Geometry I: Comprehensive Solutions & Audit

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1 Part I: Final Answers Summary

Use this section to quickly check your results before diving into the workings.

1. **Equation of Circle:** $x^2 + y^2 = 25$
2. **Equation of Line:** $y = -x + 4$ (or $x + y - 4 = 0$)
3. **Intersection Points:** $(2.48, -1.96)$ and $(-0.08, 3.16)$
4. **Point Position:** The point $(3, 4)$ lies **OUTSIDE** the circle.
5. **Parallelogram Test:** $ABCD$ is **NOT** a parallelogram (Opposite sides are unequal).
6. **Intersection Angle:** Acute angle $\theta \approx 84.61°$ (Obtuse $\approx 95.39°$).
7. **Midpoint:** $(2, 6)$
8. **Distance Between Lines:** 3 units.
9. **Centre and Radius:** Centre $(4, 0)$, Radius $r \approx 3.61$ units ($\sqrt{13}$).

2 Part II: Detailed Worked Solutions

Question 1: Diameter of a Circle

Given: Diameter endpoints $A(3, -4)$ and $B(-3, 4)$.

Step-by-Step Derivation

Step 1: Find the Centre (Midpoint Formula)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $x_1 = 3, x_2 = -3, y_1 = -4, y_2 = 4$:

$$M = \left(\frac{3 + (-3)}{2}, \frac{-4 + 4}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Centre: $C(0, 0)$.

Step 2: Find the Radius (Distance from Centre to Point A)

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute Centre $(0, 0)$ and Point $A(3, -4)$:

$$r = \sqrt{(3 - 0)^2 + (-4 - 0)^2}$$

$$r = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 3: Write the Standard Equation

$$(x - h)^2 + (y - k)^2 = r^2$$

Substitute $h = 0, k = 0, r = 5$:

$$(x - 0)^2 + (y - 0)^2 = 5^2$$

$$\boxed{x^2 + y^2 = 25}$$

Alternative Method: Diameter Form

The equation of a circle with diameter endpoints (x_1, y_1) and (x_2, y_2) is:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Substitute points:

$$(x - 3)(x - (-3)) + (y - (-4))(y - 4) = 0$$

$$(x - 3)(x + 3) + (y + 4)(y - 4) = 0$$

Expand (difference of squares):

$$(x^2 - 9) + (y^2 - 16) = 0$$

$$x^2 + y^2 - 25 = 0 \implies x^2 + y^2 = 25$$

Question 2: Equation of a Line

Given: Points $P_1(-3, 7)$ and $P_2(5, -1)$.

Step-by-Step Derivation

Step 1: Calculate Gradient (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 7}{5 - (-3)}$$

$$m = \frac{-8}{5 + 3} = \frac{-8}{8} = -1$$

Step 2: Use Point-Slope Form Using $P_1(-3, 7)$ and $m = -1$:

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -1(x - (-3))$$
$$y - 7 = -1(x + 3)$$

Expand the brackets:

$$y - 7 = -x - 3$$

Add 7 to both sides:

$$y = -x - 3 + 7$$
$$\boxed{y = -x + 4}$$

Question 3: Line-Circle Intersection

Given: Line $2x + y = 3$ and Circle $x^2 + y^2 = 10$.

Step-by-Step Derivation

Step 1: Isolate a variable in the linear equation From $2x + y = 3$, subtract $2x$ from both sides:

$$y = 3 - 2x$$

Step 2: Substitute into the Circle Equation

$$x^2 + (3 - 2x)^2 = 10$$

Step 3: Expand the squared term Recall $(a - b)^2 = a^2 - 2ab + b^2$. Here $a = 3, b = 2x$.

$$(3 - 2x)^2 = 3^2 - 2(3)(2x) + (2x)^2$$
$$= 9 - 12x + 4x^2$$

Substitute back:

$$x^2 + (9 - 12x + 4x^2) = 10$$

Step 4: Form the Quadratic Equation Combine like terms ($x^2 + 4x^2 = 5x^2$):

$$5x^2 - 12x + 9 = 10$$

Subtract 10 from both sides:

$$5x^2 - 12x - 1 = 0$$

Step 5: Solve for x (Quadratic Formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 5, b = -12, c = -1.$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-1)}}{2(5)}$$

$$x = \frac{12 \pm \sqrt{144 + 20}}{10} = \frac{12 \pm \sqrt{164}}{10}$$

$$\sqrt{164} \approx 12.806$$

$$x_1 = \frac{12 + 12.806}{10} = 2.4806 \quad \text{and} \quad x_2 = \frac{12 - 12.806}{10} = -0.0806$$

Step 6: Solve for y Using $y = 3 - 2x$:

- If $x \approx 2.48$: $y = 3 - 2(2.48) = 3 - 4.96 = -1.96$.
- If $x \approx -0.08$: $y = 3 - 2(-0.08) = 3 + 0.16 = 3.16$.

Answers: $(2.48, -1.96)$ and $(-0.08, 3.16)$.

Common Error: The Square Root Trap

Avoid writing $y = \sqrt{10 - x^2}$. Why? The symbol $\sqrt{\ldots}$ (principal root) only outputs positive numbers. If you use it, you delete the bottom half of the circle ($y < 0$) and will miss the intersection point at $y = -1.96$.

Question 4: Point Position Check

Goal: Check if $(3, 4)$ is inside $x^2 + y^2 - 14x + 16y = 0$.

Step-by-Step Derivation

Step 1: Evaluate the function at the point Let $f(x, y) = x^2 + y^2 - 14x + 16y$. Substitute $x = 3, y = 4$:

$$f(3, 4) = (3)^2 + (4)^2 - 14(3) + 16(4)$$

Calculate terms:

$$= 9 + 16 - 42 + 64$$
$$= 25 - 42 + 64$$
$$= -17 + 64$$
$$= 47$$

Step 2: Interpret the result

- If result < 0: Point is Inside.
- If result $= 0$: Point is On the Circle.
- If result > 0: Point is Outside.

Since $47 > 0$, the point $(3, 4)$ is **OUTSIDE** the circle (not inside as may have been expected).

Question 5: Parallelogram Verification

Given: $A(0, -3)$, $B(5, 0)$, $C(4, -3)$, $D(-4, -1)$.

Step-by-Step Derivation

Theory: A parallelogram must have two pairs of equal opposite sides ($AB = CD$ and $BC = AD$). We only need to disprove one pair to fail the test.

Step 1: Length of AB

$$d = \sqrt{(5-0)^2 + (0-(-3))^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34} \approx 5.83$$

Step 2: Length of CD (Opposite to AB)

$$d = \sqrt{(-4-4)^2 + (-1-(-3))^2} = \sqrt{(-8)^2 + (2)^2} = \sqrt{64+4} = \sqrt{68} \approx 8.25$$

Conclusion: Since $\sqrt{34} \neq \sqrt{68}$, the opposite sides are not equal. Therefore, **ABCD is not a parallelogram**.

Question 6: Angle of Intersection

Line 1: $(4, 3)$ and $(-6, 0)$. **Line 2:** $(0, 0)$ and $(-1, 5)$.

Step-by-Step Derivation

Step 1: Calculate Gradients

$$m_1 = \frac{0-3}{-6-4} = \frac{-3}{-10} = 0.3$$

$$m_2 = \frac{5-0}{-1-0} = \frac{5}{-1} = -5$$

Step 2: Apply the Angle Formula

$$\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

Substitute values:

$$\text{Numerator} = -5 - 0.3 = -5.3$$

$$\text{Denominator} = 1 + (0.3)(-5) = 1 - 1.5 = -0.5$$

$$\tan\theta = \left| \frac{-5.3}{-0.5} \right| = |10.6| = 10.6$$

Step 3: Solve for θ

$$\theta = \tan^{-1}(10.6) \approx 84.61^\circ$$

Note: Two Angles

Intersecting lines create two angles that sum to 180°.

- Acute Angle: 84.61° (Found by using absolute value).
- Obtuse Angle: $180° - 84.61° = 95.39°$.

Question 7: Midpoint Calculation

Given: $A(-7, 12)$ and $B(11, 0)$.

Step-by-Step Derivation

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute:

$$M = \left(\frac{-7 + 11}{2}, \frac{12 + 0}{2}\right)$$
$$M = \left(\frac{4}{2}, \frac{12}{2}\right)$$
$$M = (2, 6)$$

Question 8: Distance Between Parallel Lines

Lines: $L_1 : 4x - 3y - 9 = 0$ and $L_2 : 4x - 3y - 24 = 0$.

Step-by-Step Derivation

Step 1: Identify Constants The lines are in the form $Ax + By + C = 0$. $A = 4$, $B = -3$. $C_1 = -9$, $C_2 = -24$.

Step 2: Use the Distance Formula

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Substitute:

$$d = \frac{|-9 - (-24)|}{\sqrt{4^2 + (-3)^2}}$$

Simplify numerator (watch the signs!):

$$|-9 + 24| = |15| = 15$$

Simplify denominator:

$$\sqrt{16 + 9} = \sqrt{25} = 5$$

Final Division:

$$d = \frac{15}{5} = 3 \text{ units}$$

Question 9: Centre and Radius

Equation: $(x-4)^2 + y^2 = 13$.

Step-by-Step Derivation

Step 1: Match with Standard Form Standard form: $(x-h)^2 + (y-k)^2 = r^2$. Given: $(x-4)^2 + (y-0)^2 = 13$.
Step 2: Extract Values $h = 4$, $k = 0 \implies$ Centre $(4, 0)$. $r^2 = 13 \implies r = \sqrt{13}$.
Step 3: Calculate Radius Value

$$r = \sqrt{13}$$

$$r \approx 3.60555...$$

Round to 2 decimal places: ≈ 3.61.