Theoretical calculation of the droplet induced agitation (or pseudoturbulence) in mono disperse buoyant emulsions for low inertia and dilute regime.

Nicolas Fintzi[1], Daniel Lhuillier[2], Jean-Lou Pierson[1]
[1]*IFP Energies Nouvelles, Solaize, 69360, France,jean-lou.pierson@ifpen.fr*
[2]*Affiliation of second author: University, School, Department, Institute, City, Country, E-mail*

Abstract:

Buoyancy-driven droplet flows are encountered in many chemical engineering processes such as gravity separators and liquid-liquid extractors. The usual engineering practice to model such facilities is to use the averaged Navier-Stokes equations. The present work focuses on the velocity fluctuations tensor, also known as the *Reynolds stress* tensor, or Pseudo-turbulent tensor, which is a crucial closure term in these equations. Formally, this stress is defined as $\langle \mathbf{u}'\mathbf{u}' \rangle_f$, where $\mathbf{u}'$ is the fluid velocity fluctuation, and $\langle \ldots \rangle_f$ denotes an ensemble average procedure applied over the continuous phase. The *Reynolds stress* can be separated into two contributions : (1) The agitation generated due to the averaged wakes around the particles; (2) All other sources of fluctuations, such as those generated through particle interactions, and single-phase turbulence [**?**]. This study only considers the first contribution. Specifically we compute $\langle \mathbf{u}'\mathbf{u}' \rangle_f$, for buoyant rising droplets in an otherwise quiescent fluid. The derivation is restricted to spherical droplets (of radius a), at small particle Reynolds number, and low particle volume fraction (ϕ).

Keywords: PTKE equations, hybrid model, Averaged equations, dispersed multiphase flows

For buoyant inviscid bubbly flows under the dilute assumption, it is known since the study of [**?**] that the *Reynolds stress* can be written as

$$\langle \mathbf{u}'\mathbf{u}' \rangle_f (\mathbf{x}) \approx n_p \int_{|\mathbf{r}|>a} \mathbf{v}\mathbf{v} d\mathbf{r} = \phi \left(\frac{3}{20} U^2 \mathbf{I} + \frac{1}{20} \mathbf{U}\mathbf{U} \right), \tag{1}$$

where n_p is the number of particles per unit of volume, $\mathbf{U}$ is the averaged particle velocity, and $\mathbf{v}$ is the disturbance velocity field of an isolated particle. The integral in Equation (1) could be computed since $\mathbf{v} \sim \mathcal{O}(r^{-3})$, with $r = |\mathbf{r}|$ in the case of potential flows. However, the disturbance velocity field of a translating droplet in Stokes flows (see Figure 1) decays as $\sim \mathcal{O}(r^{-1})$. Therefore, the integral in Equation (1) cannot be computed for Stokes flows. This is the main reason why no theoretical model exists for $\langle \mathbf{u}'\mathbf{u}' \rangle_f$ in the limit of low Reynolds number.

To bypass the problem of divergent integrals we make use of the *Nearest Particle Statistics* framework recently revisited by [**?**]. In this context we can express the *Reynolds stress* in terms of nearest-particles averaged quantities, which yields

$$\langle \mathbf{u}'\mathbf{u}' \rangle_f (\mathbf{x}, t) \approx \int_{r>a} \mathbf{v}\mathbf{v} P^f_{\text{nst}}(\mathbf{r}|\mathbf{x}, t) d\mathbf{r}.$$

maybe include v nst insteadWhere $P^f_{\text{nst}}(\mathbf{r}|\mathbf{x}, t)$ is the probability that the nearest particle's center of mass is located at $\mathbf{r}$, conditionally on the point $\mathbf{x}$ being occupied by the continuous phase. In the homogeneous and dilute regime $P^f_{\text{nst}}(\mathbf{r}|\mathbf{x}, t) = n_p e^{-\phi[(r/a)^3-8]}$ [**?**]. The rapid decay of P^f_{nst} at large r enables us to compute the integral of the disturbance velocity fields even in low inertia regimes. Carrying out the integration yields directly

$$\frac{\left\langle \mathbf{u}'_y \mathbf{u}'_y \right\rangle_f}{U^2} = \frac{\phi}{5(\lambda+1)^2} \left[\frac{7\Gamma(1/3)}{12} (3\lambda+2)^2 \phi^{-1/3} - (17\lambda^2 + 22\lambda + 7) \right] + \mathcal{O}(\phi^{4/3}) \tag{2}$$

results fr the trace where $\Gamma(1/3) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function with $\Gamma(1/3) = 2.678$, λ is the viscosity ratio between the dispersed and continuous phase, and y is the direction of gravity. As can be observed on Figure 1, the present theory is in very good agreement with the experimental results of [**?**]. Direct numerical

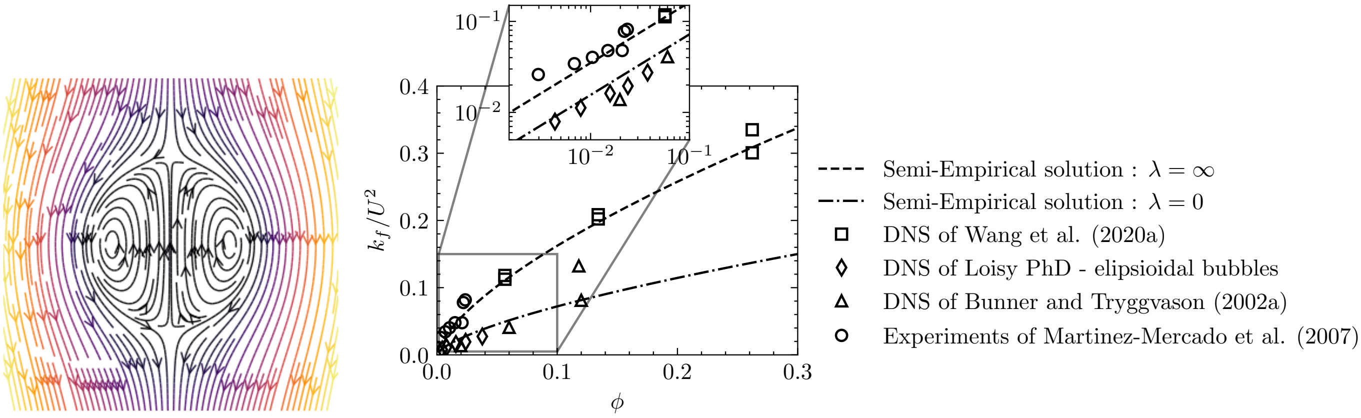

Figure 1: (left) Streamlines of a translating droplet in Stokes flow. (right) Dimensionless Reynolds stress for rising bubbly flows in the direction of gravity. (dots) Experimental results of [**?**]. Equation (1) : Potential flow theory [**?**]. Equation (2) Stokes flow theory.

simulations of rising buoyant emulsions have also been performed (with the code `http://basilisk.fr`) for $\lambda = 0.1, 1, 10$. Good agreement is also obtained.

In conclusion we provided a *Reynolds stress* closure valid in the dilute Stokes flow regime for arbitrary viscosity ratios.