2497	
8	CL14	Windsor	3	0.2977	
7	CL11	CL13	8	0.3246	
6	CL9	CL12	6	0.3842	
5	CL7	CL8	11	0.4606	
4	CL6	CL10	8	0.6431	
3	CL5	CL4	19	0.7445	
2	SaultSte	ThunderB	2	1.1694	
1	CL3	CL2	21	1.9625	

The tree

```
                         Average Distance Between Clusters

            2    1.8   1.6   1.4   1.2    1    0.8   0.6   0.4   0.2    0
            +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
     Barrie  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX......
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   OwenSoun  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX......
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   Brantfor  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX..
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   Hamilton  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX..
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   Kitchene  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX...
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    Toronto  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX....
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   NiagaraF  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   StCathar  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
     London  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
t    Sarnia  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
e            XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
a   Windsor  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.........
m            XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

The rest

```
          2     1.8   1.6   1.4   1.2    1    0.8   0.6   0.4   0.2    0
          +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
Bellevil  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX....
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
Kingston  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX....
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
Peterbor  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
Brockvil  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
Cornwall  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  Ottawa  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
Huntsvil  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
NorthBay  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.......
          X
SaultSte  XXXXXXXXXXXXXXXXXXXXXXXXX...................................
          XXXXXXXXXXXXXXXXXXXXXXXXX
ThunderB  XXXXXXXXXXXXXXXXXXXXXXXXX...................................
```

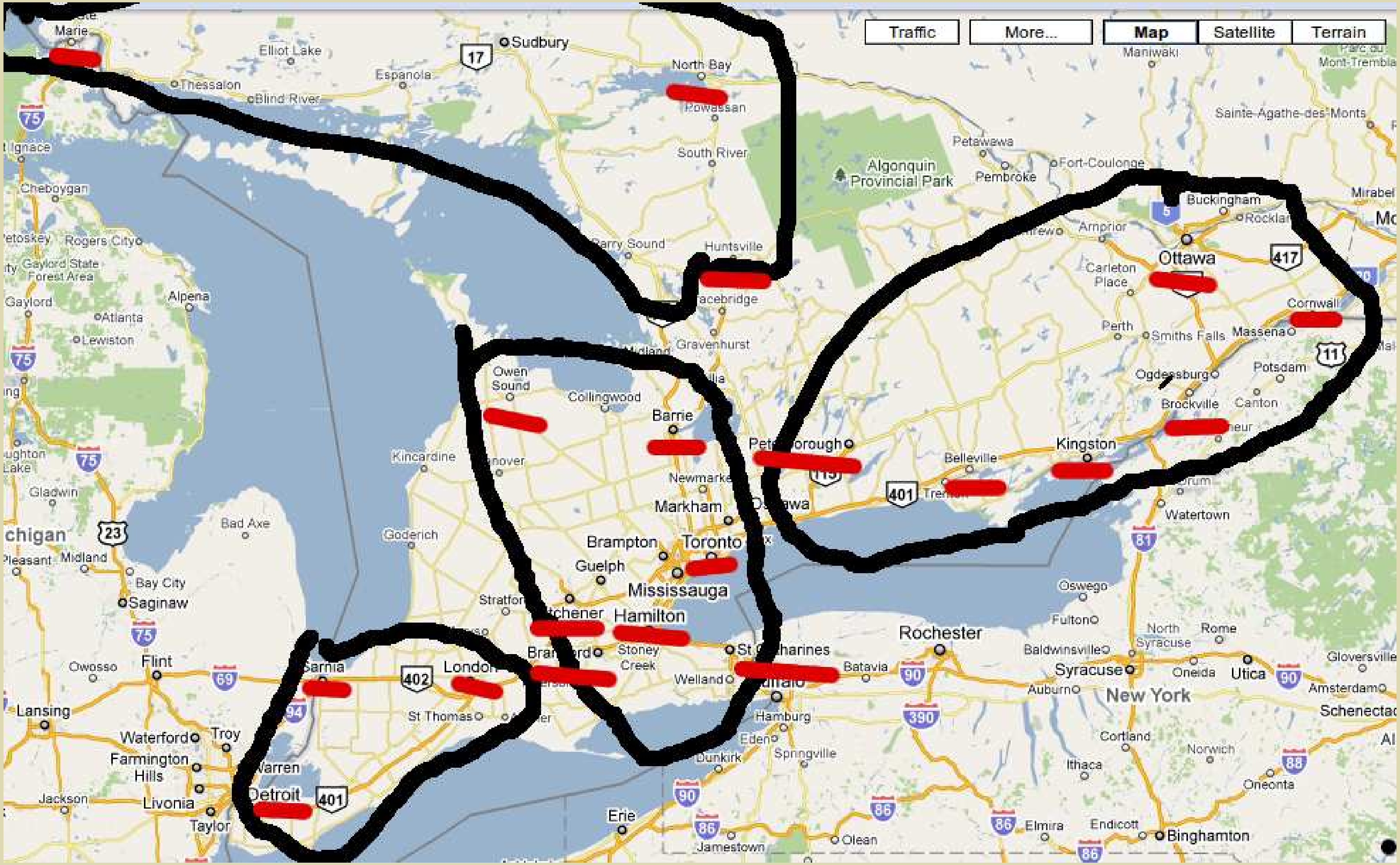
Splitting into divisions, 1st try

- Sault Ste Marie and Thunder Bay are very distant from everywhere else.
- Clustering history says 4 clusters between distance 0.6431 and 0.7445, so “chop tree” there, to get:
  - Sault Ste Marie
  - Thunder Bay
  - Belleville, Kingston, Peterborough, Brockville, Cornwall, Ottawa, Huntsville, North Bay (8 teams)
  - the rest (11 teams)
- Divisions of 1 team make no sense, so try splitting big divisions and placing 2 northernmost teams somewhere.

2nd try

- Next split at distance 0.6431 splits Huntsville and North Bay from the eastern teams. Place them in a northern division with Sault Ste Marie and Thunder Bay.
- Next split at distance 0.4606 splits London, Sarnia and Windsor off from the big group. That leaves us with this:
  - (north, 4) Huntsville, North Bay, Sault Ste Marie, Thunder Bay
  - (east, 6) Belleville, Kingston, Peterborough, Brockville, Cornwall, Ottawa
  - (west, 3) London, Sarnia, Windsor
  - (south, 8) Niagara Falls, St Catharines, Brantford, Hamilton, Kitchener, Toronto, Barrie, Owen Sound
- That's not too bad. Getting the divisions to be the same size is beyond our scope!

Another map


Traffic
More...
Map
Satellite
Terrain
Sudbury
North Bay
Powassan
South River
Elliot Lake
Espanola
Thessalon
Blind River
Marie
Maniwaki
Sainte-Agathe-des-Monts
Petawawa
Fort-Coulonge
Pembroke
Algonquin Provincial Park
Buckingham
Rockland
Arnprior
Ottawa
Carleton Place
Mirabel
Cornwall
Massena
Perth
Smiths Falls
Ogdensburg
Potsdam
Brockville
Canton
Kingston
Belleville
Watertown
Parry Sound
Huntsville
Gravenhurst
Owen Sound
Collingwood
Barrie
Newmarket
Markham
Peterborough
Kincardine
Goderich
Brampton
Toronto
Guelph
Mississauga
Hamilton
Stoney Creek
St Catharines
Welland
Batavia
Rochester
Oswego
Fulton
North Syracuse
Rome
Baldwinsville
Syracuse
Oneida
Utica
Auburn
New York
Gloversville
Amsterdam
Schenectady
Cortland
Norwich
Ithaca
Oneonta
Elmira
Endicott
Binghamton
Hamburg
Eden
Springville
Dunkirk
Jamestown
Olean
Erie
London
St Thomas
Sarnia
Warren
Detroit
Taylor
Livonia
Farmington Hills
Waterford
Troy
Jackson
Lansing
Owosso
Flint
Saginaw
Bay City
Midland
Bad Axe
Gladwin
Gaylord
Atlanta
Lewiston
Alpena
Rogers City
Cheboygan
Gaylord State Forest Area
Elmira
Stratford
Brantford
Kitchener