# The intcalc package

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### Abstract

This package provides expandable arithmetic operations with integers.

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<sup>\*</sup>Please report any issues at https://github.com/ho-tex/oberdiek/issues

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# 1 Documentation

### 1.1 Introduction

Package intcalc defines arithmetic operations that deal with integers. Integers mean numbers in TeX. The same restrictions apply, the range is limited to [-2147483647, 2147483647].

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \intcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in TeX, even inside \number, \csname, file names, or other expandable contexts.

The package contains two implementations of the operations. If  $\varepsilon$ -TEX is detected then the macros are implemented using its features (\numexpr). Otherwise the slower implementation without  $\varepsilon$ -TEX's help is choosen.

### 1.2 Conditions

### 1.2.1 Preconditions

• Arguments can be anything that TEX interprets as "number". Examples: plain numbers, count or length register, macros that expands to a number.

- The arguments are limited to the range -2147483647 until 2147483647. These numbers belong to the range. Note that some operations have additionals restrictions to the range.
- The argument may be expressions that  $\numexpr$  understands if  $\varepsilon$ -TeX is available.
- The resulting number must fit in the allowed range.

### 1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, after \number, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one to ten digits. The first digit is larger than zero for numbers that consists of more than one digit.

In short, the number format is exactly the same as \number generates. And the tokens (minus sign, digits) have catcode 12 (other).

• Call by value is simulated. First the arguments are converted to numbers. Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or  $\varepsilon$ -TEX expressions. This strategy avoids multiple evaluations of such arguments.

# 1.3 Error handling

There are two kinds of errors if a precondition is violated: Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \IntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TeX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

If a number exceeds the range of -2147483647 until 2147483647, then TeX throws an error "Number too big" and recovers by using biggest allowed value. Example for the negative number -3000000000 is replaced by -2147483647.

### 1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$Int(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \ge 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

### 1.4.1 Num

# \intcalcNum $\{\langle x \rangle\}$

Macro \intcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

### 1.4.2 Inv, Abs, Sgn

# \intcalcInv $\{\langle x \rangle\}$

Macro \intcalcInv switches the sign.

$$\operatorname{Inv}(x) := -x$$

### \intcalcAbs $\{\langle x \rangle\}$

Macro \intcalcAbs returns the absolute value of integer  $\langle x \rangle$ .

$$Abs(x) := |x|$$

# \intcalcSgn $\{\langle x \rangle\}$

Macro \intcalcSgn encodes the sign of  $\langle x \rangle$  as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

# 1.4.3 Min, Max, Cmp

# \intcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

# \intcalcMax $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcMax returns the larger of the two integers.

$$\operatorname{Max}(x,y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

# \intcalcCmp $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcCmp encodes the comparison result as number:

$$\operatorname{Cmp}(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \ifcase:

```
\ifcase\intcalcCmp{<x>}{<y>}
    $x=y$
\or
    $x>y$
\else
    $x<y$
\fi</pre>
```

# 1.4.4 Inc, Dec, Add, Sub

# \intcalcInc $\{\langle x \rangle\}$

Macro \intcalcInc increments  $\langle x \rangle$  by one.

$$Inc(x) := x + 1$$

# \intcalcDec $\{\langle x \rangle\}$

Macro \intcalcDec decrements  $\langle x \rangle$  by one.

$$Dec(x) := x - 1$$

# \intcalcAdd $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcAdd adds the two numbers.

$$Add(x, y) := x + y$$

# \intcalcSub $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcSub calculates the difference.

$$Sub(x, y) := x - y$$

### 1.4.5 Shl, Shr

# \intcalcShl $\{\langle x \rangle\}$

Macro \intcalcShl implements shifting to the left that means the number is multiplied by two. Overflow is possible. The sign is preserved.

$$Shl(x) := x * 2$$

# \intcalcShr $\{\langle x \rangle\}$

Macro \intcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

$$Shr(x) := Int(x/2)$$

### 1.4.6 Mul, Sqr, Fac, Pow

# \intcalcMul $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcMul calculates the product of  $\langle x \rangle$  and  $\langle y \rangle$ .

$$Mul(x, y) := x * y$$

### \intcalcSqr $\{\langle x \rangle\}$

Macro \intcalcSqr returns the square product.

$$Sqr(x) := x^2$$

# \intcalcFac $\{\langle x \rangle\}$

Macro \intcalcFac returns the factorial of  $\langle x \rangle$ . Negative numbers are not permitted.

$$\operatorname{Fac}(x) := x! \qquad \text{for } x \ge 0$$
 
$$(0! = 1)$$

# \intcalcPow $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcPow calculates the value of  $\langle x \rangle$  to the power of  $\langle y \rangle$ . The error "division by zero" is thrown if  $\langle x \rangle$  is zero and  $\langle y \rangle$  is negative. permitted:

$$\operatorname{Pow}(x,y) := \operatorname{Int}(x^y) \qquad \text{for } x \neq 0 \text{ or } y \geq 0$$
 
$$(0^0 = 1)$$

### 1.4.7 Div, Mod

# \intcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcDiv performs an integer division. Argument  $\langle y \rangle$  must not be zero.

$$\operatorname{Div}(x,y) := \operatorname{Int}(x/y)$$
 for  $y \neq 0$ 

# \intcalcMod $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcMod gets the remainder of the integer division. The sign follows the divisor  $\langle y \rangle$ . Argument  $\langle y \rangle$  must not be zero.

$$Mod(x, y) := x \% y$$
 for  $y \neq 0$ 

The result ranges:

$$\begin{aligned} -|y| &< \operatorname{Mod}(x,y) \leq 0 & \text{for } y < 0 \\ 0 &\leq \operatorname{Mod}(x,y) < y & \text{for } y \geq 0 \end{aligned}$$

# 1.5 Interface for programmer

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- $\bullet$  Arguments and the result must fit in range 0..2147483647.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

```
\IntCalcInc \langle number \rangle!
```

Incrementation, range: 0..2147483646.

```
\IntCalcDec \langle number \rangle!
```

Decrementation, range: 1..2147483647.

```
\IntCalcAdd \langle number A \rangle ! \langle number B \rangle !
```

Addition,  $A \geq B$ .

```
\IntCalcSub \langle number A \rangle ! \langle number B \rangle !
```

Subtraction,  $A \geq B$ .

```
\IntCalcShl\ \langle number \rangle!
```

Left shift (multiplication with two), range: 0..1073741823.

Right shift (integer division by two).

\IntCalcMul 
$$\langle number A \rangle$$
!  $\langle number B \rangle$ !

Multiplication,  $A \geq B$ .

\IntCalcDiv 
$$\langle number A \rangle$$
 !  $\langle number B \rangle$  !

Division operation.

```
\IntCalcMod \langle number A \rangle ! \langle number B \rangle !
```

Modulo operation.

# 2 Implementation

1 (\*package)

# 2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
{\tt 2 \ begingroup\ catcode 61 \ catcode 48 \ catcode 32 = 10 \ relax\%}
```

- $3 \cdot catcode13=5 \% ^M$
- 4 \endlinechar=13 %
- 5 \catcode35=6 % #
- 6 \catcode39=12 % '
- $7 \ \text{catcode}44=12 \ \%$  ,
- 8 \catcode45=12 % 9 \catcode46=12 % .
- 9 \catcode46=12 \% :
- 11 \catcode64=11 % @
- 12 \catcode123=1 % {
- 13 \catcode125=2 % }

```
\expandafter\let\expandafter\x\csname ver@intcalc.sty\endcsname
 14
     \ifx\x\relax % plain-TeX, first loading
 15
 16
     \else
       \def\empty{}%
 17
       \ifx\x\empty % LaTeX, first loading,
  18
 19
        \% variable is initialized, but \ProvidesPackage not yet seen
 20
 21
        \expandafter\ifx\csname PackageInfo\endcsname\relax
          \def\x#1#2{\%}
 22
           \immediate\write-1{Package #1 Info: #2.}%
 23
         }%
 24
        \else
 25
          \def\x#1#2{\PackageInfo{#1}{#2, stopped}}%
 26
 27
        \x{intcalc}{The package is already loaded}%
 28
 29
        \aftergroup\endinput
 30
       \fi
     \fi
 31
 32 \endgroup%
Package identification:
 33 \begingroup\catcode61\catcode48\catcode32=10\relax%
 34 \catcode13=5 % ^^M
 35 \endlinechar=13 %
 36 \catcode35=6 % #
 37 \catcode39=12 % '
 38 \catcode40=12 % (
     \catcode41=12 % )
 39
     \catcode44=12 % ,
 40
 41
     \catcode45=12 % -
 42
     \catcode46=12 % .
 43
     \catcode47=12 % /
     \catcode58=12 % :
     \catcode64=11 % @
 45
     \catcode91=12 % [
 46
     \catcode93=12 % ]
 47
     \catcode123=1 % {
 48
     \catcode125=2 % }
 49
     \expandafter\ifx\csname ProvidesPackage\endcsname\relax
 50
       \def\x#1#2#3[#4]{\endgroup}
 51
        \immediate\write-1{Package: #3 #4}%
 52
        \xdef#1{#4}%
 53
      }%
 54
 55
     \else
 56
       \def\x#1#2[#3]{\endgroup}
 57
        #2[{#3}]%
        \ifx#1\@undefined
 58
         \xdef#1{#3}%
 59
        \fi
 60
 61
        \int \frac{1}{r} dx
          \xdef#1{#3}%
 62
        \fi
 63
 64
       }%
 65
     \fi
 66 \expandafter\x\csname ver@intcalc.sty\endcsname
 67 \ProvidesPackage{intcalc}%
 68 [2016/05/16 v1.2 Expandable calculations with integers (HO)]%
      Catcodes
 69 \begingroup\catcode61\catcode48\catcode32=10\relax%
 70 \catcode13=5 % ^^M
```

```
71 \endlinechar=13 %
```

```
72 \catcode123=1 % {
          73 \catcode125=2 % }
                                \catcode64=11 % @
                                \def\x{\endgroup
          75
          76
                                         \expandafter\edef\csname InCa@AtEnd\endcsname{%
          77
                                                   \endlinechar=\the\endlinechar\relax
          78
                                                   \catcode13=\the\catcode13\relax
          79
                                                  \catcode32=\the\catcode32\relax
                                                  \color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\t
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          81
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          84
                                       }%
          85
          86 }%
          87 \x \catcode61\catcode48\catcode32=10\relax\%
         88 \catcode13=5 % ^^M
         89 \endlinechar=13 %
         90 \catcode35=6 \% #
         91 \catcode64=11 % @
         92 \catcode123=1 % {
         93 \catcode125=2 % }
         94 \def\TMP@EnsureCode#1#2{%
         95 \edef\InCa@AtEnd{%
                                         \InCa@AtEnd
         97
                                          \color=\the\color=1\
         98 }%
         99 \catcode#1=#2\relax
     100 }
     101 \TMP@EnsureCode{33}{12}%!
     102 \TMP@EnsureCode{40}{12}% (
     103 \TMP@EnsureCode{41}{12}%)
     104 \TMP@EnsureCode{42}{12}% *
     105 \TMP@EnsureCode{43}{12}% +
     106 \TMP@EnsureCode{45}{12}% -
     107 \TMP@EnsureCode{47}{12}% /
     108 \TMP@EnsureCode{58}{11}%: (letter!)
     109 \TMP@EnsureCode{60}{12}% <
     110 \TMP@EnsureCode\{62\}\{12\}\% >
     111 \TMP@EnsureCode{63}{14}% ? (comment!)
     113 \begingroup\expandafter\expandafter\expandafter\endgroup
     114 \expandafter\ifx\csname InCa@TestMode\endcsname\relax
     115 \else
     116 \catcode63=9 % ? (ignore)
     117 \fi
     118 ? \\ let \\ In Ca@@Test \\ Mode \\ In Ca@Test \\ Mode \\
                                 Macros independent of \varepsilon-T<sub>E</sub>X
2.3.1 Abs, Sgn
     119 \def\InCa@Abs#1#2!{%
     120 \ifx#1-%
                                        #2%
     121
     122 \else
     123
                                         #1#2%
     124 \fi
     125 }
```

\InCa@Abs

\InCa@Sgn

126 \def\InCa@Sgn#1#2!{%

```
127 \ifx#1-%
      -1%
128
     \else
129
130
      \ifx#10%
131
       0%
132
      \else
133
        1%
      \fi
134
135 \fi
136 }
```

### 2.3.2 Min, Max, Cmp

\InCa@Min

```
137 \def\InCa@Min#1!#2!{%

138 \ifnum#1<#2 %

139 #1%

140 \else

141 #2%

142 \fi

143 }
```

 $\InCa@Max$ 

```
144 \def\InCa@Max#1!#2!{%

145 \ifnum#1>#2 %

146 #1%

147 \else

148 #2%

149 \fi

150 }
```

\InCa@Cmp

```
151 \def\InCa@Cmp#1!#2!{%
    \ifnum#1=#2 %
152
     0%
153
    \else
154
      \ifnum#1<#2 %
155
156
       -%
157
      \fi
158
     1%
159 \fi
160 }
```

### 2.3.3 Fac

\InCa@Fac It does not make much sense to calculate the faculty by an general algorithm. The allowed range of arguments is too low because of the limited integer domain.

```
161 \def\InCa@Fac#1!{%
162 \ifcase#1 1% 0!
163 \or 1% 1!
164 \or 2% 2!
165 \or 6% 3!
166 \or 24% 4!
167 \or 120% 5!
168 \or 720% 6!
169 \or 5040% 7!
170 \or 40320% 8!
171 \or 362880% 9!
172 \or 3628800% 10!
173 \or 39916800% 11!
174 \or 479001600% 12!
175 \else
```

```
\lim #1<\z@
 176
         0\IntCalcError:FacNegative%
 177
 178
       \else
         0\IntCalcError:FacOverflow%
 179
 180
       \fi
 181
     \fi
 182 }
      Implementation based on \varepsilon-TeX
2.4
Only \numexpr is used from \varepsilon-T<sub>E</sub>X.
 183 \begingroup\expandafter\expandafter\expandafter\endgroup
```

# 2.4.1 Num

### \intcalcNum

```
186 \def\intcalcNum#1{%
187 \the\numexpr#1\relax
188 }%
```

# 2.4.2 Inv, Abs, Sgn

#### \intcalcInv

```
189 \def\intcalcInv#1{%
190 \number-\intcalcNum{#1} %
191 }%
```

### \intcalcAbs

```
    192 \def\intcalcAbs#1{%
    193 \number\expandafter\InCa@Abs\the\numexpr#1! %
    194 }%
```

184 \expandafter\ifx\csname numexpr\endcsname\relax

### \intcalcSgn

```
    195 \def\intcalcSgn#1{%
    196 \number\expandafter\InCa@Sgn\the\numexpr#1! %
    197 }%
```

# 2.4.3 Min, Max, Cmp

# \intcalcMin

```
198 \def\intcalcMin#1#2{%
199 \number\expandafter\InCa@Min
200 \the\numexpr#1\expandafter!%
201 \the\numexpr#2! %
202 }%
```

### \intcalcMax

```
203 \def\intcalcMax#1#2{%

204 \number\expandafter\InCa@Max

205 \the\numexpr#1\expandafter!%

206 \the\numexpr#2! %

207 }%
```

### \intcalcCmp

```
208 \def\intcalcCmp#1#2{%
209 \number\expandafter\InCa@Cmp
210 \the\numexpr#1\expandafter!\the\numexpr#2! %
211 }%
```

### 2.4.4 Inc, Dec

### $\$ intcalcInc

- 212 \def\intcalcInc#1{%
- 213 \the\numexpr#1+1\relax
- 214 }%

### \intcalcDec

- 215 \def\intcalcDec#1{\%
- 216 \the\numexpr#1-1\relax
- 217 }%

### \IntCalcInc

- 218 \def\IntCalcInc#1!{%
- 219 \the\numexpr#1+1\relax
- 220 }%

### $\IntCalcDec$

- 221 \def\IntCalcDec#1!{%
- 223 }%

### 2.4.5 Add, Sub

### \intcalcAdd

- $224 \left(\frac{4}{1}\right)$
- 225 \the\numexpr#1+(#2)\relax
- 226 }%

# \intcalcSub

- 227 \def\intcalcSub#1#2{\%
- $1228 \quad \text{he} = \frac{42}{r}$
- 229 }%

# $\verb|\IntCalcAdd|$

- 230 \def\IntCalcAdd#1!#2!{%
- $231 \qquad \verb|\the| numexpr#1+#2| relax|$
- 232 }%

### $\IntCalcSub$

- 233 \def\IntCalcSub#1!#2!{%
- 234 \the\numexpr#1-#2\relax
- 235 }%

### 2.4.6 Shl, Shr

### \intcalcShl

- 236 \def\intcalcShl#1{%
- 238 }%

### 

- 239 \def\intcalcShr#1{\%
- $^{240}$  \number\expandafter\InCa@Shr\the\numexpr#1! %
- 241 }%

### \IntCalcShl

- 242 \def\IntCalcShl#1!{%
- 243 \the\numexpr#1\*2\relax
- 244 }%

```
\IntCalcShr
            245 \def\IntCalcShr#1!{%
                \theta = \frac{\#1-1}{else\#1 / fi/2 / elax}
            246
 \InCa@Shr
            248 \def\InCa@Shr#1#2!{%
                 \ifx#1-%
            249
                  -\InCa@Shr#2!%
            250
            251
                  \else
                   \ifodd#1#2 %
            252
            253
                    \t = \frac{\#2-1}{2\relax}
            254
                   \theta = 1#2/2 
            255
                   \fi
            256
                 \fi
            257
            258 }%
           2.4.7 Mul, Sqr, Fac
\intcalcMul
            259 \def\intcalcMul#1#2{%
            260
                \t = \frac{\#1}{\#2}\
            261 }%
\IntCalcMul
            262 \def\IntCalcMul#1!#2!{%
            263
                264 }%
 \intcalcSqr
            265 \def\intcalcSqr#1{\%
                \number\expandafter\InCa@Sqr\the\numexpr#1! %
            266
            267 }%
 \InCa@Sqr
            268 \def\InCa@Sqr#1!{%
            269
                 \the\numexpr#1*#1\relax
            270 }%
 \intcalcFac
            271 \def\intcalcFac#1{%
                 \number\expandafter\InCa@Fac\the\numexpr#1! %
            273 }%
           2.4.8 Pow
\intcalcPow
            274 \def\intcalcPow#1#2{\%
            275
                 \number\expandafter\InCa@Pow
                 \verb|\the\numexpr#1\expandafter!%|
            276
                 \the\numexpr#2! %
            277
            278 }%
\InCa@Pow
            279 \def\InCa@Pow#1#2!#3#4!{%
            280
                ifcase#3#4 \% power = 0
                  1%
            281
                 \or \% power = 1
            282
                  #1#2%
            283
            284
                 \or \% power = 2
```

```
\theta = 1#2*#1#2\
                285
                286
                       \frac{4}{2}\% basis = 0, power <> 0
                287
                288
                        ifx#3-\% power < 0
                         {\tt 0\IntCalcError:DivisionByZero\%}
                290
                291
                        \fi
                292
                       \or
                        1\% basis = 1
                293
                       \else
                294
                        \lim #1#2=\m@ne \% basis = -1
                295
                         \ifodd#3#4 %
                296
                           -%
                297
                         \fi
                298
                         1%
                299
                        \leq \% |basis| > 1
                         ifx#3-\% power < 0
                301
                          0%
                302
                         \else % power > 2
                303
                           \InCa@PowRec#1#2!#3#4!1!%
                304
                305
                         \fi
                        \fi
                306
                307
                       \fi
                     \fi
                308
                309
\InCa@PowRec
                    Pow(b, p) {
                     PowRec(b, p, 1)
                    PowRec(b, p, r) {
                     if p == 1 then
                      return r*b
                     else
                       ifodd p then
                        return PowRec(b*b, (p-1)/2, r*b) % p div 2 = (p-1)/2
                        return PowRec(b*b, (p-1)/2, r)
                       fi
                     fi
                    \def\InCa@PowRec#1!#2!#3!{%
                311
                     \lim #2=\0
                       \theta =1*\#3\
                312
                     \else
                313
                       \ifodd#2 %
                314
                        \expandafter\InCa@PowRec
                315
                        \the\numexpr#1*#1\expandafter!%
                316
                317
                        \the\numexpr#1*#3\expandafter\expandafter\expandafter!%
                318
                319
                320
                        \expandafter\InCa@PowRec
                321
                        \the\numexpr#1*#1\expandafter!%
                322
                        323
                        \number#3\expandafter\expandafter\expandafter!%
                324
                     \fi
                325
                    }%
               326
```

# 2.4.9 Div, Mod

 $T_EX$ 's \divide truncates,  $\varepsilon$ - $T_EX$ 's \numexpr rounds the result of a division. The rounding method is called "Symmetric Arithmetic Rounding" or "Round-Half-Up" ("Kaufmännisches Runden" in German):

```
1 = 3 \text{ divide } 2 = 1.5 = \text{numexpr } 3/2 = 2
-1 = -3 divide 2 = -1.5 = numexpr -3/2 = -2
```

Macro  $\interline{\operatorname{Interline}}$  follows  $\interline{\operatorname{Interline}}$  and truncates. The calculation is done by the following formula:

$$Div(X,Y) = (X - (Y - 1)/2)/Y \quad \text{for } X, Y > 0$$
 (1)

The operator '/' is \numexpr's division.

```
\intcalcDiv
```

```
327 \def\intcalcDiv#1#2{%
328 \number\expandafter\InCa@Div
329 \the\numexpr#1\expandafter!%
330 \the\numexpr#2! %
331 }%
```

### \InCa@Div

```
\def\InCa@Div#1!#2!{%
332
      \ifcase#2 %
333
        0\IntCalcError:DivisionByZero%
334
335
336
        \ifcase#1 %
337
         0%
338
        \else
         \expandafter\InCa@@Div
339
         \romannumeral 0%
340
         \lim #1<\z@
341
           \verb|\expandafter-\number-#1%|
342
343
         \else
           \ensuremath{\texttt{w}}
344
345
346
         \expandafter!%
347
         \romannumeral 0%
         \lim #2<\z@
348
           \verb|\expandafter-\number-#2%| \\
349
         \else
350
           \verb|\expandafter+\number#2%|
351
         \fi
352
353
         !%
        \fi
354
      \fi
355
356
    }%
```

### \IntCalcDiv

```
\def\InCa@Temp#1{%
357
      \def\IntCalcDiv##1!##2!{%
358
359
       \number
       \ifcase##2 %
360
        0\IntCalcError:DivisionByZero%
361
362
363
        \ifcase##1 %
          0%
364
        \else
365
          \theta = \frac{\#1-(\#\#2-1)}{2}/\#2 
366
        \fi
367
368
       \fi
369
       #1%
     }%
370
371
    \InCa@Temp{}%
```

```
\InCa@@Div
               373
                   \def\InCa@@Div#1#2!#3#4!{%
                     #1#3%
               374
               375
                     376 }%
 \intcalcMod
                   \def\intcalcMod#1#2{%
               377
                     \number\expandafter\InCa@Mod
               378
                     \the\numexpr#1\expandafter!%
               379
                     \the\numexpr#2! %
               380
               381 }%
 \InCa@Mod
                   \def\InCa@Mod#1!#2!{%
               382
                     \ifcase#2 %
               383
                      0\IntCalcError:DivisionByZero%
               384
               385
                      \ifcase#1 %
               386
               387
                        0%
               388
                      \else
                        \expandafter\InCa@@Mod
               389
                        \romannumeral 0%
               390
                        \lim #1<\z@
               391
                         \verb|\expandafter-\number-\#1\%|
               392
                        \else
               393
                         \ensuremath{\texttt{w}}
               394
               395
               396
                        \expandafter!%
               397
                        \romannumeral 0%
                        \lim #2<\z@
               398
                         \expandafter-\number-#2%
               399
               400
                        \else
                         \verb|\expandafter+\number#2%|
               401
                        \fi
               402
                        !%
               403
               404
                      \fi
                     \fi
               405
                   }%
               406
\IntCalcMod
                   \def\InCa@Temp#1{%
               407
                     \label{lintCalcMod} $$ \left( \frac{1!}{2!} \right) = \frac{1!}{2!} 
               408
                      \number
               409
                      \ifcase##2 %
               410
                        0\IntCalcError:DivisionByZero%
               411
               412
                      \else
               413
                        \ifcase##1 %
               414
                         0%
               415
                        \else
               416
                         \theta = \frac{\#1-(\#\#1-(\#\#2-1)/2)}{\#2*\#2}
               417
                        \fi
               418
                      \fi
                      #1%
               419
                     }%
               420
               421
                   }%
                   \InCa@Temp{}%
               422
\InCa@@Mod
                   \def\InCa@@Mod#1#2!#3#4!{%
               423
                     if#3+%
               424
               425
                      \if#1+%
```

```
\t \norm \#2-\InCa@@Div+\#2!+\#4!*\#4\relax
                     426
                     427
                              \expandafter\InCa@ModX
                     428
                     429
                              \t = \sum_{y=0}^{2} \ln Ca@@Div + \#2! + \#4! * \#4! \%
                     430
                     431
                           \else
                     432
                            -%
                            \inf #1+\%
                     433
                              \verb|\expandafter| In Ca@ModX|
                     434
                              \label{linear_property} $$ \theta^-= 4!+4!*#4!#4!\% $$
                     435
                     436
                              \label{localize} $$ \ \pi^2-\ln Ca@@Div+\#2!+\#4!*\#4\ relax $$
                     437
                            \fi
                     438
                     439
                           \fi
                     440 }%
    \InCa@ModX
                         \def\InCa@ModX#1!#2!{%
                     441
                           \ifcase#1 %
                     442
                            0%
                     443
                     444
                           \else
                     445
                            <text> \the\numexpr#1+#2\relax
                     446
                           \fi
                     447 }%
                     448 \expandafter\InCa@AtEnd
                    449 \fi%
                          Implementation without \varepsilon-TeX
                   2.5
                   2.5.1 Num
      \intcalcNum
                     450 \left\ \frac{1}{\%}\right
                    451 \number\expandafter\InCa@FirstOfOne\number#1! \%
                    452 }
                   2.5.2 Inv, Abs, Sgn
       \intcalcInv
                    453 \left| def \right| 110
                    454 \number\expandafter\InCa@FirstOfOne\number-#1! %
                     455 }
\InCa@FirstOfOne
                     456 \def\InCa@FirstOfOne#1!{#1}
       \intcalcAbs
                     457 \left( \frac{457}{e} \right)
                     458 \number\expandafter\InCa@Abs\number#1! %
                    459 }
       \intcalcSgn
                     460 \def\intcalcSgn#1{%
                     461 \number\expandafter\InCa@Sgn\number#1! %
                     462 }
```

### 2.5.3 Min, Max, Cmp

```
\intcalcMin
                                                       463 \def\intcalcMin#1#2{\%
                                                       464 \necent length \necent length \necent \n
                                                       465 \number\number#1\expandafter!\number#2! \%
                                                       466 }
               \ intcalcMax
                                                       467 \left( \frac{467}{\text{mtcalcMax}} #1#2 \right)
                                                       469 \number\number#1\expandafter!\number#2! \%
                                                       470 }
              \intcalcCmp
                                                       471 \def\intcalcCmp#1#2{%
                                                       472 \number\expandafter\InCa@Cmp
                                                       473 \number\number#1\expandafter!\number#2! \%
                                                      474 }%
                                                    2.5.4 Inc, Dec
                   \intcalcInc
                                                       475 \left| \frac{475}{f}\right|
                                                      476 \number\expandafter\InCa@IncSwitch\number#1! \%
                                                      477 }
 \InCa@IncSwitch
                                                       478 \def\InCa@IncSwitch#1#2!{%
                                                      479 \ifx#1-%
                                                                     -%
                                                       480
                                                                       \csname InCa@Empty%
                                                       481
                                                                    \InCa@Dec#2!%
                                                       482
                                                       483 \else
                                                                      \csname InCa@Empty%
                                                       484
                                                       485
                                                                       \InCa@Inc#1#2!%
                                                       486 \fi
                                                       487 }
                 \intcalcDec
                                                       488 \def\intcalcDec#1{%
                                                       489 \number\expandafter\InCa@DecSwitch\number#1! %
                                                       490 }
\InCa@DecSwitch
                                                       491 \def\InCa@DecSwitch#1#2!{\%
                                                       492 \ifx#1-%
                                                       493
                                                                      -%
                                                                       \csname InCa@Empty%
                                                       494
                                                                       \expandafter\InCa@Inc#2!%
                                                       495
                                                       496 \else
                                                       497
                                                                        \ifx#10%
                                                       498
                                                                           -1%
                                                       499
                                                                        \else
                                                                            \csname InCa@Empty%
                                                       500
                                                                           \InCa@Dec#1#2!%
                                                       501
                                                                   \fi
                                                       502
                                                       503 \fi
                                                       504 }
```

```
\IntCalcInc
                    505 \def\IntCalcInc#1!{%
                    506 \number\csname InCa@Empty\InCa@Inc#1! \%
                    507 }
       \IntCalcDec
                    508 \def\IntCalcDec#1!{%
                    509 \number\csname InCa@Empty\InCa@Dec#1! %
                    510 }
        \InCa@Inc
                    511 \def\InCa@Inc#1#2{\%}
                    512 \ifx#2!%
                         \csname InCa@IncDigit#1\endcsname1%
                    513
                    514 \else
                          \csname InCa@IncDigit#1%
                    515
                    516 \expandafter\InCa@Inc\expandafter#2%
                    517 \fi
                    518 }
\InCa@IncDigit[0-8]
                    519 \def\InCa@Temp#1#2{\%}
                    520 \quad \texttt{\expandafter\def\csname\ InCa@IncDigit\#1\endcsname\#\#1\{\%, 1\}} \\
                    521
                          \endcsname
                          0%
                    522
                    523 \ifcase##1 %
                    524
                           #1%
                    525
                         \else
                    526
                           #2%
                          \fi
                    527
                    528 }%
                    529 }
                    530 \InCa@Temp 01
                    531 \InCa@Temp 12
                    532 \InCa@Temp 23
                    533 \InCa@Temp 34
                    534 \InCa@Temp 45
                    535 \InCa@Temp 56
                    536 \InCa@Temp 67
                    537 \InCa@Temp 78
                    538 \InCa@Temp 89
  \InCa@IncDigit9
                    539 \expandafter\def\csname InCa@IncDigit9\endcsname#1{%
                    540 \expandafter\endcsname
                    541 \ifcase#1 \%
                    542
                          09%
                    543 \else
                          10%
                    544
                    545 \fi
                    546 }
       \InCa@Dec
                    547 \ensuremath{\mbox{lnCa@Dec\#1\#2}}\%
                    548 \ifx#2!%
                          \csname InCa@DecDigit#1\endcsname1%
                    549
                    550 \else
                          \csname InCa@DecDigit#1%
                    551
                          \expandafter\InCa@Dec\expandafter#2%
                    552
                    553 \fi
                    554 }
```

```
\InCa@DecDigit[1-9]
                      555 \def\InCa@Temp#1#2{%
                           \expandafter\def\csname InCa@DecDigit#1\endcsname##1{%
                      557
                      558
                            0%
                            \left| \frac{\#1 \%}{\%} \right|
                      559
                              #1%
                      560
                            \else
                      561
                              #2%
                      562
                            \fi
                      563
                      564 }%
                      565 }
                      566 \InCa@Temp 98
                      567 \InCa@Temp 87
                      568 \InCa@Temp 76
                      569 \InCa@Temp 65
                      570 \InCa@Temp 54
                      571 \InCa@Temp 43
                      572 \InCa@Temp 32
                      573 \InCa@Temp 21
                      574 \InCa@Temp 10
  \InCa@DecDigit0
                      575 \expandafter\def\csname InCa@DecDigit0\endcsname#1{%
                           \expandafter\endcsname
                      576
                           \ifcase#1 %
                      577
                            00%
                      578
                           \else
                      579
                            19%
                      580
                      581 \fi
                      582 }
                     2.5.5 Add, Sub
        \intcalcAdd
                      583 \def\intcalcAdd#1#2{\%}
                      584 \number
                            \verb|\expandafter| In Ca@AddSwitch|
                      585
                            \verb|\number| 1 \le number #1 \le number! \%
                      586
                            \number#2! %
                      587
                      588 }
```

### \intcalcSub

```
589 \def\intcalcSub#1#2{%
590 \number
591 \expandafter\InCa@AddSwitch
592 \number\number#1\expandafter!%
593 \number-\number#2! %
594 }
```

\InCa@AddSwitch Decision table for \InCa@AddSwitch. The sign of negative numbers can be removed by a simple \@gobble instead of the more expensive \number-.

x < 0	y < 0	x < y	_	Add $(-x, -y)$
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	Sub(x, -y)
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)

```
595 \def\InCa@AddSwitch#1!#2!{%
                   \lim #1<\z@
               596
                     \lim #2<\z@
               597
               598
                      -%
               599
                      \ifnum#1<#2 %
               600
                        \expandafter\InCa@Add\number-#1\expandafter!%
               601
                        \@gobble#2!%
               602
                      \else
                        \verb|\expandafter| In Ca@Add \\| number-\#2 \\| expandafter! \\| \%
               603
                        \@gobble#1!%
               604
               605
                      \fi
               606
                     \else
                      \ifnum-#1>#2 %
               607
               608
               609
                        \expandafter\InCa@Sub\@gobble#1!#2!%
               610
                        \expandafter\InCa@Sub\number#2\expandafter!%
               611
                        \@gobble#1!\%
               612
               613
                      \fi
               614
                     \fi
               615
                   \else
                     \lim #2<\z@
               616
                      \ifnum#1>-#2 %
               617
                        \expandafter\InCa@Sub\number#1\expandafter!%
               618
               619
                        \@gobble#2!%
               620
                      \else
                       -%
               621
                        \verb|\expandafter| In Ca@Sub|@gobble#2!#1!%|
               622
               623
                      \fi
               624
                     \else
                      \ifnum#1>#2 %
               625
               626
                       \InCa@Add#1!#2!%
               627
                        \InCa@Add#2!#1!%
               628
               629
                      \fi
               630
                     \fi
               631 \fi
              632 }
\IntCalcAdd
               633 \def\IntCalcAdd#1!#2!{%
               634 \number\InCa@Add#1!#2! %
               635 }
\IntCalcSub
               636 \def\IntCalcSub#1!#2!{%
                   \number\InCa@Sub#1!#2! %
               637
               638 }
\InCa@Space
               639 \begingroup
               640 \def\x#1{\endgroup}
                     \let\InCa@Space= #1%
               641
               642 }%
               643 \x{ }
 \InCa@Add
               644 \def\InCa@Add#1!#2!{%
               645 \ifcase#2 %
                    #1%
               646
               647 \else
                    \InCa@@Add#1!#2!00000000\InCa@Space
               648
```

```
649 \fi
               650 }
  \InCa@Sub
               651 \def\InCa@Sub#1!#2!{%
               652 \ifnum#1=#2 %
               653
                    0%
               654 \else
               655
                    \InCa@@Sub#1!#2!00000000\InCa@Space
               656 \fi
               657 }
 \InCa@@Add
               658 \def\InCa@@Add#1!#2#3!{%
               659 \ifx\InCa@Empty#3\InCa@Empty
               660
                    \@ReturnAfterElseFi{%
               661
                      \InCa@@@Add!!#1!#2%
               662
               663 \else
                    \@ReturnAfterFi{%
               664
                      \InCa@@Add#1!#3!#2%
               665
                    }%
               666
               667 \fi
               668 }
 \InCa@@Sub
               669 \def\InCa@@Sub#1!#2#3!{%
               670 \ifx\InCa@Empty#3\InCa@Empty
                     \@ReturnAfterElseFi{%
               671
                      \InCa@@@Sub!!#1!#2%
               672
                    ٦%
               673
               674 \else
                     \@ReturnAfterFi{%
               675
                      \InCa@@Sub#1!#3!#2%
               676
               677
                    }%
               678 \fi
               679 }
\InCa@@@Add
               680 \def\InCa@@@Add#1!#2!#3#4!#5{%
               681 \ifx\InCa@Empty#4\InCa@Empty
                     \csname InCa@Empty%
               682
               683
                     \@ReturnAfterElseFi{%
               684
                      \InCa@ProcessAdd#1#3!#5#2%
               685
                    }%
               686 \else
                     \ensuremath{\texttt{QReturnAfterFi}}\%
               687
                      \InCa@@@Add#1#3!#5#2!#4!%
               688
                    }%
               689
               690 \fi
               691 }
\InCa@@@Sub
               692 \def\InCa@@@Sub#1!#2!#3#4!#5{%
               693 \ifx\InCa@Empty#4\InCa@Empty
               694
                    \csname @gobble%
                     \@ReturnAfterElseFi{%
               695
                      \InCa@ProcessSub#1#3!#5#2%
               696
                    }%
               697
               698
                   \else
                     \@ReturnAfterFi{%
               699
                      \InCa@@@Sub#1#3!#5#2!#4!%
```

```
}%
                     701
                     702 \fi
                     703 }
  \InCa@ProcessAdd
                     704 \def\InCa@ProcessAdd#1#2!#3#4{%
                     705 \ifx\InCa@Empty#2\InCa@Empty
                          \csname InCa@AddDigit#1\endcsname#3%
                     706
                     707
                          \romannumeral0#4%
                     708 \else
                     709
                           \csname\ In Ca@AddDigit\#1\\csname\ In Ca@DigitCarry\#3\%
                     710
                           \CReturnAfterFi{%
                            \InCa@ProcessAdd#2!#4%
                     711
                     712
                         1%
                     713 \fi
                     714 }
   \InCa@ProcessSub
                     715 \def\InCa@ProcessSub#1#2!#3#4{%
                     716 \ifx\InCa@Empty#2\InCa@Empty
                           \csname InCa@SubDigit#1\endcsname#3%
                     717
                           \romannumeral0#4%
                     718
                     719 \else
                           \csname InCa@SubDigit#1\csname InCa@DigitCarry#3%
                     720
                           \@ReturnAfterFi{%
                     721
                            \InCa@ProcessSub#2!#4%
                     722
                         }%
                     723
                     724 \fi
                     725 }
\InCa@DigitCarry[0-9]
                     726 \left( \frac{1}{2} \right)
                         727
                           \ifcase##1 %
                     728
                            \endcsname#1%
                     729
                           \else
                     730
                     731
                            \endcsname#2%
                     732
                          \fi
                     733 }%
                     734 }
                     735 \InCa@Temp 01
                     736 \InCa@Temp 12
                     737 \InCa@Temp 23
                     738 \InCa@Temp 34
                     739 \InCa@Temp 45
                     740 \InCa@Temp 56
                     741 \InCa@Temp 67
                     742 \InCa@Temp 78
                     743 \InCa@Temp 89
                     744 \InCa@Temp 9{{10}}
    \InCa@AddDigit0
                     745 \expandafter\def\csname InCa@AddDigit0\endcsname#1{%
                     746 \ifnum#1>9 %
                     747
                          \endcsname10%
                     748
                         \else
                     749
                          \endcsname0#1%
                     750 \fi
                     751 }
 \InCa@AddDigit[1-9]
```

752 \def\InCa@Temp#1#2#3{%

```
\expandafter\def\csname InCa@AddDigit#1\endcsname##1{%
753
     \ifnum##1>#2 %
754
755
       \endcsname 1%
756
      \else
757
       \endcsname 0%
758
      \fi
759
     \ifcase##1 #1% 0
      #3%
760
     \else #1% 10
761
     \fi
762
763 }%
764 }
765 \InCa@Temp 18{%
766 \or 2% 1
767 \or 3% 2
768 \or 4% 3
769 \or 5% 4
770 \or 6% 5
771 \or 7% 6
772 \or 8% 7
773 \or 9% 8
774 \or 0% 9
775 }%
776 \InCa@Temp 27{%
777 \or 3% 1
    \or 4% 2
778
779 \or 5% 3
780 \or 6% 4
781 \or 7% 5
782 \or 8% 6
783 \or 9% 7
784 \or 0% 8
785 \or 1% 9
786 }%
787 \InCa@Temp 36{%
788 \or 4% 1
789 \or 5% 2
790 \or 6% 3
791 \or 7% 4
792 \or 8% 5
793 \or 9% 6
794 \or 0% 7
795 \or 1% 8
796 \or 2% 9
797 }%
798 \InCa@Temp 45{%
799 \or 5% 1
800 \or 6% 2
801 \or 7% 3
802 \or 8% 4
803 \or 9% 5
804 \or 0% 6
805 \or 1% 7
806 \or 2% 8
807 \or 3% 9
808 }%
809 \InCa@Temp 54{%
810 \or 6% 1
811 \or 7% 2
812 \or 8% 3
813 \or 9% 4
814 \or 0% 5
```

```
815 \or 1% 6
                      816
                           \or 2% 7
                      817
                           \or 3% 8
                      818
                           \or 4% 9
                      819 }%
                      820 \InCa@Temp 63{%
                      821 \or 7% 1
                           \or 8% 2
                      822
                      823 \or 9% 3
                      824 \or 0% 4
                      825 \or 1% 5
                      826 \or 2% 6
                      827 \or 3% 7
                      828 \or 4% 8
                      829 \or 5% 9
                      830 }%
                      831 \InCa@Temp 72{%
                      832 \or 8% 1
                      833 \or 9% 2
                      834 \or 0% 3
                           \or 1% 4
                      835
                      836
                           \or 2% 5
                           \or 3% 6
                      837
                           \or 4% 7
                      838
                      839
                           \or 5% 8
                      840 \or 6% 9
                      841 }%
                      842 \InCa@Temp 81{\%}
                      843 \or 9% 1
                      844 \or 0% 2
                      845 \or 1% 3
                      846 \or 2% 4
                      847 \or 3% 5
                      848 \or 4% 6
                      849 \or 5% 7
                      850 \or 6% 8
                      851 \or 7% 9
                      852 }%
                      853 \label{lineal} InCa@Temp 90{\%}
                      854 \setminus 00\% 1
                      855 \or 1% 2
                      856 \or 2% 3
                      857
                           \or 3% 4
                      858
                           \or 4% 5
                      859
                           \or 5% 6
                           \or 6% 7
                           \or 7% 8
                      862 \or 8% 9
                      863 }%
\label{local_subDigit} $$ \nCa@SubDigit[0-9] $$
                      864 \def\InCa@Temp#1#2\%
                           \expandafter\def\csname InCa@SubDigit#1\endcsname##1{%
                      865
                             \ifnum##1>#1 %
                      866
                      867
                              \endcsname 1%
                      868
                             \else
                      869
                              \endcsname 0%
                      870
                             \fi
                      871
                             \ifcase##1 #1% 0
                             #2%
                      872
                             \else #1% 10
                      873
                             \fi
                      874
```

875 }%

```
876 }
877 \InCa@Temp 0{%
878 \or 9% 1
879 \or 8% 2
880 \or 7% 3
881 \or 6% 4
882 \or 5% 5
883 \or 4% 6
884 \or 3% 7
885 \or 2% 8
886 \or 1% 9
887 }
888 \InCa@Temp 1{%
889 \or 0% 1
890 \or 9% 2
891 \or 8% 3
892 \or 7% 4
893 \or 6% 5
894 \or 5% 6
895 \or 4% 7
896 \or 3% 8
897 \or 2% 9
898 }
899 \InCa@Temp 2{%
900 \or 1% 1
901 \or 0% 2
902 \or 9% 3
903 \or 8% 4
904 \or 7% 5
905 \or 6% 6
906 \or 5% 7
907 \or 4% 8
908 \or 3% 9
909 }
910 \InCa@Temp 3{%
911 \or 2% 1
912 \or 1% 2
913 \or 0% 3
914 \or 9% 4
915 \or 8% 5
916 \or 7% 6
917 \or 6% 7
918 \or 5% 8
919 \or 4% 9
920 }
921 \InCa@Temp 4{%
922 \or 3% 1
923 \or 2% 2
924 \or 1% 3
925 \or 0% 4
926 \or 9% 5
927 \or 8% 6
928 \or 7% 7
929 \or 6% 8
930 \or 5% 9
931 }
932 \InCa@Temp 5{%
933 \or 4% 1
934 \or 3% 2
935 \or 2% 3
936 \or 1% 4
937 \or 0% 5
```

```
938 \or 9% 6
939 \or 8% 7
940 \or 7% 8
941 \or 6% 9
942 }
943 \InCa@Temp 6{\%
944 \or 5% 1
945 \or 4% 2
946 \or 3% 3
947 \or 2% 4
948 \or 1% 5
949 \or 0% 6
950 \or 9% 7
951 \or 8% 8
952 \or 7% 9
953 }
954 \InCa@Temp 7{%
955 \or 6% 1
956 \or 5% 2
957 \or 4% 3
958 \or 3% 4
959 \or 2% 5
960 \or 1% 6
961 \or 0% 7
962 \or 9% 8
963 \or 8% 9
964 }
965 \InCa@Temp 8{%
966 \or 7% 1
967 \or 6% 2
968 \or 5% 3
969 \or 4% 4
970 \or 3% 5
971 \or 2% 6
972 \or 1% 7
973 \or 0% 8
974 \or 9% 9
975 }
976 \InCa@Temp 9{\%
977 \or 8% 1
978 \or 7% 2
979 \or 6% 3
980 \or 5% 4
981 \or 4% 5
982 \or 3% 6
983 \or 2% 7
984 \or 1% 8
985 \or 0% 9
986 }
2.5.6 Shl, Shr
987 \def\intcalcShl#1{\%
988 \number\expandafter\InCa@ShlSwitch\number#1! %
989 }
990 \def\InCa@ShlSwitch#1#2!{%
991 \ifx#1-%
992 -\csname InCa@Empty%
993 \InCa@Shl#2!%
```

\intcalcShl

\InCa@ShlSwitch

```
994 \else
                          \csname InCa@Empty%
                    995
                          \InCa@Shl#1#2!%
                    996
                    997 \fi
                    998 }
       \IntCalcShl
                    999 \def\IntCalcShl#1!{%
                    1000 \number
                   1001 \csname InCa@Empty%
1002 \InCa@Shl#1! %
                   1003 }
  \IntCal@ShlDigit
                   1004 \def\InCa@Shl#1#2{%
                   1005 \ifx#2!%
                   1006
                          \csname InCa@ShlDigit#1\endcsname0%
                   1007 \else
                          \csname InCa@ShlDigit#1%
                   1008
                   1009
                          \@ReturnAfterFi{%
                           \InCa@Shl#2%
                   1010
                          }%
                   1011
                   1012 \fi
                   1013 }
  \InCa@ShlDigit0
                    1014 \expandafter\def\csname InCa@ShlDigit0\endcsname{%
                   1015 \endcsname0%
                   1016 }
\InCa@ShlDigit[1-9]
                    1017 \def\InCa@Temp#1#2#3#4#5{%
                    1018 \expandafter\def\csname InCa@ShlDigit#1\endcsname##1{\%}
                          \expandafter\endcsname
                   1019
                          \ifcase##1 %
                   1020
                           #2#3%
                   1021
                          \else
                   1022
                   1023
                           #4#5%
                   1024
                          \fi
                   1025 }%
                   1026 }
                   1027 \InCa@Temp 10203
                   1028 \InCa@Temp 20405
                   1029 \InCa@Temp 30607
                   1030 \InCa@Temp 40809
                   1031 \InCa@Temp 51011
                   1032 \InCa@Temp 61213
                   1033 \InCa@Temp 71415
                   1034 \InCa@Temp 81617
                   1035 \InCa@Temp 91819
        \intcalcShr
                   1036 \def\intcalcShr#1{\%
                   1037 \number\expandafter\InCa@ShrSwitch\number#1! \%
                   1038 }
  \InCa@ShrSwitch
                   1039 \def\InCa@ShrSwitch#1#2!{%
                   1040 \ifx#1-%
                   1041
                          -\InCa@Shr#2!%
                   1042 \else
                   1043 \InCa@Shr#1#2!%
```

```
1044 \fi
            1045 }
\IntCalcShr
            1046 \def\IntCalcShr#1!{\%
            1047 \number\InCa@Shr#1! %
            1048 }
\InCa@Shr
            1049 \def\InCa@Shr#1#2{\%
            1050 \InCa@ShrDigit#1!%
            1051 \ifx#2!%
            1052 \else
                  \@ReturnAfterFi{%
            1053
            1054
                    \ifodd#1 %
            1055
                     \@ReturnAfterElseFi{%
            1056
                      \InCa@Shr{1#2}%
            1057
                     }%
            1058
                     \expandafter\InCa@Shr\expandafter#2%
            1059
            1060
                    \fi
                  }%
            1061
            1062 \fi
            1063 }
            1064 \def\InCa@ShrDigit#1!{%
            1065 \ifcase#1 0% 0
            1066 \or 0% 1
            1067 \or 1% 2
            1068 \or 1% 3
            1069 \or 2% 4
            1070 \or 2% 5
            1071 \or 3% 6
            1072 \or 3% 7
            1073 \or 4% 8
            1074 \or 4% 9
            1075 \or 5% 10
            1076 \or 5% 11
            1077 \or 6% 12
            1078 \or 6% 13
            1079 \or 7% 14
            1080 \or 7% 15
            1081 \or 8% 16
            1082 \or 8% 17
            1083 \or 9% 18
            1084 \or 9% 19
            1085 \fi
            1086 }
            2.5.7 \InCa@Tim
\InCa@Tim Macro \InCa@Tim implements "Number times digit".
            1087 \def\InCa@Temp#1{%
            1088 \def\InCa@Tim##1##2{%
                   \number
            1089
            1090
                    \ifcase##2 % 0
            1091
                     0%
                    \or % 1
            1092
                     ##1%
            1093
                    \else % 2-9
            1094
                     \csname InCa@Empty%
            1095
```

\InCa@ProcessTim##2##1!%

1096

```
1097
                           \fi
                           #1%
                    1098
                    1099 }%
                    1100 }
                    1101 \InCa@Temp{ }
 \InCa@ProcessTim
                    1102 \def\InCa@ProcessTim#1#2#3{%
                    1103 \ifx#3!%
                    1104
                          \csname InCa@TimDigit#2\endcsname#10%
                    1105
                         \else
                    1106
                          \csname InCa@TimDigit#2\csname InCa@Param#1%
                    1107
                          \@ReturnAfterFi{%
                           \InCa@ProcessTim#1#3%
                    1108
                         }%
                    1109
                    1110 \fi
                    1111 }
  \InCa@Param[0-9]
                    1112 \def\InCa@Temp#1{\%}
                    1113 \expandafter\def\csname InCa@Param#1\endcsname{%
                          \endcsname#1%
                    1115 }%
                    1116 }
                    1117 \InCa@Temp 0%
                    1118 \InCa@Temp 1%
                    1119 \InCa@Temp 2%
                    1120 \InCa@Temp 3%
                    1121 \InCa@Temp 4%
                    1122 \InCa@Temp 5%
                    1123 \InCa@Temp 6%
                    1124 \InCa@Temp 7%
                    1125 \InCa@Temp 8%
                    1126 \InCa@Temp 9%
   \InCa@TimDigit0
                    1127 \expandafter\def\csname InCa@TimDigit0\endcsname#1#2{%
                    1128 \endcsname
                    1129 0#2%
                    1130 }
  \InCa@TimDigit1
                    1131 \expandafter\def\csname InCa@TimDigit1\endcsname#1#2{%
                    1132 \ifcase#2 %
                    1133
                          \endcsname 0#1%
                    1134 \else
                    1135
                          \csname InCa@AddDigit#1\endcsname #2%
                    1136 \fi
                    1137 }
\InCa@TimDigit[2-9]
                    1138 \def\InCa@Temp#1#2{%
                    1139 \expandafter\def\csname InCa@TimDigit#1\endcsname##1{\%}
                    1140
                          \expandafter\InCa@TimDigitCarry
                    1141
                          \number
                           \ifcase##1 0% 0
                    1142
                            #2%
                    1143
                           \fi
                    1144
                          !%
                    1145
                    1146 }%
                    1147 }
                    1148 \InCa@Temp 2{%
```

```
1149 \or 2% 1
1150 \or 4% 2
1151 \or 6% 3
1152 \or 8% 4
1153 \or 10% 5
1154 \or 12% 6
1155 \or 14% 7
1156 \or 16% 8
1157 \or 18% 9
1158 }
1159 \InCa@Temp 3{%
1160 \or 3% 1
1161 \or 6% 2
1162 \or 9% 3
1163 \or 12% 4
1164 \or 15% 5
1165 \or 18% 6
1166 \or 21% 7
1167 \or 24% 8
1168 \or 27% 9
1169 }
1170 \InCa@Temp 4{%
1171 \or 4% 1
1172 \or 8% 2
1173 \or 12% 3
1174 \or 16% 4
1175 \or 20% 5
1176 \or 24% 6
1177 \or 28% 7
1178 \or 32% 8
1179 \or 36% 9
1180 }
1181 \InCa@Temp 5{%
1182 \or 5% 1
1183 \or 10% 2
1184 \or 15% 3
1185 \or 20% 4
1186 \or 25% 5
1187 \or 30% 6
1188 \or 35% 7
1189 \or 40% 8
1190 \or 45% 9
1191 }
1192 \InCa@Temp 6{%
1193 \or 6% 1
1194 \or 12% 2
1195 \or 18% 3
1196 \or 24% 4
1197 \or 30% 5
1198 \or 36% 6
1199 \or 42% 7
1200 \or 48% 8
1201 \or 54% 9
1202 }
1203 \InCa@Temp 7{%
1204 \or 7% 1
1205 \or 14% 2
1206 \or 21% 3
1207 \or 28% 4
1208 \or 35% 5
1209 \or 42% 6
```

1210 \or 49% 7

```
1211 \or 56% 8
                                                                           1212 \or 63% 9
                                                                           1213 }
                                                                           1214 \InCa@Temp 8{%
                                                                           1215 \or 8% 1
                                                                          1216 \or 16% 2
                                                                           1217 \or 24% 3
                                                                           1218 \or 32% 4
                                                                          1219 \or 40% 5
                                                                          1220 \or 48% 6
                                                                          1221 \or 56% 7
                                                                          1222 \or 64% 8
                                                                          1223 \or 72% 9
                                                                          1224 }
                                                                          1225 \InCa@Temp 9{%
                                                                          1226 \or 9% 1
                                                                          1227 \or 18% 2
                                                                          1228 \or 27% 3
                                                                          1229 \or 36% 4
                                                                           1230 \or 45% 5
                                                                           1231 \or 54% 6
                                                                           1232 \or 63% 7
                                                                           1233 \or 72% 8
                                                                           1234 \or 81% 9
                                                                           1235 }
    \InCa@TimDigitCarry
                                                                           1236 \ensuremath{\mbox{\sc loss}} 1236 \ensuremath{\mbox{\sc loss}} 11236 \ensuremat
                                                                           1237 \ifnum#1<10 %
                                                                                             \csname InCa@AddDigit#1\expandafter\endcsname
                                                                           1238
                                                                           1239 \else
                                                                                            \@ReturnAfterFi{%
                                                                          1240
                                                                           1241
                                                                                                  \InCa@@TimDigitCarry#1!%
                                                                                          }%
                                                                           1242
                                                                          1243 \fi
                                                                           1244 }
\InCa@@TimDigitCarry
                                                                           1245 \def\InCa@@TimDigitCarry#1#2!#3{%
                                                                           1246 \csname InCa@DigitCarry#1%
                                                                           1247 \csname InCa@AddDigit#2\endcsname #3%
                                                                           1248 }
                                                                           2.5.8 Mul
                                    \intcalcMul
                                                                           1249 \def\intcalcMul#1#2{\%
                                                                           1250 \number
                                                                           1251
                                                                                                 \expandafter\InCa@MulSwitch
                                                                                                 \number\number#1\expandafter!%
                                                                           1252
                                                                                                 \number#2! %
                                                                           1253
                                                                           1254 }
```

 $\verb|\InCa@MulSwitch| Decision table for \verb|\InCa@MulSwitch|.$ 

x < 0	y < 0	x < y	+	Mul(-x, -y)
		else		Mul(-y, -x)
	else	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
else	y < 0	x > -y	_	Mul(x, -y)
		else		Mul(-y,x)
	else	x > y	+	Mul(x, y)
		else		Mul(y, x)

```
1255 \def\InCa@MulSwitch#1!#2!{%
1256 \ifnum#1<\z@
       \lim #2<\z@
1257
         \ifnum#1<#2 %
1258
          \verb|\expandafter| In Ca@Mul \number-\#1 \expandafter!\%
1259
          \@gobble#2!%
1260
1261
1262
          \verb|\expandafter| In Ca@Mul \number-#2 \expandafter! \%
1263
          \verb|@gobble#1!%|
1264
         \fi
1265
       \else
1266
        -%
1267
         \ifnum-#1>#2 %
1268
          \verb|\expandafter\InCa@Mul\@gobble#1!#2!%|
1269
          \verb|\expandafter| In Ca@Mul\number #2\expandafter!\%
1270
          \@gobble#1!%
1271
1272
         \fi
1273
       \fi
1274
      \else
       \lim #2<\z@
1275
1276
         \ifnum#1>-#2 %
1277
          \verb|\expandafter| In Ca@Mul \number #1 \expandafter! \%
1278
          \@gobble#2!%
1279
         \else
1280
          \verb|\expandafter\InCa@Mul\@gobble#2!#1!%|
1281
         \fi
1282
1283
       \else
        \ifnum#1>#2 %
1284
          \InCa@Mul#1!#2!%
1285
1286
1287
          \InCa@Mul#2!#1!%
1288
        \fi
       \fi
1289
1290 \fi
1291 }
1292 \def\IntCalcMul#1!#2!{%
1293 \number\InCa@Mul#1!#2! %
1294 }
1295 \def\InCa@Mul#1!#2!{%
     \ifcase#2 %
1296
1297
1298
1299
       #1%
1300 \or
       \csname InCa@Empty%
1301
       \verb|\expandafter| InCa@Shl#1!%|
1302
```

\IntCalcMul

\InCa@Mul

```
1303
                  \else
                   \ifnum#2<10 %
            1304
                     \InCa@Tim{#1}#2%
            1305
            1306
             1307
                     \InCa@ProcessMul!#2!#1!%
            1308
                   \fi
            1309 \fi
            1310 }
\InCa@Mul
            1311 \def\InCa@ProcessMul#1!#2#3!#4!{%
            1312 \ifx\InCa@Empty#3\InCa@Empty
            1313
                   \verb|\expandafter\InCa@Add\number||
            1314
                   \#10\ensuremath{\mbox{expandafter}\mbox{expandafter}}\%
                   \InCa@Tim{#4}#2!%
            1315
                  \else
            1316
                   \label{linca@Empty#1} In Ca@Empty \\
            1317
                     \verb|\expandafter| expandafter| In Ca@Process Mul|
            1318
            1319
                     \InCa@Tim{#4}#2!%
                     #3!#4!%
            1320
            1321
                   \else
            1322
                     \expandafter\InCa@ProcessMul\number
            1323
                     \verb|\expandafter\InCa@Add\number\%| \\
            1324
                     #10\expandafter\expandafter!%
            1325
                     \InCa@Tim{#4}#2!!%
                     #3!#4!%
            1326
                   \fi
            1327
            1328 \fi
            1329 }
            2.5.9 Sqr, Fac
 \intcalcSqr
            1330 \def\intcalcSqr#1{%
            1331 \number\expandafter\InCa@Sqr\number#1! %
            1332 }
 \InCa@Sqr
            1333 \def\InCa@Sqr#1#2!{%
            1334 \ifx#1-%
                   \InCa@Mul#2!#2!%
            1335
            1336
                  \else
            1337
                   \InCa@Mul#1#2!#1#2!%
            1338 \fi
            1339 }
 \intcalcFac
            1340 \left| \frac{4}{1340} \right|
            1341 \number\expandafter\InCa@Fac\number#1! %
            1342 }
            2.5.10 Pow
\intcalcPow
            1343 \def\intcalcPow#1#2{\%
            1344 \number\expandafter\InCa@Pow
            1345 \number\number#1\expandafter!\%
            1346 \number#2! %
            1347 }
```

\InCa@Pow

34

```
1348 \def\InCa@Pow#1#2!#3#4!{%
                     \frac{3\#4 \% \text{ power} = 0}{1}
                1349
                       1%
                1350
                1351
                     1352
                       #1#2%
                1353
                     1354
                       \ifx#1-%
                        \InCa@Mul#2!#2!%
                1355
                1356
                       \else
                        \InCa@Mul#1#2!#1#2!%
                1357
                       \fi
                1358
                     \else
                1359
                       \frac{1}{2}\% basis = 0, power <> 0
                1360
                1361
                        \ifx#3-% power < 0
                1362
                          0\IntCalcError:DivisionByZero%
                1363
                        \fi
                1364
                1365
                       \or
                        1\% basis = 1
                1366
                       \else
                1367
                        \lim#1#2=\m@ne \% basis = -1
                1368
                          \ifodd#3#4 %
                1369
                           -%
                1370
                          \fi
                1371
                1372
                          1%
                        \else % | basis | > 1
                1373
                          ifx#3-\% power < 0
                1374
                           0%
                1375
                          \else % power > 2
                1376
                           \inf #1-\% basis < 0
                1377
                             \ifodd#3#4 %
                1378
                              -%
                1379
                             \fi
                1380
                             \InCa@PowRec#2!#3#4!1!%
                1381
                1382
                             \InCa@PowRec#1#2!#3#4!1!%
                1383
                1384
                           \fi
                1385
                          \fi
                1386
                        \fi
                       \fi
                1387
                1388
                     \fi
                1389 }
\InCa@PowRec
                     Pow(b, p) {
                       PowRec(b, p, 1)
                     PowRec(b, p, r) {
                       if p == 1 then
                        return r
                       else
                        ifodd p then
                          return PowRec(b*b, p div 2, r*b) % p div 2 = (p-1)/2
                          return PowRec(b*b, p div 2, r)
                        fi
                       fi
                     }
                1390 \def\InCa@PowRec#1!#2!#3!{%
                1391
                     \lim #2=\@ne
                       \ifnum#1>#3 %
                1392
                        \InCa@Mul#1!#3!%
                1393
                1394
                       \else
                        \InCa@Mul#3!#1!%
                1395
```

```
\fi
             1396
             1397
                  \else
                    \expandafter\InCa@PowRec
             1398
                    \number\InCa@Mul#1!#1!\expandafter!%
             1399
             1400
                    \number\intcalcShr{#2}\expandafter!%
             1401
                    \number
                    \ifodd#2 %
             1402
                     \ifnum#1>#3 %
             1403
                      \InCa@Mul#1!#3!%
             1404
             1405
                     \else
                       \InCa@Mul#3!#1!%
             1406
                     \fi
             1407
                    \else
             1408
             1409
                     #3%
             1410
                    \fi
             1411
                    \expandafter!%
             1412 \fi
             1413 }
             2.5.11 Div
\intcalcDiv
             1414 \def\intcalcDiv#1#2{\%
             1415 \number\expandafter\InCa@Div
             1416 \number\number#1\expandafter!%
             1417 \number#2! %
             1418 }
\InCa@Div
             1419 \def\InCa@Div#1!#2!{%
             1420 \ifcase#2 %
                    {\tt 0\IntCalcError:DivisionByZero\%}
             1421
             1422 \else
             1423
                    \ifcase#1 %
             1424
                     0%
             1425
                    \else
             1426
                     \verb|\expandafter| In Ca@DivSwitch|
             1427
                     \number#1\expandafter!%
                     \number#2!%
             1428
                    \fi
             1429
             1430 \fi
             1431 }
\IntCalcDiv
             1432 \def\InCa@Temp#1{\%
             1433 \def\IntCalcDiv##1!##2!{%
             1434
                    \number
                    \ifcase##2 %
             1435
                     0\IntCalcError:DivisionByZero%
             1436
                    \else
             1437
                     \ifcase##1 %
             1438
                      0%
             1439
             1440
                     \else
                      \InCa@@Div##1!##2!%
             1441
             1442
                     \fi
                    \fi
             1443
                    #1%
             1444
             1445 }%
             1446 }
             1447 \InCa@Temp{ }\%
```

\InCa@DivSwitch Decision table for \InCa@DivSwitch.

x < 0	y < 0	+	$\operatorname{Div}(-x, -y)$
	else	_	$\operatorname{Div}(-x,y)$
else	y < 0	_	$\operatorname{Div}(x, -y)$
	else	+	$\mathrm{Div}(x,y)$

```
1448 \def\InCa@DivSwitch#1!#2!{%
                 1449 \ifnum#1<\z@
                        \lim #2<\z@
                 1450
                         \expandafter\InCa@@Div\number-#1\expandafter!%
                 1451
                 1452
                         \@gobble#2!%
                        \else
                 1453
                 1454
                         \expandafter\InCa@@Div\@gobble#1!#2!%
                 1455
                 1456
                        \fi
                 1457
                      \else
                        \lim #2<\z@
                 1458
                 1459
                         \expandafter\InCa@@Div\number#1\expandafter!%
                 1460
                 1461
                         \ensuremath{\texttt{@gobble\#2!\%}}
                 1462
                         \InCa@@Div#1!#2!%
                 1463
                 1464
                 1465 \fi
                 1466 }
   \InCa@@Div
                 1467 \def\InCa@@Div#1!#2!{%
                 1468 \ifnum#1>#2 %
                       \ifcase#2 % 0 already catched
                 1469
                         \IntCalcError:ThisCannotHappen%
                 1470 ?
                       \or % 1
                 1471
                         #1%
                 1472
                       \or % 2
                 1473
                         \InCa@Shr#1!%
                 1474
                 1475
                        \else
                 1476
                         \InCa@DivStart!#1!#2!#2!%
                 1477
                        \fi
                 1478
                      \else
                        \ifnum#1=#2 %
                 1479
                 1480
                         1%
                        \else
                 1481
                         0%
                 1482
                        \fi
                 1483
                 1484 \fi
                 1485 }
\InCa@DivStart
                 1486 \def\InCa@DivStart#1!#2#3!#4#5{%
                      \ifx#5!%
                 1487
                        \@ReturnAfterElseFi{%
                 1488
                 1489
                         \InCa@DivStartI{#1#2}#3=!%
                 1490
                       }%
                 1491
                      \else
                 1492
                        \@ReturnAfterFi{%
                         \InCa@DivStart{#1#2}!#3!#5%
                 1493
                       }%
                 1494
                 1495 \fi
                 1496 }
  \InCa@StartI
                 1497 \def\InCa@DivStartI#1!#2!{%
                 1498 \quad \verb|\expandafter| In Ca@DivStartII
```

```
\number#2\expandafter\expandafter\expandafter!%
                  1499
                        \intcalcShl{#2}!%
                   1500
                        #1!%
                  1501
                   1502 }
    \InCa@StartII
                  1503 \def\InCa@DivStartII#1!#2!{%
                   1504 \expandafter\InCa@DivStartIII
                   1505
                        \number#1\expandafter!%
                   1506 \number#2\expandafter\expandafter\expandafter!%
                   1507 \intcalcShl{#2}!%
                  1508 }
   \InCa@StartIII
                  1509 \def\InCa@DivStartIII#1!#2!#3!{%
                  1510 \expandafter\InCa@DivStartIV
                  1511 \number#1\expandafter!%
                  1512 \number#2\expandafter!%
                  1513 \number#3\expandafter!%
                  1514 \number\InCa@Add#3!#2!\expandafter\expandafter\expandafter!%
                  1515 \intcalcShl{#3}!%
                  1516 }
   \InCa@StartIV
                   1517 \def\InCa@DivStartIV#1!#2!#3!#4!#5!#6!{%
                   1518 \InCa@ProcessDiv#6!#1!#2!#3!#4!#5!/%
                  1519 }
\InCa@ProcessDiv
                   1520 \def\InCa@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9/{%
                   1521 #9%
                  1522
                        \ifnum#1<#4 % 0
                  1523
                         0%
                   1524
                         \int x#2=\%
                   1525
                          \else
                  1526
                           \InCa@ProcessDiv{#1#2}#3!#4!#5!#6!#7!#8!%
                         \fi
                  1527
                        \else % 1-9
                  1528
                         \ifnum#1<#5 % 1
                  1529
                           1%
                  1530
                           \ifx#2=%
                  1531
                  1532
                           \else
                            \expandafter\InCa@ProcessDiv\expandafter{%
                  1533
                              \number\InCa@Sub#1!#4!%
                  1534
                   1535
                   1536
                            }#3!#4!#5!#6!#7!#8!%
                  1537
                           \fi
                  1538
                          \else % 2-9
                           \ifnum#1<#7 % 2 3 4 5
                  1539
                            \ifnum#1<#6 % 2 3
                  1540
                              \@ReturnAfterElseFi{%
                  1541
                               \expandafter\InCa@@ProcessDiv
                  1542
                               \number\InCa@Sub#1!#5!!%
                  1543
                               23%
                  1544
                  1545
                              }%
                            \else % 4 5
                  1546
                              \CReturnAfterFi{%
                  1547
                               \expandafter\InCa@@ProcessDiv
                  1548
                               \label{local_sub} $$\prod_{n\in\mathbb{N}} 1!\#6!!\%$
                  1549
                               45%
                  1550
                             ት%
                  1551
                            \fi
                  1552
```

```
#2#3!#4!#5!#6!#7!#8!%
                       1553
                                 \else % 6 7 8 9
                       1554
                                  \ifnum#1<#8 % 6 7
                       1555
                       1556
                                    \@ReturnAfterElseFi{%
                       1557
                                     \expandafter\InCa@@ProcessDiv
                       1558
                                     \label{local_sub} $$\prod_{n\in\mathbb{N}} 1!\#7!!\%$
                       1559
                                     67%
                                   }%
                       1560
                                  \else % 8 9
                       1561
                                    \@ReturnAfterFi{%
                       1562
                                     \verb|\expandafter| In Ca@@ProcessDiv|
                       1563
                                     \number\InCa@Sub#1!#8!!%
                       1564
                       1565
                       1566
                                    }%
                       1567
                                  #2#3!#4!#5!#6!#7!#8!%
                       1568
                       1569
                                 \fi
                               \fi
                       1570
                             \fi
                       1571
                             \ifx#2=%
                       1572
                               \verb|\expandafter|@gobble|
                       1573
                             \fi
                       1574
                       1575
                             /%
                       1576 }
\InCa@@ProcessDiv
                       1577 \ensuremath{\mbox{\mbox{$1$}}\mbox{$1$}} 1577 \ensuremath{\mbox{\mbox{$1$}}\mbox{$2$}} 4\#5!\#6!\%
                       1578 \ifnum#1<#6 %
                       1579
                               #2%
                               \@ReturnAfterElseFi{%
                       1580
                                 \ifx#4=%
                       1581
                       1582
                                  \expandafter\InCa@CleanupIV
                       1583
                                 \else
                                  \@ReturnAfterFi{%
                       1584
                                    \label{linCa@ProcessDiv} $$ \prod_{0 \le 1 \le 4} \#5! \#6! % $$
                       1585
                                  }%
                       1586
                       1587
                                 \fi
                               }%
                       1588
                       1589
                             \else
                               #3%
                       1590
                               \@ReturnAfterFi{%
                       1591
                       1592
                                 \ifx#4=%
                                  \expandafter\InCa@CleanupIV
                       1593
                       1594
                                  \@ReturnAfterFi{%
                       1595
                                    \verb|\expandafter\InCa@ProcessDiv\expandafter{%}|
                       1596
                                     \number\InCa@Sub#1!#6! %
                       1597
                                     #4%
                       1598
                                    }#5!#6!%
                       1599
                       1600
                                  }%
                       1601
                                 \fi
                               }%
                       1602
                       1603 \fi
                       1604 }
  \InCa@CleanupIV
                       1605 \def\InCa@CleanupIV#1!#2!#3!#4!{}
```

#### 2.5.12 Mod

 $\$  intcalcMod

1606 \def\intcalcMod#1#2{%

```
\number\expandafter\InCa@Mod
               1607
                     \number\number#1\expandafter!%
               1608
                     \number#2! %
               1609
               1610 }
\intcalc@Mod Pseudocode/decision table for \intcalc@Mod.
                            y = 0
                     if
                                     DivisionByZero
                     elsif
                            y < 0
                                     -\operatorname{Mod}(-x, -y)
                     elsif
                            x = 0
                                     0
                     elsif
                            y = 1
                                     0
                     elsif
                            y = 2 ifodd(x)? 1:0
                            x<0 \quad z\leftarrow x-(x/y)*y; \quad (z<0)\ ?\ z+y:z
                     elsif
                                     x-(x/y)*y
                     else
               1611 \def\InCa@Mod#1!#2!{%
                     \ifcase#2 %
               1612
               1613
                       0\IntCalcError:DivisionByZero%
               1614
                      \else
               1615
                       \lim #2<\z@
               1616
                        -%
               1617
                         \expandafter\InCa@Mod
               1618
                         \number-#1\expandafter!%
                         \number-#2!%
               1619
                       \else
               1620
                         \ifcase#1 %
               1621
                          0%
               1622
               1623
                         \else
                          \ifcase#2 % 0 already catched
               1624
                            \IntCalcError:ThisCannotHappen%
               1625 ?
               1626
                          \or % 1
               1627
                           0%
                          \or % 2
               1628
                           \ifodd#1 1\else 0\fi
               1629
               1630
                          \else
                            \lim #1<\z@
               1631
                             \verb|\expandafter| In Ca@ModShift|
               1632
                             \number-%
               1633
                               \expandafter\InCa@Sub
               1634
               1635
                               \number\@gobble#1\expandafter!%
               1636
                               \mbox{number\intcalcMul}{\#2}{\%}
               1637
                                \expandafter\InCa@Div\@gobble#1!#2!%
               1638
                              }!%
                             !#2!%
               1639
               1640
                             \verb|\expandafter| In Ca@Sub \number #1 \expandafter! % |
               1641
                             \label{linear} $$\prod_{\ell=1}^{\ell} \prod_{\ell=1}^{\ell} \frac{1!\#2!}{!\%} $
               1642
                            \fi
               1643
                          \fi
               1644
                         \fi
               1645
               1646
                       \fi
               1647
                     \fi
               1648 }
\IntCalcMod
               1649 \def\InCa@Temp#1{\%
               1650
                     \def\IntCalcMod##1!##2!{%
               1651
                       \number
                       \ifcase##2 %
               1652
                        {\tt 0\IntCalcError:DivisionByZero\%}
               1653
```

1654

1655

\else

\ifcase##1 %

```
0%
                                                             1656
                                                                                     \else
                                                             1657
                                                                                         \ifcase##2 % 0 already catched
                                                             1658
                                                                                              \IntCalcError:ThisCannotHappen
                                                             1659 ?
                                                             1660
                                                                                         \or % 1
                                                             1661
                                                                                            0%
                                                             1662
                                                                                         \or % 2
                                                                                             \ifodd ##1 1\else 0\fi
                                                             1663
                                                             1664
                                                                                         \else
                                                             1665
                                                                                             \expandafter\InCa@Sub\number##1\expandafter!%
                                                                                             \label{limits} $$\operatorname{\operatorname{LimCa@Div}\#1!\#2!}!\%$
                                                             1666
                                                             1667
                                                                                     \fi
                                                             1668
                                                             1669
                                                                                 \fi
                                                             1670
                                                                                 #1%
                                                             1671 }%
                                                             1672 }
                                                             1673 \InCa@Temp{ }\%
           \InCa@ModShift
                                                             1674 \ensuremath{\mbox{\mbox{$1$}}\lower.21} 1674 \ensuremath{\mbox{$1$}}\lower.21 \ensuremath{\m
                                                             1675 \ifnum#1<\z@
                                                             1676
                                                                                 \expandafter\InCa@Sub\number#2\expandafter!%
                                                             1677
                                                                                 \@gobble#1!%
                                                             1678
                                                                            \else
                                                                                #1%
                                                             1679
                                                             1680 \fi
                                                             1681 }
                                                             2.5.13 Help macros
                 \InCa@Empty
                                                             1682 \def\InCa@Empty{}
                               \@gobble
                                                             1683 \expandafter\ifx\csname @gobble\endcsname\relax
                                                             1685 \fi
          \@ReturnAfterFi
                                                             1686 \long\def\@ReturnAfterFi#1\fi{\fi#1}%
\@ReturnAfterElseFi
                                                             1687 \long\def\@ReturnAfterElseFi#1\else#2\fi{\fi#1}%
                                                             1688 \InCa@AtEnd%
                                                             1689 (/package)
                                                             3
                                                                           Test
                                                                               Catcode checks for loading
                                                             3.1
                                                             1690 (*test1)
                                                             1691 \catcode`\{=1 %
                                                             1692 \catcode`\}=2 %
                                                             1693 \catcode \#=6 \%
                                                             1694 \catcode`\@=11 %
                                                             1695 \expandafter\ifx\csname count@\endcsname\relax
                                                             1696 \countdef\count@=255 %
```

1697 **\fi** 

```
1698 \expandafter\ifx\csname @gobble\endcsname\relax
              \ \left( \frac{2}{2} \right) = 1{}
1700 \fi
1701 \expandafter\ifx\csname @firstofone\endcsname\relax
1702 \long\def\@firstofone#1{#1}%
1703 \fi
1704 \expandafter\ifx\csname loop\endcsname\relax
1705 \expandafter\@firstofone
1706 \else
1707 \expandafter\@gobble
1708 \fi
1709 {%
              \def\loop#1\repeat{%
1710
                  \def\body{#1}%
1711
                  \iterate
1712
1713 }%
1714 \def\iterate{%
1715
                  \body
                     \let\next\iterate
1716
1717
                  \else
                     \let\next\relax
1718
                  \fi
1719
1720
                  \next
              }%
1721
              \let\repeat=\fi
1722
1723 }%
1724 \def\RestoreCatcodes{}
1725 \count@=0 %
1726 \loop
             \edef\RestoreCatcodes{%
1727
                  \RestoreCatcodes
1728
1729
                  \verb|\catcode| the \catcode| count@=\the \cat
1730 }%
1731 \ifnum\count@<255 %
1732 \advance\count@ 1 %
1733 \repeat
1735 \def\RangeCatcodeInvalid#1#2\%
1736 \count@=#1\relax
1737 \loop
                \catcode\count@=15 %
1738
1739 \ifnum\count@<#2\relax
                 \advance\count@ 1 %
1740
1741 \repeat
1742 }
1743 \def\RangeCatcodeCheck#1#2#3{%
1744
             \count@=#1\relax
1745
              \loop
1746
                  \ifnum#3=\catcode\count@
1747
                  \else
                      \errmessage{%
1748
1749
                         Character \the\count@\space
                         with wrong catcode \the\catcode\count@\space
1750
1751
                        instead of \number#3%
1752
                     }%
1753
1754
              1755
                  \advance\count@ 1 %
1756 \repeat
1757 }
1758 \ensuremath{\mbox{def\space}}\
1759 \verb|\expandafter\ifx\csname\ LoadCommand\endcsname\relax|
```

```
\def\LoadCommand{\input intcalc.sty\relax}%
1760
1761 \fi
1762 \def\Test{%
1763 \RangeCatcodeInvalid{0}{47}%
     \RangeCatcodeInvalid{58}{64}%
     \RangeCatcodeInvalid{91}{96}%
1766
     \RangeCatcodeInvalid{123}{255}%
     \catcode`\@=12 %
1767
     \color=0 \%
1768
     \catcode`\%=14 %
1769
1770 \LoadCommand
1771 \RangeCatcodeCheck{0}{36}{15}%
1772 \RangeCatcodeCheck{37}{37}{14}%
1773 \RangeCatcodeCheck{38}{47}{15}%
     \RangeCatcodeCheck{48}{57}{12}%
1774
     \RangeCatcodeCheck{58}{63}{15}%
1775
1776
     \RangeCatcodeCheck{64}{64}{12}%
     \RangeCatcodeCheck{65}{90}{11}%
1777
     \RangeCatcodeCheck{91}{91}{15}%
1778
1779
     \RangeCatcodeCheck{92}{92}{0}%
1780
     \RangeCatcodeCheck{93}{96}{15}%
     \RangeCatcodeCheck{97}{122}{11}%
1781
     \RangeCatcodeCheck{123}{255}{15}%
1782
     \RestoreCatcodes
1783
1784 }
1785 \Test
1786 \csname @@end\endcsname
1787 \end
1788 (/test1)
```

#### 3.2 Macro tests

#### 3.2.1 Preamble with test macro definitions

```
1789 (*test2 j test4)
1790 \NeedsTeXFormat{LaTeX2e}
1791 \nofiles
1792 \documentclass{article}
1793 \langle noetex \rangle \setminus Iet \setminus SavedNumexpr \setminus numexpr
1794 (noetex) \let\numexpr\UNDEFINED
1795 \makeatletter
1796 \chardef\InCa@TestMode=1 %
1797 \makeatother
1798 \usepackage{intcalc}[2016/05/16]
1799 \langle noetex \rangle \setminus let \setminus numexpr \setminus Saved Numexpr
1800 \usepackage{qstest}
1801 \IncludeTests{*}
1802 \LogTests{log}{*}{*}
1803 (/test2 j test4)
1804 (*test2)
1805 \newcommand*{\TestSpaceAtEnd}[1]{\%
1806 (noetex) \let\SavedNumexpr\numexpr
1807 (noetex) \let\numexpr\UNDEFINED
1808 \edef\resultA{#1}%
1809 \edef\resultB{#1}%
1810 (noetex) \let\numexpr\SavedNumexpr
1811
     \Expect*{\resultA\space}*{\resultB}%
1812 }
1813 \newcommand*{\TestResult}[2]{\%
1814 (noetex) \let\SavedNumexpr\numexpr
1815 (noetex) \let\numexpr\UNDEFINED
1816 \edef\result{#1}%
1817 (noetex) \let\numexpr\SavedNumexpr
```

```
1818 \Expect*{\result}{#2}%
1819 }
1820 \newcommand*{\TestResultTwoExpansions}[2]{%
1821 (*noetex)
1822 \begingroup
1823
       \let\numexpr\UNDEFINED
1824
       \expandafter\expandafter\expandafter
1825 \endgroup
1826 (/noetex)
1827 \expandafter\expandafter\Expect
1828 \expandafter\expandafter\expandafter\{\#1\}\{\#2\}\%
1829 }
1830 \newcount\TestCount
1831 \langle \text{etex} \rangle \text{newcommand} {\text{TestArg}[1]} {\text{numexpr} #1 }
1832 \langle noetex \rangle \setminus mewcommand * {\TestArg}[1]{\#1}
1833 \newcommand*{\TestTeXDivide}[2]{%
1834 \text{TestCount}=\text{TestArg}\{\#1\}\
1835 \divide\TestCount by \TestArg{#2}\relax
1836 \Expect*{\intcalcDiv{#1}{#2}}*{\the\TestCount}%
1837 }
1838 \newcommand*{\Test}[2]{%
1839 \TestResult{#1}{#2}%
     \TestResultTwoExpansions{#1}{#2}%
1840
     \TestSpaceAtEnd{#1}%
1841
1842 }
1843 \ensuremath{\mbox{\mbox{$1843$ \newcommand*{\mbox{$1843$ \newcommand*{\mbox{$1843$ \newcommand*{\mbox{$1843$ \newcommand*}}}}}}
1844 \newcommand*{\TestInv}[2]{%
1845 \Test{\intcalcInv{#1}}{#2}%
1846 }
1847 \newcommand*{\TestNum}[2]{%
1848 \Test{\intcalcNum{#1}}{#2}%
1849 }
1850 \newcommand*{\TestAbs}[2]{%
1851 \Test{\intcalcAbs{#1}}{#2}%
1853 \newcommand*{\TestSgn}[2]{%
1854 \text{Test{\left( intcalcSgn{#1} \right)}{#2}}%
1855 }
1856 \newcommand*{\TestMin}[3]{\%
1857 \Test{\intcalcMin{#1}{#2}}{#3}%
1858 }
1859 \newcommand*{\TestMax}[3]{%
1860 \Test{\intcalcMax{#1}{#2}}{#3}%
1861 }
1862 \newcommand*{\TestCmp}[3]{%
1863 \Test{\intcalcCmp{#1}{#2}}{#3}%
1864 }
1865 \newcommand*{\TestInc}[2]{%
1866 \Test{\intcalcInc{#1}}{#2}%
1867
     \ifnum\intcalcNum{#1}>-1 %
       \left( x_{x}\right) 
1868
         \verb|\noexpand\Test{%}|
1869
          \label{lincalcNum} $$\operatorname{IntCalcInc\intcalcNum}$ 
1870
1871
        }{#2}%
1872
       }%
1873
       \x
1874 \fi
1876 \newcommand*{\TestDec}[2]{%
1877 \Test{\intcalcDec{#1}}{#2}%
1878 \ifnum\intcalcNum{#1}>0 %
      \left( x_{x}\right) 
1879
```

```
\noexpand\Test{%
1880
          \noexpand\IntCalcDec\intcalcNum{\#1}!\%
1881
        }{#2}%
1882
1883
1884
1885
     \fi
1886 }
1887 \newcommand*{\TestAdd}[3]{%
     \texttt{\Test{\intcalcAdd{\#1}{\#2}}{\#3}\%}
1888
      \ifnum\intcalcNum{#1}>0 %
1889
       \ifnum\intcalcNum{#2}> 0 %
1890
         1891
          \left( x_{x}\right) 
1892
1893
            \noexpand\Test{%
             \noexpand\IntCalcAdd
1894
1895
             \intcalcNum{#1}!\intcalcNum{#2}!%
1896
            }{#3}%
          }%
1897
          \mathbf{x}
1898
1899
         \else
          \left( x_{x}\right) 
1900
            \noexpand\Test{%
1901
             \noexpand\IntCalcAdd
1902
             \intcalcNum{#2}!\intcalcNum{#1}!%
1903
            }{#3}%
1904
1905
          }%
1906
          \backslash x
         \fi
1907
1908
       \fi
     \fi
1909
1910 }
1911 \newcommand*{\TestSub}[3]{%
1912
     \texttt{Test}\{\text{ub}\{\#1\}\{\#2\}\}\{\#3\}\%
     \ifnum\intcalcNum{#1}>0 %
1913
       \  \intro \{\#2\} > 0 \%
1914
1915
         \int \int \int d^2 t dt
1916
          \left( x_{x}\right) 
            \verb|\noexpand| Test{%|}
1917
1918
             \noexpand\IntCalcSub
             1919
           }{#3}%
1920
          }%
1921
1922
          \x
1923
         \fi
1924
       \fi
1925
     \fi
1926 }
1927 \newcommand*{\TestShl}[2]{%
1928
     \texttt{Test{\intcalcShl{#1}}{\#2}}\%
1929
     \left( x_{x}\right) 
       \verb|\noexpand| Test{%|}
1930
         \noexpand\IntCalcShl\intcalcAbs{#1}!%
1931
       }{\intcalcAbs{#2}}%
1932
1933
     }%
1934
     /x
1935 }
1936 \newcommand*{\TestShr}[2]{\%
1937
     \texttt{Test{\intcalcShr{#1}}{\#2}}\%
1938
     \left( x_{x}\right) 
       \noexpand\Test{%
1939
         \noexpand\IntCalcShr\intcalcAbs\{\#1\}!\%
1940
1941
       }{\intcalcAbs{#2}}%
```

```
1942 }%
1943
     \x
1944 }
1945 \newcommand*{\TestMul}[3]{%
     \texttt{Test{\intcalcMul{#1}{\#2}}{\#3}}\%
1946
1947
     \left( x_{x}\right) 
1948
       \noexpand\Test{%
         \label{lintCalcMul} $$\operatorname{Abs}${\#1}!\operatorname{CalcAbs}${\#2}!\%$
1949
       }{\intcalcAbs{#3}}%
1950
1951 }%
1952
     \x
1953 }
1954 \newcommand*{\TestSqr}[2]{%
     \Test{\intcalcSqr{#1}}{#2}%
1957 \newcommand*{\TestFac}[2]{\%
     1958
1959 }
1960 \newcommand*{\TestPow}[3]{%
1961 \Test{\intcalcPow{#1}{#2}}{#3}%
1962 }
1963 \newcommand*{\TestDiv}[3]{%
     \texttt{Test}\{\text{intcalcDiv}\{\#1\}\{\#2\}\}\{\#3\}\%
1964
      \TestTeXDivide{#1}{#2}%
1965
      \left( x_{x}\right) 
1966
1967
       \noexpand\Test{%
         \label{lintCalcDiv} $$\operatorname{LAbs}{\#1}!\operatorname{LAbs}{\#2}!\% $$
1968
1969
       }{\intcalcAbs{#3}}%
1970 }%
1971 }
1972 \newcommand*{\TestMod}[3]{%
     \texttt{Test}\{\texttt{Mod}\{\#1\}\{\#2\}\}\{\#3\}\%
1973
1974
     \ifcase\ifcase\intcalcSgn{#1} 0%
1975
1976
            \ifcase\intcalcSgn{#2} 1%
1977
            \or 0%
1978
            \else 1%
1979
            \fi
1980
           \else
            \  \int calc Sgn{\#2} 1\%
1981
            \or 1%
1982
            \else 0%
1983
            \fi
1984
1985
           fi\relax
1986
       \left( x_{x}\right) 
1987
         \noexpand\Test{%
1988
          \noexpand\IntCalcMod
1989
          \intcalcAbs{#1}!\intcalcAbs{#2}!%
1990
        }{\intcalcAbs{#3}}%
1991
       }%
1992
       \x
     \fi
1993
1994 }
1995 (/test2)
3.2.2 Time
1996 (*test2)
1997 \begingroup\expandafter\expandafter\expandafter\endgroup
1998 \expandafter\ifx\csname pdfresettimer\endcsname\relax
1999 \else
     \makeatletter
2000
2001
     \newcount\SummaryTime
     \newcount\TestTime
2002
```

```
\SummaryTime=\z@
2003
     \newcommand*{\PrintTime}[2]{%
2004
2005
      \typeout{%
        [Time #1: \strip@pt\dimexpr\number#2sp\relax\space s]%
2006
2007
2008
     }%
2009
     \newcommand*{\StartTime}[1]{%
2010
      \renewcommand*{\TimeDescription}{#1}%
2011
      \pdfresettimer
     }%
2012
     \newcommand*{\TimeDescription}{}%
2013
     \newcommand*{\StopTime}{%
2014
      \TestTime=\pdfelapsedtime
2015
      \global\advance\SummaryTime\TestTime
2016
      \PrintTime\TimeDescription\TestTime
2017
2018
     2019
     \let\saved@endqstest\endqstest
2020
     \def\qstest#1#2{\%}
2021
2022
      \square{41}{\#2}
      \mathsf{StartTime}\{\#1\}\%
2023
     }%
2024
     \def\endqstest{%
2025
      \StopTime
2026
      \saved@endqstest
2027
2028
     \AtEndDocument{%
2029
      \PrintTime{summary}\SummaryTime
2030
    }%
2031
2032
     \makeatother
2033 \fi
2034 (/test2)
3.2.3 Test 4: additional mod/div operations
2036 \newcommand*{\TestDo}[2]{%
2037 \ifcase\numexpr#2\relax
2038
      \left( \frac{\#1}{\#2} \right)
2039
2040
      \Expect*{%
2041
       \the\numexpr
        \intcalcMul{%
2042
         2043
2044
       }{\intcalcAbs{#2}}%
        +\intcalcMod{\intcalcAbs{#1}}{\intcalcAbs{#2}}\relax
2046
      }*{\the\numexpr\intcalcAbs{#1}\relax}%
2047
2048 }
2049 \newcommand*{\TestOne}[2]{%
2050
    \TestDo{#1}{#1}%
2051 }
2052 \newcommand*{\TestTwo}[3]{%
2053 \TestDo{#1}{#2}%
2054 \TestDo{#2}{#1}%
2055 }
2056 \let\TestNum\TestOne
2057 \let\TestInv\TestOne
2058 \let\TestAbs\TestOne
2059 \let\TestSgn\TestOne
2060 \left| \text{TestMin} \right|
2061 \left| \text{TestMax} \right|
2062 \let\TestCmp\TestTwo
2063 \left| \text{TestInc} \right|
```

```
2064 \let\TestDec\TestOne
2065 \let\TestAdd\TestTwo
2066 \let\TestSub\TestTwo
2067 \left| \text{TestShl} \right|
2068 \let\TestShr\TestOne
2069 \let\TestMul\TestTwo
2070 \let\TestSqr\TestOne
2071 \left( \text{TestFac} # 1 # 2 \right)
2072 \let\TestPow\TestTwo
2073 \let\TestDiv\TestTwo
2074 \left| \text{TestMod} \right|
2075 (/test4)
3.2.4 Test sets
2076 (*test2 j test4)
2077 \makeatletter
2078
2079 \begin{qstest}{num}{num}%
2080 \TestNum{0}{0}%
     \TestNum{1}{1}%
2081
2082 \TestNum{-1}{-1}%
2083
     \TestNum{10}{10}%
     \TestNum{-10}{-10}%
2084
     \TestNum{2147483647}{2147483647}%
     \TestNum{-2147483647}{-2147483647}%
2087
     \TestNum{ 0 }{0}%
2088 \TestNum{ 1 }{1}%
2089 TestNum{--1}{1}\%
2090 \text{TestNum}\{-+-+4\}\{4\}\%
2091 \TestNum{\z@}{0}%
2092 TestNum{@ne}{1}%
2093 TestNum{m@ne}{-1}%
2094 (*etex)
2095 \TestNum{-10+30}{20}%
2096 \TestNum{10-30}{-20}%
2097 (/etex)
2098 \end{qstest}
2099
2100 \begin{qstest}{inv}{inv}%
2101 \TestInv{0}{0}%
2102 \TestInv{1}{-1}%
2103 \TestInv{-1}{1}%
2104 \TestInv{10}{-10}%
     \TestInv{-10}{10}%
     \TestInv{2147483647}{-2147483647}%
     \TestInv{-2147483647}{2147483647}%
2108 \TestInv{ 0 }{0}%
2109 \TestInv{ 1 }{-1}%
2110 \TestInv\{--1\}\{-1\}\%
2111 \TestInv{\z@}{0}\%
2112 \TestInv{\@ne}{-1}%
2113 \TestInv{\m@ne}{1}%
2114 (*etex)
2115 \TestInv{-10+30}{-20}%
2116 \TestInv{10-30}{20}%
2117 (/etex)
2118 \end{qstest}
2119
2120 \left[ \left[ abs \right] \right] 
2121 \TestAbs{0}{0}%
2122 \TestAbs{1}{1}%
2123 \TestAbs{-1}{1}%
```

2124 \TestAbs{10}{10}%

```
2125 \TestAbs{-10}{10}%
            \TestAbs{2147483647}{2147483647}%
2126
            \TestAbs{-2147483647}{2147483647}%
2127
            \TestAbs{ 0 }{0}%
2129 \TestAbs{ 1 }{1}%
2130 \TestAbs{--1}{1}%
2131 \TestAbs{\z@}{0}%
2132 \text{TestAbs}(\mathbb{1})\%
2133 \TestAbs{\m@ne}{1}%
2134 (*etex)
2135 \TestAbs{-10+30}{20}%
2136 \TestAbs{10-30}{20}%
2137 (/etex)
2138 \end{qstest}
2140 \begin{qstest}{sign}{sign}%
2141 \TestSgn{0}{0}\%
2142 \TestSgn{1}{1}%
2143 \TestSgn{-1}{-1}%
2144 \TestSgn{10}{1}%
2145 \TestSgn{-10}{-1}%
2146
            \TestSgn{2147483647}{1}%
2147
            \TestSgn{-2147483647}{-1}%
            \TestSgn{ 0 }{0}%
2148
2149 \TestSgn{ 2 }{1}%
2150 \TestSgn{ -2 }{-1}%
2151 \quad \texttt{TestSgn}\{--2\}\{1\}\%
2152 \TestSgn{\z@}{0}%
2153 \TestSgn{\0ne}{1}%
2154 \TestSgn{\m@ne}{-1}%
2155 (*etex)
2156 \TestSgn{-10+30}{1}%
2157 \TestSgn{10-30}{-1}%
2158 (/etex)
2159 \end{qstest}
2160
2161 \begin{qstest}{min}{min}%
2162 \TestMin{0}{1}{0}%
2163 \TestMin{1}{0}{0}%
2164 \TestMin{-10}{-20}{-20}%
2165 \TestMin{ 1 }{ 2 }{1}%
2166 \TestMin{ 2 }{ 1 }{1}%
2167 \TestMin{1}{1}{1}%
2168 TestMin{z@}{\@ne}{0}%
2169 TestMin{Qne}{m@ne}{-1}%
2170 (*etex)
            TestMin{1+2}{3+4}{3}%
2171
2172 (/etex)
2173 \end{qstest}
2174
2175 \begin{qstest}{max}{max}%
2176 \TestMax{0}{1}{1}%
2177 \TestMax{1}{0}{1}%
2178 \TestMax{-10}{-20}{-10}%
2179 \TestMax{ 1 }{ 2 }{2}%
2180 \TestMax{ 2 }{ 1 }{2}%
2181 \TestMax{1}{1}{1}%
2182 TestMax{\z@}{\ensuremath{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20
2183 \TestMax{\encomp}{1}\%
2184 (*etex)
2185 \TestMax{1+2}{3+4}{7}%
2186 (/etex)
```

```
2187 \end{qstest}
2188
2189 \begin{qstest}{cmp}{cmp}%
           \TestCmp{0}{0}{0}\%
           \TestCmp{-21}{17}{-1}%
2192 \ \text{TestCmp}{3}{4}{-1}%
2193 \TestCmp{-10}{-10}{0}%
2194 \quad \texttt{TestCmp} \{-10\} \{-11\} \{1\} \%
           \TestCmp{100}{5}{1}%
2195
           \TestCmp{2147483647}{-2147483647}{1}%
2196
2197
           \TestCmp{-2147483647}{2147483647}{-1}%
           \TestCmp{2147483647}{2147483647}{0}%
2198
2199 \TestCmp{\z@}{\ensuremath{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{20}{\column{2
2200 \TestCmp{\@ne}{\m@ne}{1}%
2201 \TestCmp{ 4 }{ 5 }{-1}%
2202 \TestCmp{ -3 }{ -7 }{1}%
2203 (*etex)
2204 \TestCmp{1+2}{3+4}{-1}%
2205 (/etex)
2206 \end{qstest}
2207
2208 \begin{qstest}{fac}{fac}
2209 \TestFac{0}{1}%
           \TestFac{1}{1}%
2210
           \TestFac{2}{2}%
           \TestFac{3}{2*3}%
2212
2213 \TestFac{4}{2*3*4}%
2214 \TestFac{5}{2*3*4*5}%
2215 \TestFac{6}{2*3*4*5*6}%
2216 \TestFac{7}{2*3*4*5*6*7}%
2217 \TestFac{8}{2*3*4*5*6*7*8}%
2218 \TestFac{9}{2*3*4*5*6*7*8*9}%
2219 \TestFac{10}{2*3*4*5*6*7*8*9*10}%
2220 \TestFac{11}{2*3*4*5*6*7*8*9*10*11}%
2221 \TestFac{12}{2*3*4*5*6*7*8*9*10*11*12}%
2222 \end{qstest}
2223
2224 \begin{qstest}{inc}{inc}%
2225 \TestInc{0}{1}%
2226 \TestInc{1}{2}%
2227 \TestInc{-1}{0}%
2228 \TestInc{10}{11}%
2229
           \TestInc{-10}{-9}%
2230
           \TestInc{999}{1000}%
2231
           \TestInc{-1000}{-999}%
            \TestInc{129}{130}%
           \TestInc{2147483646}{2147483647}%
2234 \TestInc{-2147483647}{-2147483646}%
2235 \end{qstest}
2236
2237 \begin{qstest}{dec}{dec}%
2238 \TestDec{0}{-1}%
           \TestDec{1}{0}%
2239
2240 \TestDec{-1}{-2}%
2241 \TestDec{10}{9}%
2242 \TestDec{-10}{-11}%
2243 \TestDec{1000}{999}%
2244 \TestDec{-999}{-1000}%
2245 \TestDec{130}{129}%
2246 \TestDec{2147483647}{2147483646}%
2247 \TestDec{-2147483646}{-2147483647}%
2248 \end{qstest}
```

```
2249
2250 \geq 250 \left( \frac{3}{2} \right)
     \TestAdd{0}{0}{0}%
2251
     \TestAdd{1}{0}{1}%
     \TestAdd{0}{1}{1}%
2253
2254
     \TestAdd{1}{2}{3}%
2255
     TestAdd{-1}{-1}{-2}%
     \verb|\TestAdd{2147483646}{\{1\}}{\{2147483647\}}\%
2256
     \TestAdd{-2147483647}{2147483647}{0}%
2257
     \TestAdd{20}{-5}{15}%
2258
     \TestAdd{-4}{-1}{-5}%
2259
     \TestAdd{-1}{-4}{-5}%
2260
2261
     \TestAdd{-4}{1}{-3}%
     \TestAdd{-1}{4}{3}%
     \TestAdd{4}{-1}{3}%
2263
2264
     \TestAdd{1}{-4}{-3}%
2265
     TestAdd{-4}{-1}{-5}%
2266
     TestAdd{-1}{-4}{-5}%
     TestAdd{ -4 }{ -1 }{-5}%
2267
2268
     \TestAdd{ -1 }{ -4 }{-5}%
     \TestAdd{ -4 }{ 1 }{-3}%
2269
     \TestAdd{ -1 }{ 4 }{3}%
2270
2271
     \TestAdd{ 4 }{ -1 }{3}%
2272
     \TestAdd{ 1 }{ -4 }{-3}%
     \TestAdd{ -4 }{ -1 }{-5}%
     \TestAdd{ -1 }{ -4 }{-5}%
2274
     \verb|\TestAdd{876543210}{111111111}{987654321}\%
2275
2276
     \TestAdd{999999999}{2}{1000000001}%
2277 (*etex)
      \TestAdd{100}{50+150}{300}%
2278
     \TestAdd{2147483647}{10-2147483647}{10}%
2279
2280 (/etex)
2281 \end{qstest}
2282
2283 \begin{qstest}{sub}{sub}
2284
    \TestSub{0}{0}{0}\%
2285
     \TestSub{1}{0}{1}%
2286 \TestSub{1}{2}{-1}%
2287
     \TestSub{-1}{-1}{0}%
     \verb|\TestSub{2147483646}{{-1}}{{2147483647}}| %
2288
     \verb|\TestSub{-2147483647}{{-2147483647}}{{0}}|
2289
     \TestSub{-4}{-1}{-3}%
2290
     \TestSub{-1}{-4}{3}%
2291
2292
     \text{TestSub}\{-4\}\{1\}\{-5\}\%
2293
     \text{TestSub}\{-1\}\{4\}\{-5\}\%
     \text{TestSub}\{4\}\{-1\}\{5\}\%
2295
     \text{TestSub}\{1\}\{-4\}\{5\}\%
2296
     TestSub{-4}{-1}{-3}%
2297
     \TestSub{-1}{-4}{3}%
2298
     \TestSub{ -4 }{ -1 }{-3}%
     \TestSub{ -1 }{ -4 }{3}%
2299
     \TestSub{ -4 }{ 1 }{-5}%
2300
     \TestSub{ -1 }{ 4 }{-5}%
2301
2302
     \TestSub{ 4 }{ -1 }{5}%
2303
     \TestSub{ 1 }{ -4 }{5}%
     TestSub{ -4 }{ -1 }{-3}%
2305
     \TestSub{ -1 }{ -4 }{3}%
2306
     \TestSub{1000000000}{2}{999999998}%
2307 \TestSub{987654321}{111111111}{876543210}%
2308 (*etex)
     \TestSub{100}{50+150}{-100}%
2309
2310 \TestSub{2147483647}{-10+2147483647}{10}%
```

```
2311 \langle /\text{etex} \rangle
2312 \end{qstest}
2313
2314 \begin{qstest}{shl}{shl}
2315
     \TestShl{0}{0}%
2316
     \TestShl{1}{2}%
2317
     \TestShl{5621}{11242}%
2318
     \TestShl{1073741823}{2147483646}%
     TestShl{-1}{-2}%
2319
     \TestShl{-5621}{-11242}%
2320
2321 \end{qstest}
2322
    \begin{qstest}{shr}{shr}
2323
     TestShr{0}{0}%
2324
     \TestShr{1}{0}%
2325
2326
     \TestShr{2}{1}%
2327
     \TestShr{3}{1}%
     \TestShr{4}{2}%
2328
     TestShr{5}{2}%
2329
2330
     \TestShr{6}{3}%
2331
     \TestShr{7}{3}%
     \TestShr{8}{4}%
2332
2333
     \TestShr{9}{4}%
2334
     \TestShr{10}{5}%
     \TestShr{11}{5}%
2335
2336
     \TestShr{12}{6}%
2337
     \TestShr{13}{6}%
2338
     \TestShr{14}{7}%
2339
     \TestShr{15}{7}%
     \TestShr{16}{8}%
2340
     \TestShr{17}{8}%
2341
2342
     \TestShr{18}{9}%
2343
     \TestShr{19}{9}%
2344
     \TestShr{20}{10}%
     \TestShr{21}{10}%
2345
2346
     \TestShr{22}{11}%
2347
     \TestShr{11241}{5620}%
2348
     \TestShr{73054202}{36527101}%
2349
     \TestShr{2147483646}{1073741823}%
2350
     \TestShr{-1}{0}%
2351
     \TestShr{-2}{-1}%
     \TestShr{-3}{-1}%
2352
2353
     \TestShr{-11241}{-5620}%
2354 \end{qstest}
2355
2356 \begin{qstest}{mul}{mul}
     TestMul{0}{0}{0}%
2358
     \TestMul{1}{0}{0}%
2359
     TestMul{0}{1}{0}%
2360
     TestMul{1}{1}{1}{%}
     \TestMul{3}{1}{3}%
2361
2362
     TestMul{1}{-3}{-3}%
     TestMul{-4}{-5}{20}%
2363
2364
     \TestMul{3}{7}{21}%
2365
     \TestMul{7}{3}{21}%
2366
     TestMul{3}{-7}{-21}%
2367
     TestMul{7}{-3}{-21}%
2368
     TestMul{-3}{7}{-21}%
2369
     \TestMul{-7}{3}{-21}%
2370
     TestMul{-3}{-7}{21}%
2371 \TestMul{-7}{-3}{21}%
2372 \TestMul{12}{11}{132}%
```

```
\TestMul{999}{333}{332667}%
2373
     \TestMul{1000}{4321}{4321000}%
2374
     \TestMul{12345}{173955}{2147474475}%
2375
     TestMul{1073741823}{2}{2147483646}%
2377
     \TestMul{2}{1073741823}{2147483646}%
2378
     \TestMul{-1073741823}{2}{-2147483646}%
2379
     \TestMul{2}{-1073741823}{-2147483646}%
2380 (*etex)
     \TestMul{2+3}{5+7}{60}%
2381
     \TestMul{2147483647}{2147483647/2147483647}{2147483647}%
2382
2383 (/etex)
2384 \end{qstest}
2385
2386 \begin{qstest}{sqr}{sqr}
     \TestSqr{0}{0}%
2388
     \TestSqr{1}{1}%
     \TestSqr{2}{4}%
2389
     \TestSqr{3}{9}%
2390
     \TestSqr{4}{16}%
2391
     \TestSqr{9}{81}%
2392
     \TestSqr{10}{100}%
2393
     \TestSqr{46340}{2147395600}%
2394
2395
     \TestSqr{-1}{1}%
     \text{TestSqr}\{-2\}\{4\}\%
2396
     \TestSqr{-46340}{2147395600}%
2398 \end{qstest}
2399
2400 \ \texttt{\pow}{\text{\pow}}{\text{\pow}}
     \TestPow{-2}{0}{1}%
2401
     \TestPow{-1}{0}{1}%
2402
2403 \TestPow{0}{0}{1}%
2404 \TestPow{1}{0}{1}%
2405 \TestPow{2}{0}{1}%
2406 \TestPow{3}{0}{1}%
2407 \TestPow{-2}{1}{-2}%
2408 \TestPow{-1}{1}{-1}%
2409 \TestPow{1}{1}{1}\%
2410 \TestPow{2}{1}{2}%
2411 \TestPow{3}{1}{3}%
2412 \TestPow{-2}{2}{4}%
2413 \TestPow{-1}{2}{1}%
     \TestPow{0}{2}{0}%
2414
2415
     \TestPow{1}{2}{1}%
2416
     \TestPow{2}{2}{4}%
2417
     \TestPow{3}{2}{9}%
     \TestPow{0}{1}{0}%
2419
     \text{TestPow}\{1\}\{-2\}\{1\}\%
2420
     \TestPow{1}{-1}{1}%
2421
     TestPow{-1}{-2}{1}%
2422
     \TestPow{-1}{-1}{-1}%
     \TestPow{-1}{3}{-1}%
2423
     \TestPow{-1}{4}{1}%
2424
     \TestPow{-2}{-1}{0}%
2425
2426
     TestPow{-2}{-2}{0}%
2427
     \TestPow{2}{3}{8}%
     \TestPow{2}{4}{16}%
2429
     \TestPow{2}{5}{32}%
2430 \TestPow{2}{6}{64}%
2431 \TestPow{2}{7}{128}%
2432 \TestPow{2}{8}{256}%
2433 \TestPow{2}{9}{512}%
```

2434 \TestPow{2}{10}{1024}%

```
TestPow{-2}{3}{-8}%
2435
      TestPow{-2}{4}{16}%
2436
      TestPow{-2}{5}{-32}%
2437
      TestPow{-2}{6}{64}%
2439
      TestPow{-2}{7}{-128}%
2440
      \text{TestPow}\{-2\}\{8\}\{256\}\%
2441
      TestPow{-2}{9}{-512}%
2442
      \TestPow{-2}{10}{1024}%
2443
      \text{TestPow}{3}{3}{27}\%
      \TestPow{3}{4}{81}%
2444
      \TestPow{3}{5}{243}%
2445
      \TestPow{-3}{3}{-27}%
2446
      TestPow{-3}{4}{81}%
2447
      \TestPow{-3}{5}{-243}%
      \TestPow{2}{30}{1073741824}%
2450
      \TestPow{-3}{19}{-1162261467}%
2451
      \TestPow{5}{13}{1220703125}%
      \TestPow{-7}{11}{-1977326743}%
2452
2453 \end{qstest}
2454
2455 \verb|\div|{div}{div}|
      \TestDiv{1}{1}{1}%
2456
      \text{TestDiv}\{2\}\{1\}\{2\}\%
2457
      TestDiv{-2}{1}{-2}%
2458
      TestDiv{2}{-1}{-2}%
2460
      \text{TestDiv}\{-2\}\{-1\}\{2\}\%
2461
      \text{TestDiv}\{15\}\{2\}\{7\}\%
2462
      \text{TestDiv}\{-16\}\{2\}\{-8\}\%
2463
      \TestDiv{1}{2}{0}%
2464
      \text{TestDiv}\{1\}\{3\}\{0\}\%
      \text{TestDiv}\{2\}\{3\}\{0\}\%
2465
2466
      \text{TestDiv}\{-2\}\{3\}\{0\}\%
2467
      \TestDiv{2}{-3}{0}%
2468
      \text{TestDiv}\{-2\}\{-3\}\{0\}\%
      \TestDiv{13}{3}{4}%
2470
      \text{TestDiv}\{-13\}\{-3\}\{4\}\%
2471
      \text{TestDiv}\{-13\}\{3\}\{-4\}\%
2472
      \text{TestDiv}\{-6\}\{5\}\{-1\}\%
2473
      \text{TestDiv}\{-5\}\{5\}\{-1\}\%
2474
      \text{TestDiv}\{-4\}\{5\}\{0\}\%
      \text{TestDiv}\{-3\}\{5\}\{0\}\%
2475
      \TestDiv{-2}{5}{0}%
2476
      \TestDiv{-1}{5}{0}%
2477
2478
      \TestDiv{0}{5}{0}%
2479
      \TestDiv{1}{5}{0}%
      \TestDiv{2}{5}{0}%
      \TestDiv{3}{5}{0}%
2482
      \TestDiv{4}{5}{0}%
2483
      \TestDiv{5}{5}{1}%
2484
      \TestDiv{6}{5}{1}%
      \text{TestDiv}\{-5\}\{4\}\{-1\}\%
2485
      \text{TestDiv}_{-4}_{4}_{-1}_{\%}
2486
      \text{TestDiv}\{-3\}\{4\}\{0\}\%
2487
2488
      \TestDiv{-2}{4}{0}%
2489
      \text{TestDiv}\{-1\}\{4\}\{0\}\%
2490
      \TestDiv{0}{4}{0}%
2491
      \TestDiv{1}{4}{0}%
2492
      \TestDiv{2}{4}{0}%
2493
      \TestDiv{3}{4}{0}%
2494
      \text{TestDiv}{4}{4}{1}{\%}
      \TestDiv{5}{4}{1}%
2495
```

2496 \TestDiv{12345}{678}{18}%

```
\TestDiv{32372}{5952}{5}%
2497
     \TestDiv{284271294}{18162}{15651}%
2498
     \TestDiv{217652429}{12561}{17327}%
2499
     \TestDiv{462028434}{5439}{84947}%
2500
     \TestDiv{2147483647}{1000}{2147483}%
2501
2502
     \TestDiv{2147483647}{-1000}{-2147483}%
2503
     \TestDiv{-2147483647}{1000}{-2147483}%
2504
     \TestDiv{-2147483647}{-1000}{2147483}%
2505
    \end{qstest}
2506
    \begin{qstest}{mod}{mod}
2507
     \TestMod{-6}{5}{4}%
2508
     TestMod{-5}{5}{0}%
2509
     TestMod{-4}{5}{1}%
2510
     TestMod{-3}{5}{2}%
2511
     TestMod{-2}{5}{3}%
2512
2513
     TestMod{-1}{5}{4}%
2514
     \TestMod{0}{5}{0}%
     TestMod{1}{5}{1}%
2515
2516
     TestMod{2}{5}{2}%
2517
     \TestMod{3}{5}{3}%
     \TestMod{4}{5}{4}%
2518
2519
     \TestMod{5}{5}{0}\%
2520
     \TestMod{6}{5}{1}%
     TestMod{-5}{4}{3}%
2521
     TestMod{-4}{4}{0}%
2522
2523
     TestMod{-3}{4}{1}%
2524
     TestMod{-2}{4}{2}%
2525
     TestMod{-1}{4}{3}%
2526
     TestMod{0}{4}{0}%
     TestMod{1}{4}{1}%
2527
2528
     \TestMod{2}{4}{2}%
2529
     \TestMod{3}{4}{3}%
2530
     \TestMod{4}{4}{0}%
     \TestMod{5}{4}{1}%
2531
2532
     \TestMod{-6}{-5}{-1}\%
2533
     TestMod{-5}{-5}{0}%
2534
     \TestMod{-4}{-5}{-4}%
2535
     \TestMod{-3}{-5}{-3}\%
2536
     TestMod{-2}{-5}{-2}%
     TestMod{-1}{-5}{-1}%
2537
2538
     TestMod{0}{-5}{0}%
     TestMod{1}{-5}{-4}%
2539
2540
     TestMod{2}{-5}{-3}%
2541
     TestMod{3}{-5}{-2}%
     TestMod{4}{-5}{-1}%
2543
     TestMod{5}{-5}{0}%
2544
     TestMod{6}{-5}{-4}%
2545
     TestMod{-5}{-4}{-1}%
2546
     TestMod{-4}{-4}{0}%
     TestMod{-3}{-4}{-3}%
2547
     TestMod{-2}{-4}{-2}%
2548
     TestMod{-1}{-4}{-1}%
2549
2550
     TestMod{0}{-4}{0}%
2551
     TestMod{1}{-4}{-3}%
2552
     TestMod{2}{-4}{-2}%
2553
     TestMod{3}{-4}{-1}%
2554
     TestMod{4}{-4}{0}%
2555
     TestMod{5}{-4}{-3}%
     \verb|\TestMod{2147483647}{1000}{647}\%|
2556
     TestMod{2147483647}{-1000}{-353}%
2557
```

\TestMod{-2147483647}{1000}{353}%

```
\TestMod{-2147483647}{-1000}{-647}%
2559
     \TestMod{ 0 }{ 4 }{0}%
2560
     \TestMod{ 1 }{ 4 }{1}%
2561
     \TestMod{ -1 }{ 4 }{3}%
     \TestMod{ 0 }{ -4 }{0}%
2564 \TestMod{ 1 }{ -4 }{-3}%
2565 \TestMod{ -1 }{ -4 }{-1}%
2566 \langle *etex \rangle
     TestMod{1+2}{1+3}{3}%
2567
     \TestMod{1-2}{1+3}{3}%
2568
     \TestMod{1-2}{1-4}{-1}%
2569
2570 \TestMod{1+2}{1-4}{0}%
2571 \TestMod{1+2}{1-5}{-1}%
2572 (/etex)
2573 \end{qstest}
2574 (/test2 j test4)
2575 (*test2)
2576 \newcommand*{\TestError}[2]{%
     \begingroup
2577
      \expandafter\def\csname IntCalcError:#1\endcsname{}%
2578
2579
      \Expect*{#2}{0}%
2580
       \expandafter\def\csname IntCalcError:#1\endcsname{ERROR}%
2581
       \Expect*{#2}{0ERROR}%
2582
     \endgroup
2583 }
2584 \begin{qstest}{error}{error}
     \TestError{FacNegative}{\intcalcFac{-1}}%
2585
     2586
     \TestError{FacOverflow}{\intcalcFac{13}}%
2587
     \TestError{FacOverflow}{\intcalcFac{2147483647}}%
2588
     \TestError{DivisionByZero}{\intcalcPow{0}{-1}}%
2589
     \TestError{DivisionByZero}{\intcalcDiv{1}{0}}%
2590
     \TestError{DivisionByZero}{\intcalcMod{1}{0}}%
     \TestError{DivisionByZero}{\IntCalcDiv1!0!}%
2593 \TestError{DivisionByZero}{\IntCalcMod1!0!}%
2594 \end{qstest}
2595 (/test2)
2596 (*test2 j test4)
2597 \begin{document}
2598 \end{document}
2599 (/test2 j test4)
```

#### 4 Installation

### 4.1 Download

**Package.** This package is available on CTAN<sup>1</sup>:

CTAN:macros/latex/contrib/oberdiek/intcalc.dtx The source file.

CTAN:macros/latex/contrib/oberdiek/intcalc.pdf Documentation.

**Bundle.** All the packages of the bundle 'oberdiek' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

CTAN:install/macros/latex/contrib/oberdiek.tds.zip

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:tds/tds.pdf). Directories with texmf in their name are usually organized this way.

<sup>&</sup>lt;sup>1</sup>http://ctan.org/pkg/intcalc

#### 4.2 Bundle installation

**Unpacking.** Unpack the oberdiek.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip oberdiek.tds.zip -d ~/texmf
```

Script installation. Check the directory TDS:scripts/oberdiek/ for scripts that need further installation steps. Package attachfile2 comes with the Perl script pdfatfi.pl that should be installed in such a way that it can be called as pdfatfi. Example (linux):

```
chmod +x scripts/oberdiek/pdfatfi.pl
cp scripts/oberdiek/pdfatfi.pl /usr/local/bin/
```

### 4.3 Package installation

**Unpacking.** The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain TFX:

```
tex intcalc.dtx
```

**TDS.** Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\label{eq:control_control} intcalc.sty & \to tex/generic/oberdiek/intcalc.sty intcalc.pdf & \to doc/latex/oberdiek/intcalc.pdf \\ test/intcalc-test1.tex & \to doc/latex/oberdiek/test/intcalc-test1.tex \\ test/intcalc-test2.tex & \to doc/latex/oberdiek/test/intcalc-test2.tex \\ test/intcalc-test3.tex & \to doc/latex/oberdiek/test/intcalc-test3.tex \\ test/intcalc-test4.tex & \to doc/latex/oberdiek/test/intcalc-test4.tex \\ intcalc.dtx & \to source/latex/oberdiek/intcalc.dtx \\ \end{aligned}
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

### 4.4 Refresh file name databases

If your TEX distribution (teTEX, mikTEX, ...) relies on file name databases, you must refresh these. For example, teTEX users run texhash or mktexlsr.

### 4.5 Some details for the interested

Unpacking with LATEX. The .dtx chooses its action depending on the format:

plain T<sub>E</sub>X: Run docstrip and extract the files.

LATEX: Generate the documentation.

If you insist on using  $\LaTeX$  for docstrip (really, docstrip does not need  $\LaTeX$ ), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{intcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

\PassOptionsToClass{a4paper}{article}

An example follows how to generate the documentation with pdfLATeX:

```
pdflatex intcalc.dtx
makeindex -s gind.ist intcalc.idx
pdflatex intcalc.dtx
makeindex -s gind.ist intcalc.idx
pdflatex intcalc.dtx
```

## 5 Catalogue

The following XML file can be used as source for the TEX Catalogue. The elements caption and description are imported from the original XML file from the Catalogue. The name of the XML file in the Catalogue is intcalc.xml.

```
2600 (*catalogue)
2601 <?xml version='1.0' encoding='us-ascii'?>
2602 <!DOCTYPE entry SYSTEM 'catalogue.dtd'>
2603 <entry datestamp='$Date$' modifier='$Author$' id='intcalc'>
2604 <name>intcalc</name>
2605 <caption>Expandable arithmetic operations with integers.</caption>
2606 <authorref id='auth:oberdiek'/>
2607 <copyright owner='Heiko Oberdiek' year='2007'/>
2608 cense type='lppl1.3'/>
2609 <version number='1.2'/>
     <description>
2610
       This package provides expandable arithmetic operations
2612
       with integers, using the e-TeX extension <tt>\numexpr</tt> if it
2613
2614
       The package is part of the xref refid='oberdiek'>oberdiek</pref>
2615
2616
       bundle.
     </description>
2617
     <documentation details='Package documentation'</pre>
2618
        href='ctan:/macros/latex/contrib/oberdiek/intcalc.pdf'/>
2619
2620 <ctan file='true' path='/macros/latex/contrib/oberdiek/intcalc.dtx'/>
2621 <miktex location='oberdiek'/>
2622 <texlive location='oberdiek'/>
2623 <install path='/macros/latex/contrib/oberdiek/oberdiek.tds.zip'/>
2624 </entry>
2625 (/catalogue)
```

# 6 History

## [2007/09/09 v1.0]

• First version.

## [2007/09/27 v1.1]

- \intcalcNum added.
- $\bullet$  \intcalcSh1 and \intcalcShr allow negative numbers. The sign is preserved.
- Reuse \@gobble instead of own macro \IntCalc@Gobble.
- · Small fixes.

- $\bullet\,$  Shorter internal prefix.
- $\bullet\,$  Some programmer's interface.

# [2016/05/16 v1.2]

 $\bullet\,$  Documentation updates.

# 7 Index

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\TestDec 1876,	2422, 2423, 2424, 2425, 2426,
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2242, 2243, 2244, 2245, 2246, 2247	2432, 2433, 2434, 2435, 2436,
\TestDiv 1963, 2073, 2456, 2457, 2458,	2437, 2438, 2439, 2440, 2441,
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2464, 2465, 2466, 2467, 2468,	
2469, 2470, 2471, 2472, 2473,	\TestResult
2474, 2475, 2476, 2477, 2478,	\TestResultTwoExpansions . 1820, 1840
2479, 2480, 2481, 2482, 2483,	\TestSgn 1853, 2059,
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2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498	2140, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2156, 2157
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\TestDo 2036, 2050, 2053, 2054	2315, 2316, 2317, 2318, 2319, 2320
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\TestShr 1936,	330, 366, 375, 379, 380, 416,
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2343, 2344, 2345, 2346, 2347,	$\dots$ 94, 101, 102, 103, 104,
2348, 2349, 2350, 2351, 2352, 2353	105, 106, 107, 108, 109, 110, 111
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\TestSqr 1954, 2070,	
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2392, 2393, 2394, 2395, 2396, 2397	\UNDEFINED 1794, 1807, 1815, 1823
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2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2309, 2310	X
	<b>X</b> \x 14, 15, 18, 22, 26, 28, 51, 56, 66, 75,
2304, 2305, 2306, 2307, 2309, 2310	<del></del>
2304, 2305, 2306, 2307, 2309, 2310 \TestTeXDivide 1833, 1965	\x 14, 15, 18, 22, 26, 28, 51, 56, 66, 75,
2304, 2305, 2306, 2307, 2309, 2310 \TestTeXDivide 1833, 1965 \TestTime 2002, 2015, 2016, 2017	\x 14, 15, 18, 22, 26, 28, 51, 56, 66, 75, 87, 640, 643, 1868, 1873, 1879,
2304, 2305, 2306, 2307, 2309, 2310 \TestTeXDivide 1833, 1965 \TestTime 2002, 2015, 2016, 2017 \TestTwo 2052, 2060, 2061, 2062,	\x 14, 15, 18, 22, 26, 28, 51, 56, 66, 75, 87, 640, 643, 1868, 1873, 1879, 1884, 1892, 1898, 1900, 1906,
2304, 2305, 2306, 2307, 2309, 2310 \TestTexDivide 1833, 1965 \TestTime 2002, 2015, 2016, 2017 \TestTwo 2052, 2060, 2061, 2062, 2065, 2066, 2069, 2072, 2073, 2074	\x 14, 15, 18, 22, 26, 28, 51, 56, 66, 75, 87, 640, 643, 1868, 1873, 1879, 1884, 1892, 1898, 1900, 1906, 1916, 1922, 1929, 1934, 1938,
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