STAD29: Statistics for the Life and Social Sciences

Lecture notes

Section 1

Time Series

Packages

Uses my package `mkac` which is on Github. Install with:

```
library(devtools)
install_github("nxskok/mkac")
```

plus these, which you might need to install first:

```
library(ggfortify)
library(forecast)
library(tidyverse)
library(mkac)
```

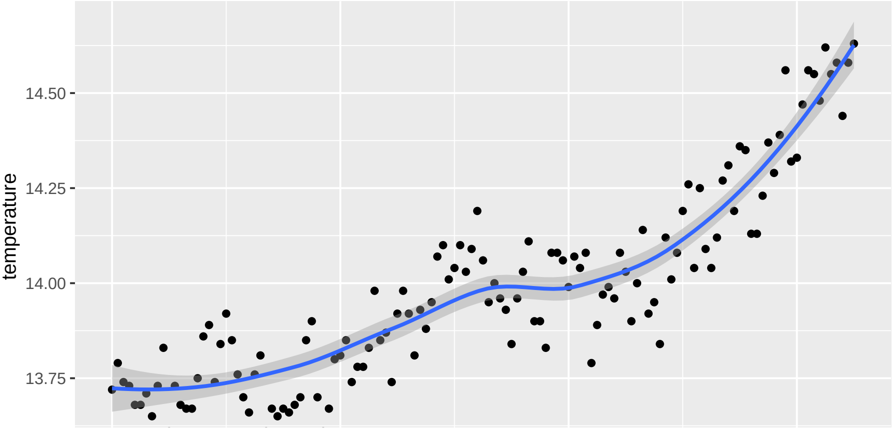
Time trends xxx

- Assess existence or nature of time trends with:
  - correlation
  - regression ideas.
  - (later) time series analysis

World mean temperatures

Global mean temperature every year since 1880: xxx

```
temp=read_csv("temperature.csv")
ggplot(temp, aes(x=year, y=temperature)) + geom_point() + geom
```



Examining trend

- Temperatures increasing on average over time, but pattern very irregular.
- Find (Pearson) correlation with time, and test for significance:

```
with(temp, cor.test(temperature,year))
```

```
##
##  Pearson's product-moment correlation
##
## data:  temperature and year
## t = 19.996, df = 129, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.8203548 0.9059362
## sample estimates:
##       cor
## 0.8695276
```

Comments xxx

- Correlation, 0.8695, significantly different from zero.
- CI shows how far from zero it is.

Tests for *linear* trend with *normal* data.

Kendall correlation

Alternative, Kendall (rank) correlation, which just tests for monotone trend (anything upward, anything downward) and is resistant to outliers:

```
with(temp, cor.test(temperature,year,method="kendall"))
```

```
##
##  Kendall's rank correlation tau
##
## data:  temperature and year
## z = 11.776, p-value < 2.2e-16
## alternative hypothesis: true tau is not equal to 0
## sample estimates:
##       tau
## 0.6992574
```

Kendall correlation usually closer to 0 for same data, but here P-values comparable. Trend again strongly significant.

Mann-Kendall

- Another way is via **Mann-Kendall**: Kendall correlation with time.
- Use my package `mkac`:

```
kendall_Z_adjusted(temp$temperature)
```

```
## $z
## [1] 11.77267
##
## $z_star
## [1] 4.475666
##
## $ratio
## [1] 6.918858
##
## $P_value
## [1] 0
##
## $P_value_adj
## [1] 7.617357e-06
```

Comments

P-value is very small, but adjusted one not as small as before because of *autocorrelation* (see later). Idea: observations close together in time are correlated with each other, so observations not independent. This is correction for that.

Examining rate of change

- Having seen that there *is* a change, question is “how fast is it?”
- Examine slopes:
    - regular regression slope, if you believe straight-line regression
    - Theil-Sen slope: resistant to outliers, based on medians

Ordinary regression against time xxx

```
temp.lm=lm(temperature~year, data=temp)
summary(temp.lm)
```

```
##
## Call:
## lm(formula = temperature ~ year, data = temp)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.32496 -0.10117  0.00575  0.08355  0.28501
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.5794197  0.5703984   4.522 1.37e-05
## year        0.0058631  0.0002932  19.996  < 2e-16
##
## (Intercept) ***
```

Theil-Sen slope

also from `mkac`:

```
theil_sen_slope(temp$temperature)
```

```
## [1] 0.005675676
```

Conclusions

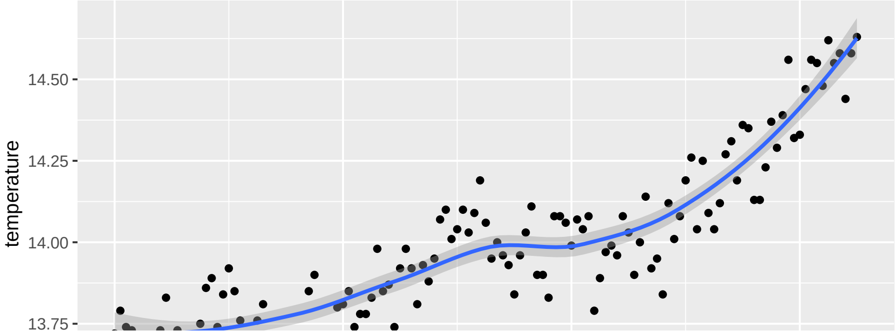
- Slopes:
  - Linear regression: 0.005863
  - Theil-Sen slope: 0.005676
  - Very close.
- Correlations:
  - Pearson 0.8675
  - Kendall 0.6993
  - Kendall correlation smaller, but P-value equally significant (often the case)

Constant rate of change? xxx

Slope assumes that the rate of change is same over all years, but trend seemed to be accelerating: xxx

```
ggplot(temp, aes(x=year, y=temperature)) +
  geom_point() + geom_smooth()
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



Pre-1970 and post-1970: xxx

```
temp %>%
  mutate(time_period=ifelse(year<=1970, "pre-1970", "post-1970
  nest(-time_period) %>%
  mutate(theil_sen=map_dbl(
    data, ~theil_sen_slope(.$temperature)))
```

```
## # A tibble: 2 x 3
##   time_period data               theil_sen
##   <chr>       <list>                 <dbl>
## 1 pre-1970    <tibble [91 × 4]>    0.00429
## 2 post-1970   <tibble [40 × 4]>    0.0168
```

Theil-Sen slope is very nearly *four times* as big since 1970 vs. before.

Actual time series: the Kings of England

- Age at death of Kings and Queens of England since William the Conqueror (1066):

```
kings=read_table("kings.txt", col_names=F)
```

```
## Parsed with column specification:
## cols(
##   X1 = col_double()
## )
```

Data in one long column `X1`, so `kings` is data frame with one column.

Turn into ts time series object

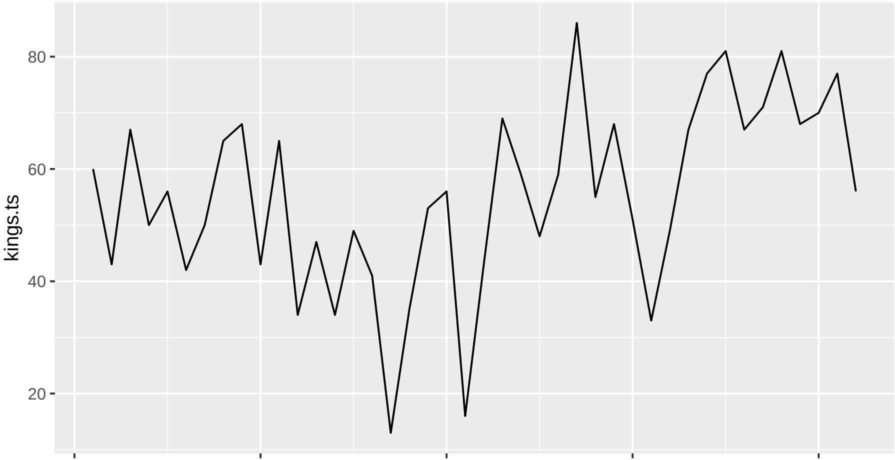
```
kings.ts=ts(kings)
kings.ts
```

```
## Time Series:
## Start = 1
## End = 42
## Frequency = 1
##       X1
##  [1,] 60
##  [2,] 43
##  [3,] 67
##  [4,] 50
##  [5,] 56
##  [6,] 42
##  [7,] 50
##  [8,] 65
```

Plotting a time series

`autoplot` from `ggfortify` gives time plot:

```
autoplot(kings.ts)
```



Comments

- "Time" here is order of monarch from William the Conqueror (1st) to George VI (last).
- Looks to be slightly increasing trend of age-at-death
- but lots of irregularity.

Stationarity

A time series is **stationary** if:

- mean is constant over time
- variability constant over time and not changing with mean.

Kings time series seems to have:

- non-constant mean
- but constant variability
- not stationary.

Getting it stationary

- Usual fix for non-stationarity is *differencing*: new series 2nd - 1st, 3rd - 2nd etc.

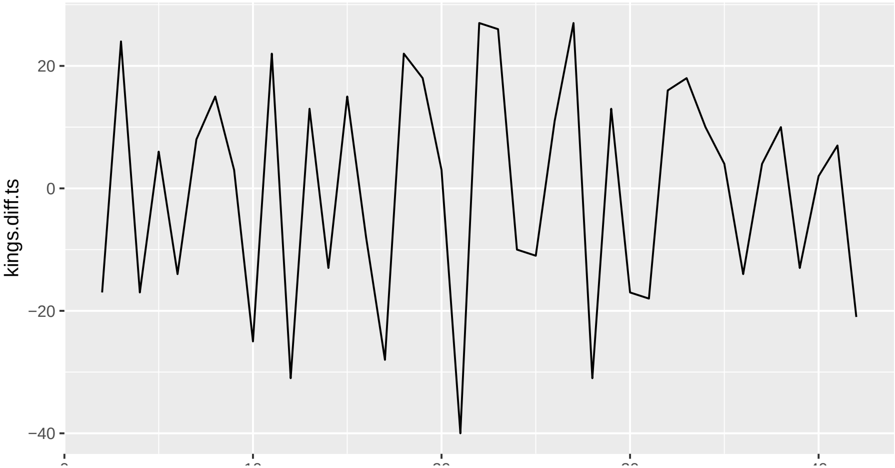
In R, `diff`:

```
kings.diff.ts=diff(kings.ts)
```

Did differencing fix stationarity?

Looks stationary now: xxx

```
autoplot(kings.diff.ts)
```



Births per month in New York City

from January 1946 to December 1959:

```
ny=read_table("nybirths.txt",col_names=F)
```

```
## Parsed with column specification:
## cols(
##   X1 = col_double()
## )
```

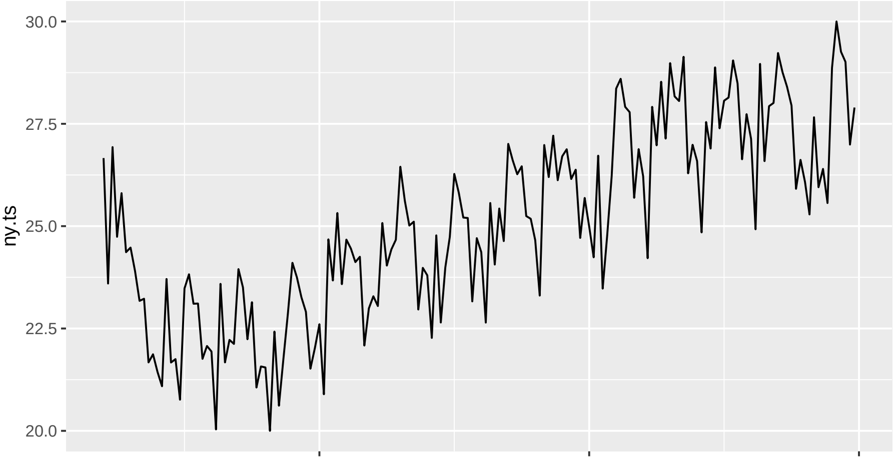
```
ny
```

```
## # A tibble: 168 x 1
##        X1
##     <dbl>
##  1  26.7
##  2  23.6
##  3  26.9
##  4  24.7
```

Time plot

- Time plot shows extra pattern: xxx

```
autoplot(ny.ts)
```



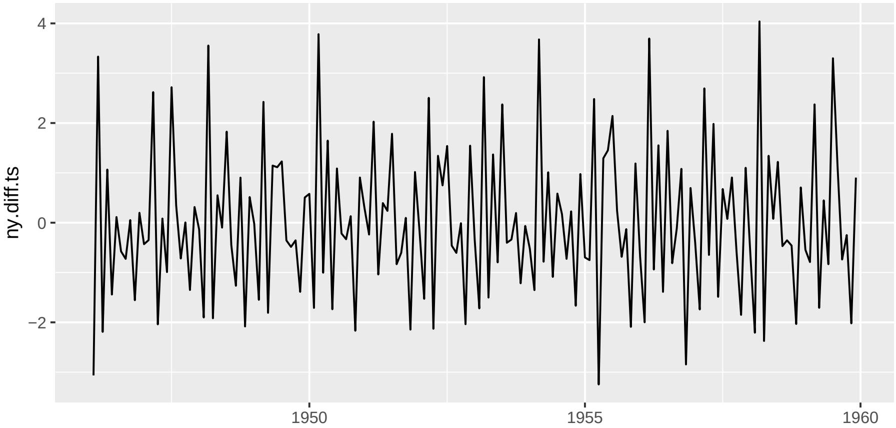
Comments on time plot

- steady increase (after initial drop)
- repeating pattern each year (seasonal component).
- Not stationary.

Differencing the New York births

Does differencing help here? xxx

```
ny.diff.ts=diff(ny.ts)
autoplot(ny.diff.ts)
```

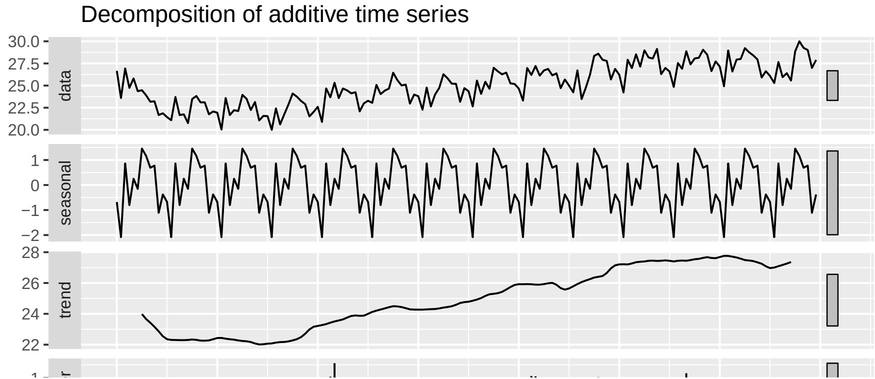


Decomposing a seasonal time series

Observations for NY births were every month. Are things the same every year?

A visual (using original data): xxx

```r
decompose(ny.ts) %>% autoplot()
```



Decomposition bits

Shows:

- original series
- a "seasonal" part: something that repeats every year
- just the trend, going steadily up (except at the start)
- random: what is left over ("remainder")

The seasonal part

Fitted seasonal part is same every year, births lowest in February and highest in July:

```
ny.d$seasonal
```

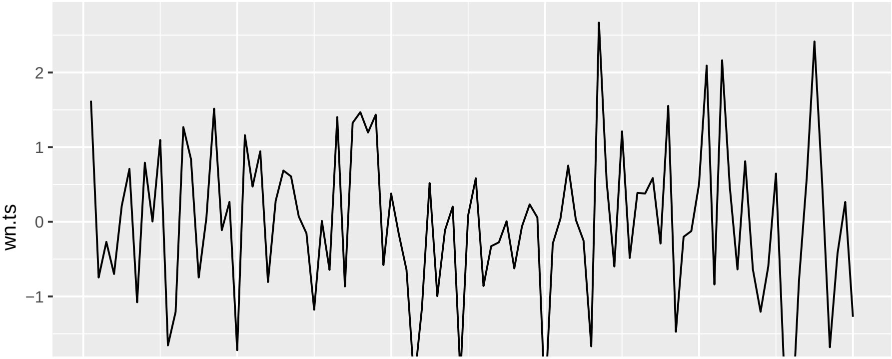
```
## Error in eval(expr, envir, enclos): object 'ny.d' not founc
```

Time series basics

White noise

Independent random normal. Knowing one value tells you nothing about the next. "Random" process. xxx

```
wn=rnorm(100)
wn.ts=ts(wn)
autoplot(wn.ts)
```



Lagging a time series

This means moving a time series one (or more) steps back in time:

```
x=rnorm(5)
tibble(x) %>% mutate(x_lagged=lag(x)) -> with_lagged
with_lagged
```

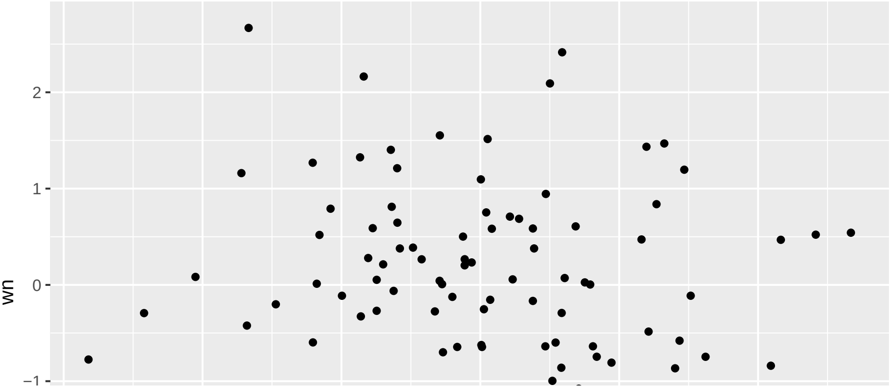
```
## # A tibble: 5 x 2
##          x x_lagged
##      <dbl>    <dbl>
## 1 -2.04      NA
## 2 -0.579     -2.04
## 3  0.608     -0.579
## 4  0.118      0.608
## 5  0.0563     0.118
```

Gain a missing because there is nothing before the first observation.

Lagging white noise

```
tibble(wn) %>% mutate(wn_lagged=lag(wn)) -> wn_with_lagged
ggplot(wn_with_lagged, aes(y=wn, x=wn_lagged))+geom_point()
```

```
## Warning: Removed 1 rows containing missing values
## (geom_point).
```



Correlation with lagged series

If you know about white noise at one time point, you know *nothing* about it at the next. This is shown by the scatterplot and the correlation.

On the other hand, this:

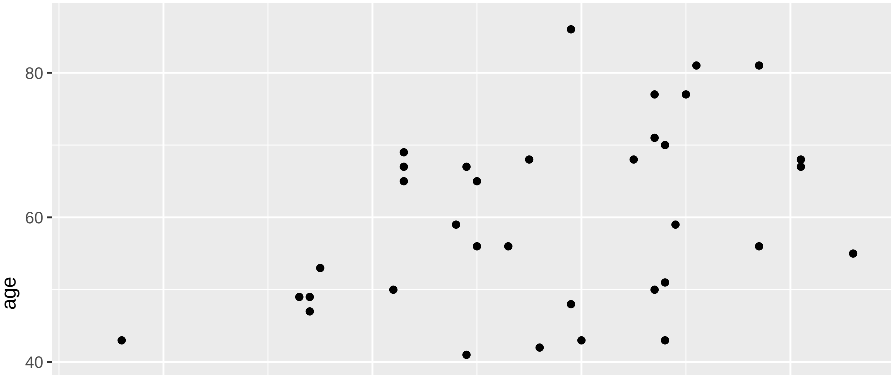
```
tibble(age=kings$X1) %>%
  mutate(age_lagged=lag(age)) -> kings_with_lagged
with(kings_with_lagged, cor.test(age, age_lagged))
```

```
##
##  Pearson's product-moment correlation
##
## data:  age and age_lagged
## t = 2.7336, df = 39, p-value = 0.00937
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.1064770 0.6308209
```

If one value larger, the next value (a bit) more likely to be larger:

```
ggplot(kings_with_lagged, aes(x=age_lagged, y=age)) + geom_poi
```

```
## Warning: Removed 1 rows containing missing values
## (geom_point).
```



Two steps back:

```
kings_with_lagged %>%
  mutate(age_lag_2=lag(age_lagged)) %>%
  with(., cor.test(age, age_lag_2))
```

```
##
##  Pearson's product-moment correlation
##
## data:  age and age_lag_2
## t = 1.5623, df = 38, p-value = 0.1265
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.07128917  0.51757510
## sample estimates:
##      cor
## 0.245676
```

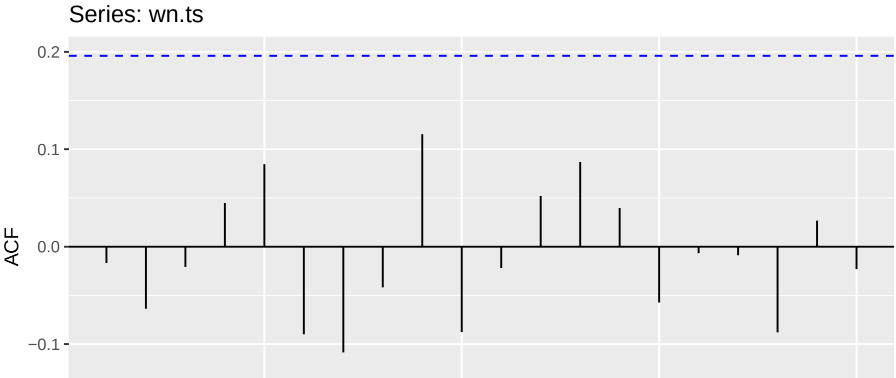
Still a correlation two steps back, but smaller (and no longer significant).

Autocorrelation

Correlation of time series with *itself* one, two,... time steps back is useful idea, called **autocorrelation**. Make a plot of it with `acf` and `autoplot`:

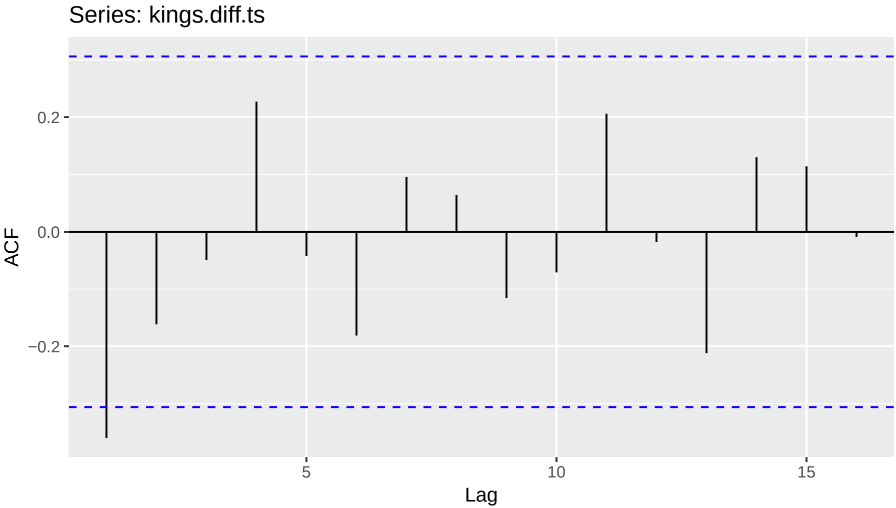
White noise: xxx

```
acf(wn.ts, plot=F) %>% autoplot()
```



Kings, differenced xxx

```
acf(kings.diff.ts, plot=F) %>% autoplot()
```



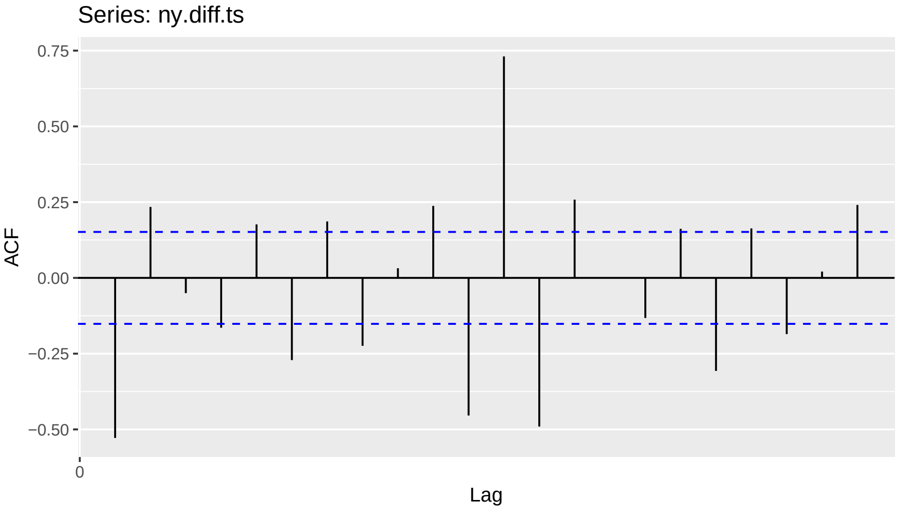
Comments on autocorrelations of kings series

Negative autocorrelation at lag 1, nothing beyond that.

- If one value of differenced series positive, next one most likely negative.
- If one king lives longer than predecessor, next one likely lives shorter.

NY births, differenced xxx

```
acf(ny.diff.ts, plot=F) %>% autoplot()
```



Lots of stuff:

- large positive autocorrelation at 1.0 years (July one year like July last year)
- large negative autocorrelation at 1 month.
- smallish but significant negative autocorrelation at 0.5 year $= 6$ months.
- Other stuff – complicated.

Souvenir sales

Monthly sales for a beach souvenir shop in Queensland, Australia:

```
souv=read_table("souvenir.txt", col_names=F)
```

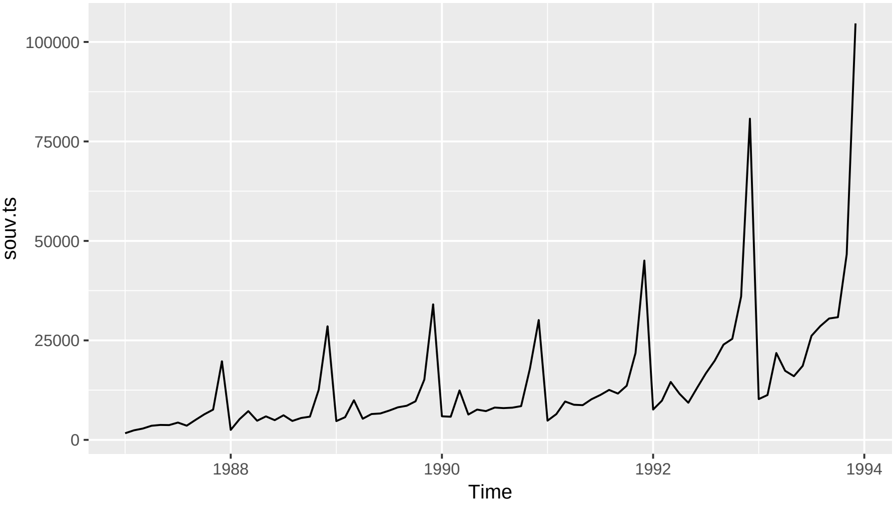
```
## Parsed with column specification:
## cols(
##   X1 = col_double()
## )
```

```
souv.ts=ts(souv,frequency=12,start=1987)
souv.ts
```

```
##              Jan       Feb       Mar       Apr
## 1987     1664.81   2397.53   2840.71   3547.29
## 1988     2499.81   5198.24   7225.14   4806.03
## 1989     4717.02   5702.63   9957.58   5304.78
## 1990     5921.10   5814.58  12421.25   6369.77
## 1991     4826.64   6470.23   9638.77   8821.17
```

Plot of souvenir sales xxx

```r
autoplot(souv.ts)
```



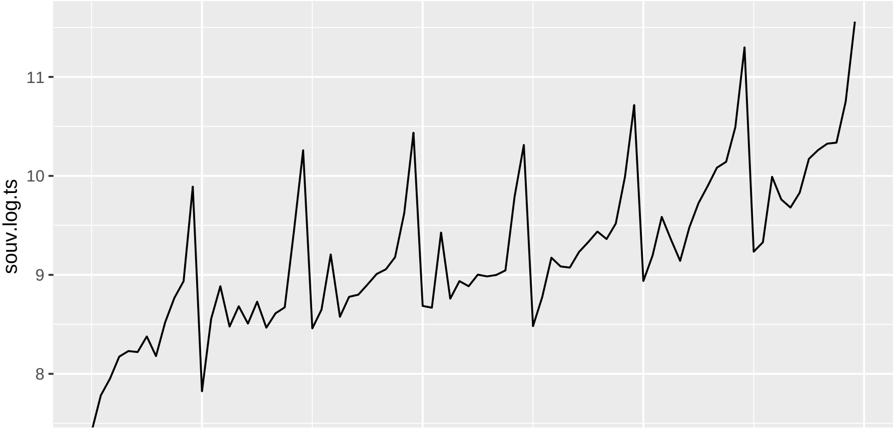
Several problems:

- Mean goes up over time
- Variability gets larger as mean gets larger
- Not stationary

Problem-fixing:

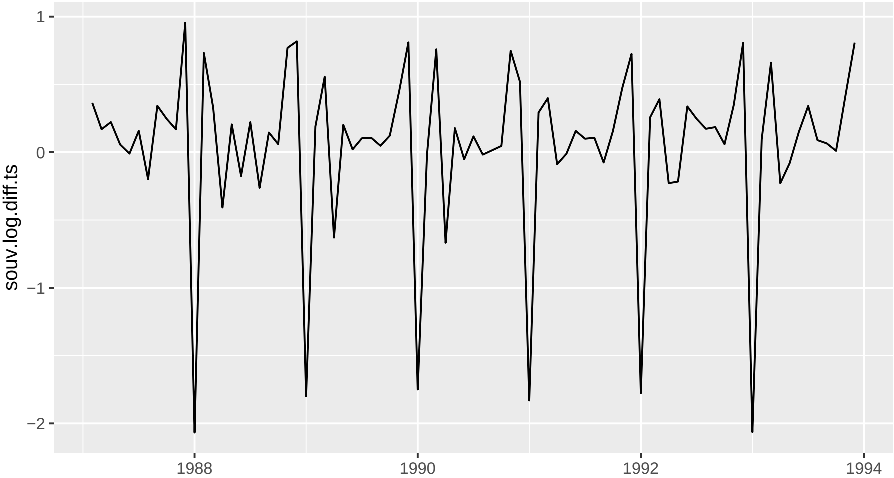
Fix non-constant variability first by taking logs: xxx

```
souv.log.ts=log(souv.ts)
autoplot(souv.log.ts)
```



Mean still not constant, so try taking differences xxx

```
souv.log.diff.ts=diff(souv.log.ts)
autoplot(souv.log.diff.ts)
```

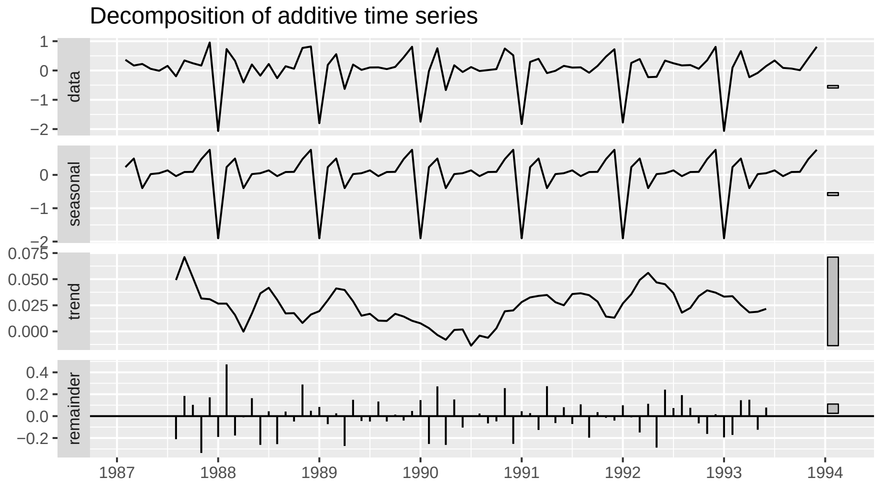


Comments

- Now stationary
- but clear seasonal effect.

Decomposing to see the seasonal effect xxx

```
souv.d=decompose(souv.log.diff.ts)
autoplot(souv.d)
```



Comments

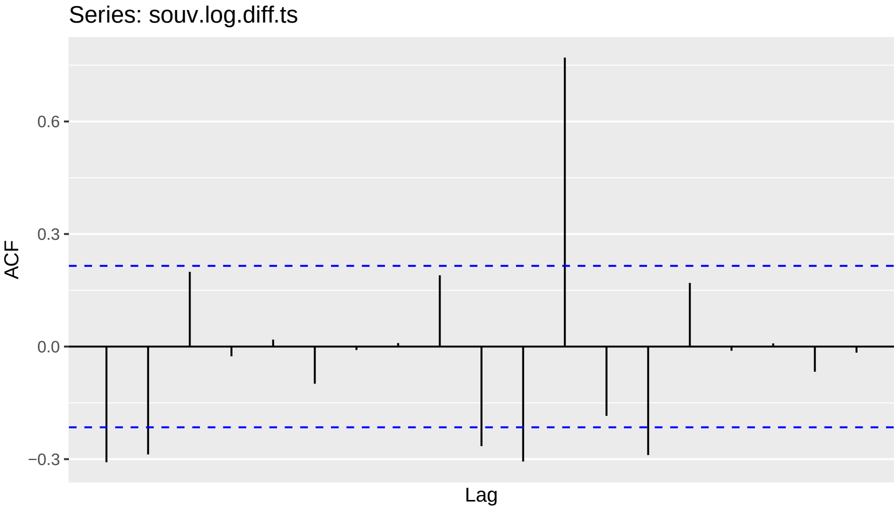
Big drop in one month's differences. Look at seasonal component to see which:

```
souv.d$seasonal
```

```
##              Jan         Feb         Mar
## 1987              0.23293343  0.49068755
## 1988 -1.90372141  0.23293343  0.49068755
## 1989 -1.90372141  0.23293343  0.49068755
## 1990 -1.90372141  0.23293343  0.49068755
## 1991 -1.90372141  0.23293343  0.49068755
## 1992 -1.90372141  0.23293343  0.49068755
## 1993 -1.90372141  0.23293343  0.49068755
##              Apr         May         Jun
## 1987 -0.39700942  0.02410429  0.05074206
## 1988 -0.39700942  0.02410429  0.05074206
## 1989 -0.39700942  0.02410429  0.05074206
```

Autocorrelations xxx

```
acf(souv.log.diff.ts, plot=F) %>% autoplot()
```



Moving average

- A particular type of time series called a **moving average** or MA process captures idea of autocorrelations at a few lags but not at others.
- Here's generation of MA(1) process, with autocorrelation at lag 1 but not otherwise:

```
beta=1
tibble(e=rnorm(100)) %>%
  mutate(e_lag=lag(e)) %>%
  mutate(y=e+beta*e_lag) %>%
  mutate(y=ifelse(is.na(y), 0, y)) -> ma
ma
```

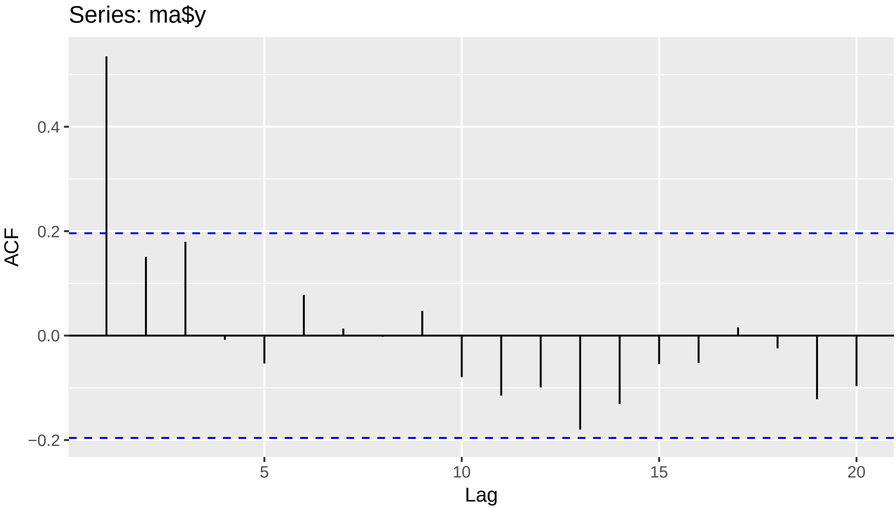
```
## # A tibble: 100 x 3
##          e   e_lag      y
##      <dbl>   <dbl>  <dbl>
##   1  0.991  NA       0
```

Comments

- e contains independent "random shocks".
- Start process at 0.
- Then, each value of the time series has that time's random shock, plus a multiple of the last time's random shock.
- `y[i]` has shock in common with `y[i-1]`; should be a lag 1 autocorrelation.
- But `y[i]` has no shock in common with `y[i-2]`, so no lag 2 autocorrelation (or beyond).

ACF for MA(1) process xxx

```
acf(ma$y, plot=F, na.rm=T) %>% autoplot()
```



AR process

Another kind of time series is AR process, where each value depends on previous one, like this (loop):

```
e=rnorm(100)
x=numeric(0)
x[1]=0
alpha=0.7
for (i in 2:100)
{
  x[i]=alpha*x[i-1]+e[i]
}
x
```

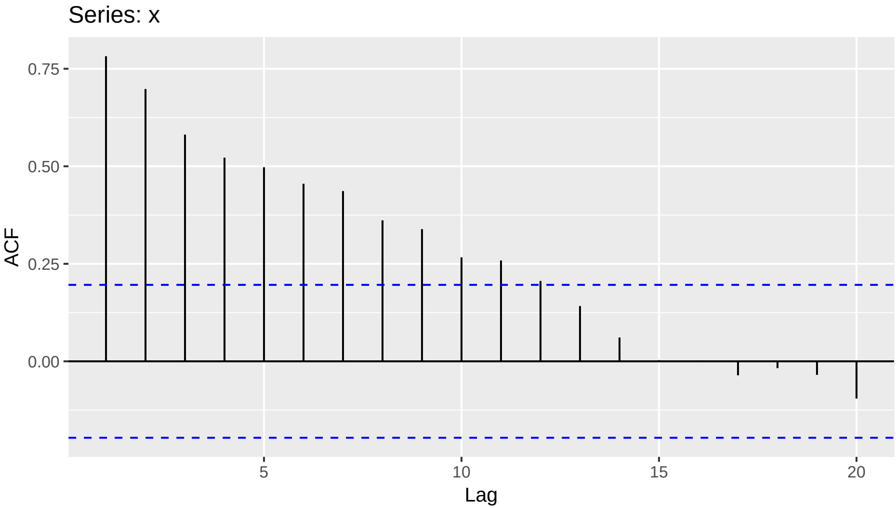
```
##   [1]  0.00000000  0.69150384 -0.27156693
##   [4] -1.69374385 -0.04624706 -0.61289729
##   [7]  0.26464756 -0.21493841 -1.31429232
##  [10]  0.44277420  0.09918044  0.19080999
```

Comments

- Each random shock now only used for its own value of `x`
- but `x[i]` also depends on previous value `x[i-1]`
- so correlated with previous value
- *but* `x[i]` also contains multiple of `x[i-2]` and previous x's
- so all x's correlated, but autocorrelation dying away.

ACF for AR(1) series xxx

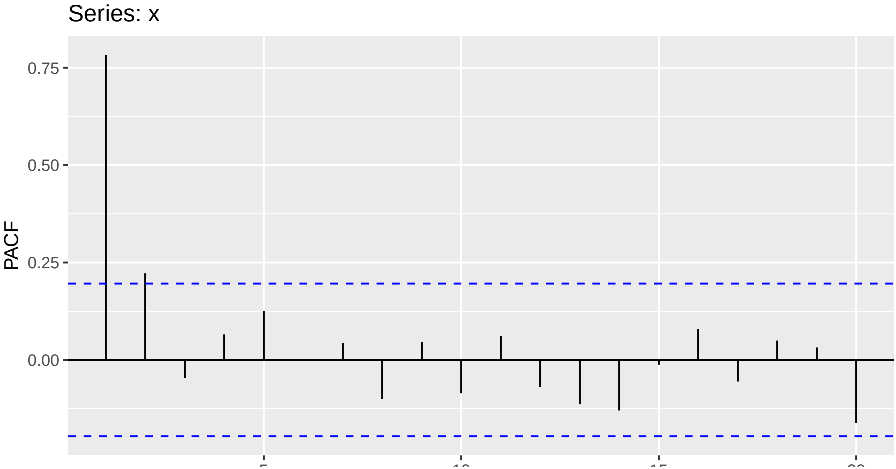
```
acf(x, plot=F) %>% autoplot()
```



Partial autocorrelation function

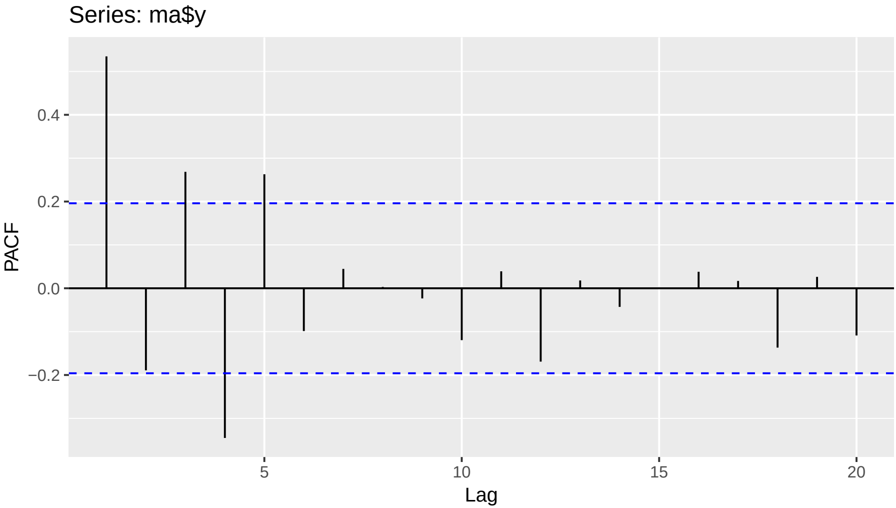
This cuts off for an AR series: xxx

```
pacf(x, plot=F) %>% autoplot()
```



PACF for an MA series decays slowly xxx

```
pacf(ma$y, plot=F) %>% autoplot()
```



The old way of doing time series analysis

Starting from a series with constant variability (eg. transform first to get it, as for souvenirs):

- Assess stationarity.
- If not stationary, take differences as many times as needed until it is.
- Look at ACF, see if it dies off. If it does, you have MA series.
- Look at PACF, see if that dies off. If it does, have AR series.
- If neither dies off, probably have a mixed “ARMA” series.
- Fit coefficients (like regression slopes).
- Do forecasts.

The new way of doing time series analysis (in R)

- Transform series if needed to get constant variability
- Use package `forecast`.
- Use function `auto.arima` to estimate what kind of series best fits data.
- Use `forecast` to see what will happen in future.

Anatomy of auto.arima output

```r
auto.arima(ma$y)
```

```
## Series: ma$y
## ARIMA(0,0,1) with zero mean
##
## Coefficients:
##          ma1
##       0.9070
## s.e.  0.0617
##
## sigma^2 estimated as 0.9878:  log likelihood=-141.64
## AIC=287.29   AICc=287.41   BIC=292.5
```

- ARIMA part tells you what kind of series you are estimated to have:
  - first number (first 0) is AR (autoregressive) part
  - second number (second 0) is amount of differencing here
  - third number (1) is MA (moving average) part

What other models were possible?

Run `auto.arima` with `trace=T`:

```
auto.arima(ma$y,trace=T)
```

```
##
##  ARIMA(2,0,2) with non-zero mean : Inf
##  ARIMA(0,0,0) with non-zero mean : 345.2328
##  ARIMA(1,0,0) with non-zero mean : 313.9535
##  ARIMA(0,0,1) with non-zero mean : 287.9463
##  ARIMA(0,0,0) with zero mean     : 346.0889
##  ARIMA(1,0,1) with non-zero mean : 290.112
##  ARIMA(0,0,2) with non-zero mean : 290.1128
##  ARIMA(1,0,2) with non-zero mean : 291.7865
##  ARIMA(0,0,1) with zero mean     : 287.4124
##  ARIMA(1,0,1) with zero mean     : 289.4909
##  ARIMA(0,0,2) with zero mean     : 289.4993
##  ARIMA(1,0,0) with zero mean     : 312.7625
```

Doing it all the new way: white noise

```
wn.aa=auto.arima(wn.ts)
wn.aa
```

```
## Series: wn.ts
## ARIMA(0,0,0) with zero mean
##
## sigma^2 estimated as 1.111:  log likelihood=-147.16
## AIC=296.32   AICc=296.36   BIC=298.93
```

Best fit *is* white noise (no AR, no MA, no differencing).

Forecasts:

```
forecast(wn.aa)
```

```
##     Point Forecast     Lo 80    Hi 80     Lo 95
## 101              0 -1.350869 1.350869 -2.065975
## 102              0 -1.350869 1.350869 -2.065975
## 103              0 -1.350869 1.350869 -2.065975
## 104              0 -1.350869 1.350869 -2.065975
## 105              0 -1.350869 1.350869 -2.065975
## 106              0 -1.350869 1.350869 -2.065975
## 107              0 -1.350869 1.350869 -2.065975
## 108              0 -1.350869 1.350869 -2.065975
## 109              0 -1.350869 1.350869 -2.065975
## 110              0 -1.350869 1.350869 -2.065975
##        Hi 95
## 101 2.065975
## 102 2.065975
## 103 2.065975
```

MA(1)

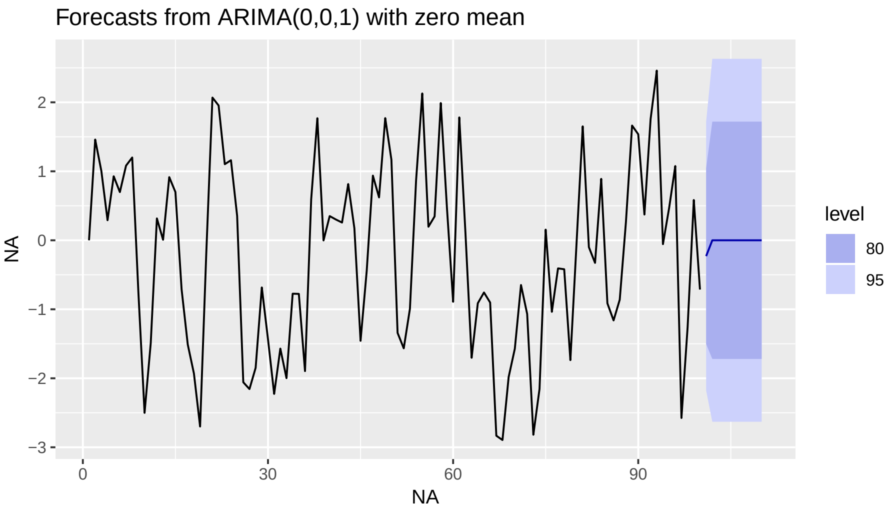
```
y.aa=auto.arima(ma$y)
y.aa
```

```
## Series: ma$y
## ARIMA(0,0,1) with zero mean
##
## Coefficients:
##          ma1
##       0.9070
## s.e.  0.0617
##
## sigma^2 estimated as 0.9878:  log likelihood=-141.64
## AIC=287.29   AICc=287.41   BIC=292.5
```

```
y.f=forecast(y.aa)
```

Plotting the forecasts for MA(1) xxx

```
autoplot(y.f)
```



AR(1)

```
x.aa=auto.arima(x)
x.aa
```

```
## Series: x
## ARIMA(0,1,1)
##
## Coefficients:
##           ma1
##       -0.3544
## s.e.   0.1062
##
## sigma^2 estimated as 0.979:  log likelihood=-138.99
## AIC=281.97   AICc=282.1   BIC=287.16
```

Oops!

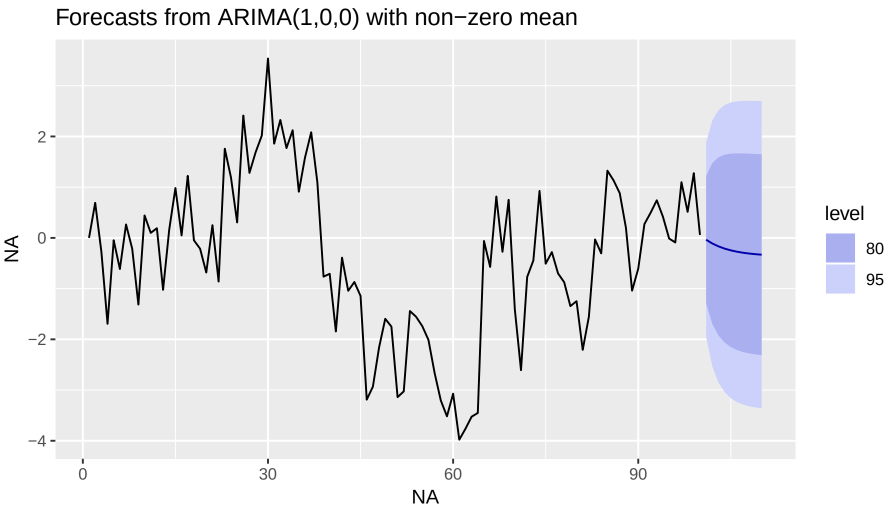
Got it wrong! Fit right AR(1) model:

```
x.arima=arima(x,order=c(1,0,0))
x.arima
```

```
##
## Call:
## arima(x = x, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.7758    -0.3646
## s.e.  0.0611     0.4220
##
## sigma^2 estimated as 0.957:  log likelihood = -140.16,  aic
```

Forecasts for x xxx

```
forecast(x.arima) %>% autoplot()
```



Kings

```
kings.aa=auto.arima(kings.ts)
kings.aa
```

```
## Series: kings.ts
## ARIMA(0,1,1)
##
## Coefficients:
##           ma1
##       -0.7218
## s.e.   0.1208
##
## sigma^2 estimated as 236.2:  log likelihood=-170.06
## AIC=344.13   AICc=344.44   BIC=347.56
```

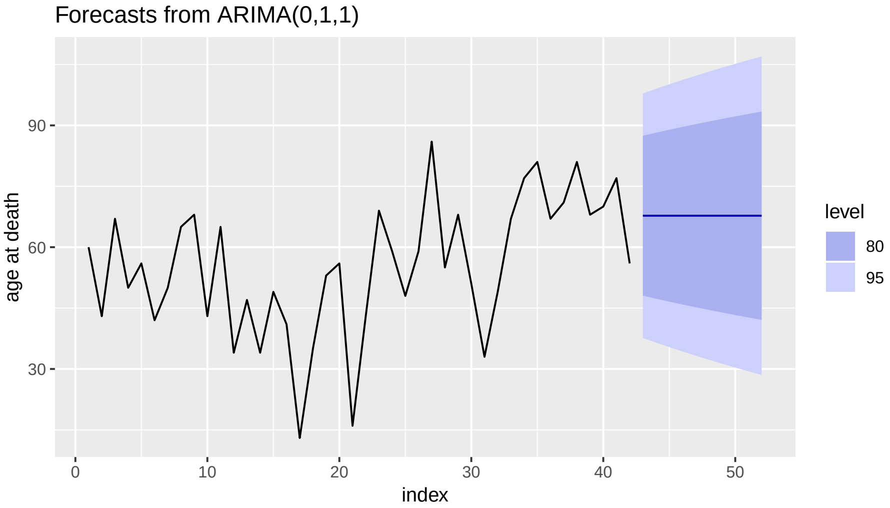
Kings forecasts:

```
kings.f=forecast(kings.aa)
kings.f
```

```
##    Point Forecast     Lo 80    Hi 80    Lo 95
## 43       67.75063 48.05479 87.44646 37.62845
## 44       67.75063 47.30662 88.19463 36.48422
## 45       67.75063 46.58489 88.91637 35.38042
## 46       67.75063 45.88696 89.61429 34.31304
## 47       67.75063 45.21064 90.29062 33.27869
## 48       67.75063 44.55402 90.94723 32.27448
## 49       67.75063 43.91549 91.58577 31.29793
## 50       67.75063 43.29362 92.20763 30.34687
## 51       67.75063 42.68718 92.81408 29.41939
## 52       67.75063 42.09507 93.40619 28.51383
##        Hi 95
## 43  97.87281
```

Kings forecasts, plotted xxx

```
autoplot(kings.f) + labs(x="index", y= "age at death")
```



NY births

```
ny.aa=auto.arima(ny.ts)
ny.aa
```

```
## Series: ny.ts
## ARIMA(2,1,2)(1,1,1)[12]
##
## Coefficients:
##          ar1      ar2      ma1     ma2     sar1
##       0.6539  -0.4540  -0.7255  0.2532  -0.2427
## s.e.  0.3003   0.2429   0.3227  0.2878   0.0985
##          sma1
##       -0.8451
## s.e.   0.0995
##
## sigma^2 estimated as 0.4076:  log likelihood=-157.45
## AIC=328.91   AICc=329.67   BIC=350.21
```

NY births forecasts

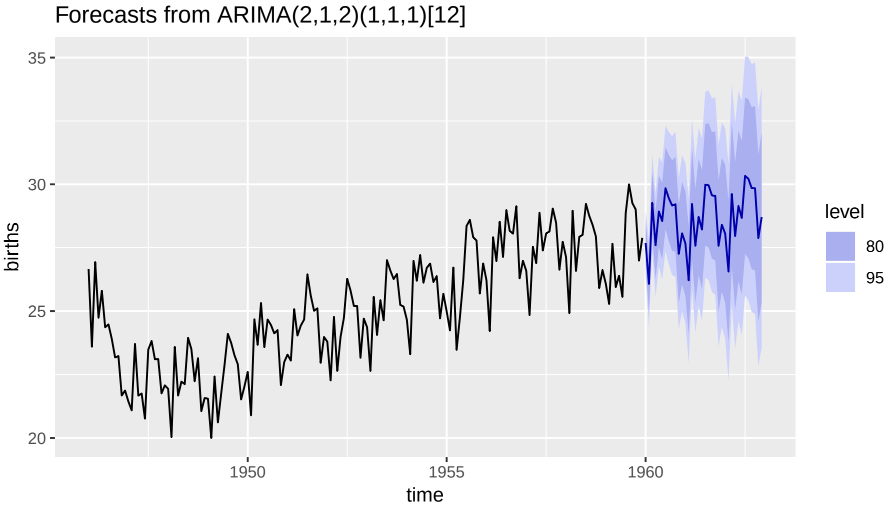
Not *quite* same every year:

```
ny.f
```

```
##          Point Forecast    Lo 80    Hi 80    Lo 95
## Jan 1960       27.69056 26.87069 28.51043 26.43668
## Feb 1960       26.07680 24.95838 27.19522 24.36632
## Mar 1960       29.26544 28.01566 30.51523 27.35406
## Apr 1960       27.59444 26.26555 28.92333 25.56208
## May 1960       28.93193 27.52089 30.34298 26.77392
## Jun 1960       28.55379 27.04381 30.06376 26.24448
## Jul 1960       29.84713 28.23370 31.46056 27.37960
## Aug 1960       29.45347 27.74562 31.16132 26.84155
## Sep 1960       29.16388 27.37259 30.95517 26.42433
## Oct 1960       29.21343 27.34498 31.08188 26.35588
## Nov 1960       27.26221 25.31879 29.20563 24.29000
## Dec 1960       28.06863 26.05137 30.08589 24.98349
```

Plotting the forecasts xxx

```
autoplot(ny.f)+labs(x="time", y="births")
```



Log-souvenir sales

```
souv.aa=auto.arima(souv.log.ts)
souv.aa
```

```
## Series: souv.log.ts
## ARIMA(2,0,0)(0,1,1)[12] with drift
##
## Coefficients:
##          ar1     ar2     sma1   drift
##       0.3470  0.3516  -0.5205  0.0238
## s.e.  0.1092  0.1115   0.1700  0.0031
##
## sigma^2 estimated as 0.02953:  log likelihood=24.54
## AIC=-39.09   AICc=-38.18   BIC=-27.71
```

```
souv.f=forecast(souv.aa,h=27)
```

The forecasts

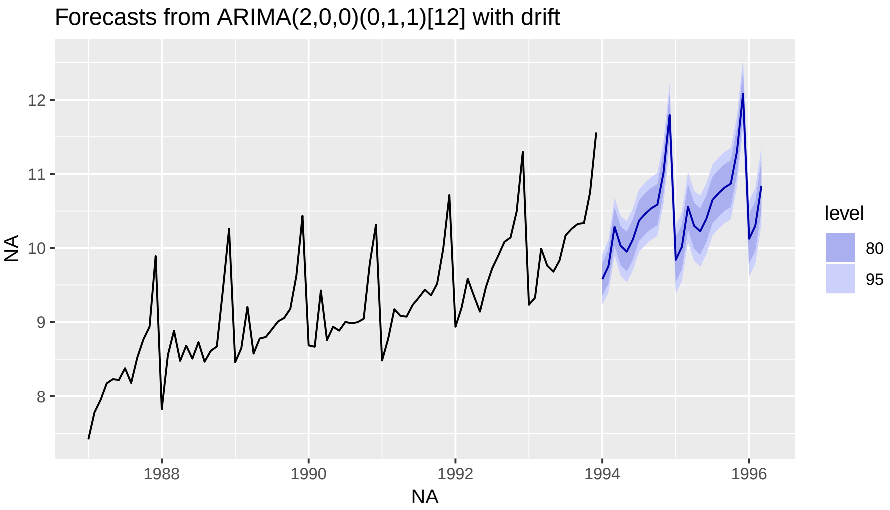
Differenced series showed low value for January (large drop). December highest, Jan and Feb lowest:

```
souv.f
```

```
##          Point Forecast     Lo 80     Hi 80
## Jan 1994       9.578291  9.358036  9.798545
## Feb 1994       9.754836  9.521700  9.987972
## Mar 1994      10.286195 10.030937 10.541453
## Apr 1994      10.028630  9.765727 10.291532
## May 1994       9.950862  9.681555 10.220168
## Jun 1994      10.116930  9.844308 10.389551
## Jul 1994      10.369140 10.094251 10.644028
## Aug 1994      10.460050 10.183827 10.736274
## Sep 1994      10.535595 10.258513 10.812677
## Oct 1994      10.585995 10.308386 10.863604
## Nov 1994      11.017734 10.739793 11.295674
```

Plotting the forecasts xxx

```r
autoplot(souv.f)
```



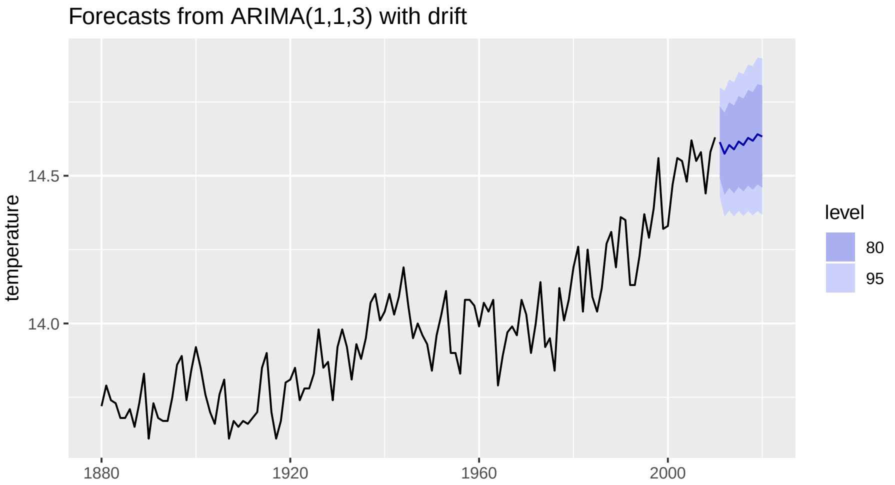
Global mean temperatures, revisited

```
temp.ts=ts(temp$temperature,start=1880)
temp.aa=auto.arima(temp.ts)
temp.aa
```

```
## Series: temp.ts
## ARIMA(1,1,3) with drift
##
## Coefficients:
##          ar1     ma1      ma2      ma3   drift
##      -0.9374  0.5038  -0.6320  -0.2988  0.0067
## s.e.  0.0835  0.1088   0.0876   0.0844  0.0025
##
## sigma^2 estimated as 0.008939:  log likelihood=124.34
## AIC=-236.67   AICc=-235.99   BIC=-219.47
```

Forecasts xxx

```
temp.f=forecast(temp.aa)
autoplot(temp.f)+labs(x="year", y="temperature")
```



Section 2

Multiway frequency tables

Packages

```r
library(tidyverse)
```

Multi-way frequency analysis

- A study of gender and eyewear-wearing finds the following frequencies:

Gender	Contacts	Glasses	None
Female	121	32	129
Male	42	37	85

- Is there association between eyewear and gender?
- Normally answer this with chisquare test (based on observed and expected frequencies from null hypothesis of no association).
- Two categorical variables and a frequency.
- We assess in way that generalizes to more categorical variables.

The data file

```
gender contacts glasses none
female 121      32      129
male   42       37      85
```

- This is *not tidy!*
- Two variables are gender and *eyewear*, and those numbers all frequencies.

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/eyewear.txt
eyewear <- read_delim(my_url, " ")
eyewear
```

```
## # A tibble: 2 x 4
##   gender contacts glasses  none
##   <chr>     <dbl>   <dbl> <dbl>
## 1 female      121      32   129
```

Tidying the data

```
eyes <- eyewear %>%
  gather(eyewear, frequency, contacts:none)
eyes
```

```
## # A tibble: 6 x 3
##   gender eyewear  frequency
##   <chr>  <chr>        <dbl>
## 1 female contacts       121
## 2 male   contacts        42
## 3 female glasses         32
## 4 male   glasses         37
## 5 female none           129
## 6 male   none            85
```

```
xt <- xtabs(frequency ~ gender + eyewear, data = eyes)
xt
```

...

Modelling

- Last table on previous page is "reconstituted" contingency table, for checking.
- Predict frequency from other factors and combos. `glm` with `poisson` family.

```
eyes.1 <- glm(frequency ~ gender * eyewear,
  data = eyes,
  family = "poisson"
)
```

def

- Called **log-linear model**.

What can we get rid of?

{

```
drop1(eyes.1, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## frequency ~ gender * eyewear
##                Df Deviance    AIC    LRT  Pr(>Chi)
## <none>               0.000 47.958
## gender:eyewear  2   17.829 61.787 17.829 0.0001345
##
## <none>
## gender:eyewear ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

def }

Conclusions

- `drop1` says what we can remove at this step. Significant = must stay.
- Cannot remove anything.
- Frequency depends on `gender-wear` *combination*, cannot be simplified further.
- Gender and eyewear are *associated*.
- Stop here.

prop.table

Original table:

{

```
xt
```

```
##          eyewear
## gender   contacts glasses none
##   female      121      32  129
##   male         42      37   85
```

} Calculate eg. row proportions like this:

{

```
prop.table(xt, margin = 1)
```

```
##          eyewear
## gender     contacts   glasses      none
##   female 0.4290780 0.1134752 0.4574468
##   male   0.2560976 0.2256098 0.5182927
```

No association

- Suppose table had been as shown below:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/eyewear2.tx
eyewear2 <- read_table(my_url)
eyes2 <- eyewear2 %>% gather(eyewear, frequency, contacts:none
xt2 <- xtabs(frequency ~ gender + eyewear, data = eyes2)
xt2
```

```
##          eyewear
## gender    contacts glasses none
##   female       150      30  120
##   male          75      16   62
```

```
prop.table(xt2, margin = 1)
```

```
##          eyewear
## gender     contacts   glasses      none
##   female 0.5000000 0.1000000 0.4000000
```

Analysis for revised data

```
eyes.2 <- glm(frequency ~ gender * eyewear,
  data = eyes2,
  family = "poisson"
)
drop1(eyes.2, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## frequency ~ gender * eyewear
##                Df Deviance    AIC      LRT Pr(>Chi)
## <none>            0.000000 47.467
## gender:eyewear  2 0.047323 43.515 0.047323   0.9766
```

No longer any association. Take out interaction.

No interaction

{

```
eyes.3 <- update(eyes.2, . ~ . - gender:eyewear)
drop1(eyes.3, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## frequency ~ gender + eyewear
##         Df Deviance     AIC     LRT  Pr(>Chi)
## <none>         0.047  43.515
## gender   1   48.624  90.091  48.577 3.176e-12 ***
## eyewear  2  138.130 177.598 138.083 < 2.2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

}

- More females (gender effect)

Chest pain, being overweight and being a smoker

- In a hospital emergency department, 176 subjects who attended for acute chest pain took part in a study.
- Each subject had a normal or abnormal electrocardiogram reading (ECG), were overweight (as judged by BMI) or not, and were a smoker or not.
- How are these three variables related, or not?

The data

In modelling-friendly format:

```
ecg bmi smoke count
abnormal overweight yes 47
abnormal overweight no 10
abnormal normalweight yes 8
abnormal normalweight no 6
normal overweight yes 25
normal overweight no 15
normal normalweight yes 35
normal normalweight no 30
```

First step

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/ecg.txt"
chest <- read_delim(my_url, " ")
chest.1 <- glm(count ~ ecg * bmi * smoke,
  data = chest,
  family = "poisson"
)
drop1(chest.1, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## count ~ ecg * bmi * smoke
##               Df Deviance    AIC    LRT Pr(>Chi)
## <none>             0.0000 53.707
## ecg:bmi:smoke  1   1.3885 53.096 1.3885   0.2387
```

That 3-way interaction comes out.

Removing the 3-way interaction

```
chest.2 <- update(chest.1, . ~ . - ecg:bmi:smoke)
drop1(chest.2, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## count ~ ecg + bmi + smoke + ecg:bmi + ecg:smoke + bmi:smoke
##           Df Deviance    AIC     LRT  Pr(>Chi)
## <none>          1.3885 53.096
## ecg:bmi    1   29.0195 78.727 27.6310 1.468e-07 ***
## ecg:smoke  1    4.8935 54.601  3.5050   0.06119 .
## bmi:smoke  1    4.4689 54.176  3.0803   0.07924 .
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

At $\alpha = 0.05$, `bmi:smoke` comes out.

Removing bmi:smoke

```
chest.3 <- update(chest.2, . ~ . - bmi:smoke)
drop1(chest.3, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## count ~ ecg + bmi + smoke + ecg:bmi + ecg:smoke
##           Df Deviance    AIC    LRT  Pr(>Chi)
## <none>          4.469 54.176
## ecg:bmi    1   36.562 84.270 32.094 1.469e-08 ***
## ecg:smoke  1   12.436 60.144  7.968  0.004762 **
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

ecg:smoke has become significant. So we have to stop.

Understanding the final model

- Thinking of `ecg` as "response" that might depend on anything else.
- What is associated with `ecg`? Both `bmi` on its own and `smoke` on its own, but *not* the combination of both.
- `ecg:bmi` table:

```
xtabs(count ~ ecg + bmi, data = chest)
```

```
##            bmi
## ecg         normalweight overweight
##   abnormal            14         57
##   normal              65         40
```

- Most normal weight people have a normal ECG, but a majority of overweight people have an *abnormal* ECG. That is, knowing about BMI says something about likely ECG.

ecg:smoke

- ecg:smoke table:

```
xtabs(count ~ ecg + smoke, data = chest)
```

```
##             smoke
## ecg          no yes
##   abnormal 16  55
##   normal   45  60
```

- Most nonsmokers have a normal ECG, but smokers are about 50–50 normal and abnormal ECG.
- Don't look at smoke:bmi table since not significant.

Simpson's paradox: the airlines example

Airport	Alaska Airlines On time	Alaska Airlines Delayed	America West On time	America West Delayed
Los Angeles	497	62	694	117
Phoenix	221	12	4840	415
San Diego	212	20	383	65
San Francisco	503	102	320	129
Seattle	1841	305	201	61
Total	3274	501	6438	787

Use `status` as variable name for "on time/delayed".

- Alaska: 13.3% flights delayed $(501/(3274 + 501))$.
- America West: 10.9% $(787/(6438 + 787))$.
- America West more punctual, right?

Arranging the data

- Can only have single thing in columns, so we have to construct column names like this: \begin{small}

```
airport     aa_ontime aa_delayed aw_ontime aw_delayed
LosAngeles    497          62        694        117
Phoenix       221          12       4840        415
SanDiego      212          20        383         65
SanFrancisco 503          102        320        129
Seattle      1841         305        201         61
```

\end{small}

- Some tidying gets us the right layout, with frequencies all in one column and the airline and delayed/on time status separated out:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/airlines.tx
airlines <- read_table2(my_url)
```

The data frame punctual

```
## # A tibble: 20 x 4
##    airport       airline status   freq
##    <chr>         <chr>   <chr>   <dbl>
##  1 LosAngeles    aa      ontime    497
##  2 Phoenix       aa      ontime    221
##  3 SanDiego      aa      ontime    212
##  4 SanFrancisco  aa      ontime    503
##  5 Seattle       aa      ontime   1841
##  6 LosAngeles    aa      delayed    62
##  7 Phoenix       aa      delayed    12
##  8 SanDiego      aa      delayed    20
##  9 SanFrancisco  aa      delayed   102
## 10 Seattle       aa      delayed   305
## 11 LosAngeles    aw      ontime    694
## 12 Phoenix       aw      ontime   4840
## 13 SanDiego      aw      ontime    383
```

Proportions delayed by airline

- Two-step process: get appropriate subtable:

```
xt <- xtabs(freq ~ airline + status, data = punctual)
xt
```

```
##          status
## airline delayed ontime
##      aa     501   3274
##      aw     787   6438
```

- and then calculate appropriate proportions:

```
prop.table(xt, margin = 1)
```

```
##          status
## airline   delayed    ontime
##      aa 0.1327152 0.8672848
##      aw 0.1089273 0.8910727
```

Proportion delayed by airport, for each airline

```
xt <- xtabs(freq ~ airline + status + airport, data = punctual
xp <- prop.table(xt, margin = c(1, 3))
ftable(xp,
  row.vars = c("airport", "airline"),
  col.vars = "status"
)
```

```
##                        status    delayed     ontime
## airport       airline
## LosAngeles    aa              0.11091234 0.88908766
##               aw              0.14426634 0.85573366
## Phoenix       aa              0.05150215 0.94849785
##               aw              0.07897241 0.92102759
## SanDiego      aa              0.08620690 0.91379310
##               aw              0.14508929 0.85491071
## SanFrancisco  aa              0.16859504 0.83140496
##               aw              0.28730512 0.71269488
```

Simpson's Paradox

Airport	Alaska	America West
Los Angeles	11.4	14.4
Phoenix	5.2	7.9
San Diego	8.6	14.5
San Francisco	16.9	28.7
Seattle	14.2	23.2
Total	13.3	10.9

- America West more punctual overall,
- but worse at *every single* airport!
- How is that possible?
- Log-linear analysis sheds some light.

Model 1 and output

```
punctual.1 <- glm(freq ~ airport * airline * status,
  data = punctual, family = "poisson"
)
drop1(punctual.1, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ airport * airline * status
##                         Df Deviance    AIC    LRT
## <none>                       0.0000 183.44
## airport:airline:status  4   3.2166 178.65 3.2166
##                         Pr(>Chi)
## <none>
## airport:airline:status    0.5223
```

def

Remove 3-way interaction

```
punctual.2 <- update(punctual.1, ~ . - airport:airline:status)
drop1(punctual.2, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ airport + airline + status + airport:airline + airpo
##     airline:status
##                 Df Deviance    AIC    LRT  Pr(>Chi)
## <none>                  3.2  178.7
## airport:airline  4   6432.5 6599.9 6429.2 < 2.2e-16
## airport:status   4    240.1  407.5  236.9 < 2.2e-16
## airline:status   1     45.5  218.9   42.2 8.038e-11
##
## <none>
## airport:airline ***
```

Understanding the significance

- airline:status:

```
xt <- xtabs(freq ~ airline + status, data = punctual)
prop.table(xt, margin = 1)
```

```
##          status
## airline    delayed    ontime
##       aa 0.1327152 0.8672848
##       aw 0.1089273 0.8910727
```

- More of Alaska Airlines' flights delayed overall.
- Saw this before.

Understanding the significance (2)

- `airport:status`:

```
xt <- xtabs(freq ~ airport + status, data = punctual)
prop.table(xt, margin = 1)
```

```
##                 status
## airport             delayed     ontime
##   LosAngeles    0.13065693 0.86934307
##   Phoenix       0.07780612 0.92219388
##   SanDiego      0.12500000 0.87500000
##   SanFrancisco  0.21916509 0.78083491
##   Seattle       0.15199336 0.84800664
```

- Flights into San Francisco (and maybe Seattle) are often late, and flights into Phoenix are usually on time.
- Considerable variation among airports.

Understanding the significance (3)

- `airport:airline`:

```
xt <- xtabs(freq ~ airport + airline, data = punctual)
prop.table(xt, margin = 2)
```

```
##                airline
## airport                 aa         aw
##   LosAngeles    0.14807947 0.11224913
##   Phoenix       0.06172185 0.72733564
##   SanDiego      0.06145695 0.06200692
##   SanFrancisco  0.16026490 0.06214533
##   Seattle       0.56847682 0.03626298
```

- What fraction of each airline's flights are to each airport.
- Most of Alaska Airlines' flights to Seattle and San Francisco.
- Most of America West's flights to Phoenix.

The resolution

- Most of America West's flights to Phoenix, where it is easy to be on time.
- Most of Alaska Airlines' flights to San Francisco and Seattle, where it is difficult to be on time.
- Overall comparison looks bad for Alaska because of this.
- But, *comparing like with like*, if you compare each airline's performance *to the same airport*, Alaska does better.
- Aggregating over the very different airports was a (big) mistake: that was the cause of the Simpson's paradox.
- Alaska Airlines is *more* punctual when you do the proper comparison.

Ovarian cancer: a four-way table

- Retrospective study of ovarian cancer done in 1973.
- Information about 299 women operated on for ovarian cancer 10 years previously.
- Recorded:
- stage of cancer (early or advanced)
- type of operation (radical or limited)
- X-ray treatment received (yes or no)
- 10-year survival (yes or no)
- Survival looks like response (suggests logistic regression).
- Log-linear model finds any associations at all.

The data

after tidying:

{

```
stage operation xray survival freq
early radical no no 10
early radical no yes 41
early radical yes no 17
early radical yes yes 64
early limited no no 1
early limited no yes 13
early limited yes no 3
early limited yes yes 9
advanced radical no no 38
advanced radical no yes 6
advanced radical yes no 64
advanced radical yes yes 11
advanced limited no no 3
advanced limited no yes 1
advanced limited yes no 13
advanced limited yes yes 5
```

Stage 1

hopefully looking familiar by now:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/cancer.txt'
cancer <- read_delim(my_url, " ")
cancer %>% print(n = 6)
```

```
## # A tibble: 16 x 5
##   stage operation xray  survival  freq
##   <chr> <chr>     <chr> <chr>    <dbl>
## 1 early radical   no    no          10
## 2 early radical   no    yes         41
## 3 early radical   yes   no          17
## 4 early radical   yes   yes         64
## 5 early limited   no    no           1
## 6 early limited   no    yes         13
## # … with 10 more rows
```

```
cancer.1 <- glm(freq ~ stage * operation * xray * survival
```

Output 1

See what we can remove:

```
drop1(cancer.1, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage * operation * xray * survival
##                            Df Deviance    AIC
## <none>                          0.00000 98.130
## stage:operation:xray:survival  1  0.60266 96.732
##                                 LRT Pr(>Chi)
## <none>
## stage:operation:xray:survival 0.60266   0.4376
```

def

Non-significant interaction can come out.

Stage 2

```
cancer.2 <- update(cancer.1, ~ .
- stage:operation:xray:survival)
drop1(cancer.2, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + operatio
##     xray:survival + stage:operation:xray + stage:operation:
##     stage:xray:survival + operation:xray:survival
##                            Df Deviance    AIC     LRT
## <none>                         0.60266 96.732
## stage:operation:xray        1  2.35759 96.487 1.75493
## stage:operation:survival    1  1.17730 95.307 0.57465
## stage:xray:survival         1  0.95577 95.085 0.35311
```

Take out stage:xray:survival

```
cancer.3 <- update(cancer.2, . ~ . - stage:xray:survival)
drop1(cancer.3, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + operatio
##     xray:survival + stage:operation:xray + stage:operation:
##     operation:xray:survival
##                          Df Deviance    AIC     LRT
## <none>                       0.95577 95.085
## stage:operation:xray      1  3.08666 95.216 2.13089
## stage:operation:survival  1  1.56605 93.696 0.61029
## operation:xray:survival   1  1.55124 93.681 0.59547
##                          Pr(>Chi)
```

Remove operation:xray:survival

```
cancer.4 <- update(cancer.3, . ~ . - operation:xray:survival)
drop1(cancer.4, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + operatio
##     xray:survival + stage:operation:xray + stage:operation:
##                           Df Deviance    AIC    LRT
## <none>                         1.5512 93.681
## xray:survival              1   1.6977 91.827 0.1464
## stage:operation:xray       1   6.8420 96.972 5.2907
## stage:operation:survival   1   1.9311 92.061 0.3799
##                           Pr(>Chi)
## <none>
```

Comments

- `stage:operation:xray` has now become significant, so won't remove that.
- Shows value of removing terms one at a time.
- There are no higher-order interactions containing both `xray` and `survival`, so now we get to test (and remove) `xray:survival`.

Remove xray:survival

```
cancer.5 <- update(cancer.4, . ~ . - xray:survival)
drop1(cancer.5, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + operatio
##     stage:operation:xray + stage:operation:survival
##                          Df Deviance    AIC    LRT
## <none>                        1.6977 91.827
## stage:operation:xray      1   6.9277 95.057 5.2300
## stage:operation:survival  1   2.0242 90.154 0.3265
##                          Pr(>Chi)
## <none>
## stage:operation:xray       0.0222 *
```

Remove stage:operation:survival

```
cancer.6 <- update(cancer.5, . ~ . - stage:operation:survival)
drop1(cancer.6, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + operatio
##     stage:operation:xray
##                       Df Deviance     AIC     LRT
## <none>                      2.024  90.154
## stage:survival         1  135.198 221.327 133.173
## operation:survival     1    4.116  90.245   2.092
## stage:operation:xray   1    7.254  93.384   5.230
##                       Pr(>Chi)
## <none>
```

Last step?

Remove `operation:survival`.

```
cancer.7 <- update(cancer.6, . ~ . - operation:survival)
drop1(cancer.7, test = "Chisq")
```

```
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operatio
##     stage:xray + operation:xray + stage:survival + stage:op
##                      Df Deviance     AIC    LRT
## <none>                    4.116  90.245
## stage:survival        1  136.729 220.859 132.61
## stage:operation:xray  1    9.346  93.475   5.23
##                      Pr(>Chi)
## <none>
## stage:survival         <2e-16 ***
```

Conclusions

- What matters is things associated with survival (survival is “response”).
- Only significant such term is stage:survival:

```
xt <- xtabs(freq ~ stage + survival, data = cancer)
prop.table(xt, margin = 1)
```

```
##           survival
## stage              no       yes
##   advanced 0.8368794 0.1631206
##   early    0.1962025 0.8037975
```

- Most people in early stage of cancer survived, and most people in advanced stage did not survive.
- This true *regardless* of type of operation or whether or not X-ray treatment was received. These things have no impact on survival.

What about that other interaction?

```
xt <- xtabs(freq ~ operation + xray + stage, data = cancer)
ftable(prop.table(xt, margin = 3))
```

```
##                     stage   advanced      early
## operation xray
## limited   no               0.02836879 0.08860759
##           yes              0.12765957 0.07594937
## radical   no               0.31205674 0.32278481
##           yes              0.53191489 0.51265823
```

- Out of the people at each stage of cancer (since `margin=3` and `stage` was listed 3rd).
- The association is between `stage` and `xray` *only for those who had the limited operation*.
- For those who had the radical operation, there was no association between `stage` and `xray`.

General procedure

- Start with "complete model" including all possible interactions.
- `drop1` gives highest-order interaction(s) remaining, remove least non-significant.
- Repeat as necessary until everything significant.
- Look at subtables of significant interactions.
- Main effects not usually very interesting.
- Interactions with "response" usually of most interest: show association with response.

```
## Error in FUN(X[[i]], ...): invalid 'name' argument
```