STAD29: Statistics for the Life and Social Sciences

Lecture notes

Section 1

Principal components

Principal Components

- Have measurements on (possibly large) number of variables on some individuals.
- Question: can we describe data using fewer variables (because original variables correlated in some way)?
- Look for direction (linear combination of original variables) in which values *most spread out*. This is *first principal component*.
- Second principal component then direction uncorrelated with this in which values then most spread out. And so on.

Principal components

- See whether small number of principal components captures most of variation in data.
- Might try to interpret principal components.
- If 2 components good, can make plot of data.
- (Like discriminant analysis, but no groups.)
- "What are important ways that these data vary?"

Packages

You might not have installed the first of these. See over for instructions.

```
library(ggbiplot) # see over
library(tidyverse)
library(ggrepel)
```

Installing ggbiplot

- ggbiplot not on CRAN, so usual install.packages will not work. This is same procedure you used for smmr in C32:
- Install package devtools first (once):

```
install.packages("devtools")
```

- Then install ggbiplot (once):

```
library(devtools)
install_github("vqv/ggbiplot")
```

Small example: 2 test scores for 8 people xxx

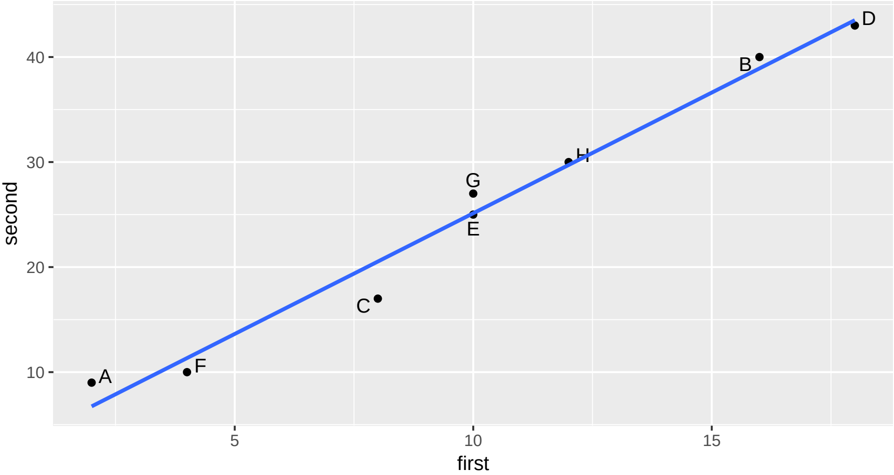
```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/test12.txt"
test12 <- read_table2(my_url)
test12
```

```
## # A tibble: 8 x 3
##   first second id
##   <dbl>  <dbl> <chr>
## 1     2      9 A
## 2    16     40 B
## 3     8     17 C
## 4    18     43 D
## 5    10     25 E
## 6     4     10 F
## 7    10     27 G
## 8    12     30 H
```

```
g <- ggplot(test12, aes(x = first, y = second, label = id)) +
  geom_point() + geom_text_repel()
```

The plot

```
g + geom_smooth(method = "lm", se = F)
```



Principal component analysis

- Grab just the numeric columns:

```
test12 %>% select_if(is.numeric) -> test12_numbers
```

- Strongly correlated, so data nearly 1-dimensional:

```
cor(test12_numbers)
```

```
##            first   second
## first  1.000000 0.989078
## second 0.989078 1.000000
```

Finding principal components

- Make a score summarizing this one dimension. Like this:

```
test12.pc <- princomp(test12_numbers, cor = T)
summary(test12.pc)
```

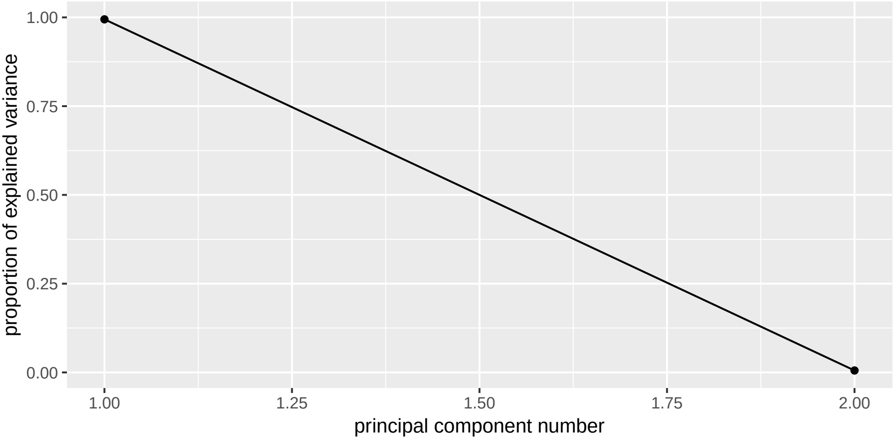
```
## Importance of components:
##                           Comp.1      Comp.2
## Standard deviation     1.410347 0.104508582
## Proportion of Variance 0.994539 0.005461022
## Cumulative Proportion  0.994539 1.000000000
```

Comments

- "Standard deviation" shows relative importance of components (as for LDs in discriminant analysis)
- Here, first one explains almost all (99.4%) of variability.
- That is, look only at first component and ignore second.
- `cor=T` standardizes all variables first. Usually wanted, because variables measured on different scales. (Only omit if variables measured on same scale and expect similar variability.)

Scree plot

```r
ggscreeplot(test12.pc)
```



xxx Component loadings

explain how each principal component depends on (standardized) original variables (test scores):

```
test12.pc$loadings
```

```
##
## Loadings:
##        Comp.1 Comp.2
## first   0.707  0.707
## second  0.707 -0.707
##
##                Comp.1 Comp.2
## SS loadings       1.0    1.0
## Proportion Var    0.5    0.5
## Cumulative Var    0.5    1.0
```

First component basically sum of (standardized) test scores. That is, person tends to score similarly on two tests, and a composite score would summarize performance.

xxx Component scores

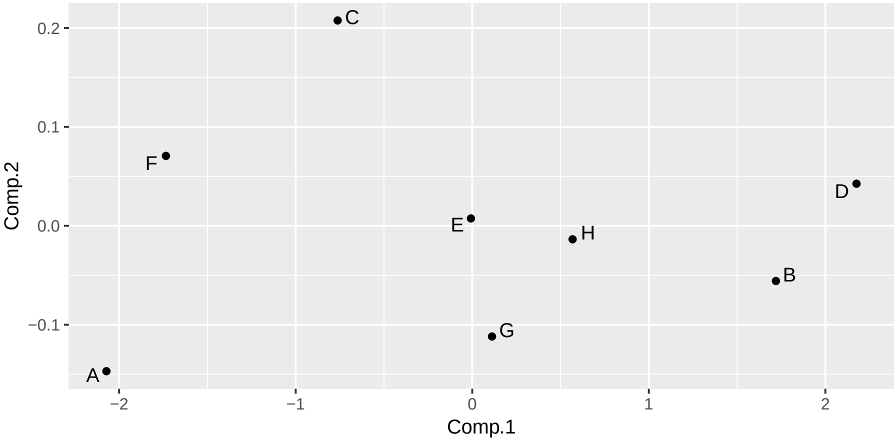
```
d <- data.frame(test12, test12.pc$scores)
d
```

```
##   first second id       Comp.1       Comp.2
## 1     2      9  A -2.071819003 -0.146981782
## 2    16     40  B  1.719862811 -0.055762223
## 3     8     17  C -0.762289708  0.207589512
## 4    18     43  D  2.176267535  0.042533250
## 5    10     25  E -0.007460609  0.007460609
## 6     4     10  F -1.734784030  0.070683441
## 7    10     27  G  0.111909141 -0.111909141
## 8    12     30  H  0.568313864 -0.013613668
```

- Person A is a low scorer, high positive `comp.1` score.
- Person D is high scorer, high negative `comp.1` score.
- Person E average scorer, near-zero `comp.1` score.
- `comp.2` says basically nothing.

Plot of scores

```
ggplot(d, aes(x = Comp.1, y = Comp.2, label = id)) +
  geom_point() + geom_text_repel()
```



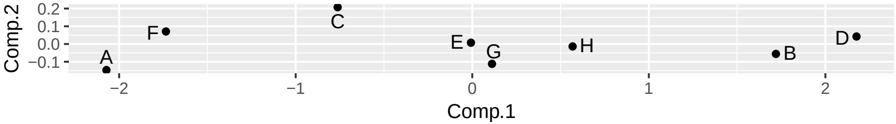
Comments

- Vertical scale exaggerates importance of `comp.2`.
- Fix up to get axes on same scale:

```
g <- ggplot(d, aes(x = Comp.1, y = Comp.2, label = id)) +
  geom_point() + geom_text_repel() +
  coord_fixed()
```

- Shows how exam scores really spread out along one dimension:

```
g
```

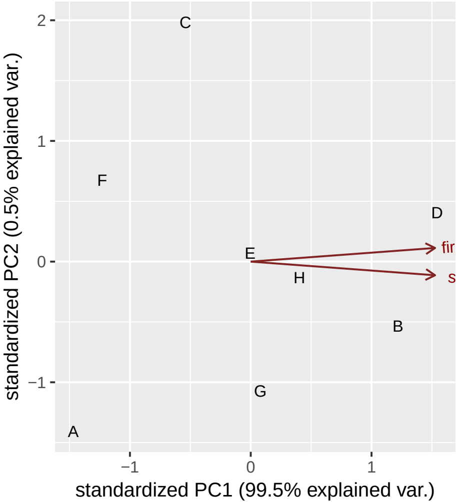


The biplot

- Plotting variables and individuals on one plot.
- Shows how components and original variables related.
- Shows how individuals score on each component, and therefore suggests how they score on each variable.
- Add `labels` option to identify individuals:

```
g <- ggbiplot(test12.pc, labels = test12$id)
```

The biplot


standardized PC2 (0.5% explained var.)
standardized PC1 (99.5% explained var.)
2
1
0
−1
−1
0
1
C
F
D
E
H
B
G
A
fir
s

Comments

- Variables point almost same direction (left). Thus very negative value on `comp.1` goes with high scores on both tests, and test scores highly correlated.
- Position of individuals on plot according to scores on principal components, implies values on original variables. Eg.:
- D very negative on `comp.1`, high scorer on both tests.
- A and F very positive on `comp.1`, poor scorers on both tests.
- C positive on `comp.2`, high score on first test relative to second.
- A negative on `comp.2`, high score on second test relative to first.

xxx Track running data

Track running records (1984) for distances 100m to marathon, arranged by country. Countries labelled by (mostly) Internet domain names (ISO 2-letter codes):

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/men_track_field.txt"
track <- read_table(my_url)
track %>% sample_n(8)
```

```
## # A tibble: 8 x 9
##    m100  m200  m400  m800 m1500 m5000 m10000 marathon country
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>    <dbl> <chr>
## 1  10.8  21.9  49    2.02  4.24  16.3   34.7     162. ws
## 2  10.3  20.7  45.0  1.73  3.6   13.2   27.4     130. be
## 3  10.4  20.6  45.6  1.76  3.58  13.4   28.2     134. cz
## 4  10.3  20.8  45.9  1.79  3.64  13.4   27.7     129. jp
## 5  10.6  21.5  47.8  1.84  3.92  14.7   30.8     149. id
## 6  10.4  20.8  46.8  1.81  3.7   14.0   29.4     138. ar
## 7  10.3  20.1  44.8  1.74  3.57  13.3   27.7     128. au
## 8  10.2  20.6  45.6  1.77  3.61  13.3   27.9     131. se
```

xxx Country names

Also read in a table to look country names up in later:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/isocodes.csv"
iso <- read_csv(my_url)
iso
```

```
## # A tibble: 251 x 4
##    Country        ISO2  ISO3    M49
##    <chr>          <chr> <chr> <dbl>
##  1 <NA>           <NA>  <NA>     NA
##  2 Afghanistan    af    afg       4
##  3 Aland Islands  ax    ala     248
##  4 Albania        al    alb       8
##  5 Algeria        dz    dza      12
##  6 American Samoa as    asm      16
##  7 Andorra        ad    and      20
##  8 Angola         ao    ago      24
##  9 Anguilla       ai    aia     660
## 10 Antarctica     aq    ata      10
## # … with 241 more rows
```

Data and aims

- Times in seconds 100m–400m, in minutes for rest (800m up).
- This taken care of by standardization.
- 8 variables; can we summarize by fewer and gain some insight?
- In particular, if 2 components tell most of story, what do we see in a plot?

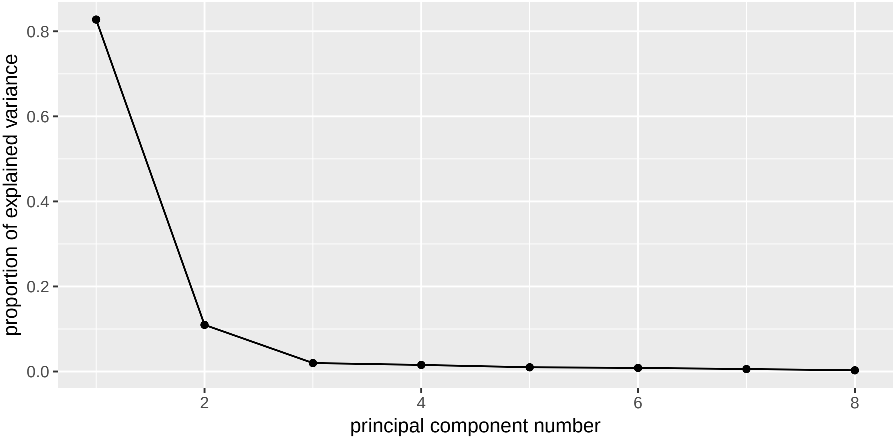
xxx Fit and examine principal components

```
track_num <- track %>% select_if(is.numeric)
track.pc <- princomp(track_num, cor = T)
summary(track.pc)
```

```
## Importance of components:
##                           Comp.1    Comp.2
## Standard deviation     2.5733531 0.9368128
## Proportion of Variance 0.8277683 0.1097023
## Cumulative Proportion  0.8277683 0.9374706
##                            Comp.3     Comp.4
## Standard deviation     0.39915052 0.35220645
## Proportion of Variance 0.01991514 0.01550617
## Cumulative Proportion  0.95738570 0.97289187
##                             Comp.5      Comp.6
## Standard deviation     0.282630981 0.260701267
## Proportion of Variance 0.009985034 0.008495644
## Cumulative Proportion  0.982876903 0.991372547
##                             Comp.7      Comp.8
## Standard deviation     0.215451919 0.150333291
## Proportion of Variance 0.005802441 0.002825012
```

Scree plot

```
ggscreeplot(track.pc)
```



How many components?

- As for discriminant analysis, look for “elbow” in scree plot.
- See one here at 3 components; everything 3 and beyond is “scree”.
- So take 2 components.
- Note difference from discriminant analysis: want “large” rather than “small”, so go 1 step left of elbow.
- Another criterion: any component with eigenvalue bigger than about 1 is worth including. 2nd one here has eigenvalue just less than 1.
- Refer back to `summary`: cumulative proportion of variance explained for 2 components is 93.7%, pleasantly high. 2 components tell almost whole story.

xxx How do components depend on original variables?

Loadings:

```
track.pc$loadings
```

```
##
## Loadings:
##          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## m100      0.318  0.567  0.332  0.128  0.263  0.594  0.136  0.106
## m200      0.337  0.462  0.361 -0.259 -0.154 -0.656 -0.113
## m400      0.356  0.248 -0.560  0.652 -0.218 -0.157
## m800      0.369        -0.532 -0.480  0.540        -0.238
## m1500     0.373 -0.140 -0.153 -0.405 -0.488  0.158  0.610  0.139
## m5000     0.364 -0.312  0.190        -0.254  0.141 -0.591  0.547
## m10000    0.367 -0.307  0.182        -0.133  0.219 -0.177 -0.797
## marathon  0.342 -0.439  0.263  0.300  0.498 -0.315  0.399  0.158
##
##                Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
## SS loadings     1.000  1.000  1.000  1.000  1.000  1.000  1.000
## Proportion Var  0.125  0.125  0.125  0.125  0.125  0.125  0.125
## Cumulative Var  0.125  0.250  0.375  0.500  0.625  0.750  0.875
##                Comp.8
```

xxx Comments

- `comp.1` loads about equally (has equal weight) on times over all distances.
- `comp.2` has large positive loading for short distances, large negative for long ones.
- `comp.3`: large negative for middle distance, large positive especially for short distances.
- Country overall good at running will have lower than average record times at all distances, so `comp.1` *small*. Conversely, for countries bad at running, `comp.1` very positive.
- Countries relatively better at sprinting (low times) will be *negative* on `comp.2`; countries relatively better at distance running *positive* on `comp.2`.

xxx Commands for plots

- Principal component scores (first two). Also need country names.

```
d <- data.frame(track.pc$scores,
  country = track$country
)
names(d)
```

```
## [1] "Comp.1"  "Comp.2"  "Comp.3"  "Comp.4"  "Comp.5"  "Comp
## [7] "Comp.7"  "Comp.8"  "country"
```

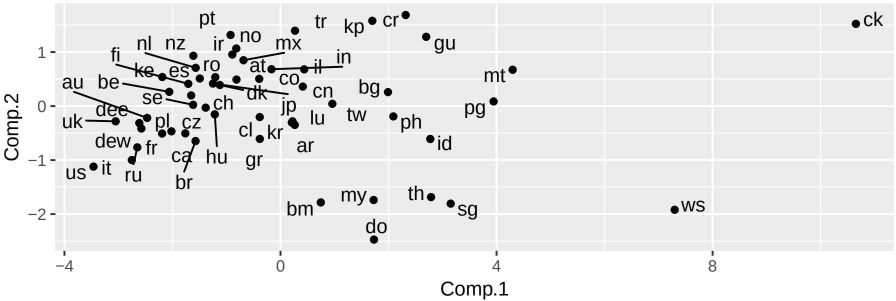
```
g1 <- ggplot(d, aes(x = Comp.1, y = Comp.2,
  label = country)) +
  geom_point() + geom_text_repel() + coord_fixed()
```

- Biplot:

```
g2 <- ggbiplot(track.pc, labels = track$country)
```

xxx Principal components plot

```
g1
```

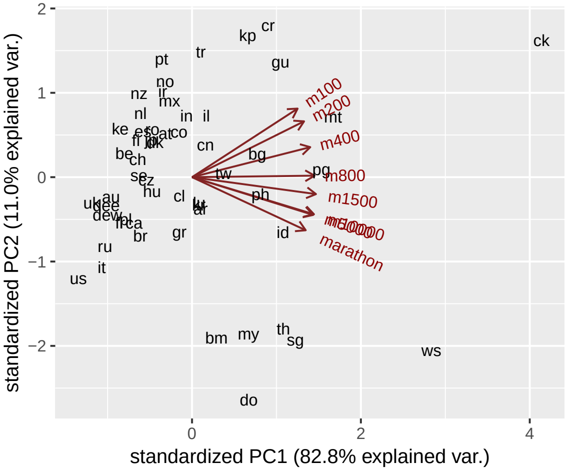


xxx Comments on principal components plot

- Good running countries at left of plot: US, UK, Italy, Russia, East and West Germany.
- Bad running countries at right: Western Samoa, Cook Islands.
- Better sprinting countries at bottom: US, Italy, Russia, Brazil, Greece. `do` is Dominican Republic, where sprinting records relatively good, distance records very bad.
- Better distance-running countries at top: Portugal, Norway, Turkey, Ireland, New Zealand, Mexico. `ke` is Kenya.

xxx Biplot

```
g2
```



xxx Comments on biplot

- Had to do some pre-work to interpret PC plot. Biplot more self-contained.
- All variable arrows point right; countries on right have large (bad) record times overall, countries on left good overall.
- Imagine that variable arrows extend negatively as well. Bottom right = bad at distance running, top left = good at distance running.
- Top right = bad at sprinting, bottom left = good at sprinting.
- Doesn't require so much pre-interpretation of components.

xxx Best 8 running countries

Need to look up two-letter abbreviations in ISO table:

xxx

```
d %>%
  arrange(Comp.1) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, country, Country) %>%
  slice(1:8)
```

```
##      Comp.1 country                  Country
## 1 -3.462175      us United States of America
## 2 -3.052104      uk           United Kingdom
## 3 -2.752084      it                    Italy
## 4 -2.651062      ru       Russian Federation
## 5 -2.613964     dee             East Germany
## 6 -2.576272     dew             West Germany
## 7 -2.468919      au                Australia
## 8 -2.191917      fr                   France
```

xxx Worst 8 running countries

```
d %>%
  arrange(desc(Comp.1)) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, country, Country) %>%
  slice(1:8)
```

```
##      Comp.1 country          Country
## 1 10.652914      ck     Cook Islands
## 2  7.297865      ws            Samoa
## 3  4.297909      mt            Malta
## 4  3.945224      pg Papua New Guinea
## 5  3.150886      sg        Singapore
## 6  2.787273      th         Thailand
## 7  2.773125      id        Indonesia
## 8  2.697066      gu             Guam
```

xxx Better at distance running

```
d %>%
  arrange(desc(Comp.2)) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.2, country, Country) %>%
  slice(1:10)
```

```
##        Comp.2 country                    Country
## 1   1.6860391      cr                 Costa Rica
## 2   1.5791490      kp              Korea (North)
## 3   1.5226742      ck               Cook Islands
## 4   1.3957839      tr                     Turkey
## 5   1.3167578      pt                   Portugal
## 6   1.2829272      gu                       Guam
## 7   1.0663756      no                     Norway
## 8   0.9547437      ir Iran, Islamic Republic of
## 9   0.9318729      nz                New Zealand
## 10  0.8495104      mx                     Mexico
```

xxx Better at sprinting

```
d %>%
  arrange(Comp.2) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.2, country, Country) %>%
  slice(1:10)
```

```
##        Comp.2 country                  Country
## 1  -2.4715736      do       Dominican Republic
## 2  -1.9196130      ws                    Samoa
## 3  -1.8055052      sg                Singapore
## 4  -1.7832229      bm                  Bermuda
## 5  -1.7386063      my                 Malaysia
## 6  -1.6851772      th                 Thailand
## 7  -1.1204235      us United States of America
## 8  -0.9989821      it                    Italy
## 9  -0.7639385      ru       Russian Federation
## 10 -0.6470634      br                   Brazil
```

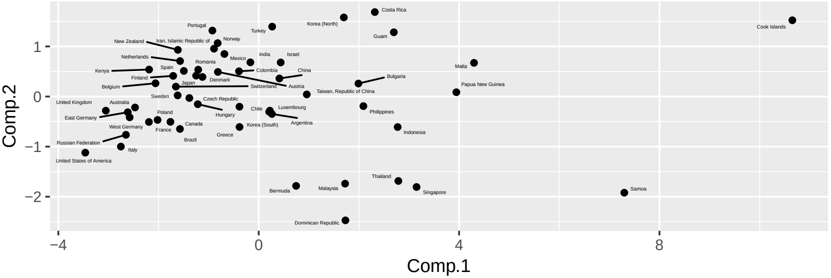
Plot with country names

```
g <- d %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, Comp.2, Country) %>%
  ggplot(aes(x = Comp.1, y = Comp.2, label = Country)) +
  geom_point() + geom_text_repel(size = 1) +
  coord_fixed()
```

```
## Warning: Column `country`/`ISO2` joining factor and charact
## vector, coercing into character vector
```

The plot

```
g
```


Costa Rica
Korea (North)
Cook Islands
Portugal
Turkey
Guam
New Zealand
Iran, Islamic Republic of
Norway
Netherlands
India
Israel
Mexico
Romania
Malta
Kenya
Spain
Colombia
China
Finland
Denmark
Bulgaria
Japan
Belgium
Switzerland
Austria
Papua New Guinea
Taiwan, Republic of China
Sweden
Czech Republic
United Kingdom
Australia
Chile
Luxembourg
Philippines
Poland
Hungary
East Germany
Argentina
Canada
West Germany
Korea (South)
France
Indonesia
Greece
Brazil
Russian Federation
Italy
United States of America
Thailand
Samoa
Bermuda
Malaysia
Singapore
Dominican Republic
Comp.2
Comp.1
1
0
−1
−2
−4
0
4
8

xxx Principal components from correlation matrix

Create data file like this:

```
 1         0.9705 -0.9600
 0.9705    1      -0.9980
-0.9600   -0.9980  1
```

and read in like this:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/cov.txt"
mat <- read_table(my_url, col_names = F)
mat
```

```
## # A tibble: 3 x 3
##       X1     X2     X3
##    <dbl>  <dbl>  <dbl>
## 1  1      0.970 -0.96
## 2  0.970  1     -0.998
## 3 -0.96  -0.998  1
```

Pre-processing

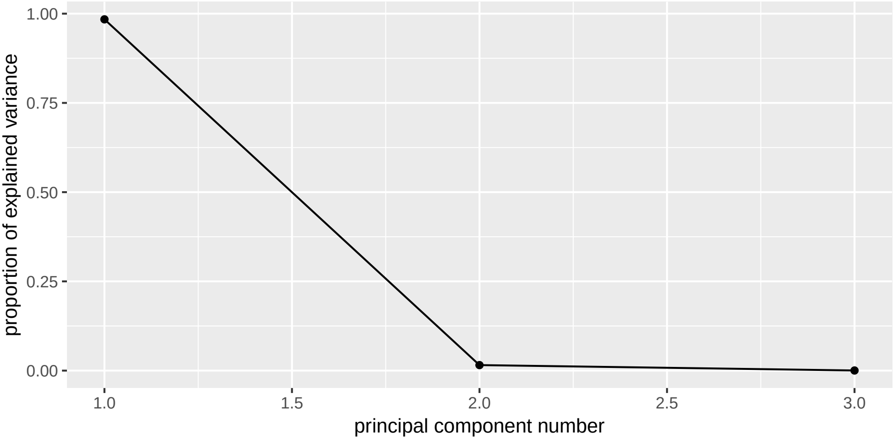
A little pre-processing required:

- Turn into matrix (from data frame)
- Feed into `princomp` as `covmat=`

```
mat.pc <- mat %>%
  as.matrix() %>%
  princomp(covmat = .)
```

Scree plot: one component fine

```
ggscreeplot(mat.pc)
```



xxx Component loadings

Compare correlation matrix:

```
mat
```

```
## # A tibble: 3 x 3
##       X1     X2     X3
##    <dbl>  <dbl>  <dbl>
## 1  1      0.970 -0.96
## 2  0.970  1     -0.998
## 3 -0.96  -0.998  1
```

with component loadings

```
mat.pc$loadings
```

```
##
## Loadings:
##    Comp.1 Comp.2 Comp.3
## X1  0.573  0.812  0.112
## X2  0.581 -0.306 -0.755
## X3 -0.578  0.498 -0.646
##
##                Comp.1 Comp.2 Comp.3
## SS loadings     1.000  1.000  1.000
```

Comments xxx for sign

- When X1 large, X2 also large, X3 small.
- Then `comp.1` *negative*.
- When X1 small, X2 small, X3 large.
- Then `comp.1` *positive*.

xxx No scores

- With correlation matrix rather than data, no component scores
- So no principal component plot
- and no biplot.