STAD29: Statistics for the Life and Social Sciences

Lecture notes

Section 1

Principal components

Principal Components

- Have measurements on (possibly large) number of variables on some individuals.
- Question: can we describe data using fewer variables (because original variables correlated in some way)?
- Look for direction (linear combination of original variables) in which values *most spread out*. This is *first principal component*.
- Second principal component then direction uncorrelated with this in which values then most spread out. And so on.

Principal components

- See whether small number of principal components captures most of variation in data.
- Might try to interpret principal components.
- If 2 components good, can make plot of data.
- (Like discriminant analysis, but no groups.)
- "What are important ways that these data vary?"

Packages

You might not have installed the first of these. See over for instructions.

```r
library(ggbiplot) # see over
library(tidyverse)
library(ggrepel)
```

Installing ggbiplot

- ggbiplot not on CRAN, so usual install.packages will not work. This is same procedure you used for smmr in C32:
- Install package devtools first (once):

```
install.packages("devtools")
```

- Then install ggbiplot (once):

```
library(devtools)
install_github("vqv/ggbiplot")
```

Small example: 2 test scores for 8 people

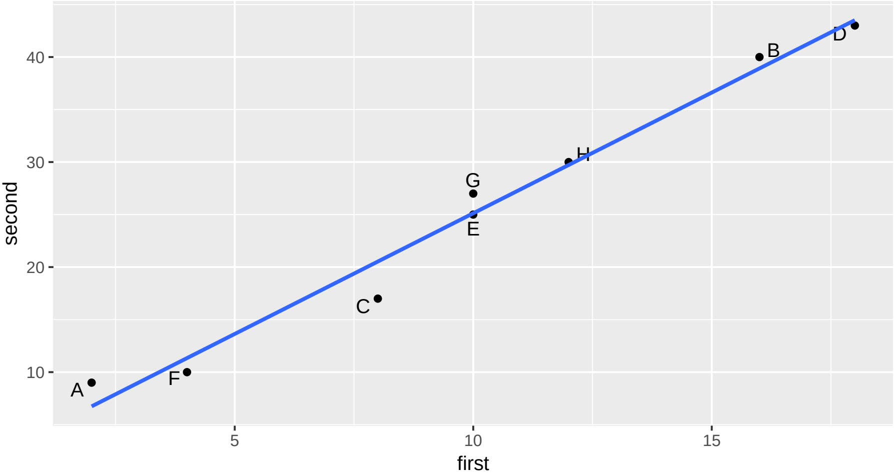
```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/test12.txt"
test12 <- read_table2(my_url)
test12
```

```
## # A tibble: 8 x 3
##   first second id
##   <dbl>  <dbl> <chr>
## 1     2      9 A
## 2    16     40 B
## 3     8     17 C
## 4    18     43 D
## 5    10     25 E
## 6     4     10 F
## 7    10     27 G
## 8    12     30 H
```

```
g <- ggplot(test12, aes(x = first, y = second, label = id)) +
  geom_point() + geom_text_repel()
```

The plot

```
g + geom_smooth(method = "lm", se = F)
```



Principal component analysis

- Grab just the numeric columns:

```
test12 %>% select_if(is.numeric) -> test12_numbers
```

- Strongly correlated, so data nearly 1-dimensional:

```
cor(test12_numbers)
```

```
##            first   second
## first  1.000000 0.989078
## second 0.989078 1.000000
```

Finding principal components

- Make a score summarizing this one dimension. Like this:

```
test12.pc <- princomp(test12_numbers, cor = T)
summary(test12.pc)
```

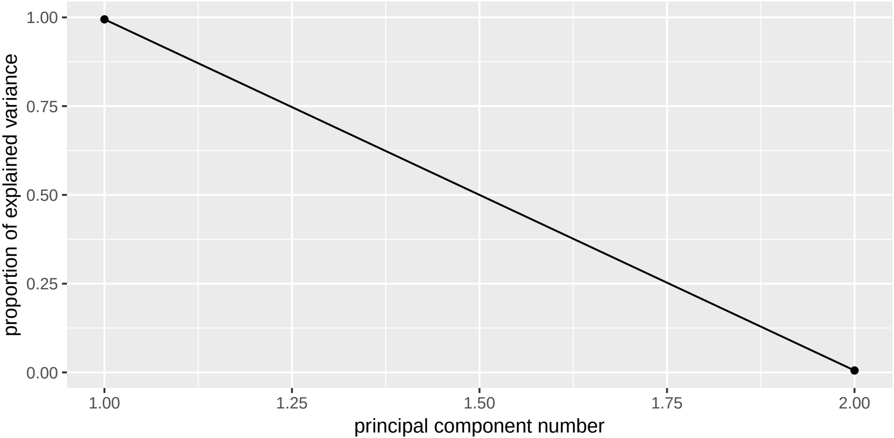
```
## Importance of components:
##                           Comp.1      Comp.2
## Standard deviation     1.410347 0.104508582
## Proportion of Variance 0.994539 0.005461022
## Cumulative Proportion  0.994539 1.000000000
```

Comments

- “Standard deviation” shows relative importance of components (as for LDs in discriminant analysis)
- Here, first one explains almost all (99.4%) of variability.
- That is, look only at first component and ignore second.
- `cor=T` standardizes all variables first. Usually wanted, because variables measured on different scales. (Only omit if variables measured on same scale and expect similar variability.)

Scree plot

```r
ggscreeplot(test12.pc)
```



Component loadings

explain how each principal component depends on (standardized) original variables (test scores):

```
test12.pc$loadings
```

```
##
## Loadings:
##        Comp.1 Comp.2
## first   0.707  0.707
## second  0.707 -0.707
##
##                Comp.1 Comp.2
## SS loadings       1.0    1.0
## Proportion Var    0.5    0.5
## Cumulative Var    0.5    1.0
```

First component basically sum of (standardized) test scores. That is, person tends to score similarly on two tests, and a composite score would summarize performance.

Component scores

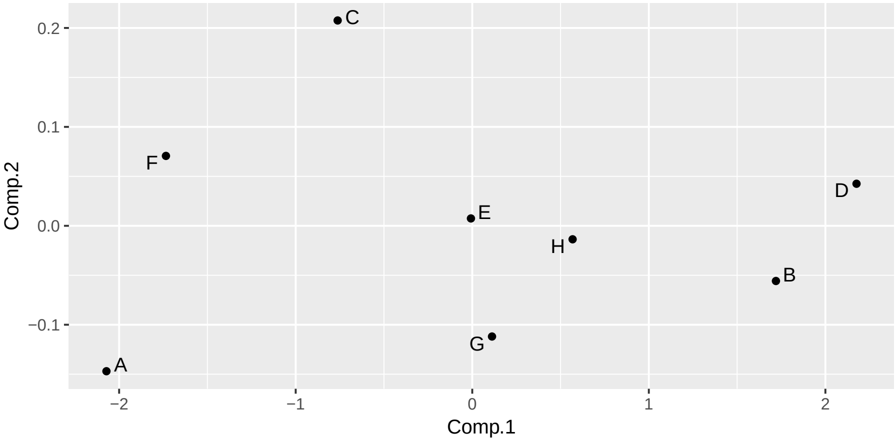
```
d <- data.frame(test12, test12.pc$scores)
d
```

```
##   first second id       Comp.1       Comp.2
## 1     2      9  A -2.071819003 -0.146981782
## 2    16     40  B  1.719862811 -0.055762223
## 3     8     17  C -0.762289708  0.207589512
## 4    18     43  D  2.176267535  0.042533250
## 5    10     25  E -0.007460609  0.007460609
## 6     4     10  F -1.734784030  0.070683441
## 7    10     27  G  0.111909141 -0.111909141
## 8    12     30  H  0.568313864 -0.013613668
```

- Person A is a low scorer, very negative `comp.1` score.
- Person D is high scorer, high positive `comp.1` score.
- Person E average scorer, near-zero `comp.1` score.
- `comp.2` says basically nothing.

Plot of scores

```
ggplot(d, aes(x = Comp.1, y = Comp.2, label = id)) +
  geom_point() + geom_text_repel()
```



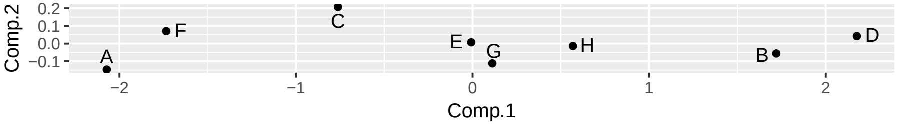
Comments

- Vertical scale exaggerates importance of `comp.2`.
- Fix up to get axes on same scale:

```
g <- ggplot(d, aes(x = Comp.1, y = Comp.2, label = id)) +
  geom_point() + geom_text_repel() +
  coord_fixed()
```

- Shows how exam scores really spread out along one dimension:

```
g
```

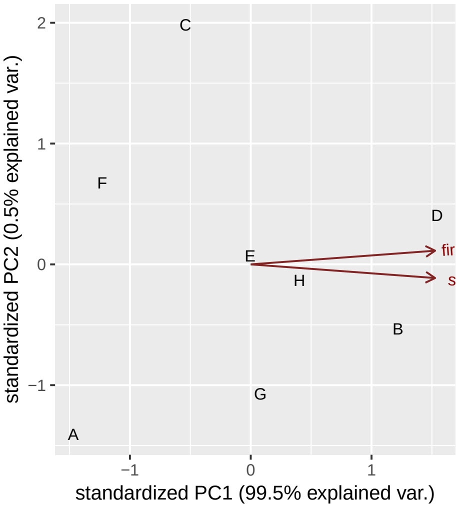


The biplot

- Plotting variables and individuals on one plot.
- Shows how components and original variables related.
- Shows how individuals score on each component, and therefore suggests how they score on each variable.
- Add `labels` option to identify individuals:

```
g <- ggbiplot(test12.pc, labels = test12$id)
```

The biplot


C
F
D
E
fir
s
H
B
G
A
2
1
0
−1
−1
0
1
standardized PC1 (99.5% explained var.)
standardized PC2 (0.5% explained var.)

Comments

- Variables point almost same direction (left). Thus very negative value on `comp.1` goes with high scores on both tests, and test scores highly correlated.
- Position of individuals on plot according to scores on principal components, implies values on original variables. Eg.:
- D very negative on `comp.1`, high scorer on both tests.
- A and F very positive on `comp.1`, poor scorers on both tests.
- C positive on `comp.2`, high score on first test relative to second.
- A negative on `comp.2`, high score on second test relative to first.

Track running data

Track running records (1984) for distances 100m to marathon, arranged by country. Countries labelled by (mostly) Internet domain names (ISO 2-letter codes):

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/men_track_field.txt"
track <- read_table(my_url)
track %>% sample_n(10)
```

```
## # A tibble: 10 x 9
##     m100  m200  m400  m800 m1500 m5000 m10000 marathon country
##    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>    <dbl> <chr>
##  1  12.2  23.2  52.9  2.02  4.24  16.7   35.4     165. ck
##  2  10.2  20.7  46.6  1.78  3.64  14.6   28.4     135. gr
##  3  10.8  21.9  49    2.02  4.24  16.3   34.7     162. ws
##  4  10.3  20.9  46.9  1.79  3.77  14.0   29.2     136. kr
##  5  10.2  20.6  45.6  1.77  3.61  13.3   27.9     131. se
##  6  10.4  21.3  46.1  1.8   3.65  13.5   28.0     129. mx
##  7  10.1  20.2  44.9  1.7   3.51  13.0   27.5     129. uk
##  8  11.0  21.8  47.9  1.9   4.01  14.7   31.4     148. pg
##  9  10.2  20.2  45.4  1.76  3.6   13.3   27.9     132. pl
## 10  10.4  20.6  45.6  1.76  3.58  13.4   28.2     134. cz
```

Country names

Also read in a table to look country names up in later:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/isocodes.csv"
iso <- read_csv(my_url)
iso
```

```
## # A tibble: 250 x 4
##    Country             ISO2  ISO3    M49
##    <chr>               <chr> <chr> <dbl>
##  1 Afghanistan         af    afg       4
##  2 Aland Islands       ax    ala     248
##  3 Albania             al    alb       8
##  4 Algeria             dz    dza      12
##  5 American Samoa      as    asm      16
##  6 Andorra             ad    and      20
##  7 Angola              ao    ago      24
##  8 Anguilla            ai    aia     660
##  9 Antarctica          aq    ata      10
## 10 Antigua and Barbuda ag    atg      28
## # … with 240 more rows
```

Data and aims

- Times in seconds 100m–400m, in minutes for rest (800m up).
- This taken care of by standardization.
- 8 variables; can we summarize by fewer and gain some insight?
- In particular, if 2 components tell most of story, what do we see in a plot?

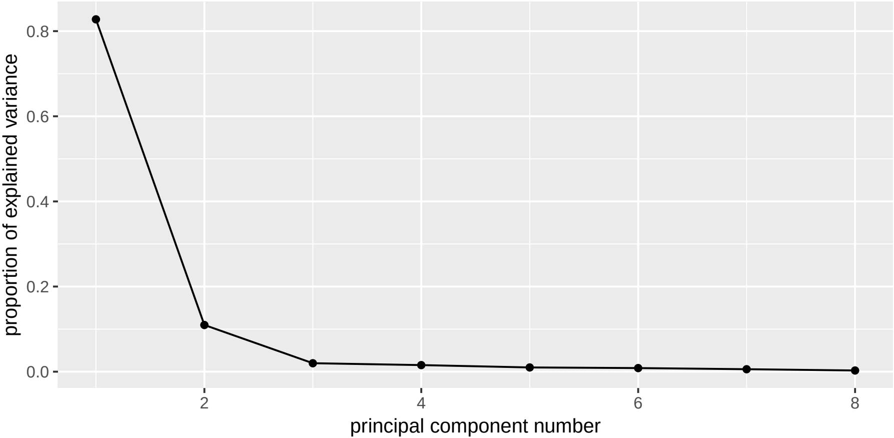
Fit and examine principal components

```
track %>% select_if(is.numeric) -> track_num
track.pc <- princomp(track_num, cor = T)
summary(track.pc)
```

```
## Importance of components:
##                           Comp.1    Comp.2
## Standard deviation     2.5733531 0.9368128
## Proportion of Variance 0.8277683 0.1097023
## Cumulative Proportion  0.8277683 0.9374706
##                            Comp.3     Comp.4
## Standard deviation     0.39915052 0.35220645
## Proportion of Variance 0.01991514 0.01550617
## Cumulative Proportion  0.95738570 0.97289187
##                             Comp.5      Comp.6
## Standard deviation     0.282630981 0.260701267
## Proportion of Variance 0.009985034 0.008495644
## Cumulative Proportion  0.982876903 0.991372547
##                             Comp.7      Comp.8
## Standard deviation     0.215451919 0.150333291
## Proportion of Variance 0.005802441 0.002825012
```

Scree plot

```
ggscreeplot(track.pc)
```



How many components?

- As for discriminant analysis, look for "elbow" in scree plot.
- See one here at 3 components; everything 3 and beyond is "scree".
- So take 2 components.
- Note difference from discriminant analysis: want "large" rather than "small", so go 1 step left of elbow.
- Another criterion: any component with eigenvalue bigger than about 1 is worth including. 2nd one here has eigenvalue just less than 1.
- Refer back to `summary`: cumulative proportion of variance explained for 2 components is 93.7%, pleasantly high. 2 components tell almost whole story.

How do components depend on original variables?

Loadings:

```
track.pc$loadings
```

```
##
## Loadings:
##          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## m100      0.318  0.567  0.332  0.128  0.263  0.594  0.136  0.106
## m200      0.337  0.462  0.361 -0.259 -0.154 -0.656 -0.113
## m400      0.356  0.248 -0.560  0.652 -0.218 -0.157
## m800      0.369        -0.532 -0.480  0.540        -0.238
## m1500     0.373 -0.140 -0.153 -0.405 -0.488  0.158  0.610  0.139
## m5000     0.364 -0.312  0.190        -0.254  0.141 -0.591  0.547
## m10000    0.367 -0.307  0.182        -0.133  0.219 -0.177 -0.797
## marathon  0.342 -0.439  0.263  0.300  0.498 -0.315  0.399  0.158
##
##                Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
## SS loadings     1.000  1.000  1.000  1.000  1.000  1.000  1.000
## Proportion Var  0.125  0.125  0.125  0.125  0.125  0.125  0.125
## Cumulative Var  0.125  0.250  0.375  0.500  0.625  0.750  0.875
##                Comp.8
```

Comments

- `comp.1` loads about equally (has equal weight) on times over all distances.
- `comp.2` has large positive loading for short distances, large negative for long ones.
- `comp.3`: large negative for middle distance, large positive especially for short distances.
- Country overall good at running will have lower than average record times at all distances, so `comp.1` *small*. Conversely, for countries bad at running, `comp.1` very positive.
- Countries relatively better at sprinting (low times) will be *negative* on `comp.2`; countries relatively better at distance running *positive* on `comp.2`.

Commands for plots

- Principal component scores (first two). Also need country IDs.

```
d <- data.frame(track.pc$scores,
  country = track$country
)
names(d)
```

```
## [1] "Comp.1"  "Comp.2"  "Comp.3"  "Comp.4"  "Comp.5"  "Comp
## [7] "Comp.7"  "Comp.8"  "country"
```

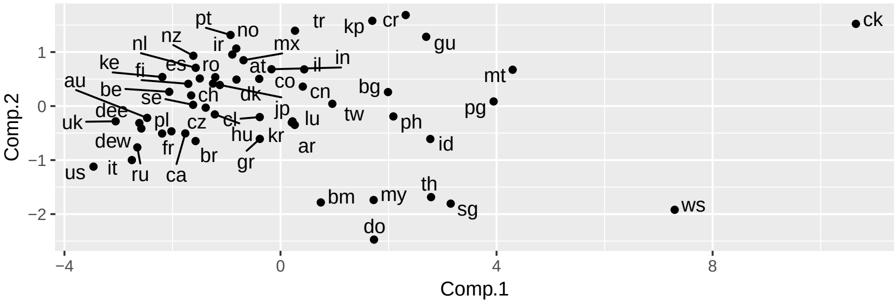
```
g1 <- ggplot(d, aes(x = Comp.1, y = Comp.2,
  label = country)) +
  geom_point() + geom_text_repel() + coord_fixed()
```

- Biplot:

```
g2 <- ggbiplot(track.pc, labels = track$country)
```

Principal components plot

```
g1
```

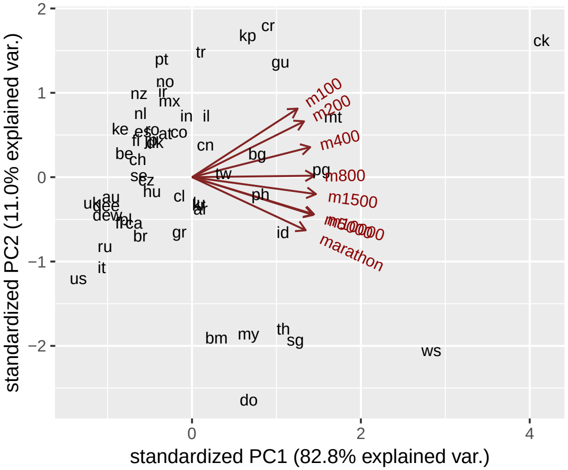


Comments on principal components plot

- Good running countries at left of plot: US, UK, Italy, Russia, East and West Germany.
- Bad running countries at right: Western Samoa, Cook Islands.
- Better sprinting countries at bottom: US, Italy, Russia, Brazil, Greece. `do` is Dominican Republic, where sprinting records relatively good, distance records very bad.
- Better distance-running countries at top: Portugal, Norway, Turkey, Ireland, New Zealand, Mexico. `ke` is Kenya.

Biplot

```
g2
```


standardized PC2 (11.0% explained var.)
standardized PC1 (82.8% explained var.)
m100
m200
m400
m800
m1500
m5000
m10000
marathon
ck
ws
do
us
it
ru
cr
kp
tr
gu
pt
no
nz
mx
in
il
mt
nl
ke
co
cn
bg
be
ch
tw
pg
se
cz
hu
cl
ph
au
br
gr
id
bm
my
th
sg

Comments on biplot

- Had to do some pre-work to interpret PC plot. Biplot more self-contained.
- All variable arrows point right; countries on right have large (bad) record times overall, countries on left good overall.
- Imagine that variable arrows extend negatively as well. Bottom right = bad at distance running, top left = good at distance running.
- Top right = bad at sprinting, bottom left = good at sprinting.
- Doesn't require so much pre-interpretation of components.

Best 8 running countries

Need to look up two-letter abbreviations in ISO table:

```
d %>%
  arrange(Comp.1) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, country, Country) %>%
  slice(1:8)
```

```
##      Comp.1 country                  Country
## 1 -3.462175      us United States of America
## 2 -3.052104      uk           United Kingdom
## 3 -2.752084      it                    Italy
## 4 -2.651062      ru       Russian Federation
## 5 -2.613964     dee             East Germany
## 6 -2.576272     dew             West Germany
## 7 -2.468919      au                Australia
## 8 -2.191917      fr                   France
```

Worst 8 running countries

```
d %>%
  arrange(desc(Comp.1)) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, country, Country) %>%
  slice(1:8)
```

```
##      Comp.1 country          Country
## 1 10.652914      ck     Cook Islands
## 2  7.297865      ws            Samoa
## 3  4.297909      mt            Malta
## 4  3.945224      pg Papua New Guinea
## 5  3.150886      sg        Singapore
## 6  2.787273      th         Thailand
## 7  2.773125      id        Indonesia
## 8  2.697066      gu             Guam
```

Better at distance running

```
d %>%
  arrange(desc(Comp.2)) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.2, country, Country) %>%
  slice(1:10)
```

```
##       Comp.2 country                   Country
## 1  1.6860391      cr                Costa Rica
## 2  1.5791490      kp             Korea (North)
## 3  1.5226742      ck              Cook Islands
## 4  1.3957839      tr                    Turkey
## 5  1.3167578      pt                  Portugal
## 6  1.2829272      gu                      Guam
## 7  1.0663756      no                    Norway
## 8  0.9547437      ir Iran, Islamic Republic of
## 9  0.9318729      nz               New Zealand
## 10 0.8495104      mx                    Mexico
```

Better at sprinting

```
d %>%
  arrange(Comp.2) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.2, country, Country) %>%
  slice(1:10)
```

```
##         Comp.2 country                  Country
## 1   -2.4715736      do       Dominican Republic
## 2   -1.9196130      ws                    Samoa
## 3   -1.8055052      sg                Singapore
## 4   -1.7832229      bm                  Bermuda
## 5   -1.7386063      my                 Malaysia
## 6   -1.6851772      th                 Thailand
## 7   -1.1204235      us United States of America
## 8   -0.9989821      it                    Italy
## 9   -0.7639385      ru       Russian Federation
## 10 -0.6470634      br                   Brazil
```

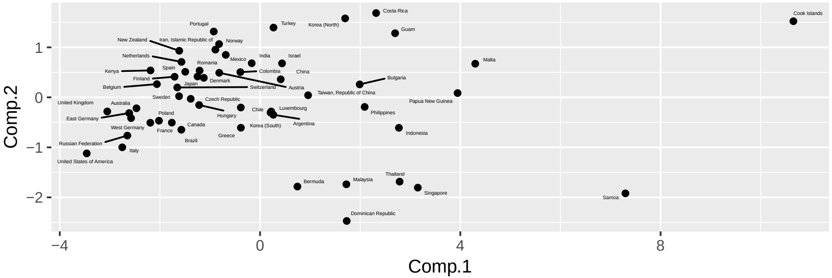
Plot with country names

```
g <- d %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Comp.1, Comp.2, Country) %>%
  ggplot(aes(x = Comp.1, y = Comp.2, label = Country)) +
  geom_point() + geom_text_repel(size = 1) +
  coord_fixed()
```

```
## Warning: Column `country`/`ISO2` joining factor and charact
## vector, coercing into character vector
```

The plot

```
g
```


Costa Rica
Cook Islands
Turkey
Korea (North)
Portugal
Guam
New Zealand
Iran, Islamic Republic of
Norway
Netherlands
India
Israel
Mexico
Malta
Romania
Kenya
Spain
Colombia
China
Finland
Denmark
Bulgaria
Belgium
Japan
Switzerland
Austria
Taiwan, Republic of China
Sweden
Czech Republic
Papua New Guinea
United Kingdom
Australia
Luxembourg
Chile
Poland
Hungary
Philippines
East Germany
Canada
Argentina
Korea (South)
West Germany
France
Indonesia
Greece
Russian Federation
Brazil
Italy
United States of America
Thailand
Bermuda
Malaysia
Singapore
Samoa
Dominican Republic
Comp.2
Comp.1
1
0
−1
−2
−4
0
4
8

Principal components from correlation matrix

Create data file like this:

```
 1         0.9705 -0.9600
 0.9705    1      -0.9980
-0.9600   -0.9980  1
```

and read in like this:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/cov.txt"
mat <- read_table(my_url, col_names = F)
mat
```

```
## # A tibble: 3 x 3
##        X1     X2     X3
##     <dbl>  <dbl>  <dbl>
## 1  1      0.970 -0.96
## 2  0.970  1     -0.998
## 3 -0.96  -0.998  1
```

Pre-processing

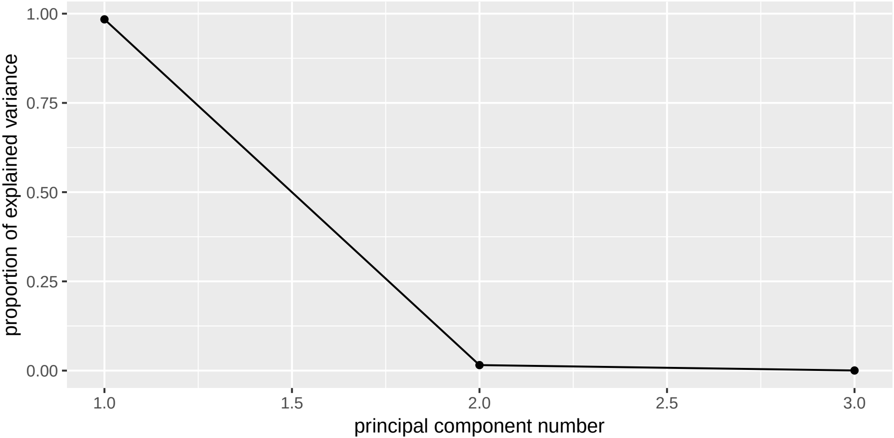
A little pre-processing required:

- Turn into matrix (from data frame)
- Feed into `princomp` as `covmat=`

```
mat.pc <- mat %>%
  as.matrix() %>%
  princomp(covmat = .)
```

Scree plot: one component fine

```
ggscreeplot(mat.pc)
```



Component loadings

Compare correlation matrix:

```
mat
```

```
## # A tibble: 3 x 3
##       X1     X2     X3
##    <dbl>  <dbl>  <dbl>
## 1  1      0.970 -0.96
## 2  0.970  1     -0.998
## 3 -0.96  -0.998  1
```

with component loadings

```
mat.pc$loadings
```

```
##
## Loadings:
##    Comp.1 Comp.2 Comp.3
## X1  0.573  0.812  0.112
## X2  0.581 -0.306 -0.755
## X3 -0.578  0.498 -0.646
##
##                Comp.1 Comp.2 Comp.3
## SS loadings     1.000  1.000  1.000
```

Comments

- When X1 large, X2 also large, X3 small.
  - Then `comp.1` *positive*.
- When X1 small, X2 small, X3 large.
  - Then `comp.1` *negative*.

No scores

- With correlation matrix rather than data, no component scores
  - So no principal component plot
  - and no biplot.

Section 2

Exploratory factor analysis

Principal components and factor analysis

- Principal components:
  - Purely mathematical.
  - Find eigenvalues, eigenvectors of correlation matrix.
  - No testing whether observed components reproducible, or even probability model behind it.
- Factor analysis:
  - some way towards fixing this (get test of appropriateness)
  - In factor analysis, each variable modelled as: “common factor” (eg. verbal ability) and “specific factor” (left over).
  - Choose the common factors to “best” reproduce pattern seen in correlation matrix.
  - Iterative procedure, different answer from principal components.

Packages

```
library(lavaan) # for confirmatory, later
library(ggbiplot)
library(tidyverse)
```

Example

- 145 children given 5 tests, called PARA, SENT, WORD, ADD and DOTS. 3 linguistic tasks (paragraph comprehension, sentence completion and word meaning), 2 mathematical ones (addition and counting dots).
- Correlation matrix of scores on the tests:

```
para 1     0.722 0.714 0.203 0.095
sent 0.722 1     0.685 0.246 0.181
word 0.714 0.685 1     0.170 0.113
add  0.203 0.246 0.170 1     0.585
dots 0.095 0.181 0.113 0.585 1
```

- Is there small number of underlying "constructs" (unobservable) that explains this pattern of correlations?

To start: principal components

Using correlation matrix. Read that first:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/rex2.txt"
kids <- read_delim(my_url, " ")
kids
```

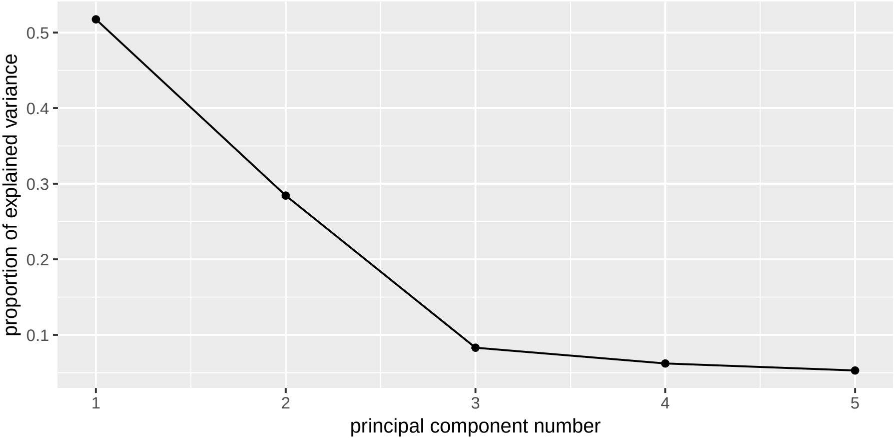
```
## # A tibble: 5 x 6
##   test   para  sent  word   add  dots
##   <chr> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 para  1     0.722 0.714 0.203 0.095
## 2 sent  0.722 1     0.685 0.246 0.181
## 3 word  0.714 0.685 1     0.17  0.113
## 4 add   0.203 0.246 0.17  1     0.585
## 5 dots  0.095 0.181 0.113 0.585 1
```

Principal components on correlation matrix

```
kids %>%
  select_if(is.numeric) %>%
  as.matrix() %>%
  princomp(covmat = .) -> kids.pc
```

Scree plot

```
ggscreeplot(kids.pc)
```



Principal component results

- Need 2 components. Loadings:

```
kids.pc$loadings
```

```
##
## Loadings:
##      Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## para  0.534  0.245  0.114         0.795
## sent  0.542  0.164         0.660 -0.489
## word  0.523  0.247 -0.144 -0.738 -0.316
## add   0.297 -0.627  0.707
## dots  0.241 -0.678 -0.680         0.143
##
##                Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## SS loadings       1.0    1.0    1.0    1.0    1.0
## Proportion Var    0.2    0.2    0.2    0.2    0.2
## Cumulative Var    0.2    0.4    0.6    0.8    1.0
```

Comments

- First component has a bit of everything, though especially the first three tests.
- Second component rather more clearly `add` and `dots`.
- No scores, plots since no actual data.

Factor analysis

- Specify number of factors first, get solution with exactly that many factors.
- Includes hypothesis test, need to specify how many children wrote the tests.
- Works from correlation matrix via `covmat` or actual data, like `princomp`.
- Introduces extra feature, *rotation*, to make interpretation of loadings (factor-variable relation) easier.

Factor analysis for the kids data

- Create "covariance list" to include number of children who wrote the tests.
- Feed this into `factanal`, specifying how many factors (2).

```
km <- kids %>%
  select_if(is.numeric) %>%
  as.matrix()
km2 <- list(cov = km, n.obs = 145)
kids.f2 <- factanal(factors = 2, covmat = km2)
```

Uniquenesses

```
kids.f2$uniquenesses
```

```
##      para      sent      word       add      dots
## 0.2424457 0.2997349 0.3272312 0.5743568 0.1554076
```

- Uniquenesses say how "unique" a variable is (size of specific factor). Small uniqueness means that the variable is summarized by a factor (good).
- Very large uniquenesses are bad; `add`'s uniqueness is largest but not large enough to be worried about.
- Also see "communality" for this idea, where *large* is good and *small* is bad.

Loadings

```
kids.f2$loadings
```

```
##
## Loadings:
##      Factor1 Factor2
## [1,] 0.867
## [2,] 0.820   0.166
## [3,] 0.816
## [4,] 0.167   0.631
## [5,]         0.918
##
##                Factor1 Factor2
## SS loadings      2.119   1.282
## Proportion Var   0.424   0.256
## Cumulative Var   0.424   0.680
```

- Loadings show how each factor depends on variables. Blanks indicate "small", less than 0.1.

Comments

- Factor 1 clearly the "linguistic" tasks, factor 2 clearly the "mathematical" ones.
- Two factors together explain 68% of variability (like regression R-squared).
- Which variables belong to which factor is *much* clearer than with principal components.

Are 2 factors enough?

```
kids.f2$STATISTIC
```

```
## objective
## 0.5810578
```

```
kids.f2$dof
```

```
## [1] 1
```

```
kids.f2$PVAL
```

```
## objective
##  0.445898
```

P-value not small, so 2 factors OK.

1 factor

```
kids.f1 <- factanal(factors = 1, covmat = km2)
kids.f1$STATISTIC
```

```
## objective
##  58.16534
```

```
kids.f1$dof
```

```
## [1] 5
```

```
kids.f1$PVAL
```

```
##    objective
## 2.907856e-11
```

1 factor rejected (P-value small). Definitely need more than 1.

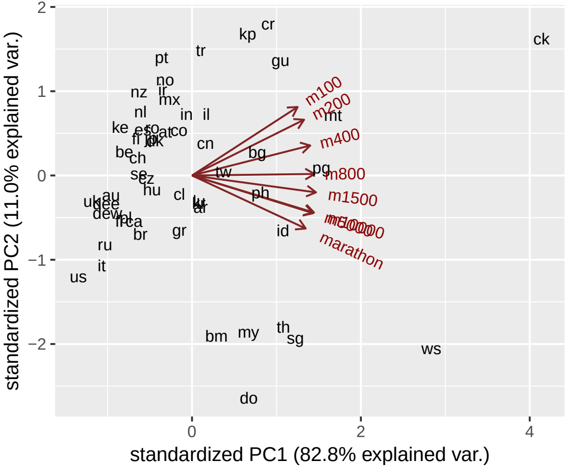
Track running records revisited

Read the data, run principal components, get biplot:

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/men_track_f
track <- read_table(my_url)
track %>% select_if(is.numeric) -> track_num
track.pc <- princomp(track_num, cor = T)
g2 <- ggbiplot(track.pc, labels = track$country)
```

The biplot

```
g2
```



Benefit of rotation

- 100m and marathon arrows almost perpendicular, but components don't match anything much:
- sprinting: bottom left and top right
- distance running: top left and bottom right.
- Can we arrange things so that components (factors) correspond to something meaningful?

Track records by factor analysis

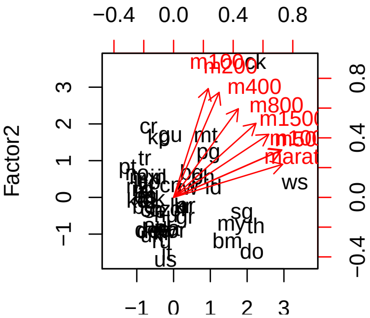
Obtain factor scores (have actual data):

```
track %>%
  select_if(is.numeric) %>%
  factanal(2, scores = "r") -> track.f
```

Track data biplot

Not so nice-looking:

```
biplot(track.f$scores, track.f$loadings,
  xlabs = track$country
)
```



Comments

- This time 100m "up" (factor 2), marathon "right" (factor 1).
- Countries most negative on factor 2 good at sprinting.
- Countries most negative on factor 1 good at distance running.

Rotated factor loadings

```
track.f$loadings
```

```
##
## Loadings:
##          Factor1 Factor2
## m100     0.291   0.914
## m200     0.382   0.882
## m400     0.543   0.744
## m800     0.691   0.622
## m1500    0.799   0.530
## m5000    0.901   0.394
## m10000   0.907   0.399
## marathon 0.915   0.278
##
##                Factor1 Factor2
## SS loadings      4.112   3.225
## Proportion Var   0.514   0.403
## Cumulative Var   0.514   0.917
```

Which countries are good at sprinting or distance running?

Make a data frame with the countries and scores in:

```
scores <- data.frame(
  country = track$country,
  track.f$scores
)
scores %>% slice(1:6)
```

```
##   country     Factor1    Factor2
## 1      ar  0.33633782 -0.2651512
## 2      au -0.49395787 -0.8121335
## 3      at -0.74199914  0.1764151
## 4      be -0.79602754 -0.2388525
## 5      bm  1.46541593 -1.1704466
## 6      br  0.07780163 -0.8871291
```

The best sprinting countries

Most negative on factor 2:

```
scores %>%
  arrange(Factor2) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Country, Factor1, Factor2) %>%
  slice(1:10)
```

```
##                     Country     Factor1    Factor2
## 1  United States of America -0.21942697 -1.7251036
## 2                     Italy -0.18436705 -1.4990521
## 3        Dominican Republic  2.12906546 -1.4666402
## 4        Russian Federation -0.32473110 -1.2236590
## 5                   Bermuda  1.46541593 -1.1704466
## 6            United Kingdom -0.58969058 -1.0139983
## 7                    France -0.25301846 -0.9519162
## 8              West Germany -0.46748876 -0.9079005
## 9                    Canada -0.13690160 -0.8920777
## 10                   Brazil  0.07780163 -0.8871291
```

The best distance-running countries

Most negative on factor 1:

```
scores %>%
  arrange(Factor1) %>%
  left_join(iso, by = c("country" = "ISO2")) %>%
  select(Country, Factor1, Factor2) %>%
  slice(1:10)
```

```
##                         Country    Factor1     Factor2
## 1                      Portugal -1.2509805  0.78366889
## 2                        Norway -0.9920727  0.62299560
## 3                   New Zealand -0.9813348  0.26603491
## 4                         Kenya -0.9749696 -0.07099477
## 5  Iran, Islamic Republic of -0.9231505  0.50271208
## 6                   Netherlands -0.9078661  0.23948200
## 7                       Romania -0.8178386  0.18555001
## 8                        Mexico -0.8096291  0.51446762
## 9                       Finland -0.8094725 -0.05705220
## 10                      Belgium -0.7960275 -0.23885253
```

A bigger example: BEM sex role inventory

- 369 women asked to rate themselves on 60 traits, like “self-reliant” or “shy”.
- Rating 1 “never or almost never true of me” to 7 “always or almost always true of me”.
- 60 personality traits is a lot. Can we find a smaller number of factors that capture aspects of personality?
- The whole BEM sex role inventory on next page.

The whole inventory

1. self reliant
2. yielding
3. helpful
4. defends own beliefs
5. cheerful
6. moody
7. independent
8. shy
9. conscientious
10. athletic
11. affectionate
12. theatrical
13. assertive
14. flatterable
15. happy
16. strong personality
17. loyal
18. unpredictable
19. forceful
20. feminine
21. reliable
22. analytical
23. sympathetic
24. jealous
25. leadership ability
26. sensitive to other's needs
27. truthful
28. willing to take risks
29. understanding
30. secretive
31. makes decisions easily
32. compassionate
33. sincere
34. self-sufficient
35. eager to soothe hurt feelings
36. conceited
37. dominant
38. soft spoken
39. likable
40. masculine
41. warm
42. solemn
43. willing to take a stand
44. tender
45. friendly
46. aggressive
47. gullible
48. inefficient
49. acts as a leader
50. childlike
51. adaptable
52. individualistic
53. does not use harsh language
54. unsystematic
55. competitive
56. loves children
57. tactful
58. ambitious
59. gentle
60. conventional

Some of the data

```
my_url <- "http://www.utsc.utoronto.ca/~butler/d29/factor.txt"
bem <- read_tsv(my_url)
bem
```

```
## # A tibble: 369 x 45
##    subno helpful reliant defbel yielding cheerful indpt athlet
##    <dbl>   <dbl>   <dbl>  <dbl>    <dbl>    <dbl> <dbl>  <dbl>
##  1     1       7       7      5        5        7     7      7
##  2     2       5       6      6        6        2     3      3
##  3     3       7       6      4        4        5     5      2
##  4     4       6       6      7        4        6     6      3
##  5     5       6       6      7        4        7     7      7
##  6     7       5       6      7        4        6     6      2
##  7     8       6       4      6        6        6     3      1
##  8     9       7       6      7        5        6     7      5
##  9    10       7       6      6        4        4     5      2
## 10    11       7       4      7        4        7     5      2
## # … with 359 more rows, and 37 more variables: shy <dbl>,
## #   assert <dbl>, strpers <dbl>, forceful <dbl>, affect <dbl>,
## #   flatter <dbl>, loyal <dbl>, analyt <dbl>, feminine <dbl>,
## #   sympathy <dbl>, moody <dbl>, sensitiv <dbl>, undstand <dbl>,
## #   compass <dbl>, leaderab <dbl>, soothe <dbl>, risk <dbl>,
## #   decide <dbl>, selfsuff <dbl>, conscien <dbl>,
## #   dominant <dbl>, masculin <dbl>, stand <dbl>, happy <dbl>,
```

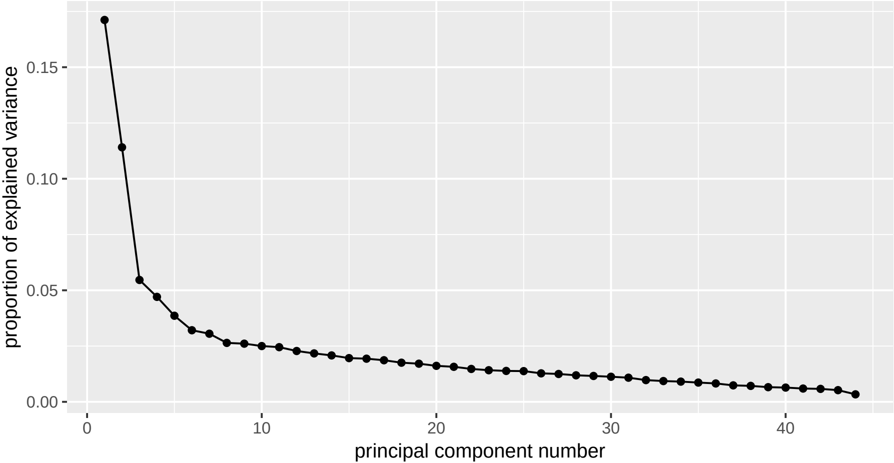
Principal components first

…to decide on number of factors:

```
bem.pc <- bem %>%
  select(-subno) %>%
  princomp(cor = T)
```

The scree plot

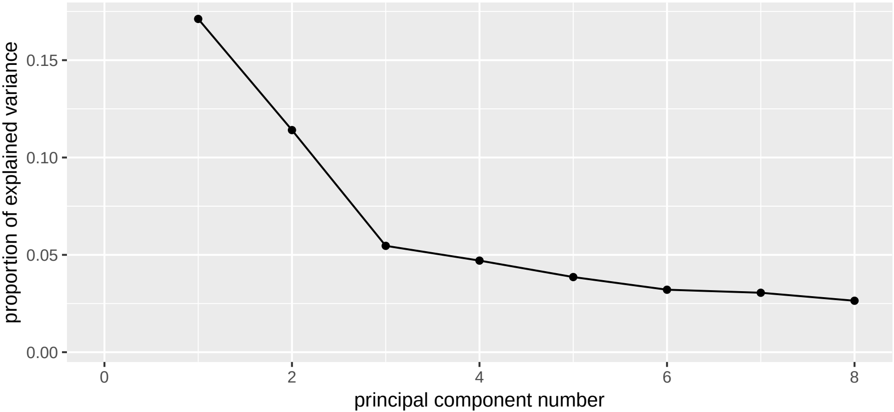
```
(g <- ggscreeplot(bem.pc))
```



Zoom in to search for elbow

Possible elbows at 3 (2 factors) and 6 (5):

```
g + scale_x_continuous(limits = c(0, 8))
```



but is 2 really good?

```
summary(bem.pc)
```

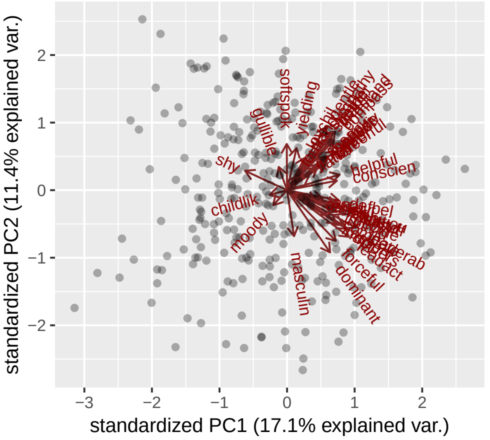
```
## Importance of components:
##                           Comp.1    Comp.2     Comp.3     Comp.4
## Standard deviation     2.7444993 2.2405789 1.55049106 1.43886350
## Proportion of Variance 0.1711881 0.1140953 0.05463688 0.04705291
## Cumulative Proportion  0.1711881 0.2852834 0.33992029 0.38697320
##                            Comp.5     Comp.6     Comp.7
## Standard deviation     1.30318840 1.18837867 1.15919129
## Proportion of Variance 0.03859773 0.03209645 0.03053919
## Cumulative Proportion  0.42557093 0.45766738 0.48820657
##                            Comp.8     Comp.9    Comp.10
## Standard deviation     1.07838912 1.07120568 1.04901318
## Proportion of Variance 0.02643007 0.02607913 0.02500974
## Cumulative Proportion  0.51463664 0.54071577 0.56572551
##                           Comp.11    Comp.12    Comp.13
## Standard deviation     1.03848656 1.00152287 0.97753974
## Proportion of Variance 0.02451033 0.02279655 0.02171782
## Cumulative Proportion  0.59023584 0.61303238 0.63475020
##                           Comp.14   Comp.15    Comp.16
## Standard deviation     0.95697572 0.9287543 0.92262649
## Proportion of Variance 0.02081369 0.0196042 0.01934636
## Cumulative Proportion  0.65556390 0.6751681 0.69451445
##                           Comp.17   Comp.18    Comp.19
```

Comments

- Want overall fraction of variance explained ("cumulative proportion") to be reasonably high.
- 2 factors, 28.5%. Terrible!
- Even 56% (10 factors) not that good!
- Have to live with that.

Biplot

```
ggbiplot(bem.pc, alpha = 0.3)
```



Comments

- Ignore individuals for now.
- Most variables point to 10 o'clock or 7 o'clock.
- Suggests factor analysis with rotation will get interpretable factors (rotate to 6 o'clock and 9 o'clock, for example).
- Try for 2-factor solution (rough interpretation, will be bad):

```
bem.2 <- bem %>%
  select(-subno) %>%
  factanal(factors = 2)
```

- Show output in pieces (just print `bem.2` to see all of it).

Uniquenesses, sorted

```
sort(bem.2$uniquenesses)
```

```
##  leaderab   leadact      warm    tender  dominant    gentle
## 0.4091894 0.4166153 0.4764762 0.4928919 0.4942909 0.5064551
##  forceful   strpers   compass     stand  undstand    assert
## 0.5631857 0.5679398 0.5937073 0.6024001 0.6194392 0.6329347
##    soothe    affect    decide  selfsuff  sympathy     indpt
## 0.6596103 0.6616625 0.6938578 0.7210246 0.7231450 0.7282742
##   helpful    defbel      risk   reliant   individ   compete
## 0.7598223 0.7748448 0.7789761 0.7808058 0.7941998 0.7942910
##  conscien     happy  sensitiv     loyal  ambitiou       shy
## 0.7974820 0.8008966 0.8018851 0.8035264 0.8101599 0.8239496
##  softspok  cheerful  masculin  yielding  feminine  truthful
## 0.8339058 0.8394916 0.8453368 0.8688473 0.8829927 0.8889983
##   lovchil    analyt    athlet   flatter  gullible     moody
## 0.8924392 0.8968744 0.9229702 0.9409500 0.9583435 0.9730607
##  childlik  foullang
## 0.9800360 0.9821662
```

Comments

- Mostly high or very high (bad).
- Some smaller, eg.: Leadership ability (0.409), Acts like leader (0.417), Warm (0.476), Tender (0.493).
- Smaller uniquenesses captured by one of our two factors.
- Larger uniquenesses are not: need more factors to capture them.

Factor loadings, some

```
bem.2$loadings
```

```
##
## Loadings:
##          Factor1 Factor2
## helpful   0.314   0.376
## reliant   0.453   0.117
## defbel    0.434   0.193
## yielding -0.131   0.338
## cheerful  0.152   0.371
## indpt     0.521
## athlet    0.267
## shy      -0.414
## assert    0.605
## strpers   0.657
## forceful  0.649  -0.126
## affect    0.178   0.554
## flatter           0.223
## loyal     0.151   0.417
## analyt    0.295   0.127
## feminine  0.113   0.323
## sympathy          0.526
## moody            -0.162
## sensitiv  0.135   0.424
```

Making a data frame

There are too many to read easily, so make a data frame. A bit tricky:

```
loadings <- as.data.frame(unclass(bem.2$loadings)) %>%
  mutate(trait = rownames(bem.2$loadings))
loadings %>% slice(1:12)
```

```
##       Factor1      Factor2    trait
## 1   0.3137466  0.376484908  helpful
## 2   0.4532904  0.117140647  reliant
## 3   0.4336574  0.192602996   defbel
## 4  -0.1309965  0.337629288 yielding
## 5   0.1523718  0.370530549 cheerful
## 6   0.5212403  0.005870336    indpt
## 7   0.2670788  0.075542858   athlet
## 8  -0.4144579 -0.065372760      shy
## 9   0.6049588  0.033004846   assert
## 10  0.6569855  0.020777649  strpers
## 11  0.6487190 -0.126405816 forceful
## 12  0.1778911  0.553799444   affect
```

Pick out the big ones on factor 1

Arbitrarily defining > 0.4 or < -0.4 as "big":

```
loadings %>% filter(abs(Factor1) > 0.4)
```

```
##       Factor1      Factor2    trait
## 1   0.4532904  0.117140647  reliant
## 2   0.4336574  0.192602996   defbel
## 3   0.5212403  0.005870336    indpt
## 4  -0.4144579 -0.065372760      shy
## 5   0.6049588  0.033004846   assert
## 6   0.6569855  0.020777649  strpers
## 7   0.6487190 -0.126405816 forceful
## 8   0.7654924  0.069513572 leaderab
## 9   0.4416176  0.161238425     risk
## 10  0.5416796  0.112807957   decide
## 11  0.5109964  0.133626767 selfsuff
## 12  0.6676490 -0.244855780 dominant
## 13  0.6066864  0.171848896    stand
## 14  0.7627129 -0.040667202  leadact
## 15  0.4448064  0.089146147  individ
## 16  0.4504188  0.053207281  compete
## 17  0.4136498  0.136869589 ambitiou
```

Factor 2, the big ones

```
loadings %>% filter(abs(Factor2) > 0.4)
```

```
##        Factor1   Factor2    trait
## 1   0.17789112 0.5537994   affect
## 2   0.15121266 0.4166622    loyal
## 3   0.02301456 0.5256654 sympathy
## 4   0.13476970 0.4242037 sensitiv
## 5   0.09111299 0.6101294 undstand
## 6   0.11350643 0.6272223  compass
## 7   0.06061755 0.5802714   soothe
## 8   0.11893011 0.4300698    happy
## 9   0.07956978 0.7191610     warm
## 10  0.05113807 0.7102763   tender
## 11 -0.01873224 0.7022768   gentle
```

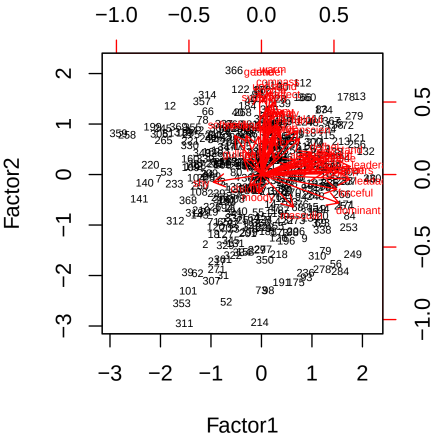
Plotting the two factors

- A bi-plot, this time with the variables reduced in size. Looking for unusual individuals.
- Have to run `factanal` again to get factor scores for plotting.

```
bem %>% select(-subno) %>%
  factanal(factors = 2, scores = "r") -> bem.2a
biplot(bem.2a$scores, bem.2a$loadings, cex = c(0.5, 0.5))
```

- Numbers on plot are row numbers of `bem` data frame.

The (awful) biplot



Comments

- Variables mostly up ("feminine") and right ("masculine"), accomplished by rotation.
- Some unusual individuals: 311, 214 (low on factor 2), 366 (high on factor 2), 359, 258 (low on factor 1), 230 (high on factor 1).

Individual 366

```
bem %>% slice(366) %>% glimpse()
```

```
## Observations: 1
## Variables: 45
## $ subno    <dbl> 755
## $ helpful  <dbl> 7
## $ reliant  <dbl> 7
## $ defbel   <dbl> 5
## $ yielding <dbl> 7
## $ cheerful <dbl> 7
## $ indpt    <dbl> 7
## $ athlet   <dbl> 7
## $ shy      <dbl> 2
## $ assert   <dbl> 1
## $ strpers  <dbl> 3
## $ forceful <dbl> 1
## $ affect   <dbl> 7
## $ flatter  <dbl> 9
## $ loyal    <dbl> 7
## $ analyt   <dbl> 7
## $ feminine <dbl> 7
## $ sympathy <dbl> 7
## $ moody    <dbl> 1
## $ sensitiv <dbl> 7
## $ undstand <dbl> 7
## $ compass  <dbl> 6
## $ leaderab <dbl> 3
## $ soothe   <dbl> 7
## $ risk     <dbl> 7
## $ decide   <dbl> 7
## $ selfsuff <dbl> 7
## $ conscien <dbl> 7
```

Comments

- Individual 366 high on factor 2, but hard to see which traits should have high scores (unless we remember).
- Idea: *tidy* original data frame to make easier to look things up.

Tidying original data

```
bem_tidy <- bem %>%
  mutate(row = row_number()) %>%
  gather(trait, score, c(-subno, -row))
bem_tidy
```

```
## # A tibble: 16,236 x 4
##    subno   row trait   score
##    <dbl> <int> <chr>   <dbl>
##  1     1     1 helpful     7
##  2     2     2 helpful     5
##  3     3     3 helpful     7
##  4     4     4 helpful     6
##  5     5     5 helpful     6
##  6     7     6 helpful     5
##  7     8     7 helpful     6
##  8     9     8 helpful     7
##  9    10     9 helpful     7
## 10    11    10 helpful     7
## # … with 16,226 more rows
```

Recall data frame of loadings

```
loadings %>% slice(1:10)
```

```
##       Factor1      Factor2    trait
## 1   0.3137466  0.376484908  helpful
## 2   0.4532904  0.117140647  reliant
## 3   0.4336574  0.192602996   defbel
## 4  -0.1309965  0.337629288 yielding
## 5   0.1523718  0.370530549 cheerful
## 6   0.5212403  0.005870336    indpt
## 7   0.2670788  0.075542858   athlet
## 8  -0.4144579 -0.065372760      shy
## 9   0.6049588  0.033004846   assert
## 10  0.6569855  0.020777649  strpers
```

Want to add the factor scores for each trait to our tidy data frame `bem_tidy`. This is a left-join (over), matching on the column `trait` that is in both data frames (thus, the default):

Looking up loadings

```
bem_tidy %>% left_join(loadings) -> bem_tidy
```

```
## Joining, by = "trait"
```

```
bem_tidy %>% sample_n(12)
```

```
## # A tibble: 12 x 6
##    subno   row trait    score Factor1 Factor2
##    <dbl> <int> <chr>    <dbl>   <dbl>   <dbl>
##  1    98    60 decide       4  0.542   0.113
##  2   247   141 compete      4  0.450   0.0532
##  3   104    64 decide       5  0.542   0.113
##  4   365   213 affect       7  0.178   0.554
##  5   266   154 shy          1 -0.414  -0.0654
##  6   528   307 helpful      6  0.314   0.376
##  7   214   123 decide       6  0.542   0.113
##  8   245   139 compass      7  0.114   0.627
##  9   146    95 yielding     3 -0.131   0.338
## 10   689   354 tender       6  0.0511  0.710
## 11   467   265 forceful     1  0.649  -0.126
## 12   461   260 truthful     6  0.109   0.315
```

Individual 366, high on Factor 2

So now pick out the rows of the tidy data frame that belong to individual 366 (`row=366`) and for which the `Factor2` score exceeds 0.4 in absolute value (our "big" from before):

```
bem_tidy %>% filter(row == 366, abs(Factor2) > 0.4)
```

```
## # A tibble: 11 x 6
##    subno   row trait    score Factor1 Factor2
##    <dbl> <int> <chr>    <dbl>   <dbl>   <dbl>
##  1   755   366 affect       7  0.178    0.554
##  2   755   366 loyal        7  0.151    0.417
##  3   755   366 sympathy     7  0.0230   0.526
##  4   755   366 sensitiv     7  0.135    0.424
##  5   755   366 undstand     7  0.0911   0.610
##  6   755   366 compass      6  0.114    0.627
##  7   755   366 soothe       7  0.0606   0.580
##  8   755   366 happy        7  0.119    0.430
##  9   755   366 warm         7  0.0796   0.719
## 10   755   366 tender       7  0.0511   0.710
## 11   755   366 gentle       7 -0.0187   0.702
```

As expected, high scorer on these.

Several individuals

Rows 311 and 214 were *low* on Factor 2, so their scores should be low. Can we do them all at once?

```
bem_tidy %>% filter(
  row %in% c(366, 311, 214),
  abs(Factor2) > 0.4
)
```

```
## # A tibble: 33 x 6
##    subno   row trait    score Factor1 Factor2
##    <dbl> <int> <chr>    <dbl>   <dbl>   <dbl>
##  1   369   214 affect       1  0.178    0.554
##  2   534   311 affect       5  0.178    0.554
##  3   755   366 affect       7  0.178    0.554
##  4   369   214 loyal        7  0.151    0.417
##  5   534   311 loyal        4  0.151    0.417
##  6   755   366 loyal        7  0.151    0.417
##  7   369   214 sympathy     4  0.0230   0.526
##  8   534   311 sympathy     4  0.0230   0.526
##  9   755   366 sympathy     7  0.0230   0.526
```

Individual by column

Un-tidy, that is, spread:

```
bem_tidy %>%
  filter(
    row %in% c(366, 311, 214),
    abs(Factor2) > 0.4
  ) %>%
  select(-subno, -Factor1, -Factor2) %>%
  spread(row, score)
```

```
## # A tibble: 11 x 4
##    trait    `214` `311` `366`
##    <chr>    <dbl> <dbl> <dbl>
##  1 affect       1     5     7
##  2 compass      5     4     6
##  3 gentle       2     3     7
##  4 happy        4     3     7
##  5 loyal        7     4     7
##  6 sensitiv     7     4     7
##  7 soothe       3     4     7
##  8 sympathy     4     4     7
##  9 tender       3     4     7
## 10 undstand     5     3     7
## 11 warm         1     3     7
```

366 high, 311 middling, 214 (sometimes) low.

Individuals 230, 258, 359

These were high, low, low on factor 1. Adapt code:

```
bem_tidy %>%
  filter(row %in% c(359, 258, 230), abs(Factor1) > 0.4) %>%
  select(-subno, -Factor1, -Factor2) %>%
  spread(row, score)
```

```
## # A tibble: 17 x 4
##    trait    `230` `258` `359`
##    <chr>    <dbl> <dbl> <dbl>
##  1 ambitiou     7     2     4
##  2 assert       7     3     1
##  3 compete      6     2     1
##  4 decide       7     1     2
##  5 defbel       7     1     1
##  6 dominant     7     1     1
##  7 forceful     7     1     1
##  8 individ      7     3     3
##  9 indpt        7     7     1
## 10 leadact      7     1     1
## 11 leaderab     7     1     1
## 12 reliant      7     4     1
## 13 risk         7     5     7
## 14 selfsuff     7     4     1
## 15 shy          2     7     5
## 16 stand        7     1     6
## 17 strpers      7     1     3
```

Is 2 factors enough?

Suspect not:

```
bem.2$PVAL
```

```
##     objective
## 1.458183e-150
```

2 factors resoundingly rejected. Need more. Have to go all the way to 15 factors to not reject:

```
bem.15 <- bem %>%
  select(-subno) %>%
  factanal(factors = 15)
bem.15$PVAL
```

```
## objective
##  0.132617
```

Even then, only just over 50% of variability explained.

Get 15-factor loadings

into a data frame, as before:

```
loadings <- as.data.frame(unclass(bem.15$loadings)) %>%
  mutate(trait = rownames(bem.15$loadings))
```

then show the highest few loadings on each factor.

Factor 1 (of 15)

```
loadings %>%
  arrange(desc(abs(Factor1))) %>%
  select(Factor1, trait) %>%
  slice(1:10)
```

```
##      Factor1    trait
## 1  0.8127595  compass
## 2  0.6756043 undstand
## 3  0.6611293 sympathy
## 4  0.6408327 sensitiv
## 5  0.5971006   soothe
## 6  0.3481290     warm
## 7  0.2797159   gentle
## 8  0.2788627   tender
## 9  0.2501505  helpful
## 10 0.2340594 conscien
```

Compassionate, understanding, sympathetic, soothing: thoughtful of others.

Factor 2

```
loadings %>%
  arrange(desc(abs(Factor2))) %>%
  select(Factor2, trait) %>%
  slice(1:10)
```

```
##       Factor2    trait
## 1   0.7615492  strpers
## 2   0.7160312 forceful
## 3   0.6981500   assert
## 4   0.5041921 dominant
## 5   0.3929344 leaderab
## 6   0.3669560    stand
## 7   0.3507080  leadact
## 8  -0.3131682 softspok
## 9  -0.2866862      shy
## 10  0.2602525   analyt
```

Strong personality, forceful, assertive, dominant: getting ahead.

Factor 3

```
loadings %>%
  arrange(desc(abs(Factor3))) %>%
  select(Factor3, trait) %>%
  slice(1:10)
```

```
##       Factor3    trait
## 1   0.6697542  reliant
## 2   0.6475496 selfsuff
## 3   0.6204018    indpt
## 4   0.3899607  helpful
## 5  -0.3393605 gullible
## 6   0.3333813  individ
## 7   0.3319003   decide
## 8   0.3294806 conscien
## 9   0.2877396 leaderab
## 10  0.2804170   defbel
```

Self-reliant, self-sufficient, independent: going it alone.

Factor 4

```
loadings %>%
  arrange(desc(abs(Factor4))) %>%
  select(Factor4, trait) %>%
  slice(1:10)
```

```
##      Factor4    trait
## 1  0.6956206   gentle
## 2  0.6920303   tender
## 3  0.5992467     warm
## 4  0.4465546   affect
## 5  0.3942568 softspok
## 6  0.2779793  lovchil
## 7  0.2444249 undstand
## 8  0.2442119    happy
## 9  0.2125905    loyal
## 10 0.2022861   soothe
```

Gentle, tender, warm (affectionate): caring for others.

Factor 5

```
loadings %>%
  arrange(desc(abs(Factor5))) %>%
  select(Factor5, trait) %>%
  slice(1:10)
```

```
##       Factor5    trait
## 1  0.6956846  compete
## 2  0.6743459 ambitiou
## 3  0.3453425     risk
## 4  0.3423456  individ
## 5  0.2808623   athlet
## 6  0.2695570 leaderab
## 7  0.2449656   decide
## 8  0.2064415 dominant
## 9  0.1928159  leadact
## 10 0.1854989  strpers
```

Ambitious, competitive (with a bit of risk-taking and individualism): Being the best.

Factor 6

```
loadings %>%
  arrange(desc(abs(Factor6))) %>%
  select(Factor6, trait) %>%
  slice(1:10)
```

```
##       Factor6    trait
## 1   0.8675651  leadact
## 2   0.6078869 leaderab
## 3   0.3378645 dominant
## 4   0.2014835 forceful
## 5  -0.1915632      shy
## 6   0.1789256     risk
## 7   0.1703440 masculin
## 8   0.1639190   decide
## 9   0.1594585  compete
## 10  0.1466037   athlet
```

Acts like a leader, leadership ability (with a bit of Dominant): Taking charge.

Factor 7

```
loadings %>%
  arrange(desc(abs(Factor7))) %>%
  select(Factor7, trait) %>%
  slice(1:10)
```

```
##       Factor7    trait
## 1   0.6698996    happy
## 2   0.6667105 cheerful
## 3  -0.5219125    moody
## 4   0.2191425   athlet
## 5   0.2126626     warm
## 6   0.1719953   gentle
## 7  -0.1640302 masculin
## 8   0.1601472  reliant
## 9   0.1472926 yielding
## 10  0.1410481  lovchil
```

Acts like a leader, leadership ability (with a bit of Dominant): Taking charge.

Factor 8

```
loadings %>%
  arrange(desc(abs(Factor8))) %>%
  select(Factor8, trait) %>%
  slice(1:10)
```

```
##       Factor8    trait
## 1   0.6296764   affect
## 2   0.5158355  flatter
## 3  -0.2512066 softspok
## 4   0.2214623     warm
## 5   0.1878549   tender
## 6   0.1846225  strpers
## 7  -0.1804838      shy
## 8   0.1801992  compete
## 9   0.1658105    loyal
## 10  0.1548617  helpful
```

Affectionate, flattering: Making others feel good.

Factor 9

```
loadings %>%
  arrange(desc(abs(Factor9))) %>%
  select(Factor9, trait) %>%
  slice(1:10)
```

```
##       Factor9    trait
## 1   0.8633171    stand
## 2   0.3403294   defbel
## 3   0.2446971  individ
## 4   0.1941110     risk
## 5  -0.1715481      shy
## 6   0.1710978   decide
## 7   0.1197126   assert
## 8   0.1157729 conscien
## 9   0.1120308   analyt
## 10 -0.1115140 gullible
```

Taking a stand.

Factor 10

```
loadings %>%
  arrange(desc(abs(Factor10))) %>%
  select(Factor10, trait) %>%
  slice(1:10)
```

```
##       Factor10    trait
## 1   0.80751267 feminine
## 2  -0.26378513 masculin
## 3   0.24507184 softspok
## 4   0.23175597 conscien
## 5   0.20192035 selfsuff
## 6   0.17584233 yielding
## 7   0.14127067   gentle
## 8   0.11282028  flatter
## 9   0.10934531   decide
## 10 -0.09407978  lovchil
```

Feminine. (A little bit of not-masculine!)

Factor 11

```
loadings %>%
  arrange(desc(abs(Factor11))) %>%
  select(Factor11, trait) %>%
  slice(1:10)
```

```
##      Factor11    trait
## 1  0.91622589    loyal
## 2  0.18949077   affect
## 3  0.15883857 truthful
## 4  0.12464529  helpful
## 5  0.10440664   analyt
## 6  0.10076794   tender
## 7  0.09720457  lovchil
## 8  0.09635223 gullible
## 9  0.09350623 cheerful
## 10 0.08207596 conscien
```

Loyal.

Factor 12

```
loadings %>%
  arrange(desc(abs(Factor12))) %>%
  select(Factor12, trait) %>%
  slice(1:10)
```

```
##      Factor12    trait
## 1   0.6106933 childlik
## 2  -0.2845004 selfsuff
## 3  -0.2786751 conscien
## 4   0.2588843    moody
## 5   0.2013245      shy
## 6  -0.1669301   decide
## 7   0.1542031 masculin
## 8   0.1455526 dominant
## 9   0.1379163  compass
## 10 -0.1297408 leaderab
```

Childlike. (With a bit of moody, shy, not-self-sufficient, not-conscientious.)

Factor 13

```
loadings %>%
  arrange(desc(abs(Factor13))) %>%
  select(Factor13, trait) %>%
  slice(1:10)
```

```
##      Factor13    trait
## 1   0.5729242 truthful
## 2  -0.2776490 gullible
## 3   0.2631046    happy
## 4   0.1885152     warm
## 5  -0.1671924      shy
## 6   0.1646031    loyal
## 7  -0.1438127 yielding
## 8  -0.1302900   assert
## 9   0.1137074   defbel
## 10 -0.1105583  lovchil
```

Truthful. (With a bit of happy and not-gullible.)

Factor 14

```
loadings %>%
  arrange(desc(abs(Factor14))) %>%
  select(Factor14, trait) %>%
  slice(1:10)
```

```
##      Factor14    trait
## 1   0.4429926   decide
## 2   0.2369714 selfsuff
## 3   0.1945034 forceful
## 4  -0.1862756 softspok
## 5   0.1604175     risk
## 6  -0.1484606  strpers
## 7   0.1461972 dominant
## 8   0.1279456    happy
## 9   0.1154479  compass
## 10  0.1054078 masculin
```

Decisive. (With a bit of self-sufficient and not-soft-spoken.)

Factor 15

```
loadings %>%
  arrange(desc(abs(Factor15))) %>%
  select(Factor15, trait) %>%
  slice(1:10)
```

```
##      Factor15    trait
## 1  -0.3244092  compass
## 2   0.2471884   athlet
## 3   0.2292980 sensitiv
## 4   0.1986878     risk
## 5  -0.1638296   affect
## 6   0.1632164    moody
## 7  -0.1118135  individ
## 8   0.1100678     warm
## 9   0.1047347 cheerful
## 10  0.1012342  reliant
```

Not-compassionate, athletic, sensitive: A mixed bag. (“Cares about self”?)

Anything left out? Uniquenesses

```
enframe(bem.15$uniquenesses, name="quality", value="uniq") %>%
  arrange(desc(uniq)) %>%
  slice(1:10)
```

```
## # A tibble: 10 x 2
##    quality   uniq
##    <chr>    <dbl>
##  1 foullang 0.914
##  2 lovchil  0.824
##  3 analyt   0.812
##  4 yielding 0.791
##  5 masculin 0.723
##  6 athlet   0.722
##  7 shy      0.703
##  8 gullible 0.700
##  9 flatter  0.663
## 10 helpful  0.652
```

Uses foul language especially, also loves children and analytical. So could use even more factors.

Section 3

Confirmatory factor analysis}

Confirmatory factor analysis

- Exploratory: what do data suggest as hidden underlying factors (in terms of variables observed)?
- Confirmatory: have *theory* about how underlying factors depend on observed variables; test whether theory supported by data:
- does theory provide *some* explanation (better than nothing)
- can we do better?
- Also can compare two theories about factors: is more complicated one significantly better than simpler one?

Children and tests again

- Previously had this correlation matrix of test scores (based on 145 children):

```
km
```

```
##       para  sent  word   add  dots
## [1,] 1.000 0.722 0.714 0.203 0.095
## [2,] 0.722 1.000 0.685 0.246 0.181
## [3,] 0.714 0.685 1.000 0.170 0.113
## [4,] 0.203 0.246 0.170 1.000 0.585
## [5,] 0.095 0.181 0.113 0.585 1.000
```

- Will use package `lavaan` for confirmatory analysis.
- Can use actual data or correlation matrix.
- Latter (a bit) more work, as we see.

Two or three steps

- Make sure correlation matrix (if needed) is handy.
- Specify factor model (from theory)
- Fit factor model: does it fit acceptably?

Terminology

- Thing you cannot observe called **latent variable**.
- Thing you *can* observe called **manifest variable**.
- Model predicts latent variables from manifest variables.
  - asserts a relationship between latent and manifest.
- We need to invent names for the latent variables.

Specifying a factor model

- Model with one factor including all the tests:

```
test.model.1 <- "ability=~para+sent+word+add+dots"
```

- and a model that we really believe, that there are two factors, a verbal and a mathematical:

```
test.model.2 <- "verbal=~para+sent+word
                 math=~add+dots"
```

- Note the format: really all one line between single quotes, but putting it on several lines makes the layout clearer.
- Also note special notation =~ for "this latent variable depends on these observed variables".

Fitting a 1-factor model

- Need to specify model, correlation matrix, n like this:

```
fit1 <- cfa(test.model.1,
  sample.cov = km,
  sample.nobs = 145
)
```

- Has `summary`, or briefer version like this:

```
fit1
```

```
## lavaan 0.6-3 ended normally after 16 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         10
##
##   Number of observations                           145
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      59.886
##   Degrees of freedom                                 5
##   P-value (Chi-square)                           0.000
```

Two-factor model

```
fit2 <- cfa(test.model.2, sample.cov = km, sample.nobs = 145)
fit2
```

```
## lavaan 0.6-3 ended normally after 25 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         11
##
##   Number of observations                           145
##
##   Estimator                                         ML
##   Model Fit Test Statistic                       2.951
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.566
```

- This fits OK: 2-factor model supported by the data.
- 1-factor model did not fit. We really need 2 factors.
- Same conclusion as from `factanal` earlier.

Comparing models

- Use anova as if this were a regression:

```
anova(fit1, fit2)
```

```
## Chi Square Difference Test
##
##      Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit2  4 1776.7 1809.4  2.9509
## fit1  5 1831.6 1861.4 59.8862     56.935       1  4.504e-14
##
## fit2
## fit1 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- 2-factor model fits significantly better than 1-factor.
- No surprise!

Track and field data, yet again

- `cfa` works easier on actual data, such as the running records:

```
track %>% print(n = 6)
```

```
## # A tibble: 55 x 9
##    m100  m200  m400  m800 m1500 m5000 m10000 marathon
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>    <dbl>
## 1  10.4  20.8  46.8  1.81  3.7   14.0   29.4     138.
## 2  10.3  20.1  44.8  1.74  3.57  13.3   27.7     128.
## 3  10.4  20.8  46.8  1.79  3.6   13.3   27.7     136.
## 4  10.3  20.7  45.0  1.73  3.6   13.2   27.4     130.
## 5  10.3  20.6  45.9  1.8   3.75  14.7   30.6     147.
## 6  10.2  20.4  45.2  1.73  3.66  13.6   28.6     133.
## # … with 49 more rows, and 1 more variable: country <chr>
```

- Specify factor model. Factors seemed to be "sprinting" (up to 800m) and "distance running" (beyond):

```
track.model <- "sprint=~m100+m200+m400+m800
                distance=~m1500+m5000+m10000+marathon"
```

Fit and examine the model

- Fit the model. The observed variables are on different scales, so we should standardize them first via `std.ov`:

```
track.1 <- track %>%
  select(-country) %>%
  cfa(track.model, data = ., std.ov = T)
track.1
```

```
## lavaan 0.6-3 ended normally after 59 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         17
##
##   Number of observations                            55
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      87.608
##   Degrees of freedom                                19
##   P-value (Chi-square)                           0.000
```

- This fits badly. Can we do better?
- Idea: move middle distance races (800m, 1500m) into a third factor.

Factor model 2

- Define factor model:

```
track.model.2 <- "sprint=~m100+m200+m400
                  middle=~m800+m1500
                  distance=~m5000+m10000+marathon"
```

- Fit:

```
track %>%
  select(-country) %>%
  cfa(track.model.2, data = ., std.ov = T) -> track.2
```

Examine

```
track.2
```

```
## lavaan 0.6-3 ended normally after 72 iterations
##
##   Optimization method                           NLMINB
##   Number of free parameters                         19
##
##   Number of observations                            55
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      40.089
##   Degrees of freedom                                17
##   P-value (Chi-square)                           0.001
```

- Fits marginally better, though still badly.

Comparing the two models

- Second model doesn't fit well, but is it better than first?

```r
anova(track.1, track.2)
```

```
## Chi Square Difference Test
##
##         Df    AIC    BIC  Chisq Chisq diff Df diff
## track.2 17 535.49 573.63 40.089
## track.1 19 579.01 613.13 87.608     47.519       2
##         Pr(>Chisq)
## track.2
## track.1  4.802e-11 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Oh yes, a lot better.