COMPX216-24A
Artificial Intelligence

Local search and optimisation

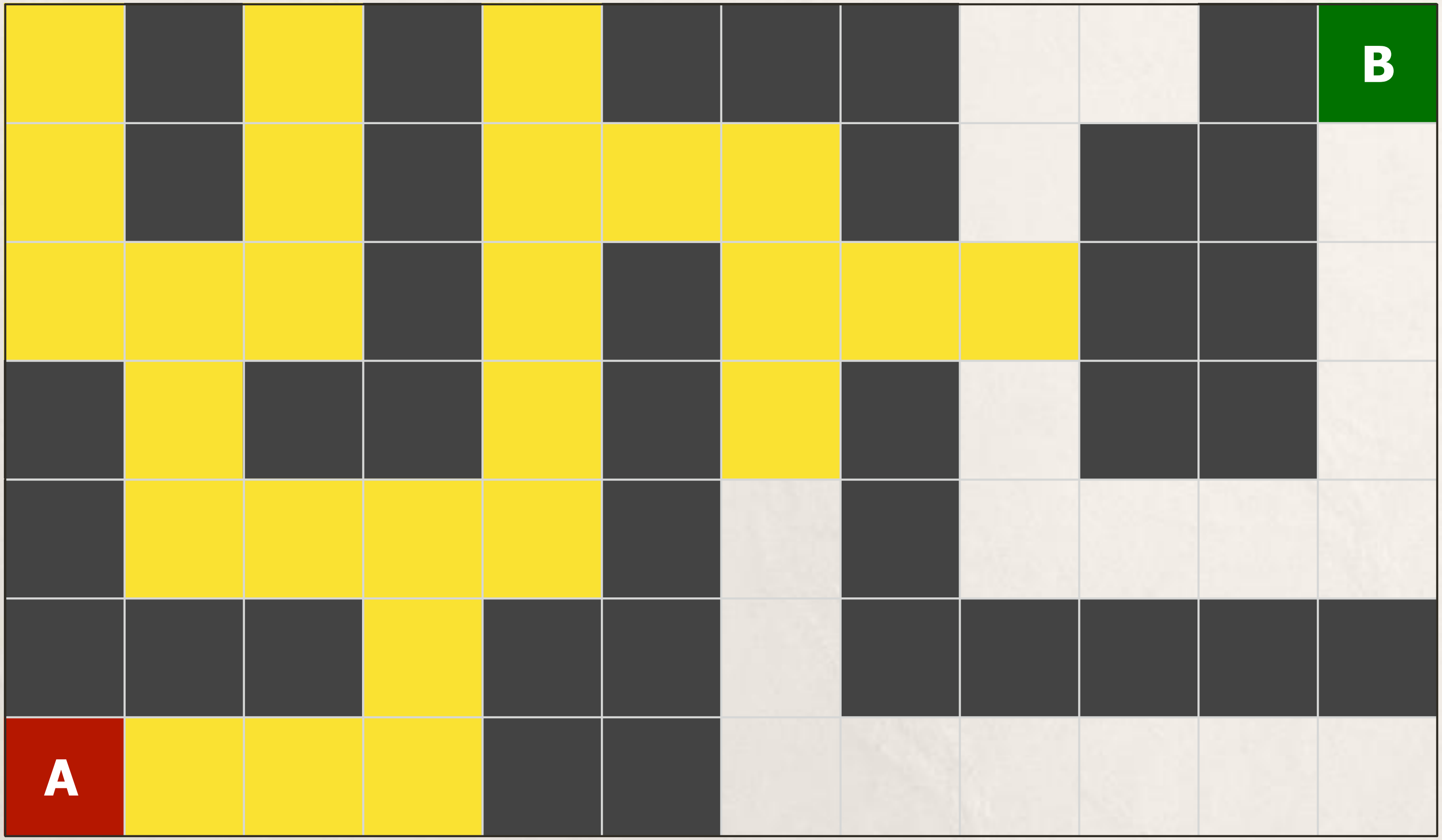
Today: Local search and optimisation

- Optimisation problems
- Local search
- Hill-climbing search
- Local and global optima
- Ridges and plateaus
- The 8-queens problem
- Variants of hill-climbing
- Simulated annealing
- Local beam search

Optimisation problems

- In many applications, we just want to find a good state, not a sequence of actions that get us to a goal from an initial state
- Example: given items of different weight, how can we distribute them as fairly as possible into two backpacks?
  - This is an instance of the *number partitioning problem*
- Finding a good state is called an **optimisation problem**
- In the context of optimization problems, the evaluation function used for states is called the **objective function**
- Depending on the problem, we may want to find a state where the objective function is **maximised** or **minimised**

Recall: Search problems

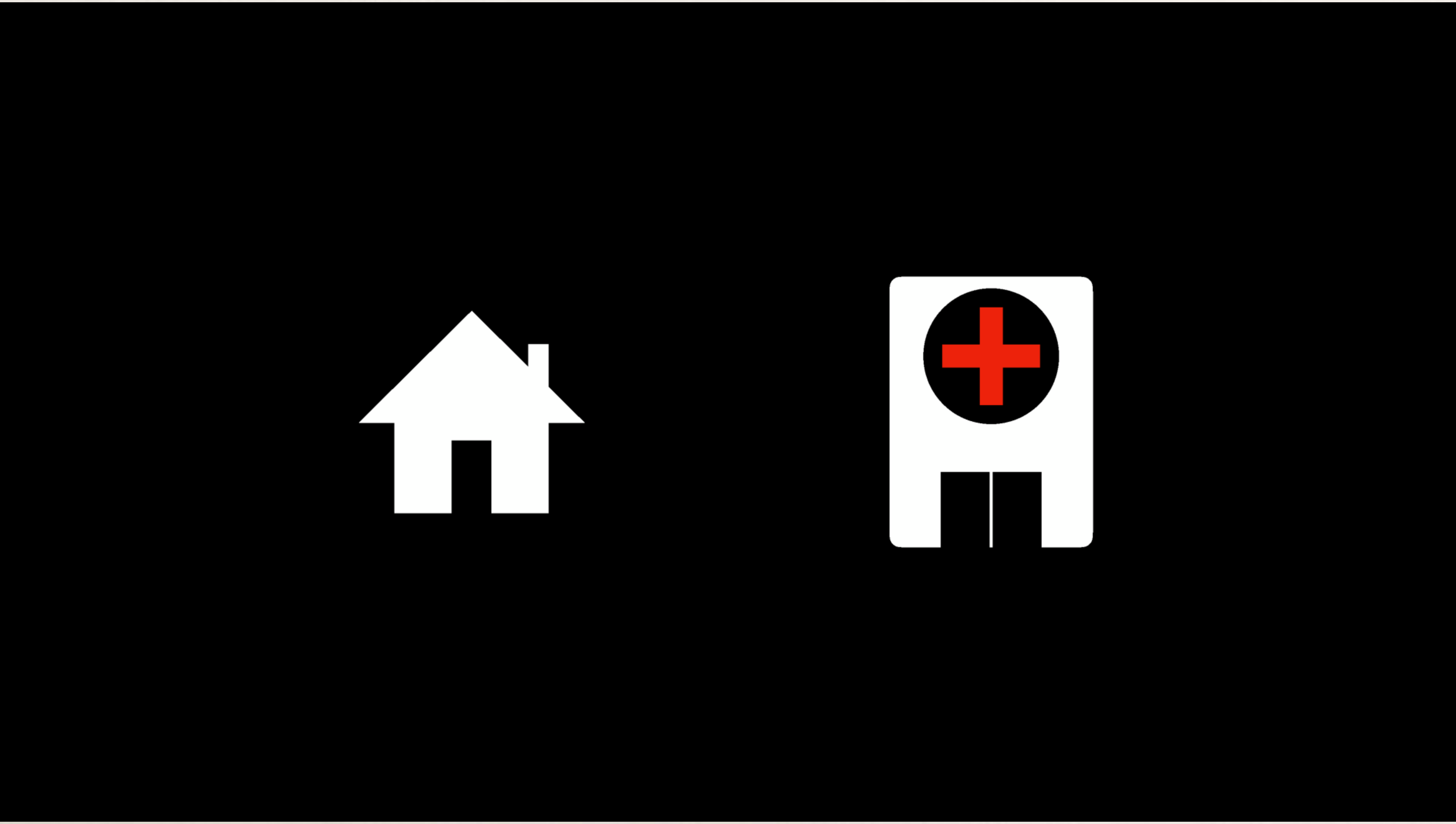


Local search problems

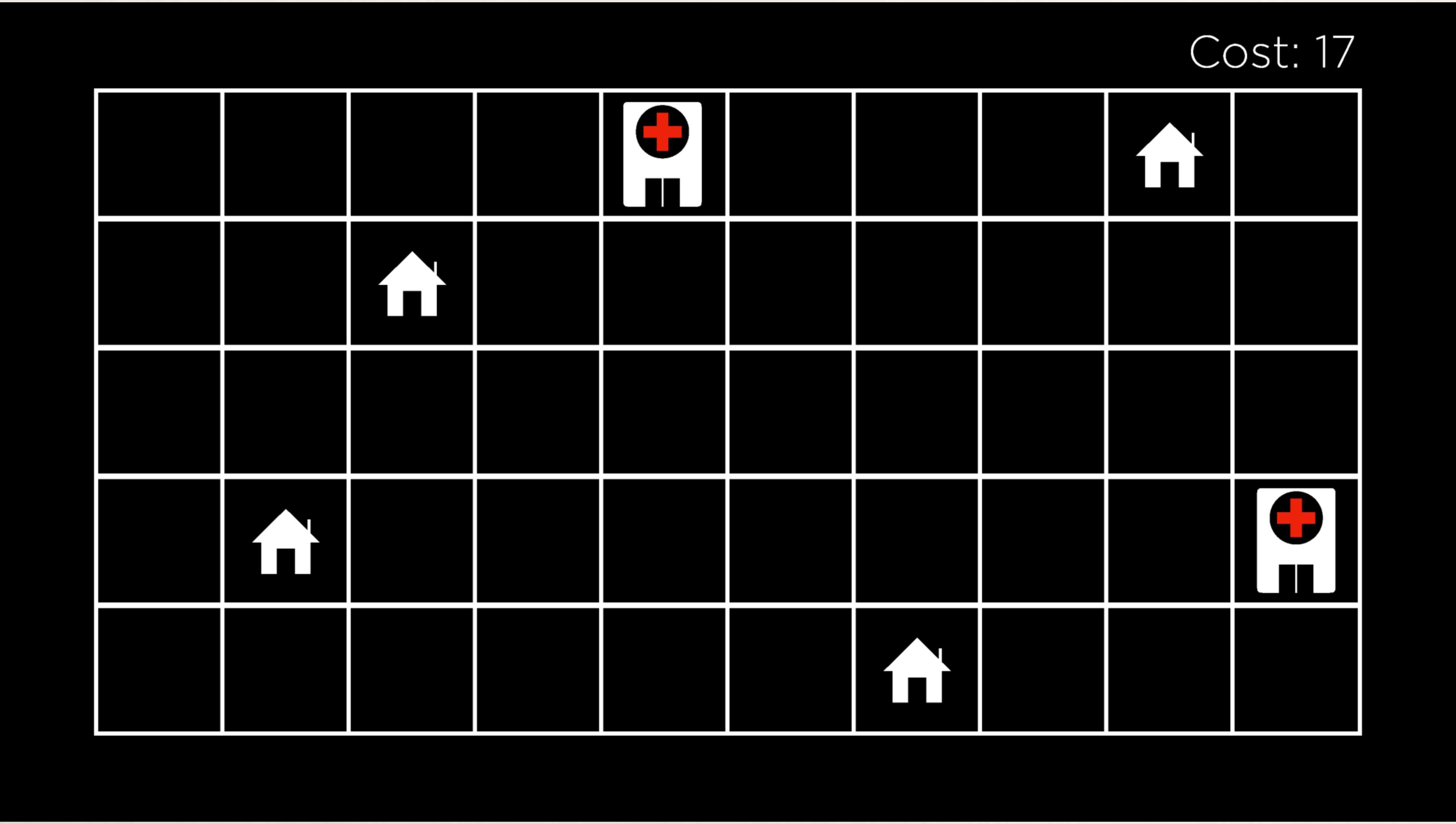
Local search

- Optimisation problems are often tackled using **local search** algorithms that may not find the best possible state
- Local search algorithms do not keep track of paths used to reach states and do not keep track of states reached previously
- In the simplest case, they move around the state space by keeping information only about the state that looks best so far
- Local search algorithms require very limited memory but are unsystematic in their search

Local search and optimisation



Local search and optimisation



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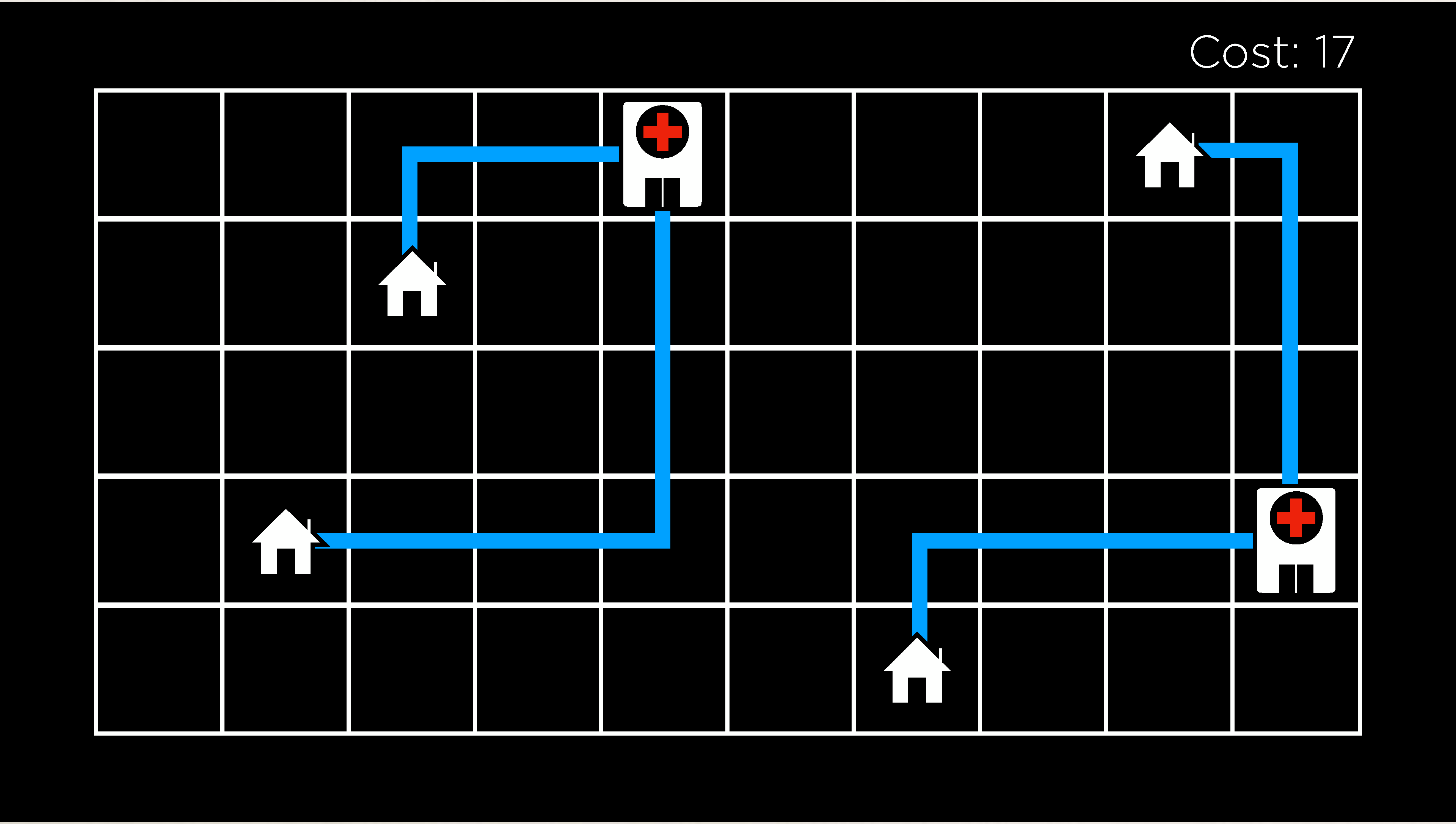

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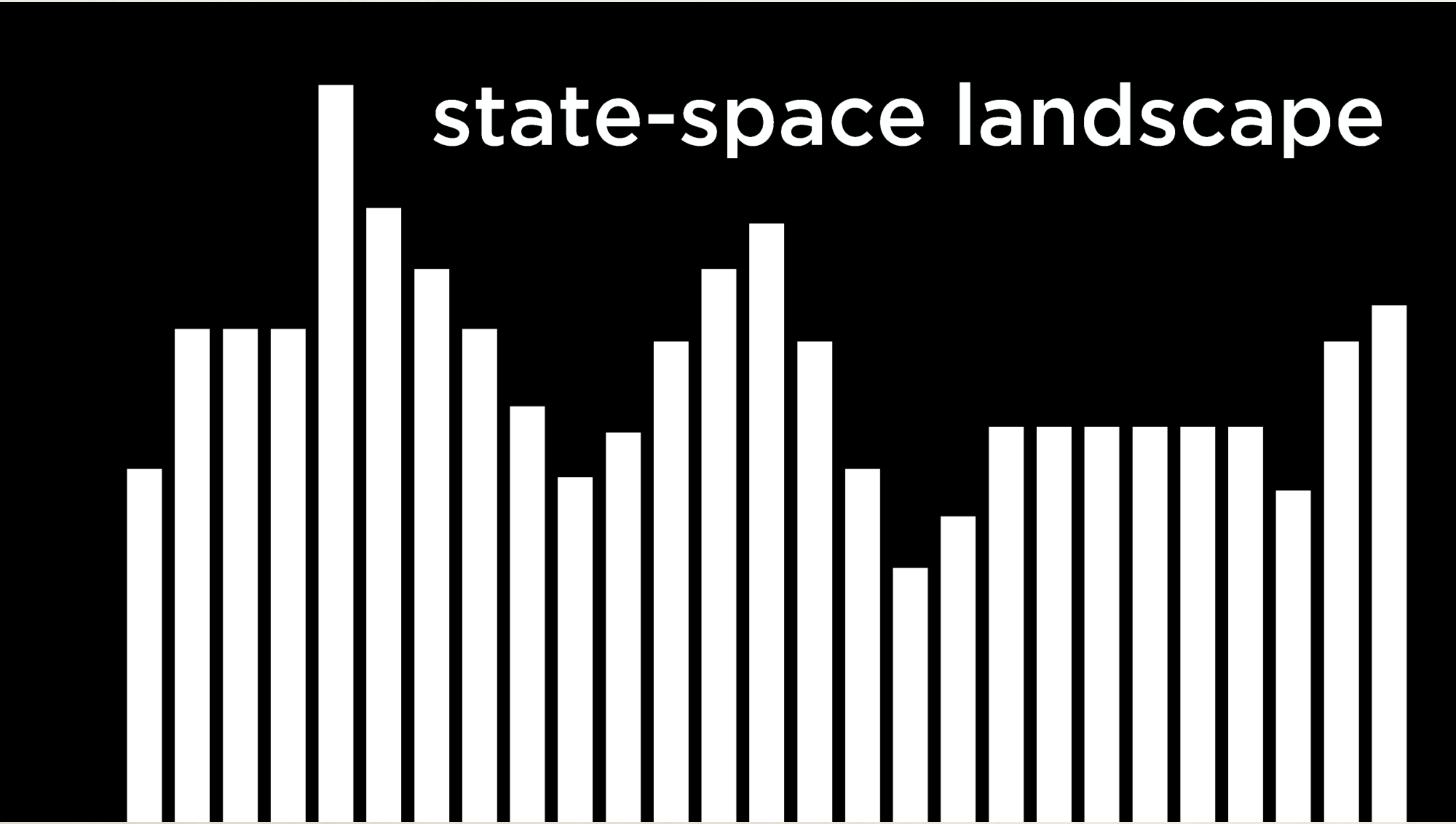

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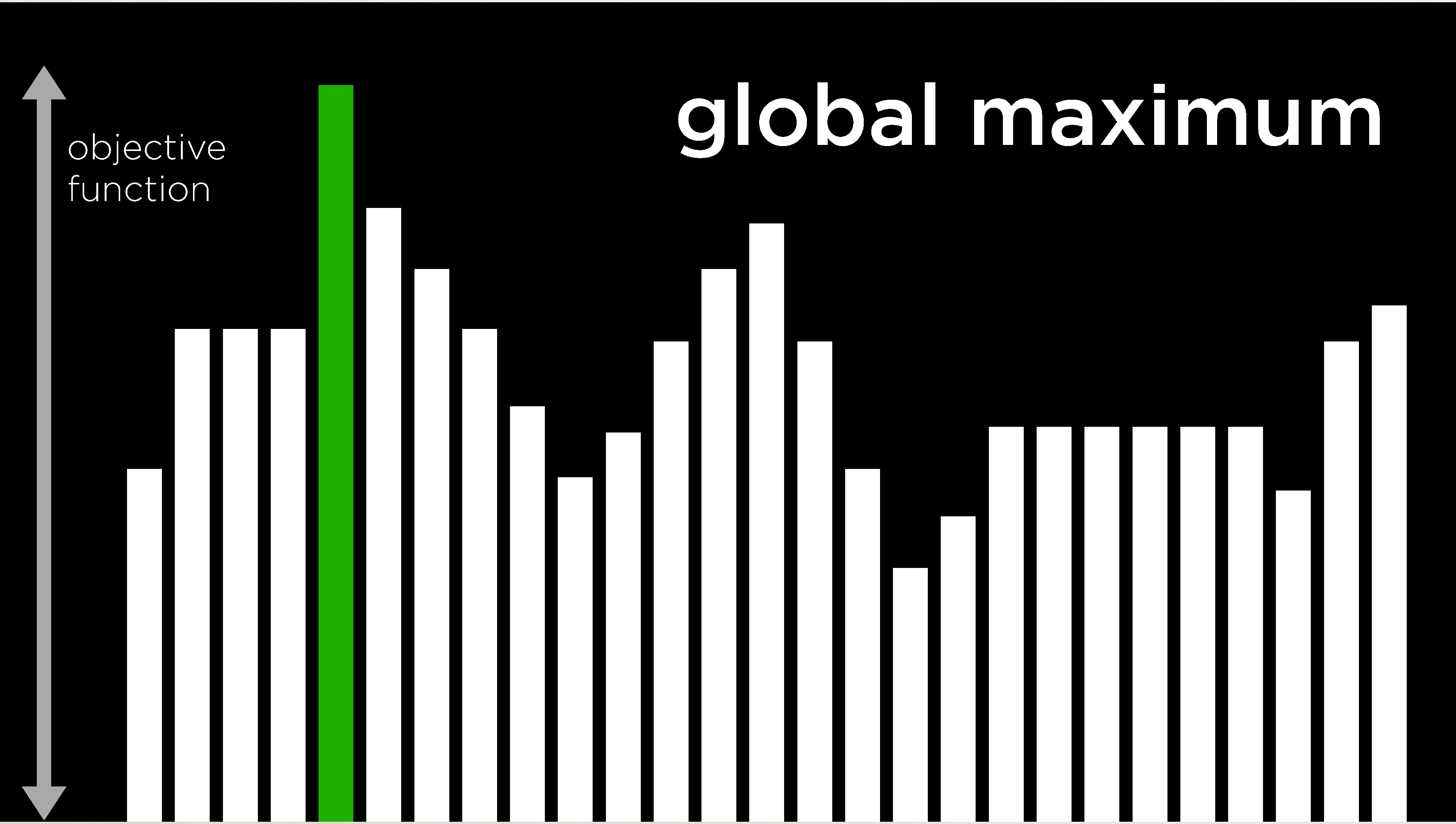

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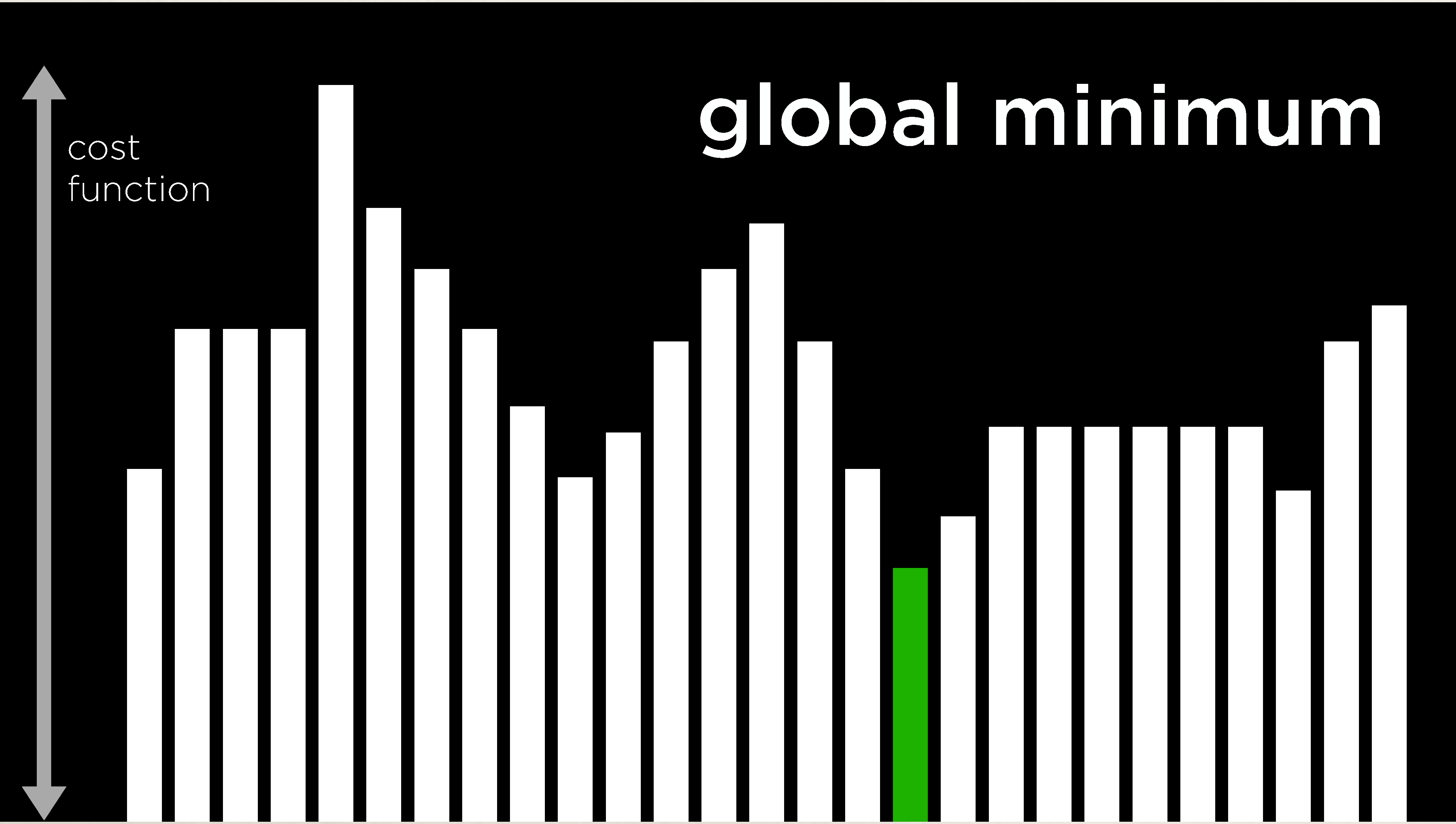

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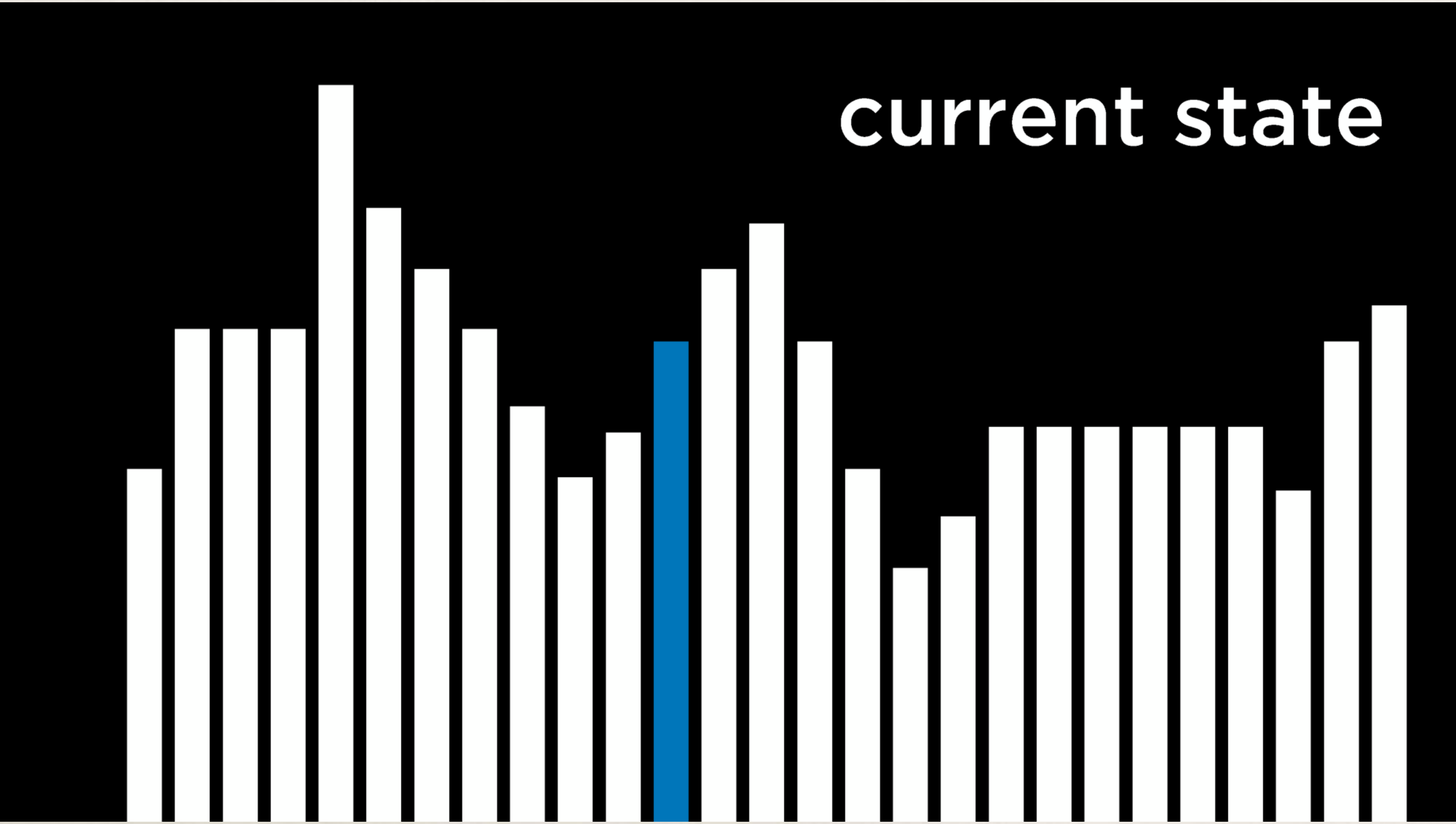

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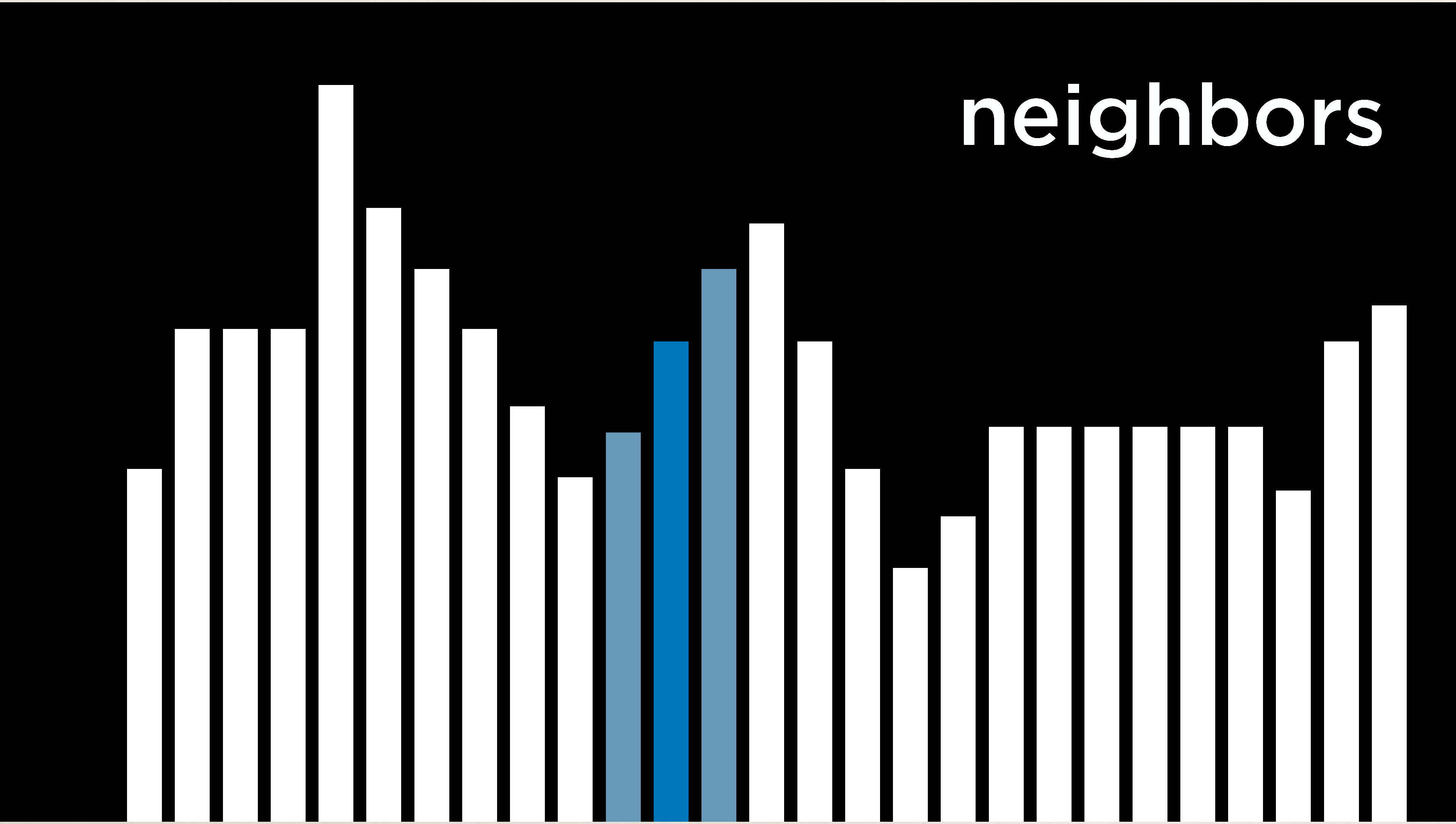
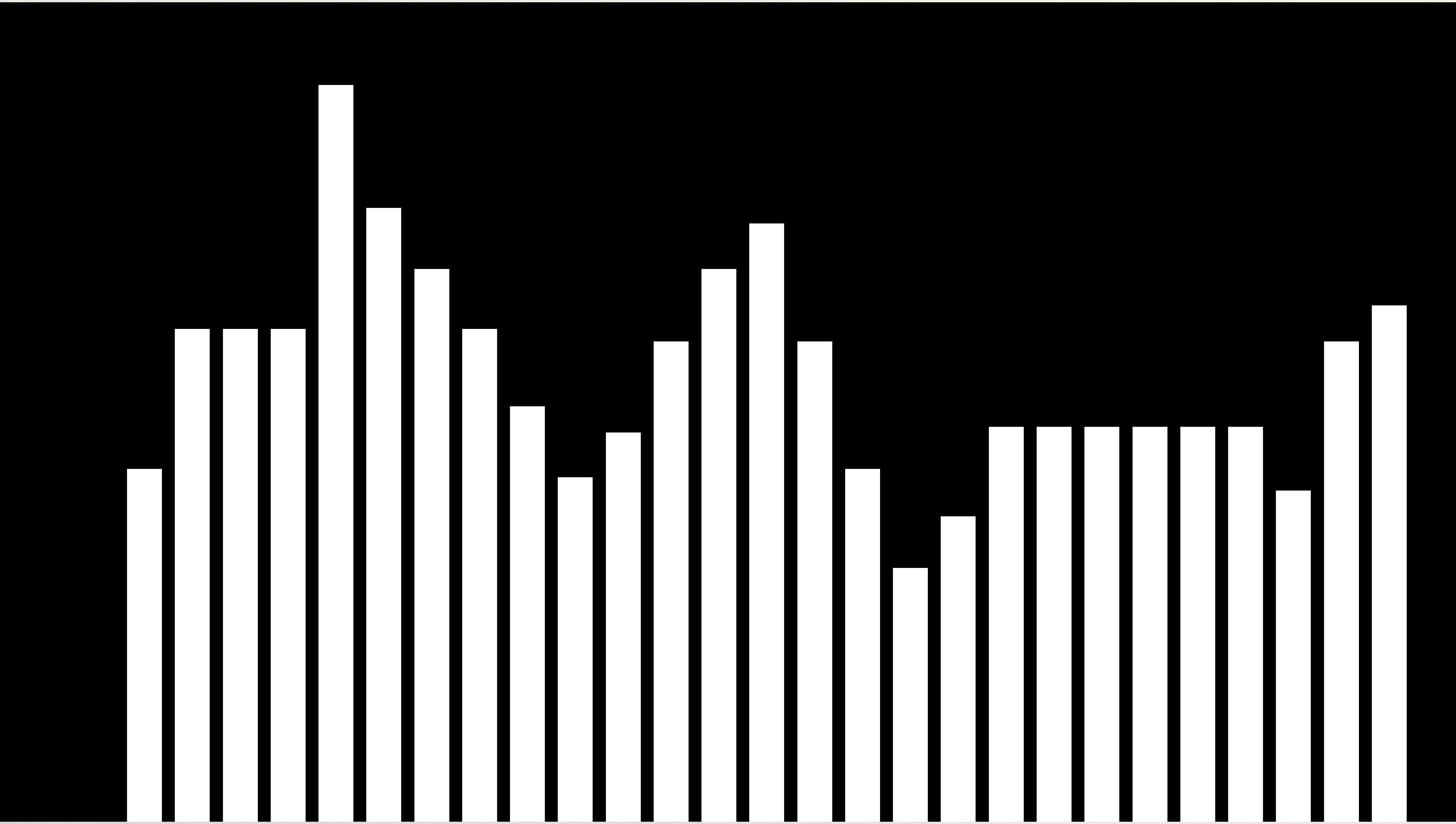


Hill-climbing search

- **Hill-climbing search** is a type of optimsation problem.
- Moves to the best neighbour in state space in each iteration until no neighbour is better
- More sophisticated variants keep information about a fixed-size set of states

https://inteligenciaartificial360.com/glosario/algoritmo-de-hill-climbing/

Hill-climbing search



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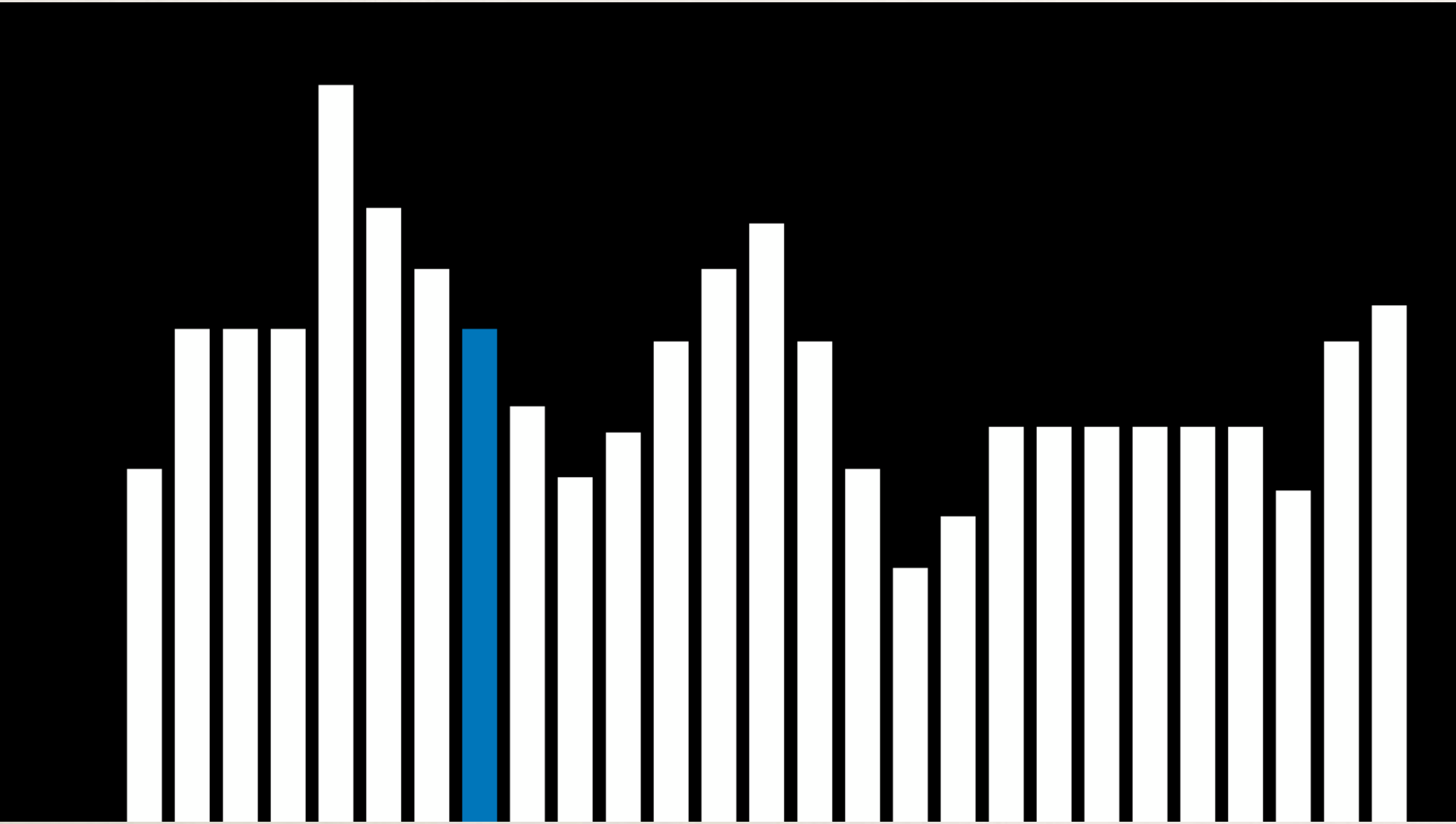

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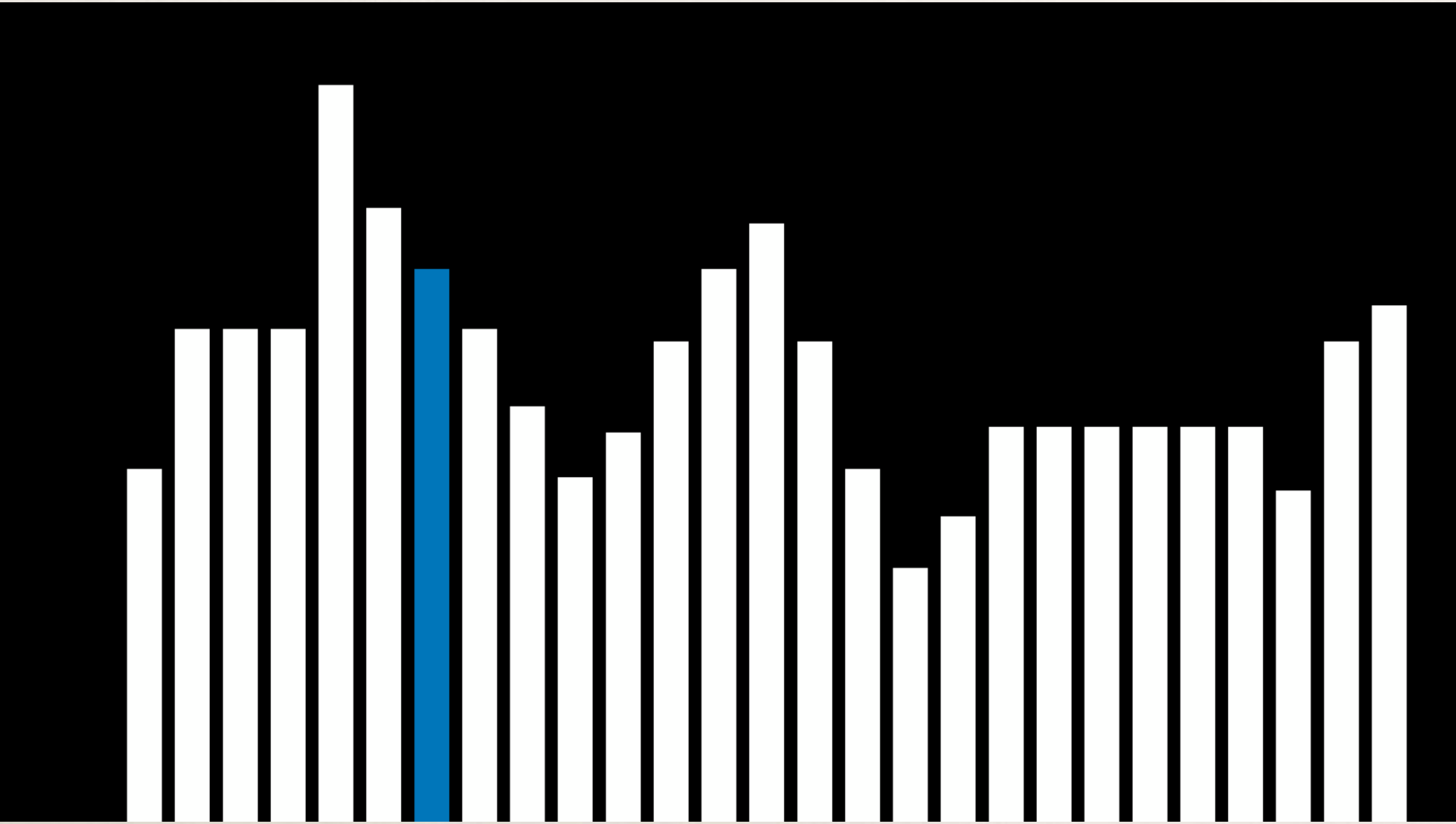

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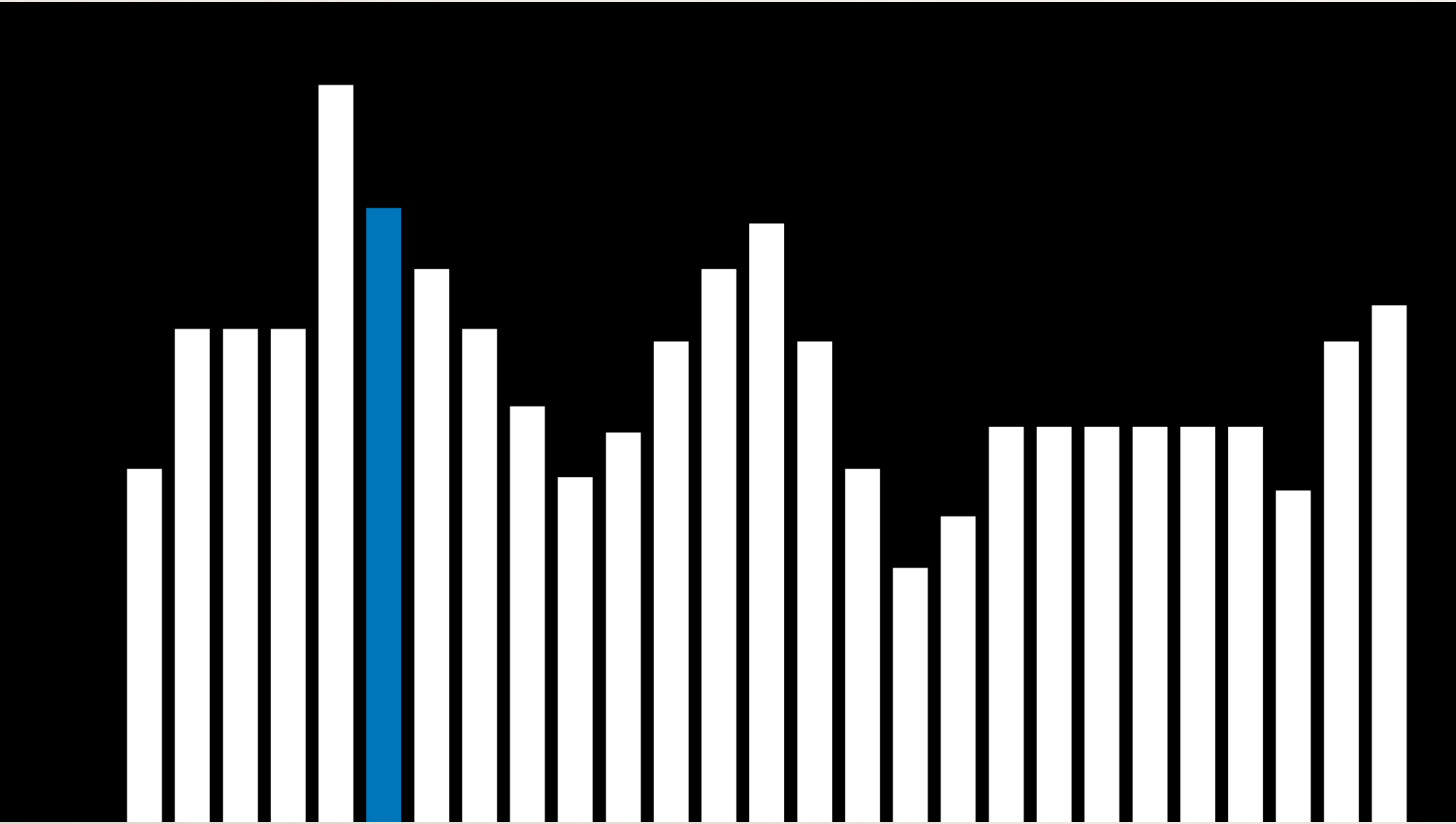

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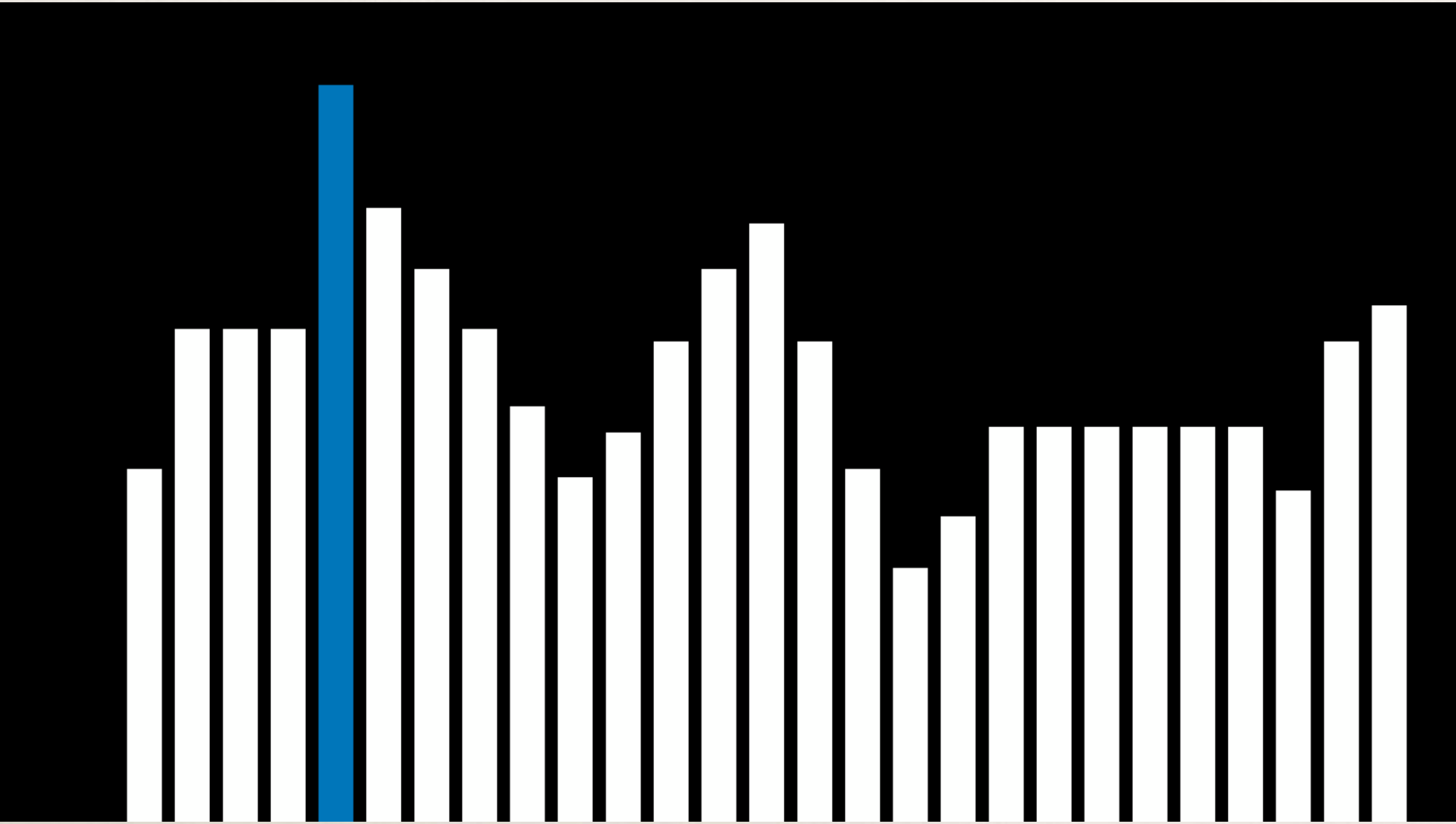

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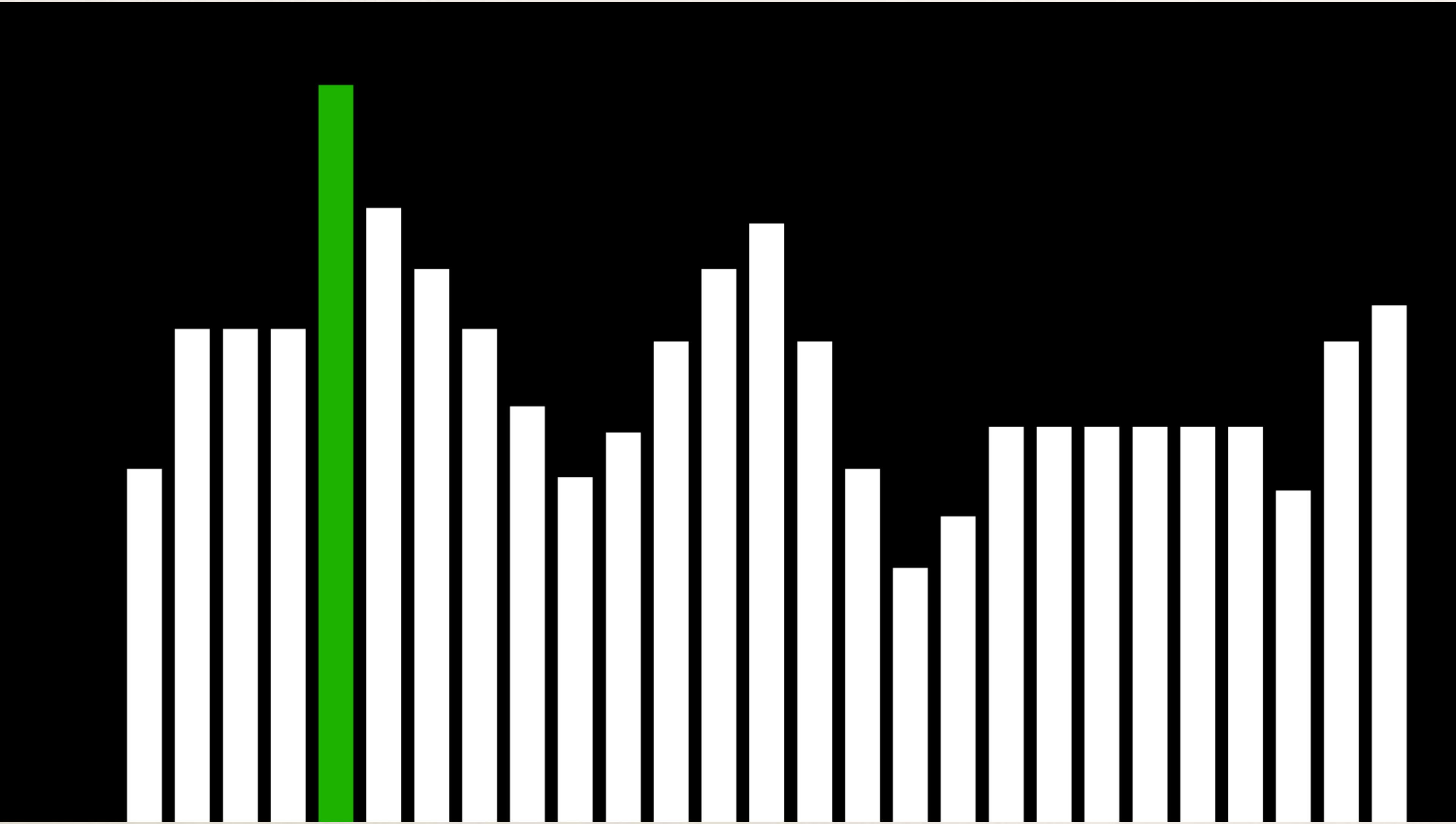

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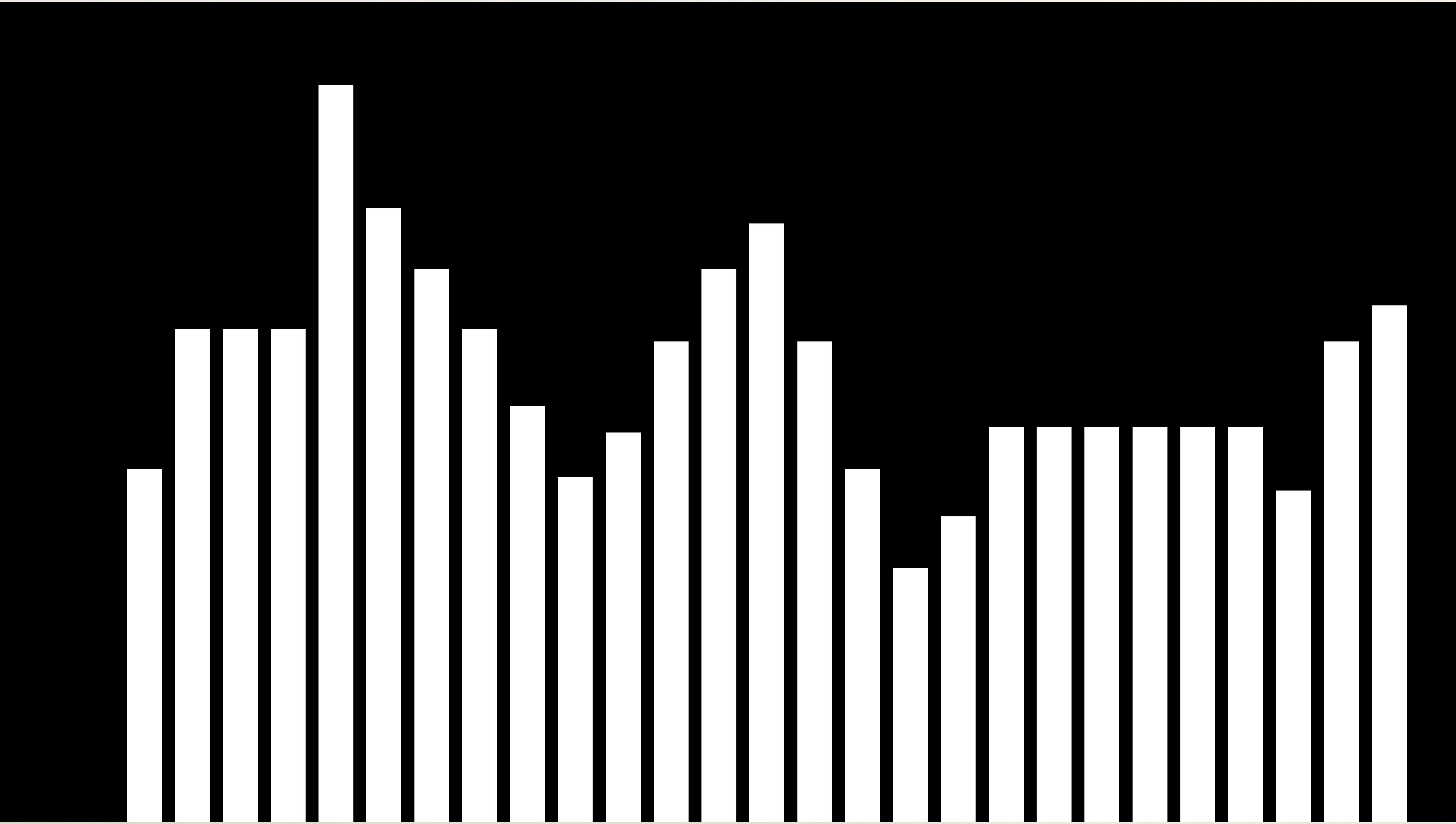

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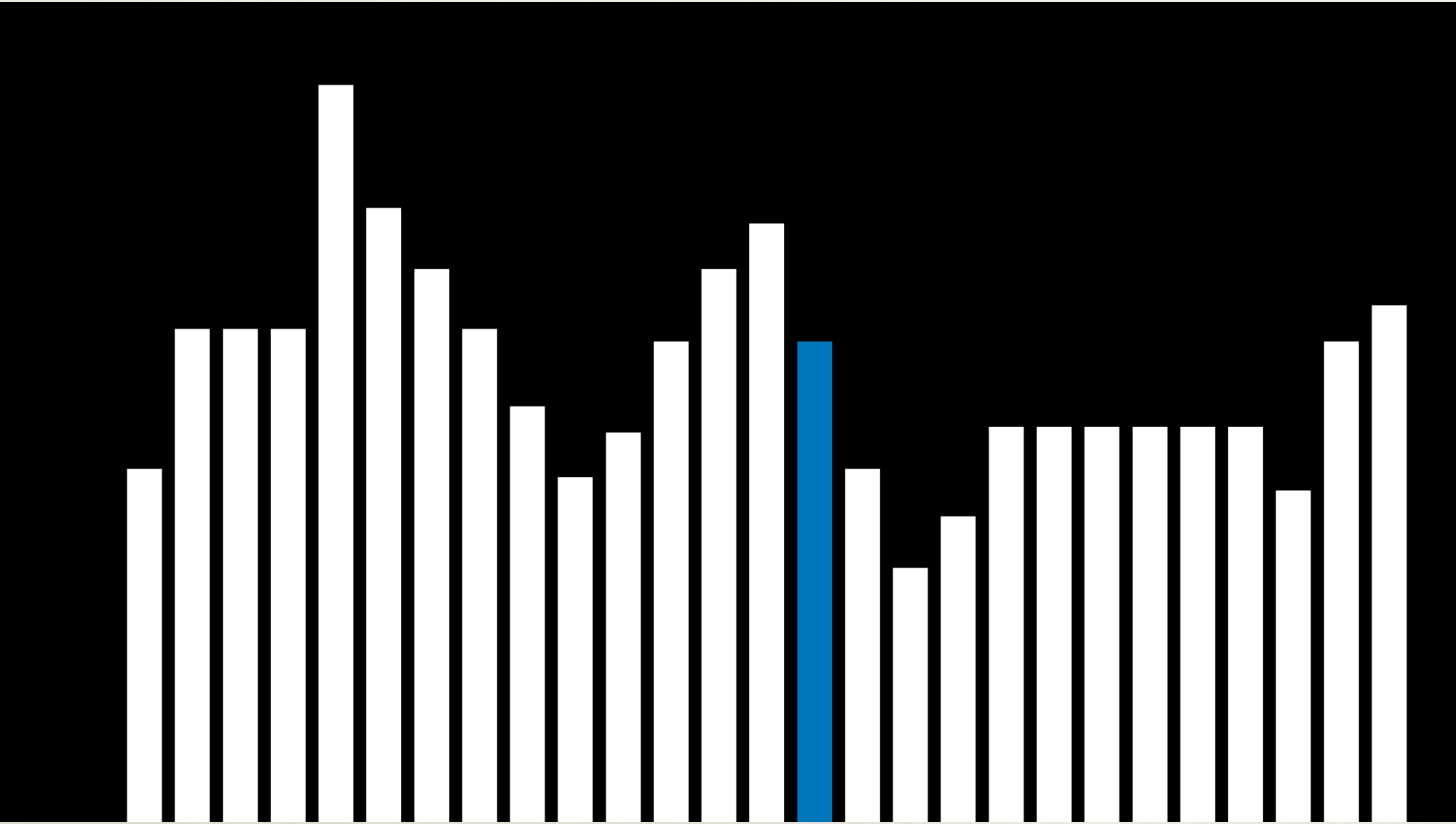

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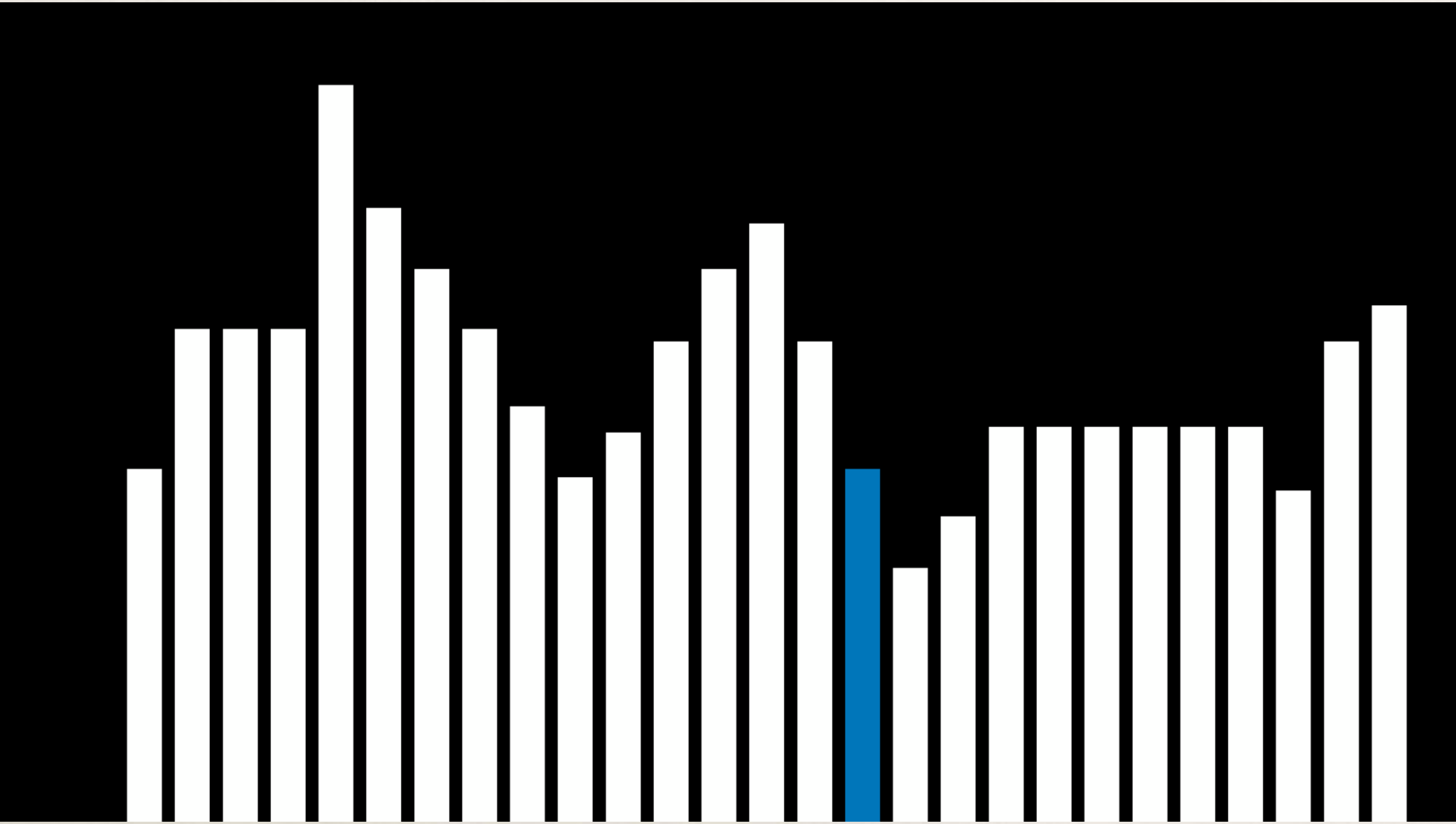

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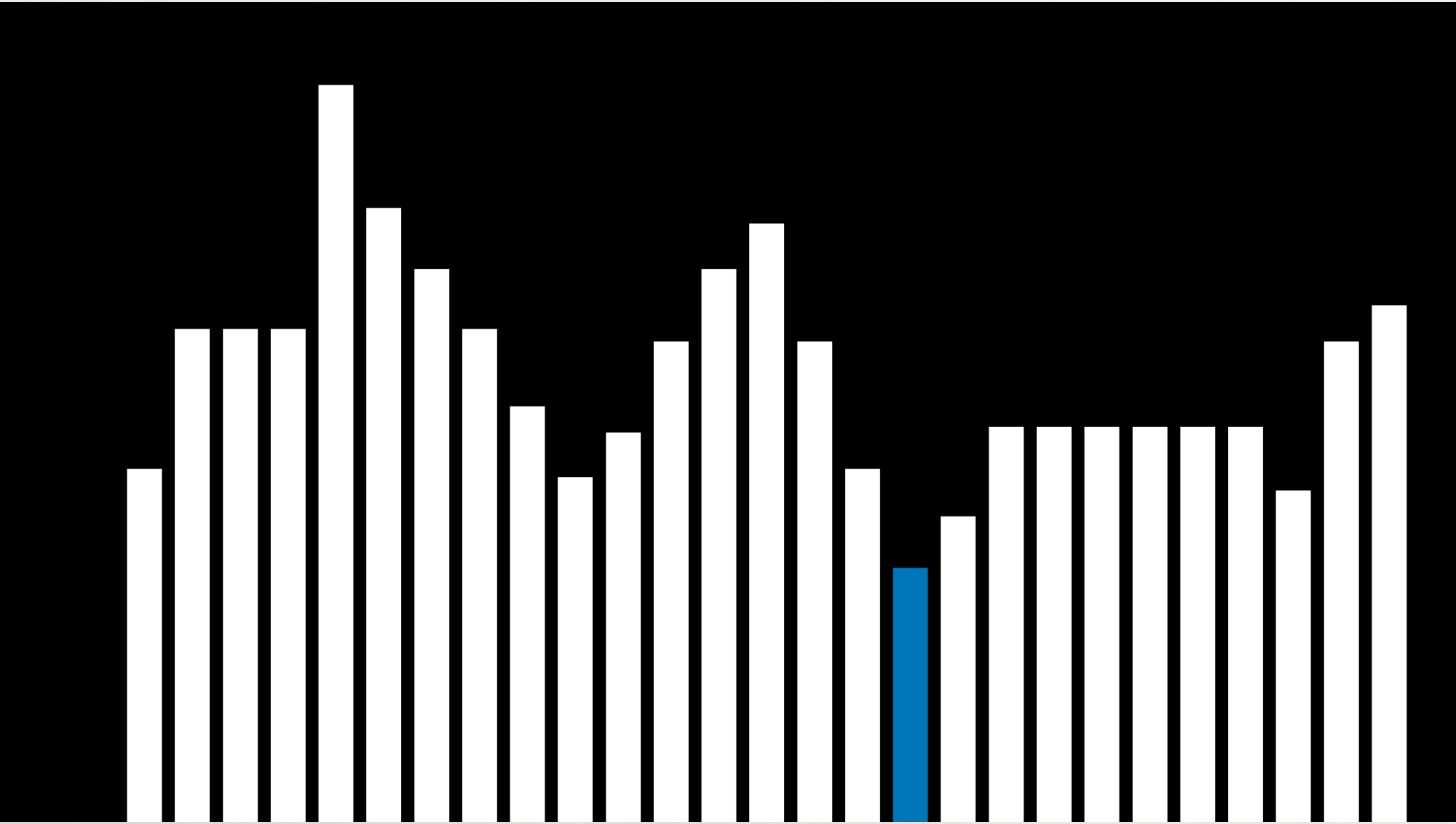

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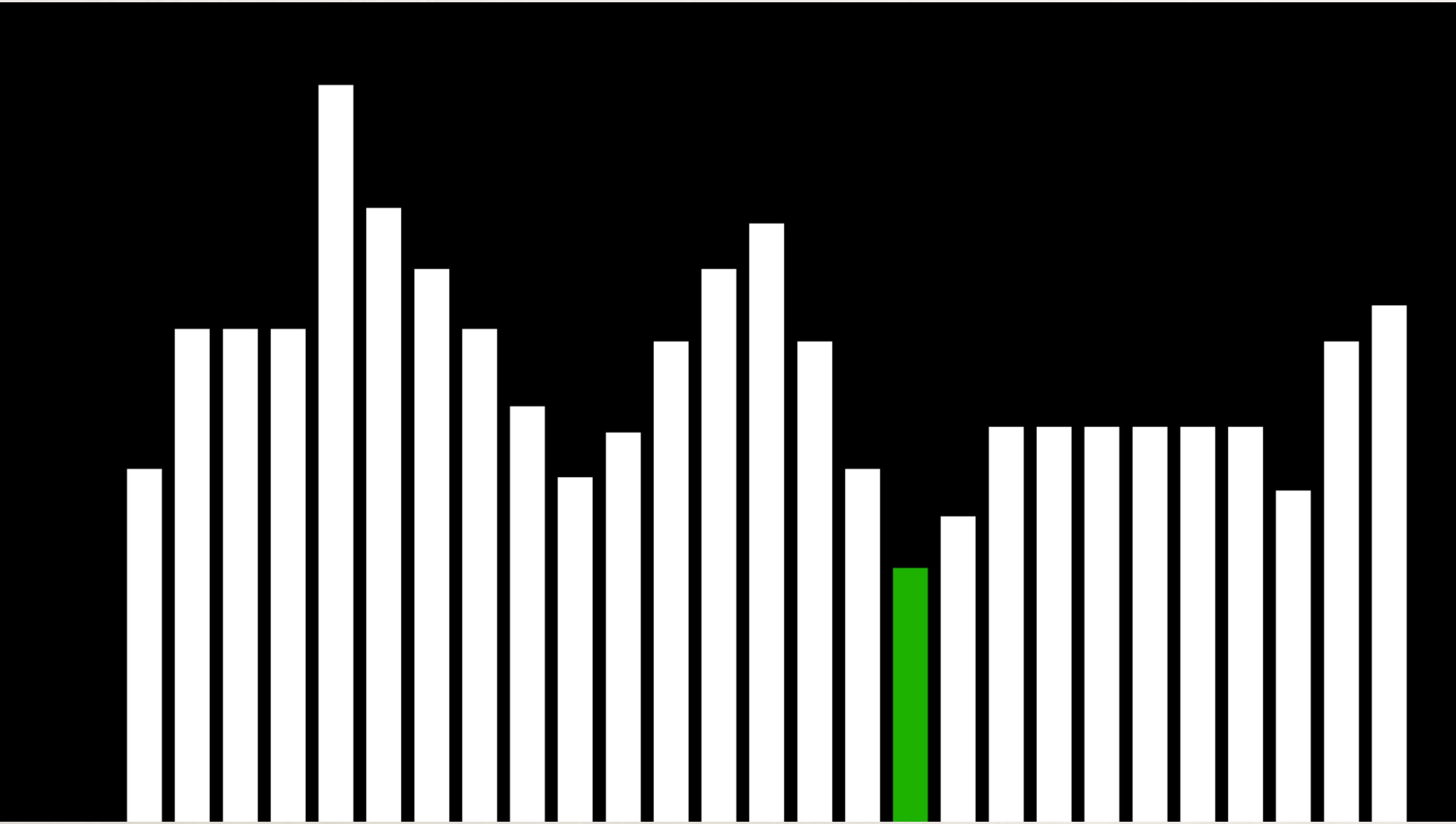
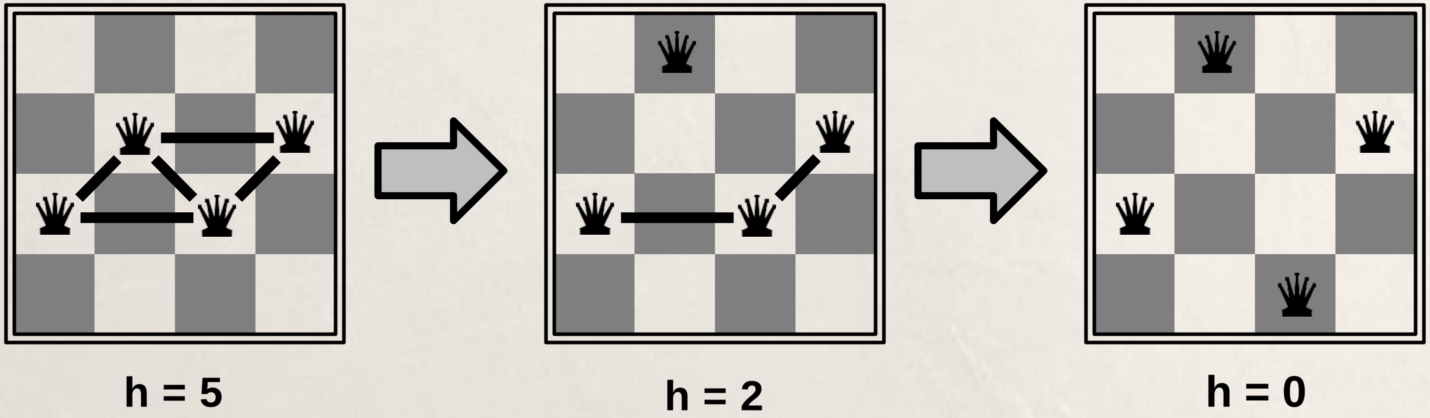


Hill-climbing search

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
    current ← *problem*.INITIAL
    while *true* **do**
        neighbor ← a highest-valued successor state of *current*
        if VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
        current ← *neighbor*

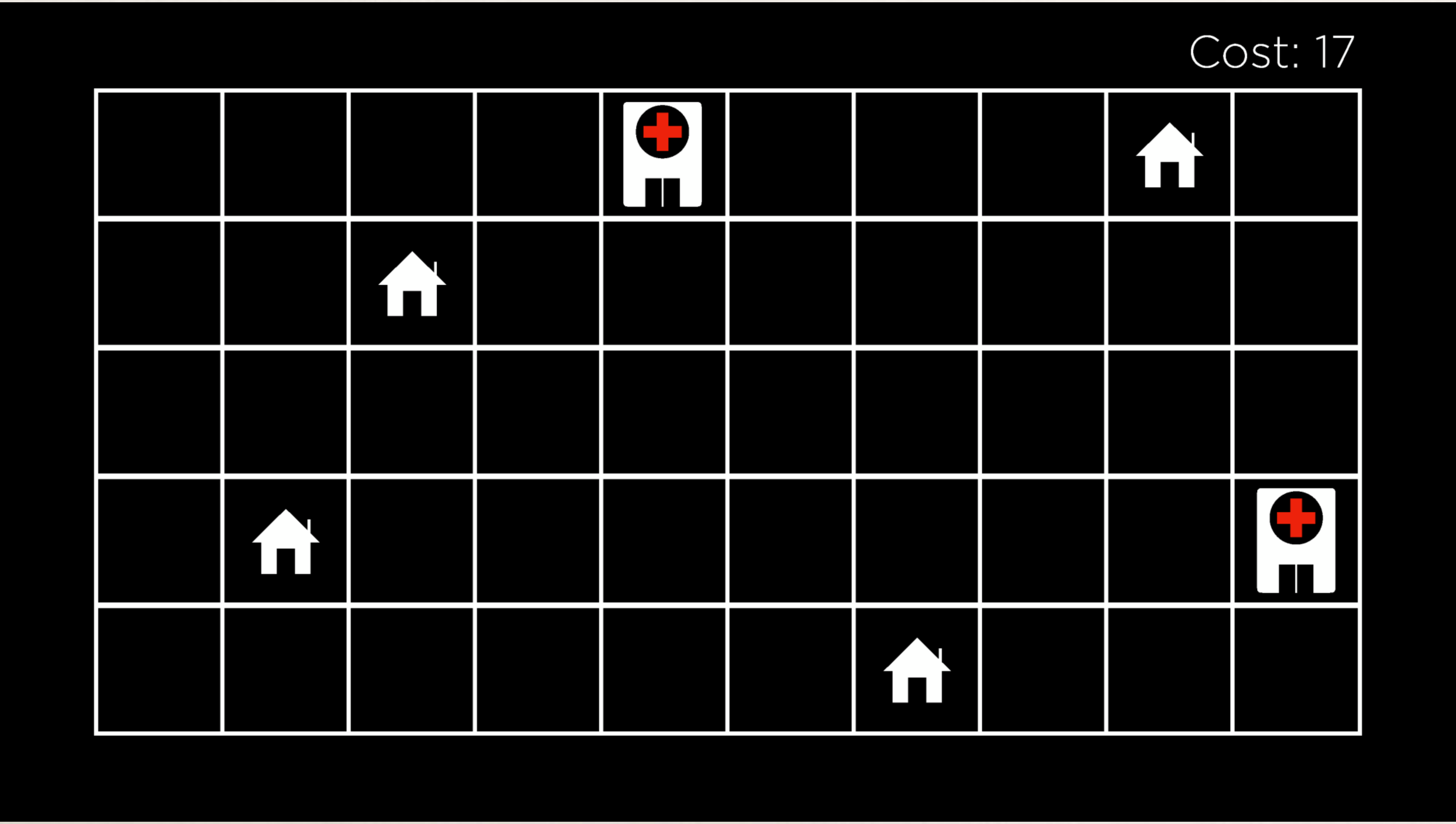
The n-queens problem

- The objective is to minimise the number of pairs of queens that attack each other
- Put *n* queens on an $n \times n$ board with no two queens on the same row, column, or diagonal
- Move a queen to reduce number of conflicts



- Almost always solves *n*-queens problems almost instantaneously for very large *n*, e.g., $n = 1$ *million*

Hill-climbing search and optimisation



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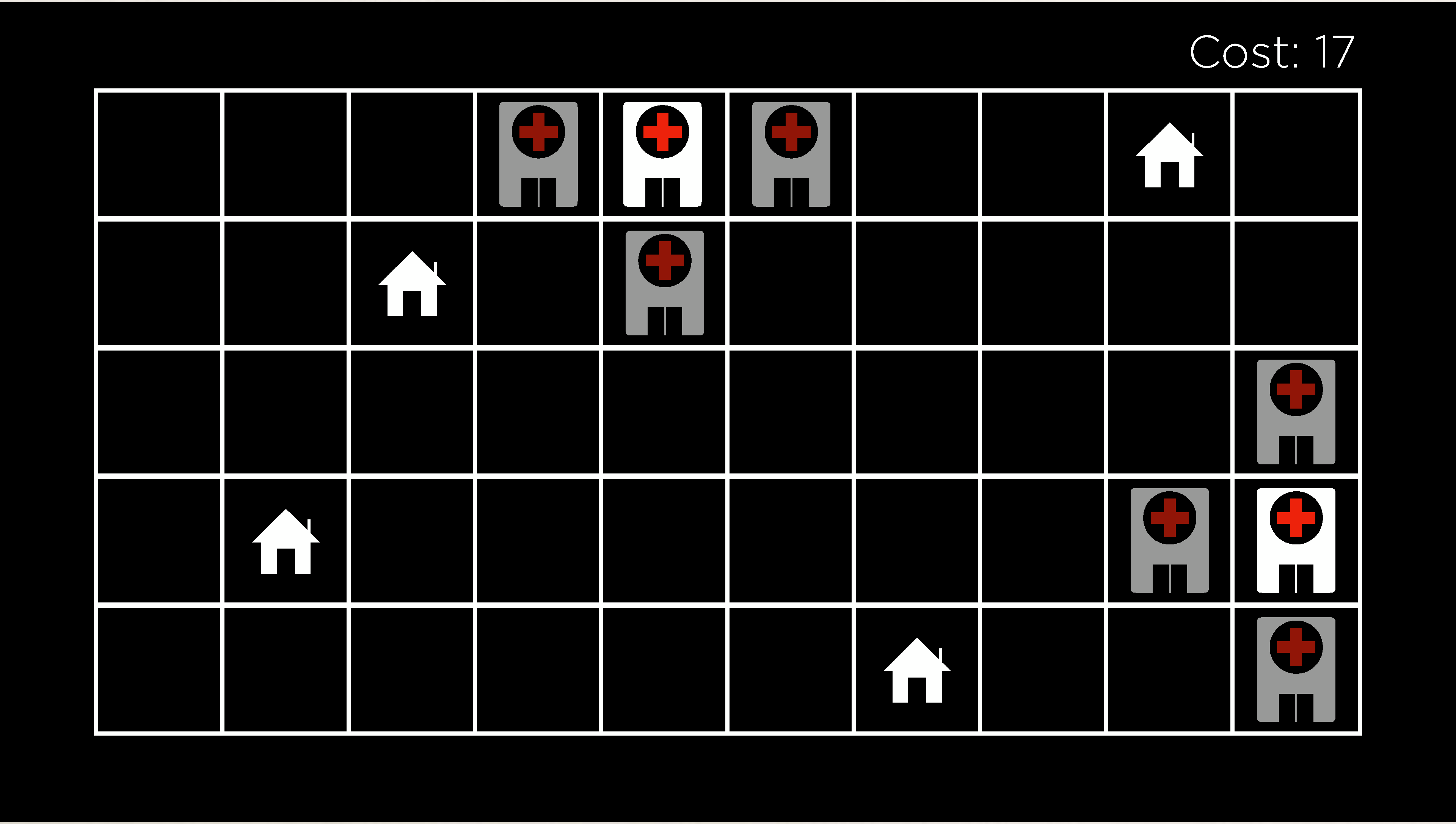

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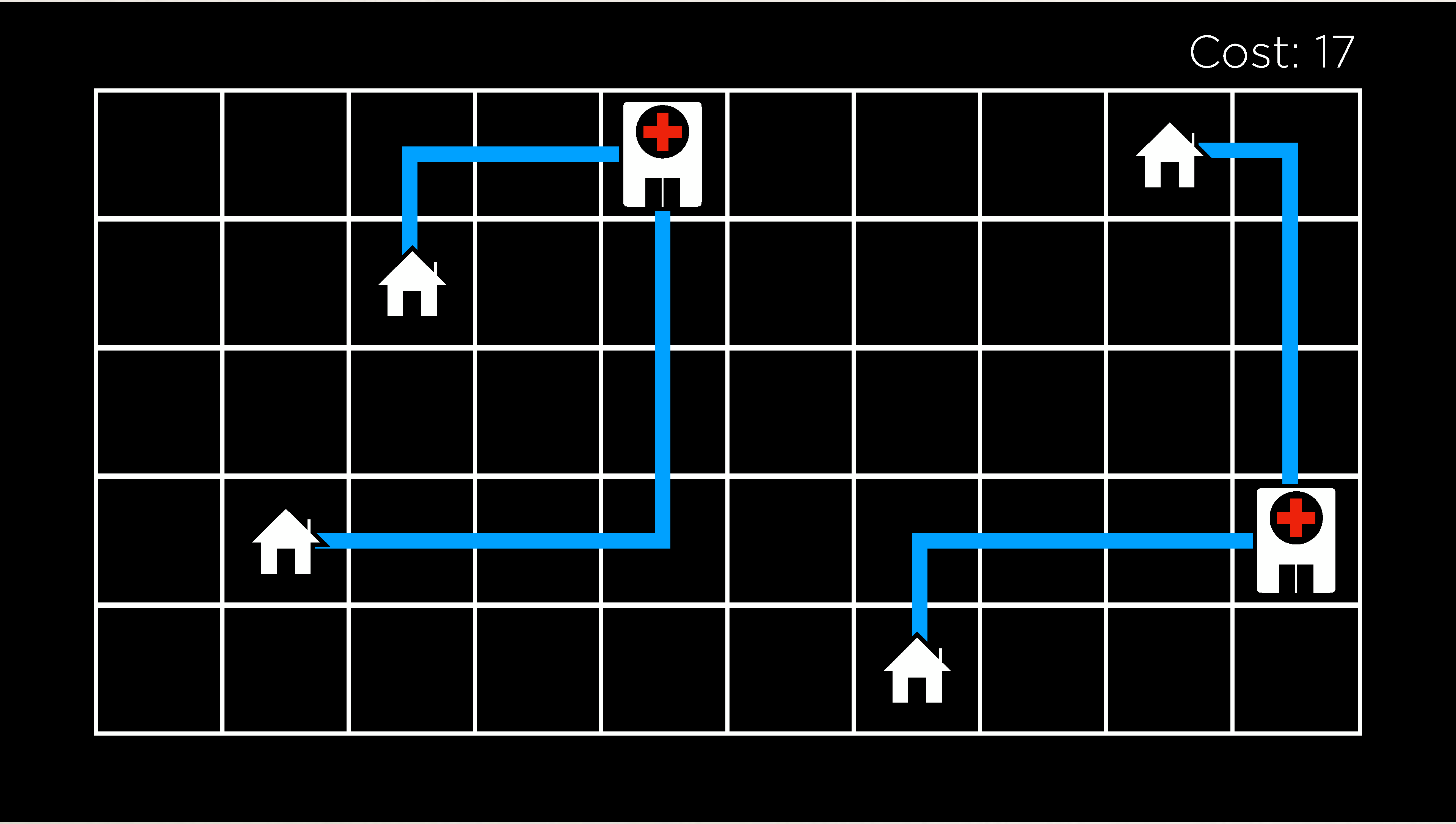

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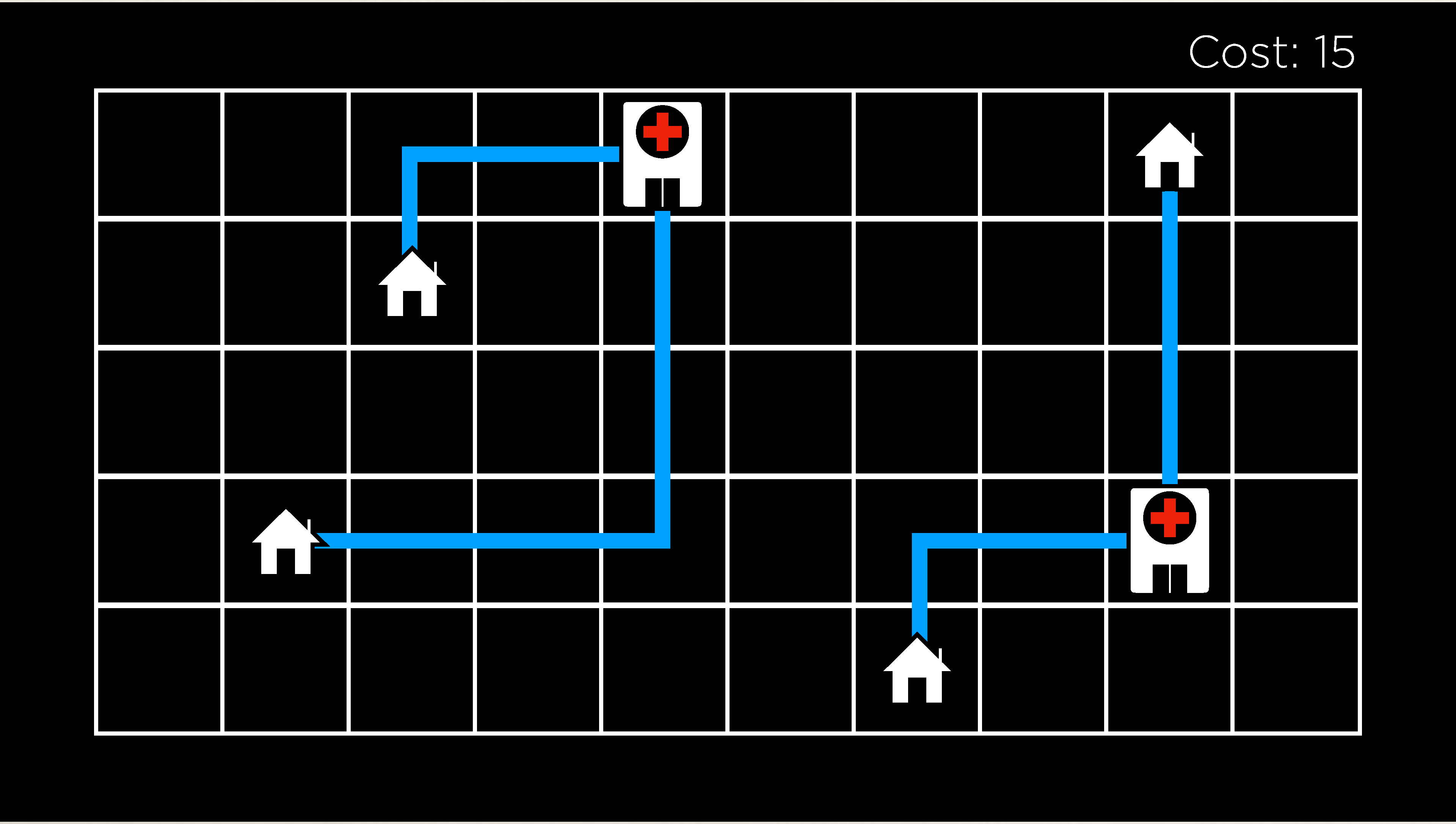

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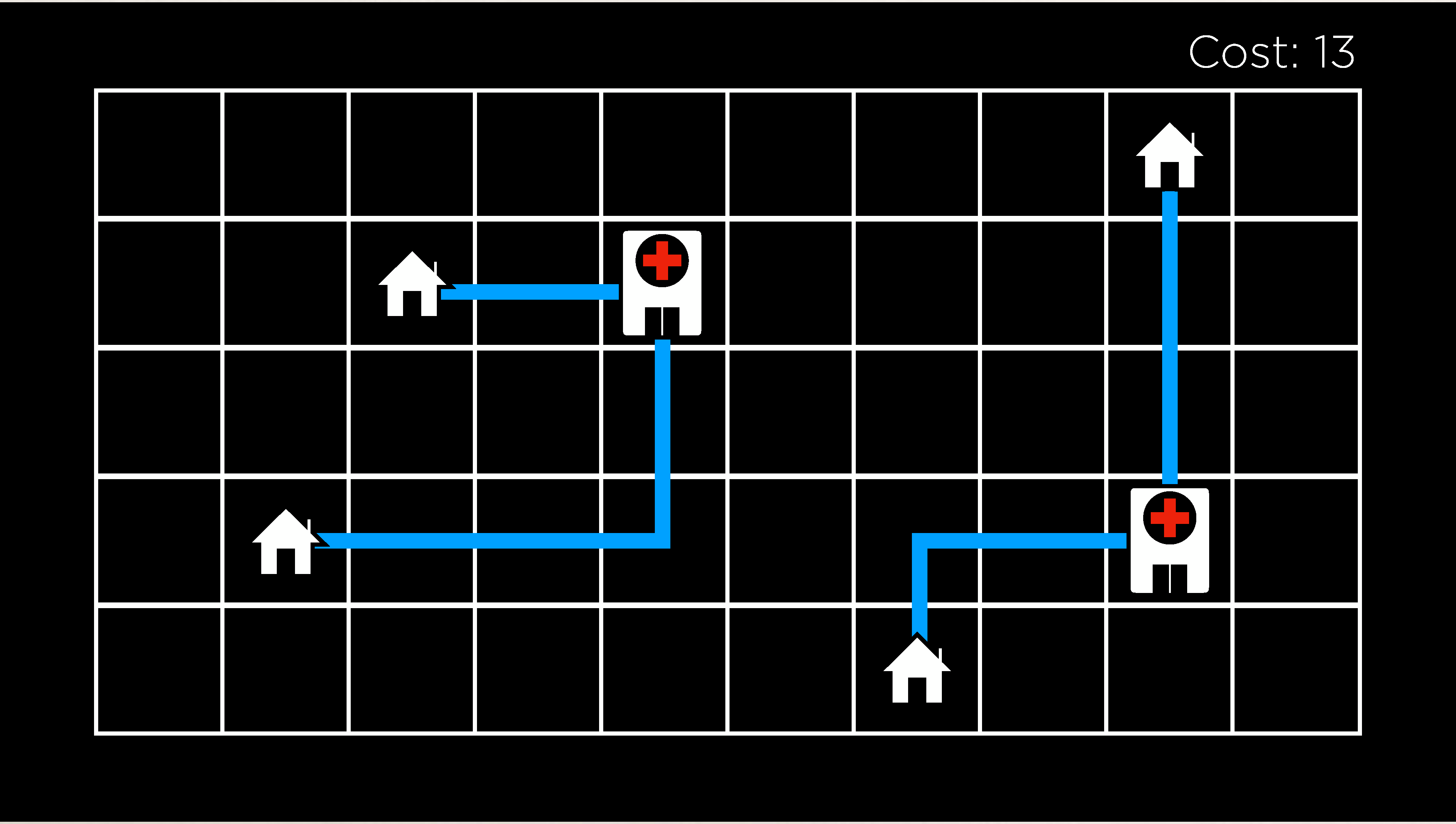

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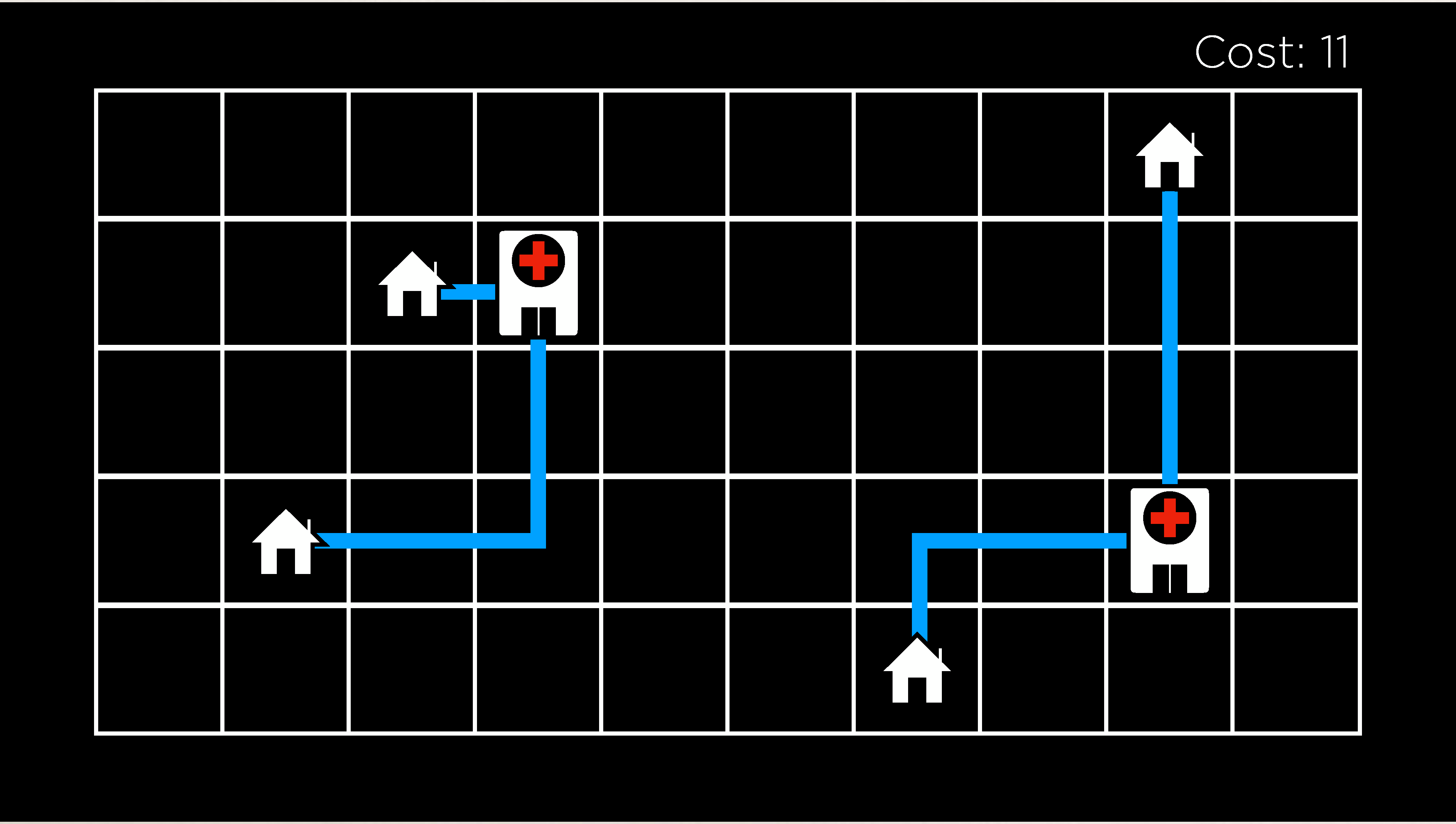

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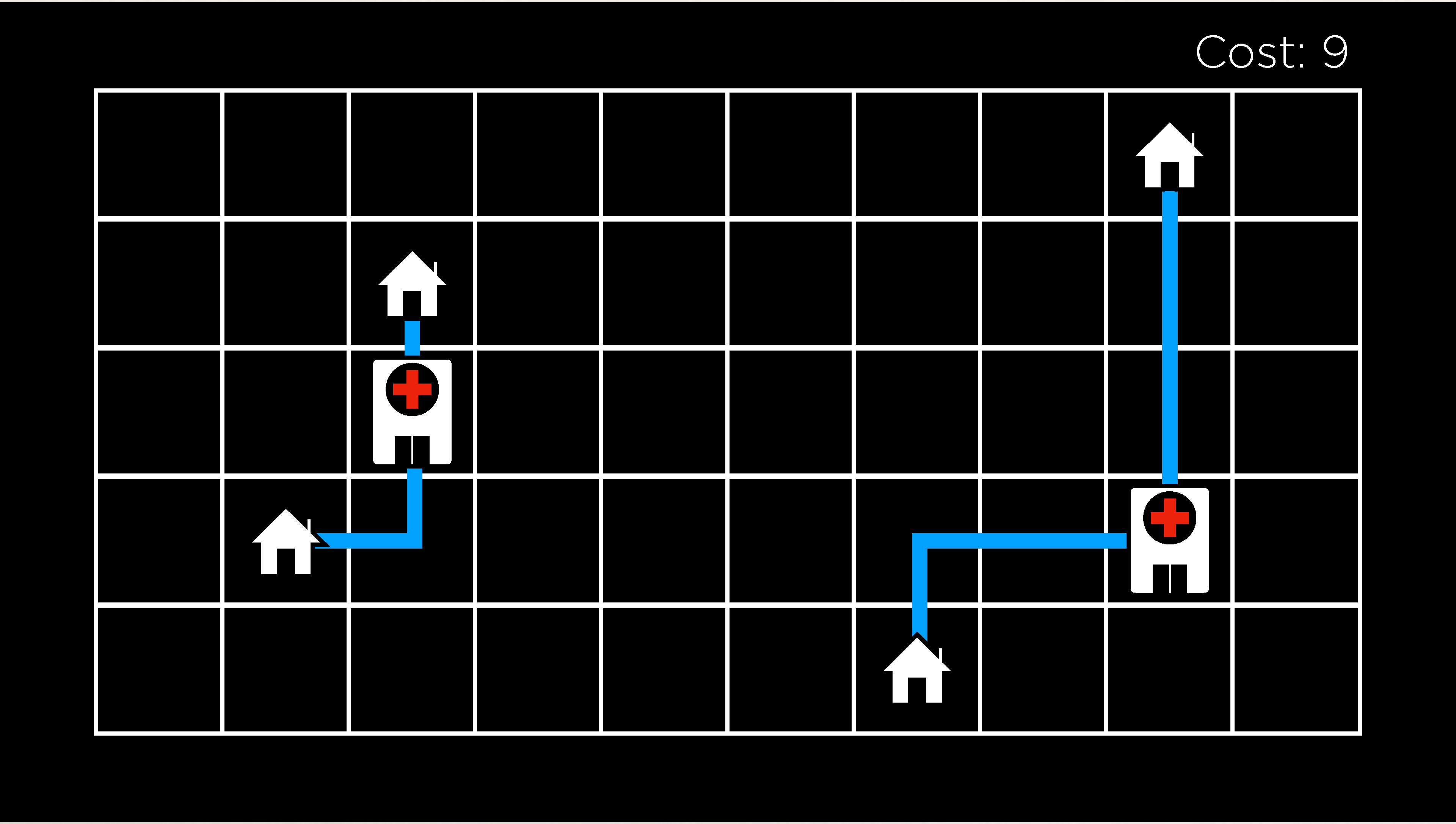

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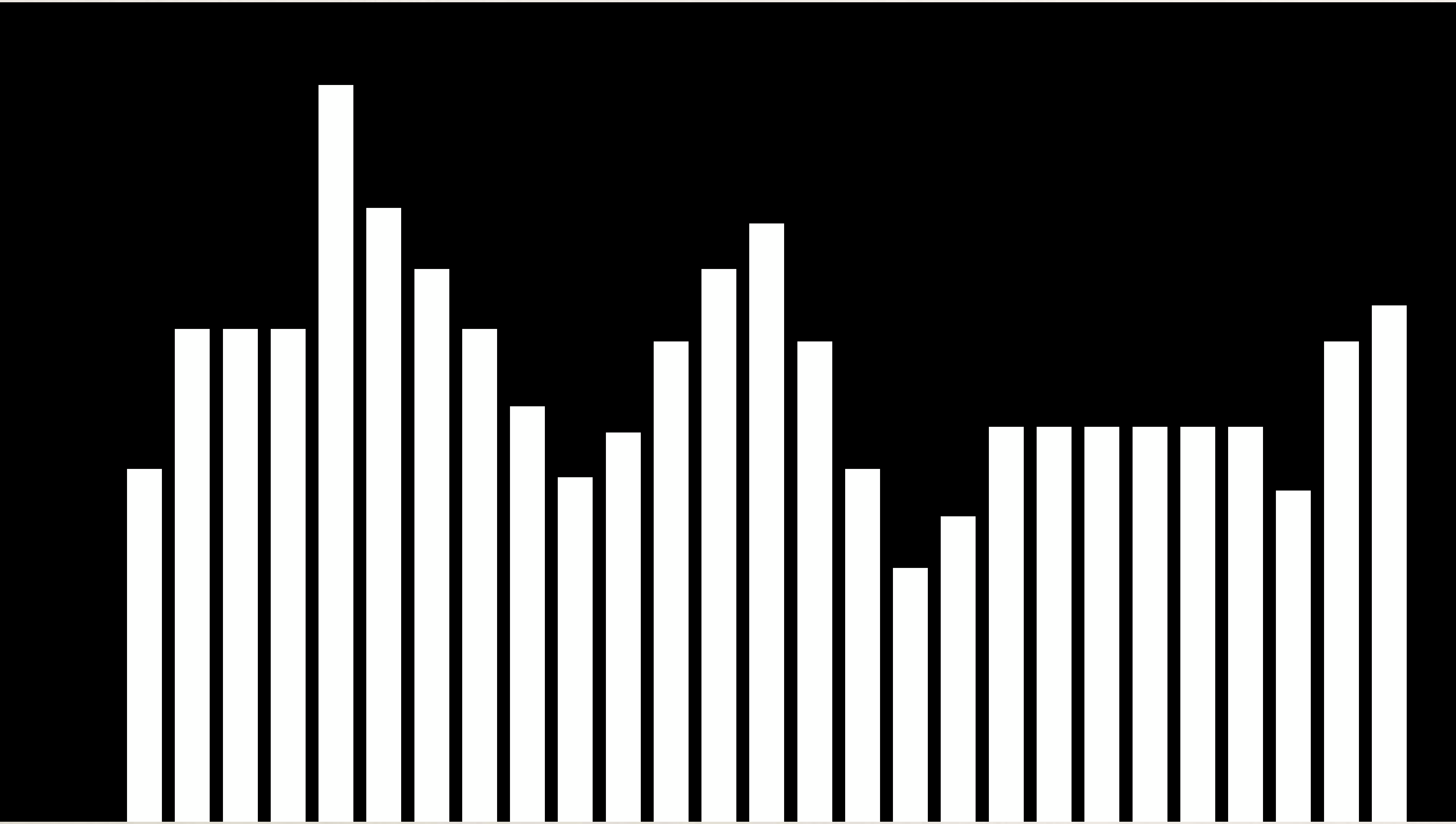

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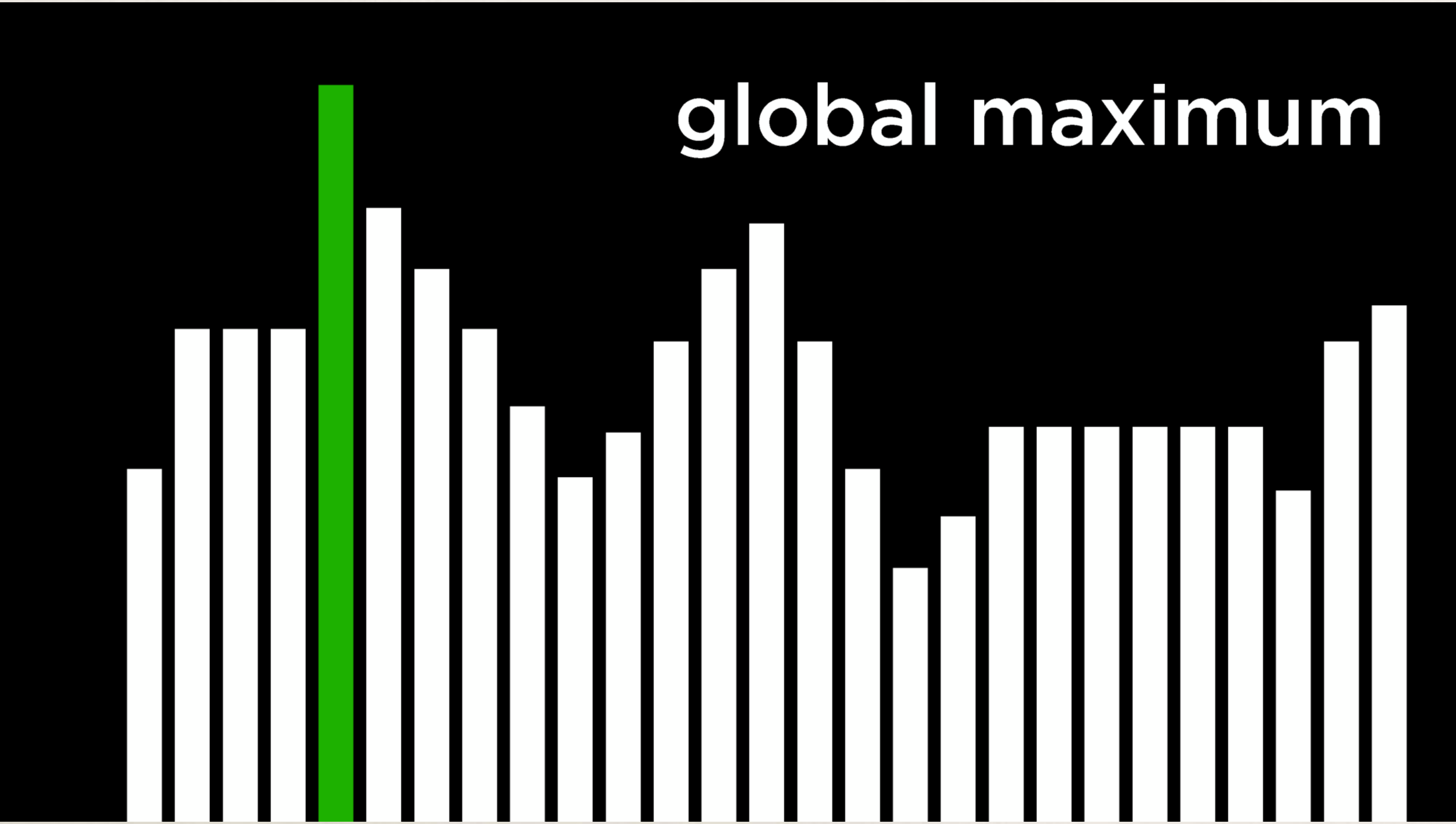

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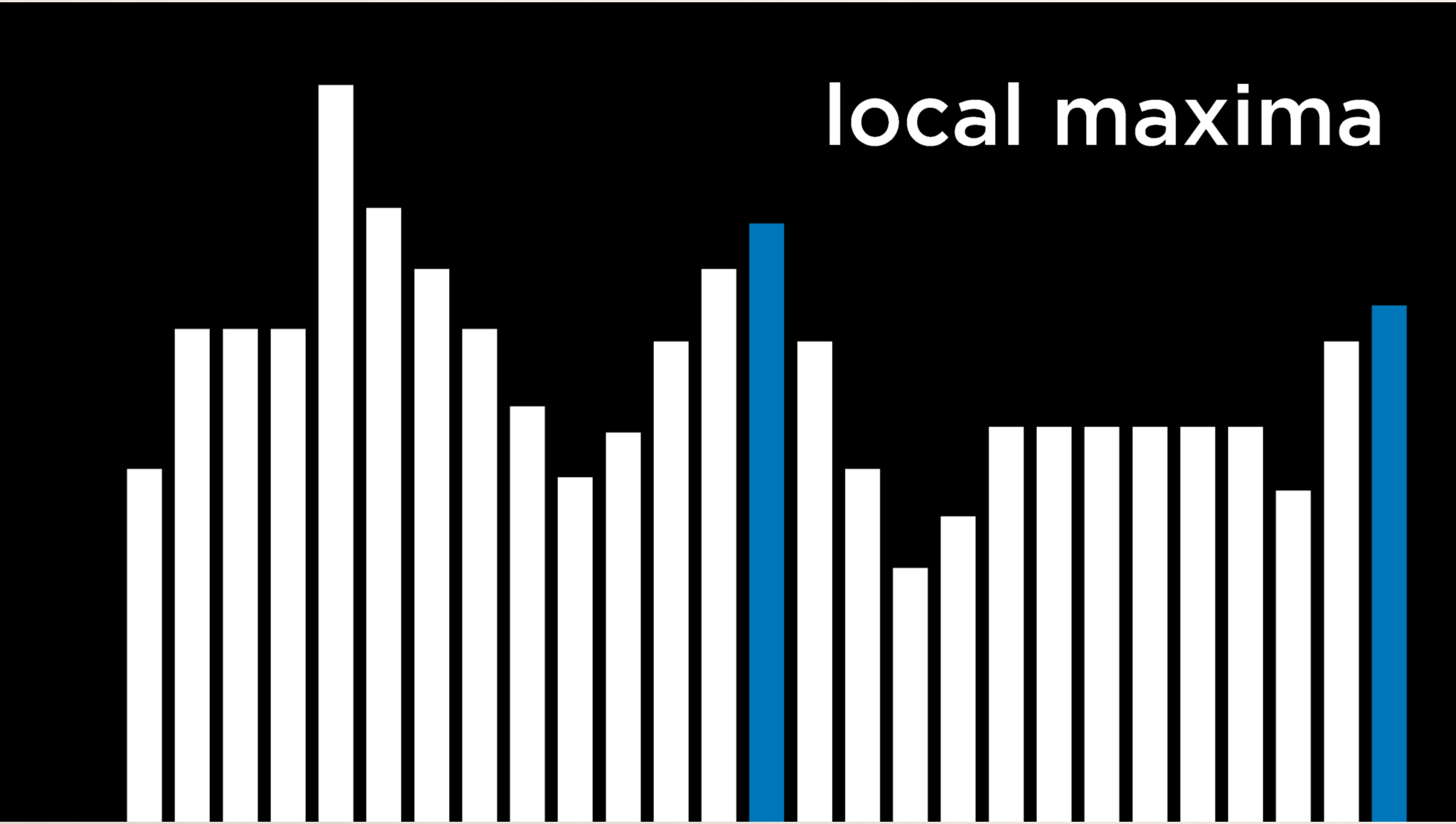

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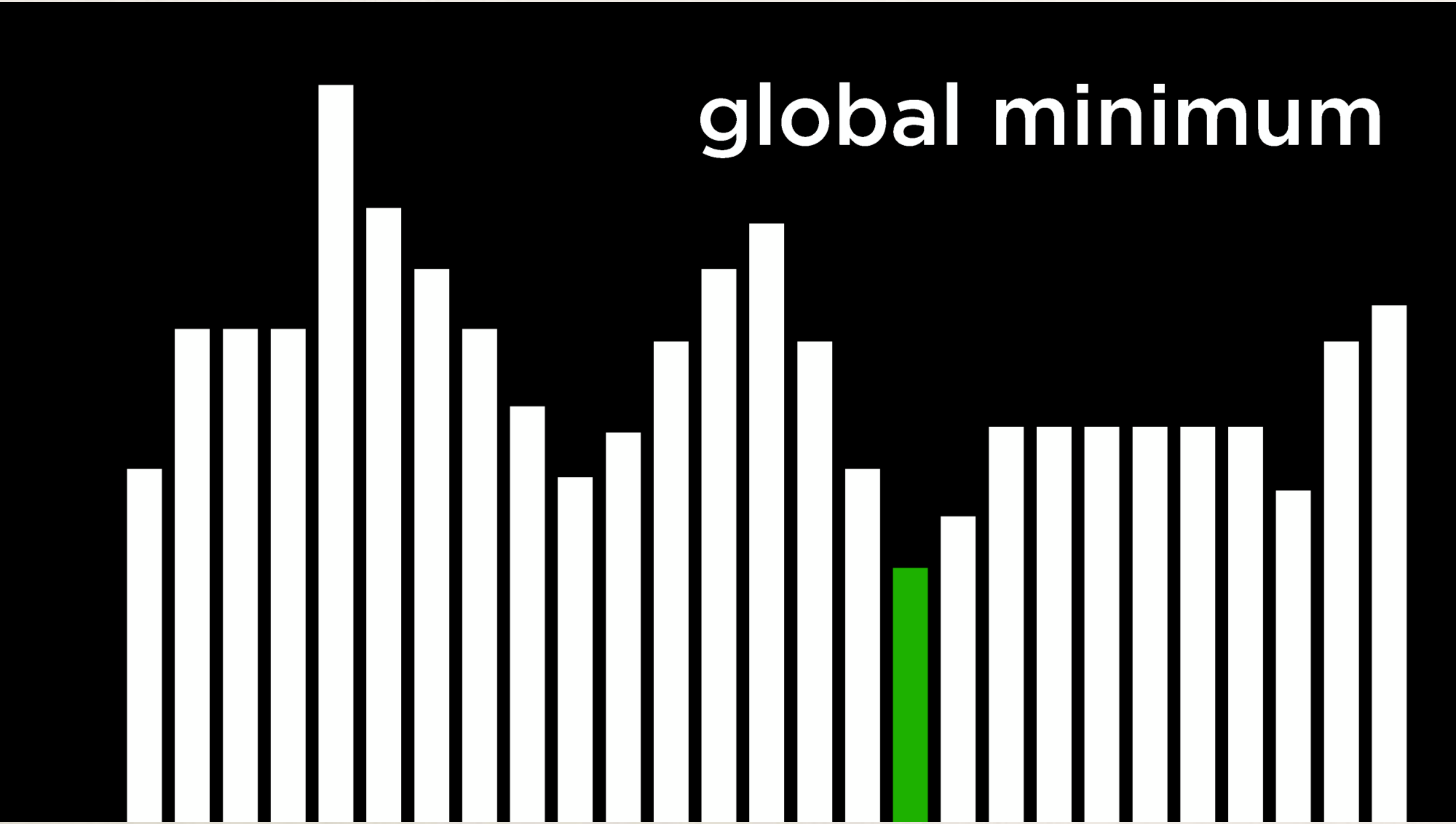

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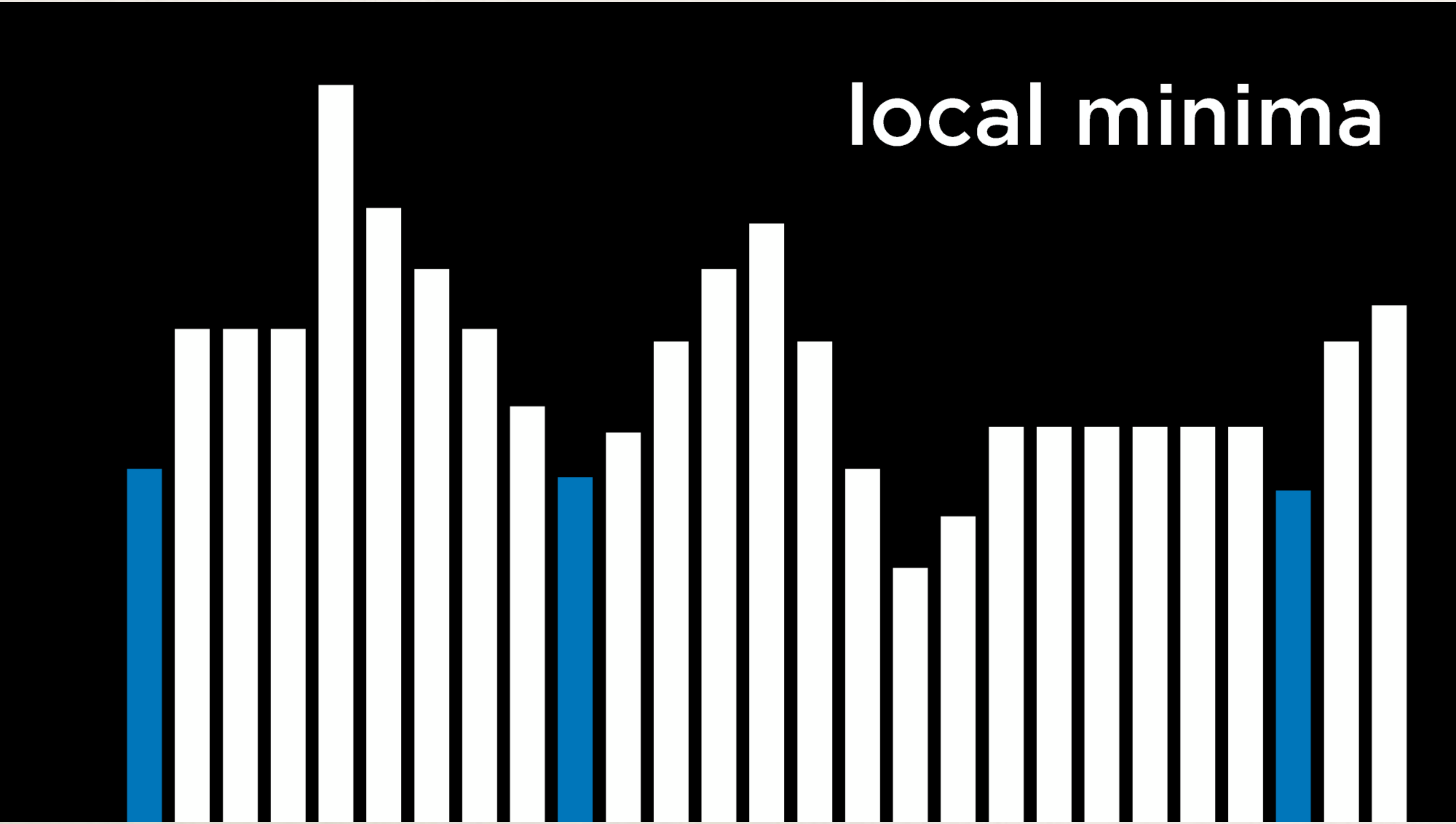

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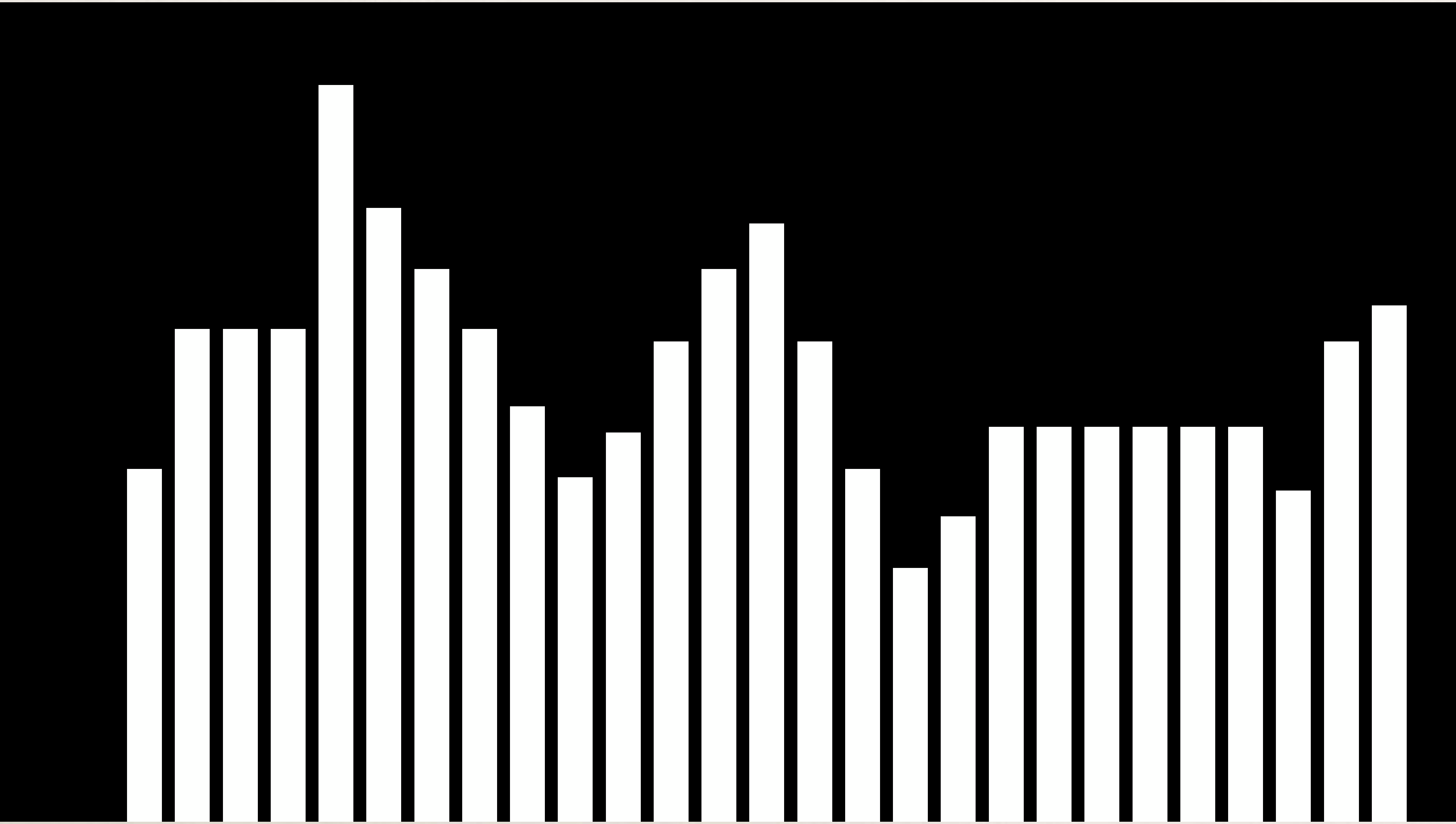

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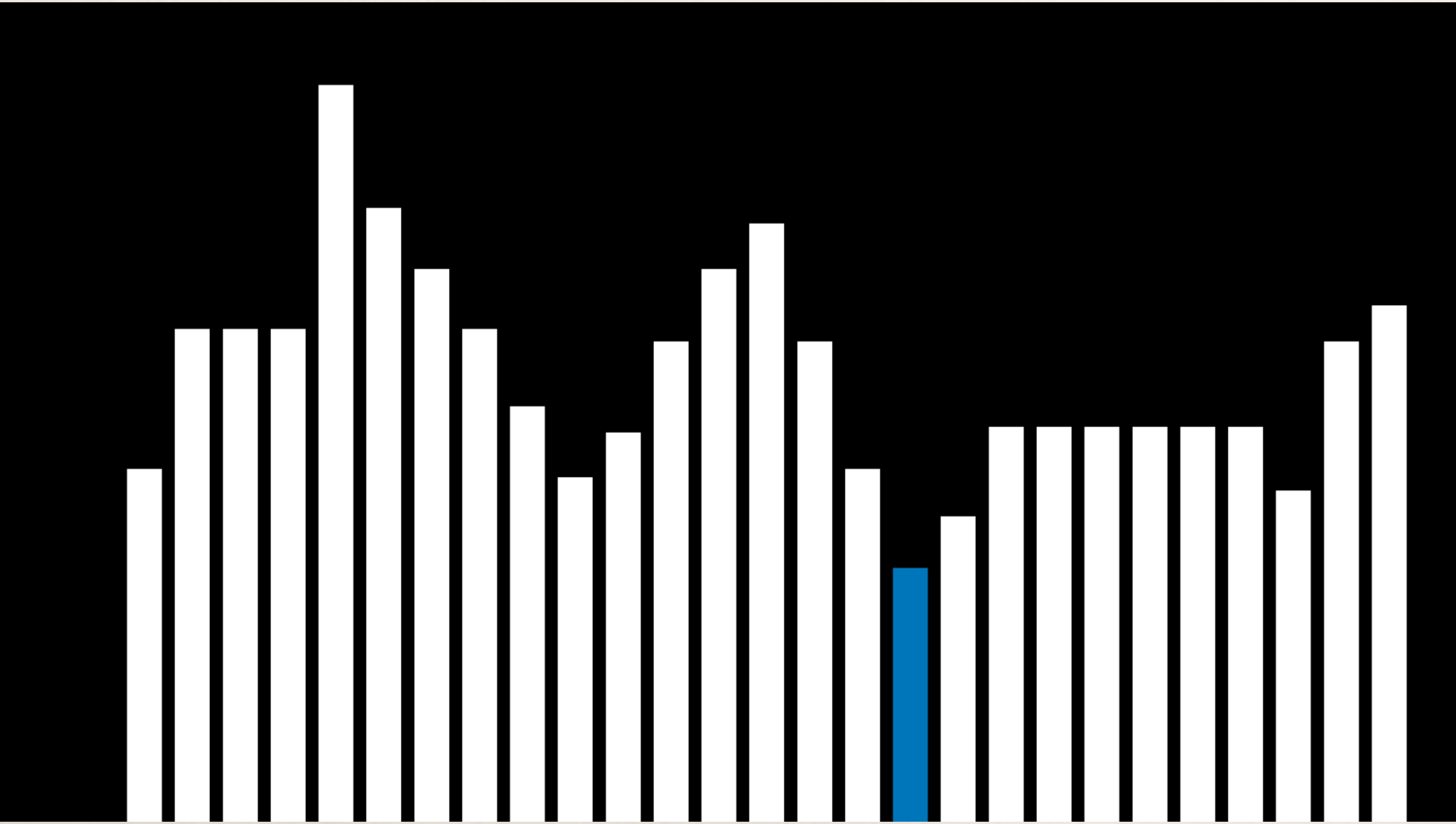

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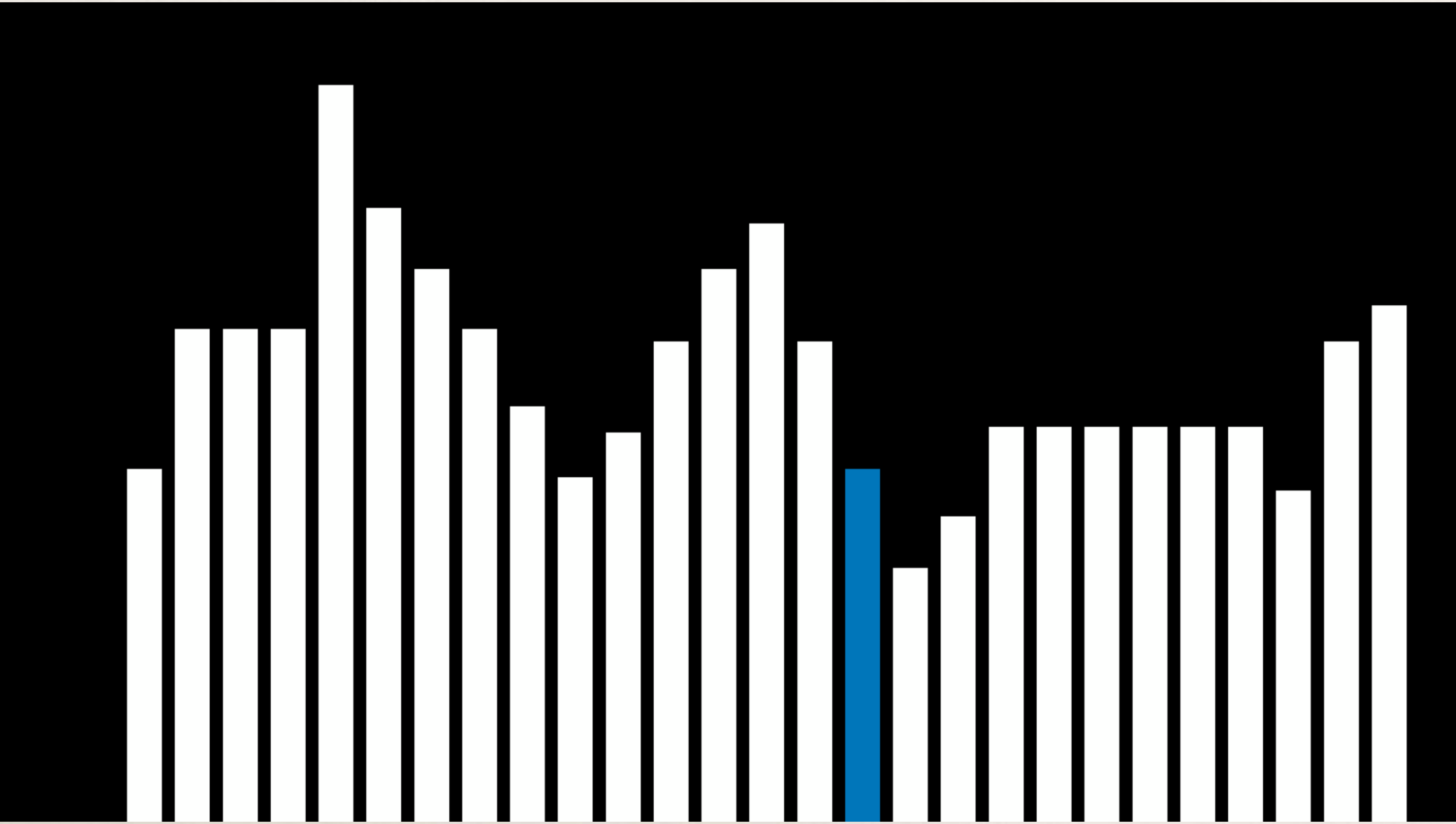

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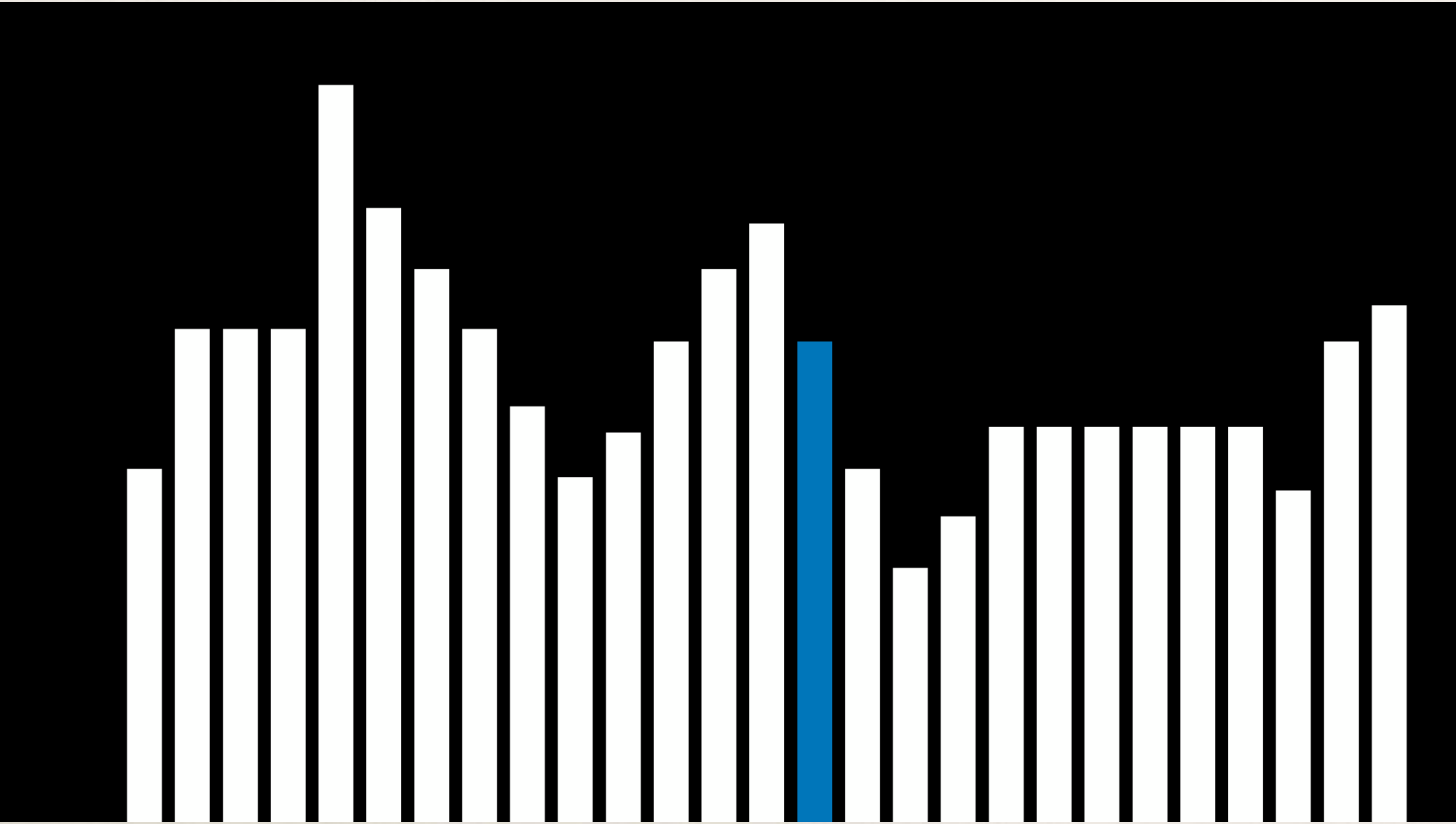

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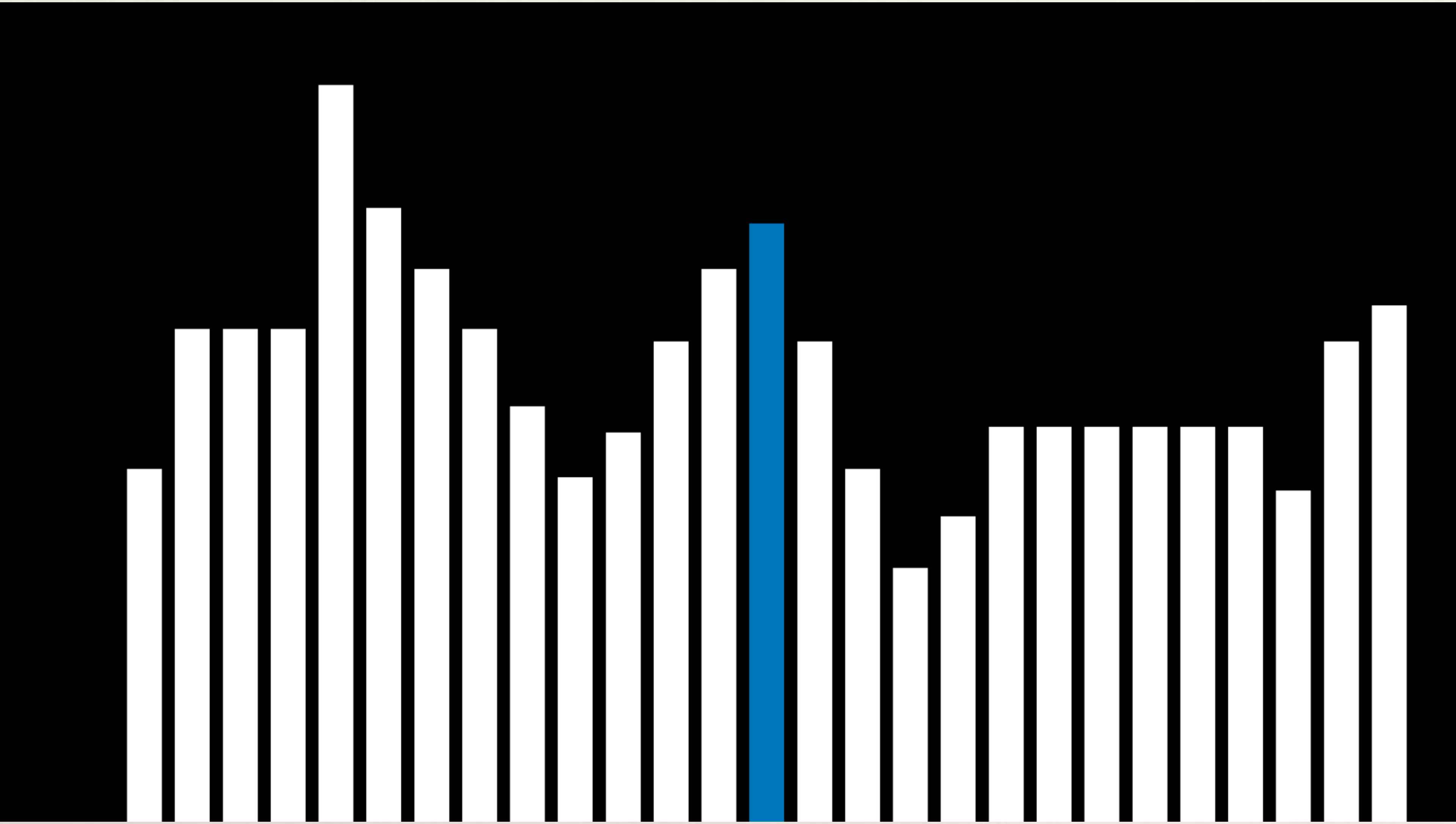

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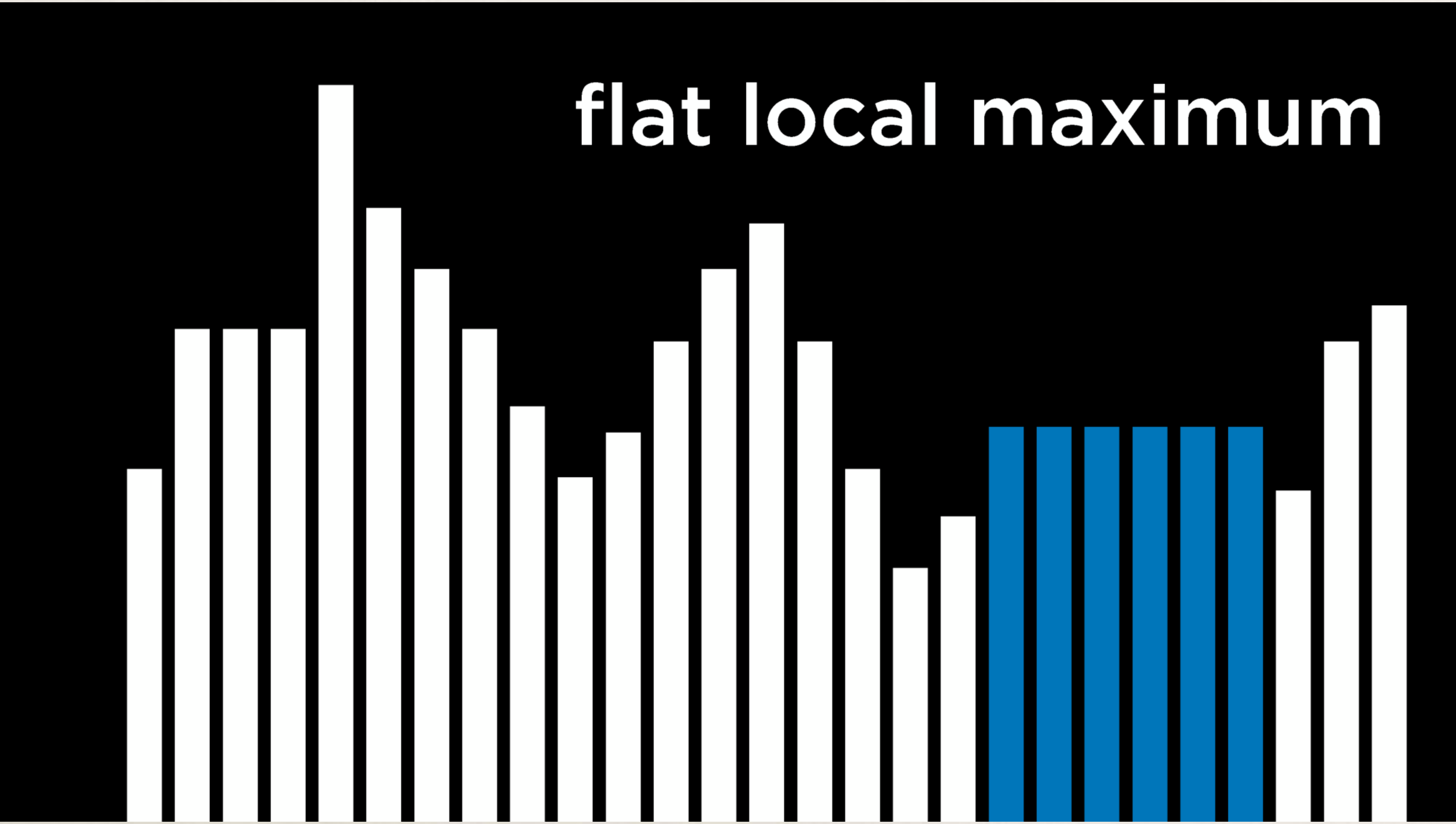

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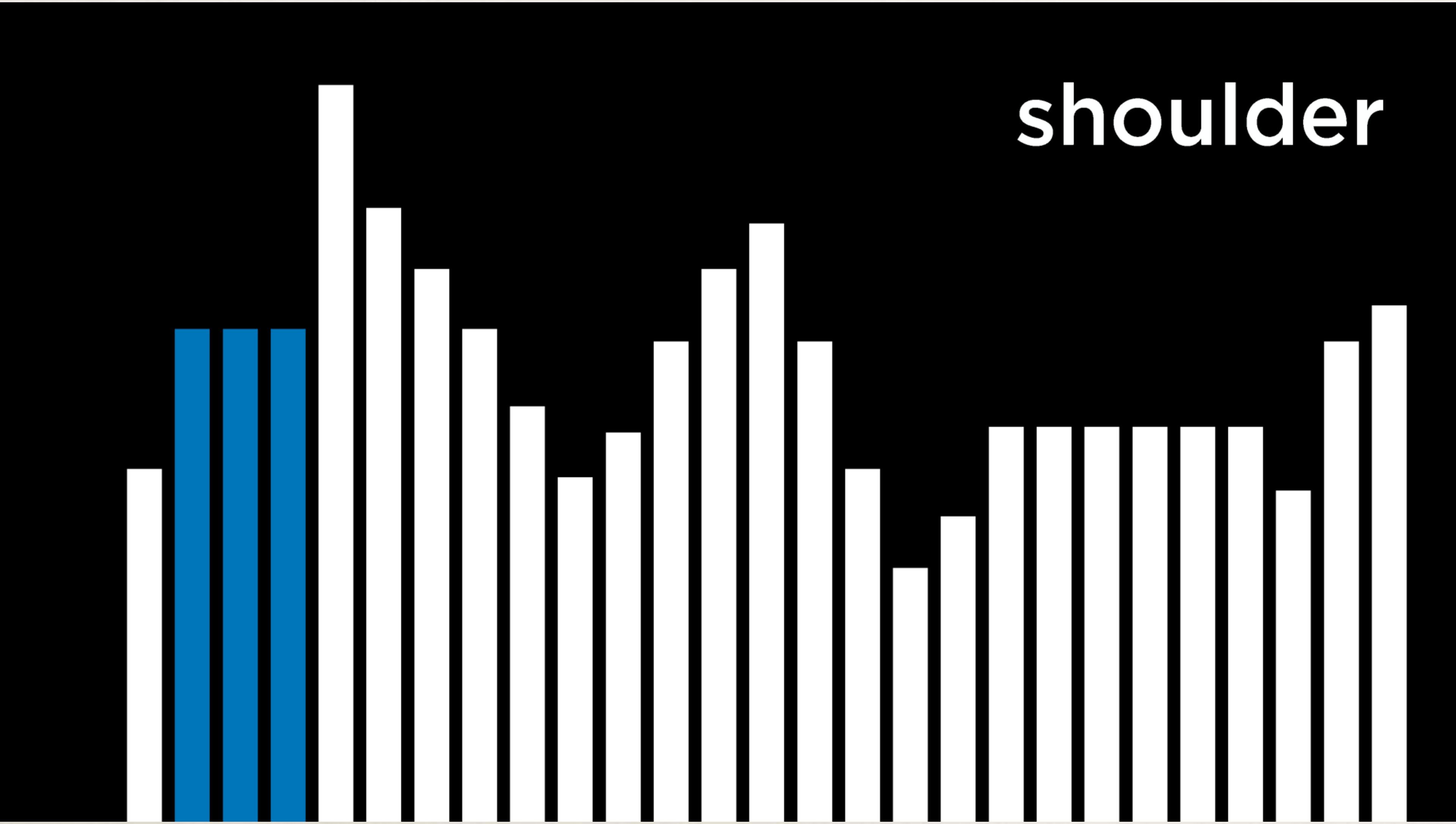

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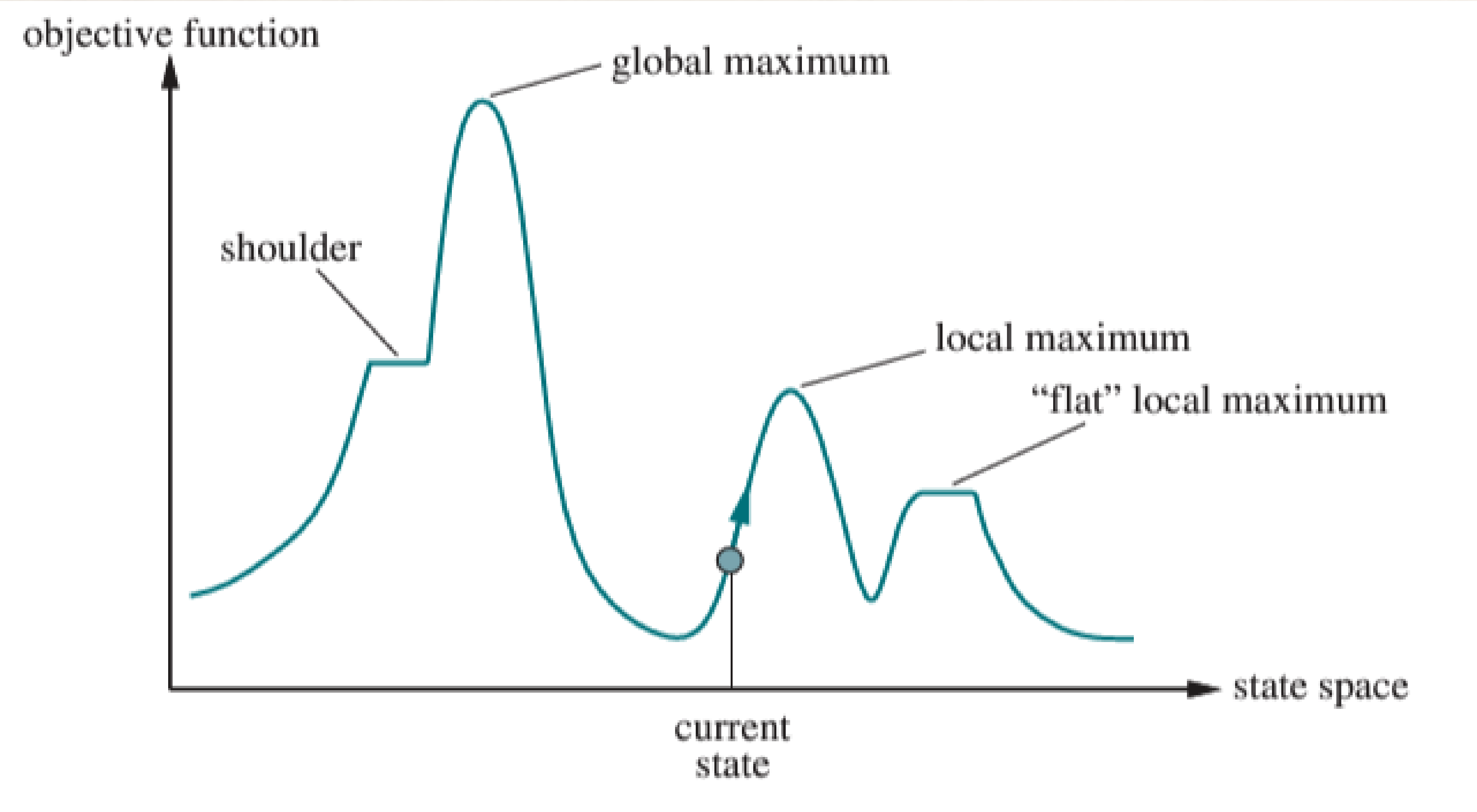


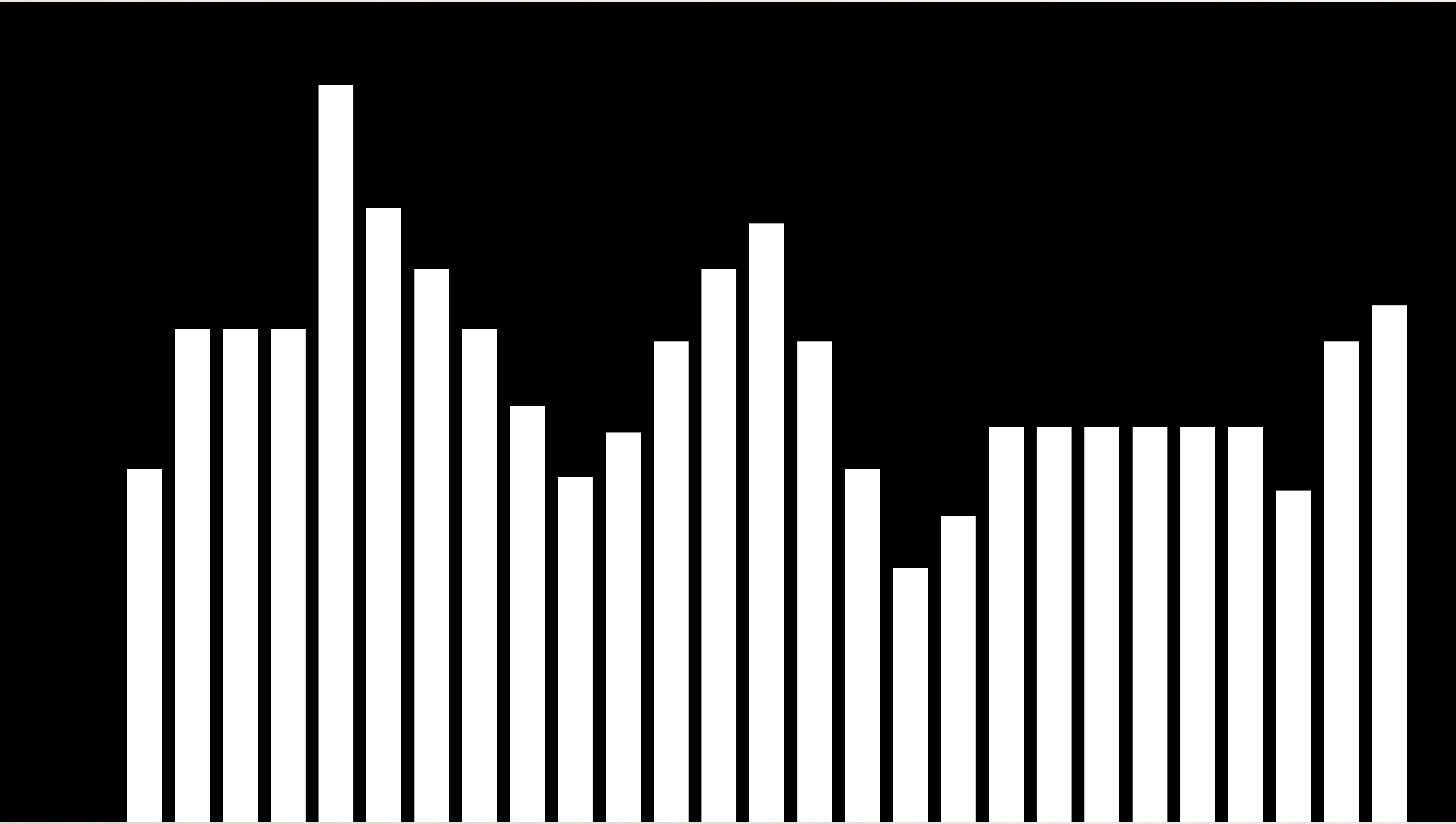
Variants of hill-climbing

- **Stochastic hill-climbing** chooses at random from uphill moves
  - Probability of selection may be based on size of improvement
- Stochastic hill-climbing finds good states more slowly but may find better solutions in some state spaces
- **First-choice hill-climbing** generates neighbours randomly and picks the first one that yields an improvement
  - Useful if the set of neighbours is very large
- Good, very simple option to improve on basic hill-climbing: **random-restart hill-climbing**
  - If each of n restarts has probability p of not finding the global maximum, the probability of failure is $1 - p^n$

Simulated annealing

- In metallurgy, **annealing** is used to increase “ductility” and decrease hardness of a material to make it easier to work with
- Hot material’s temperature is gradually lowered according to a pre-defined scheduled to achieve this
- **Simulated annealing**: stochastic hill-climbing that allows downhill moves with a probability dependent on the temperature
- Early on, higher "temperature": more likely to accept neighbors that are worse than current state
- Later on, lower "temperature": less likely to accept neighbors that are worse than current state

Simulated annealing



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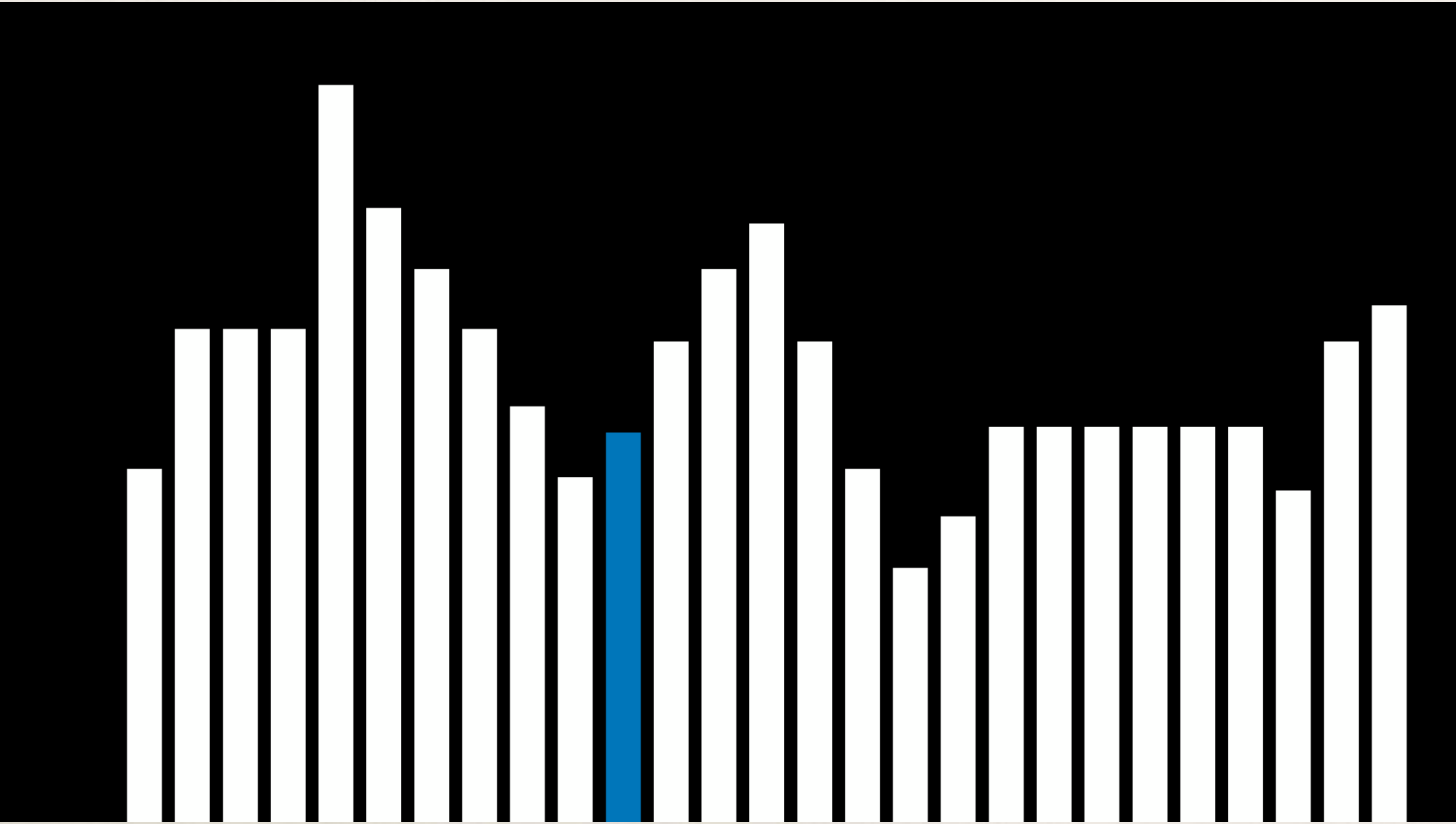

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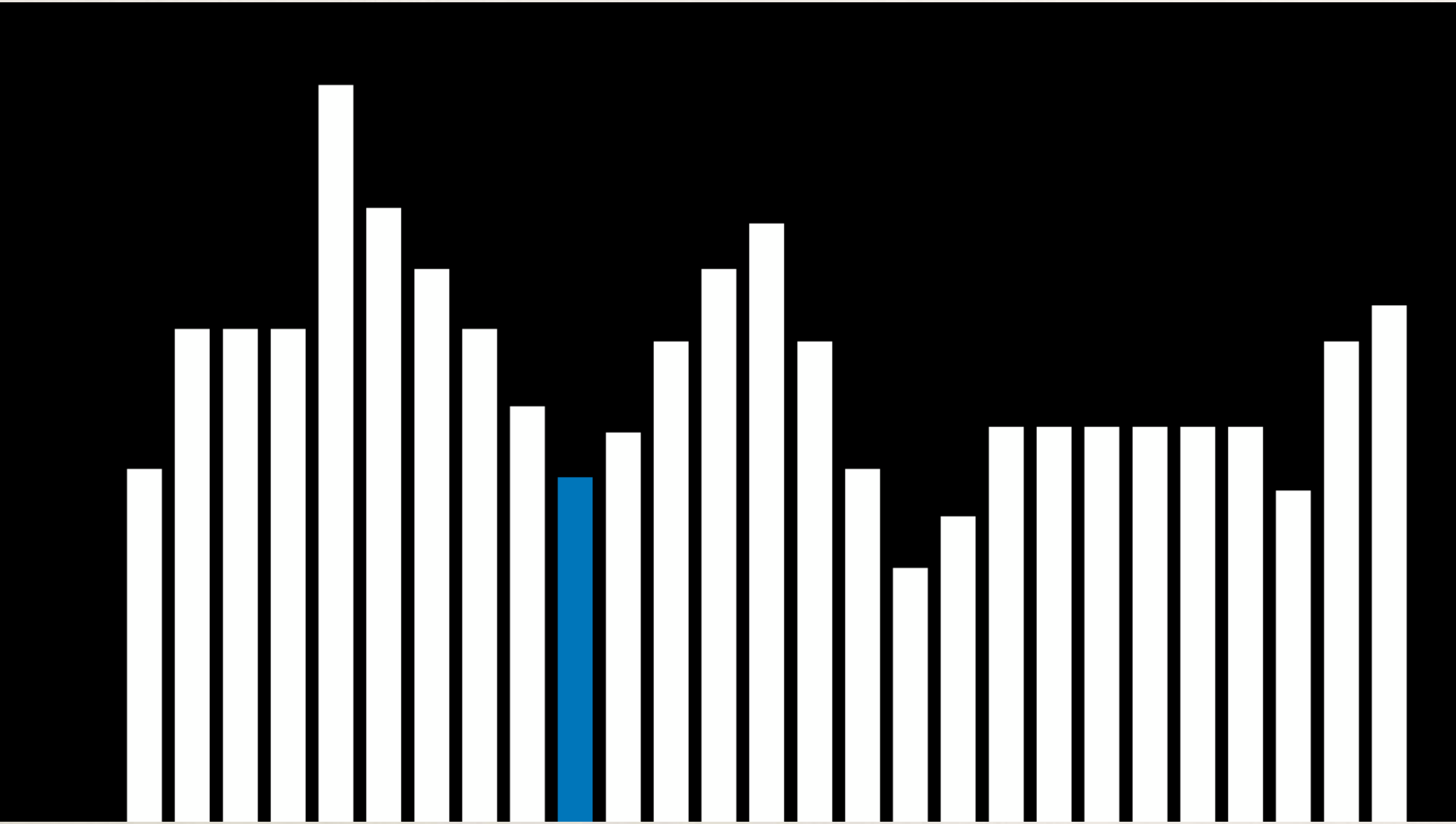

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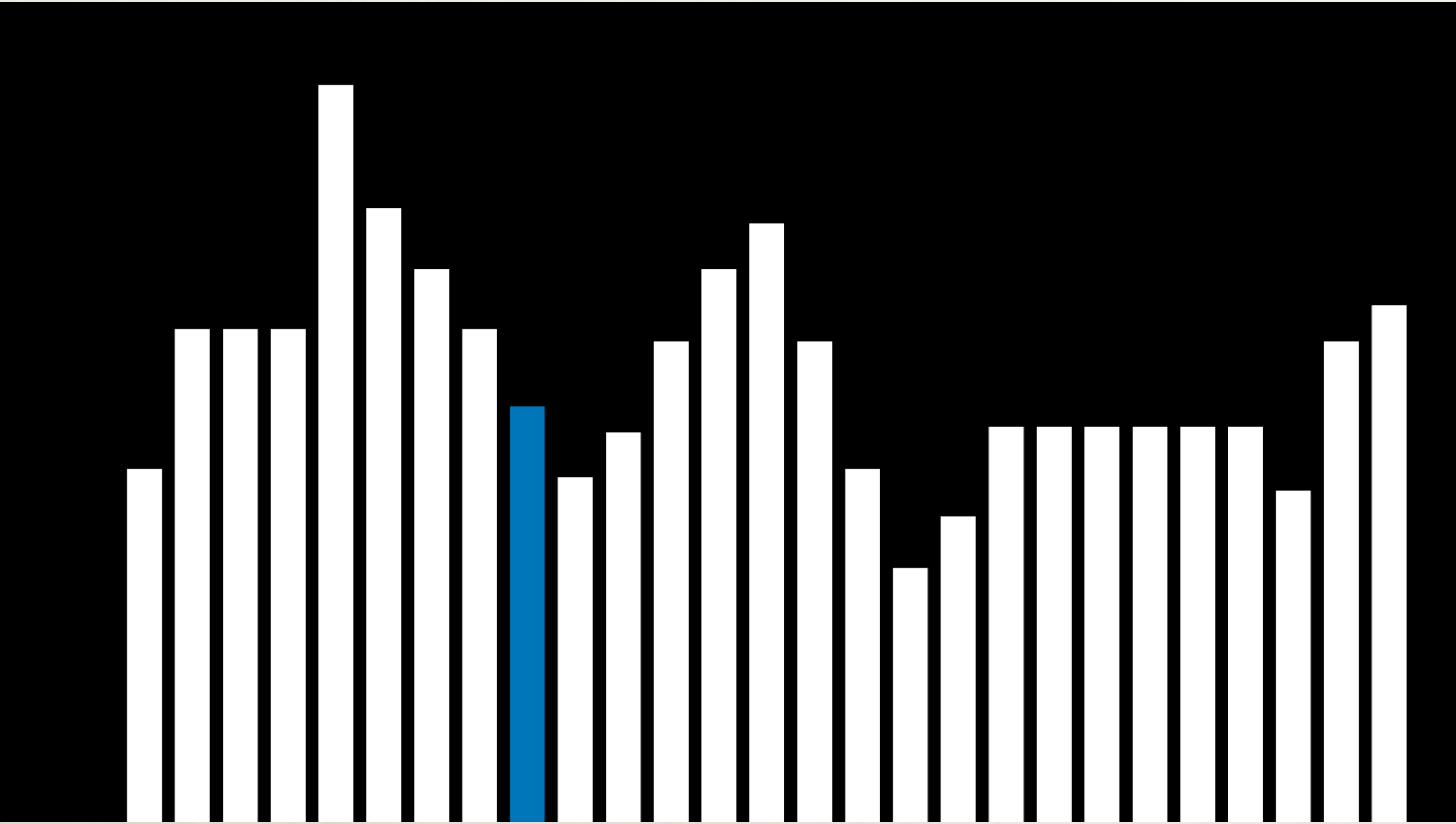

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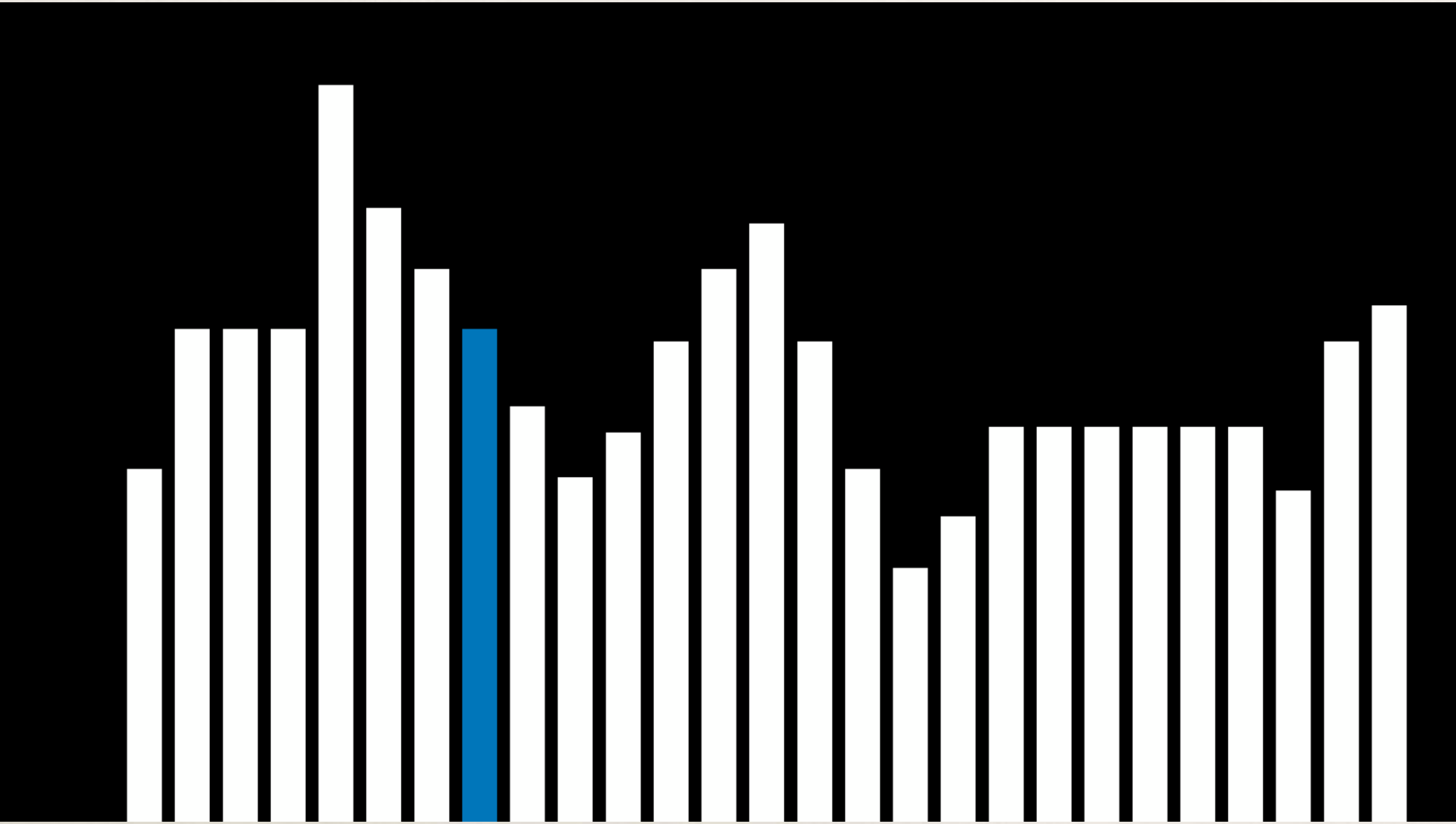

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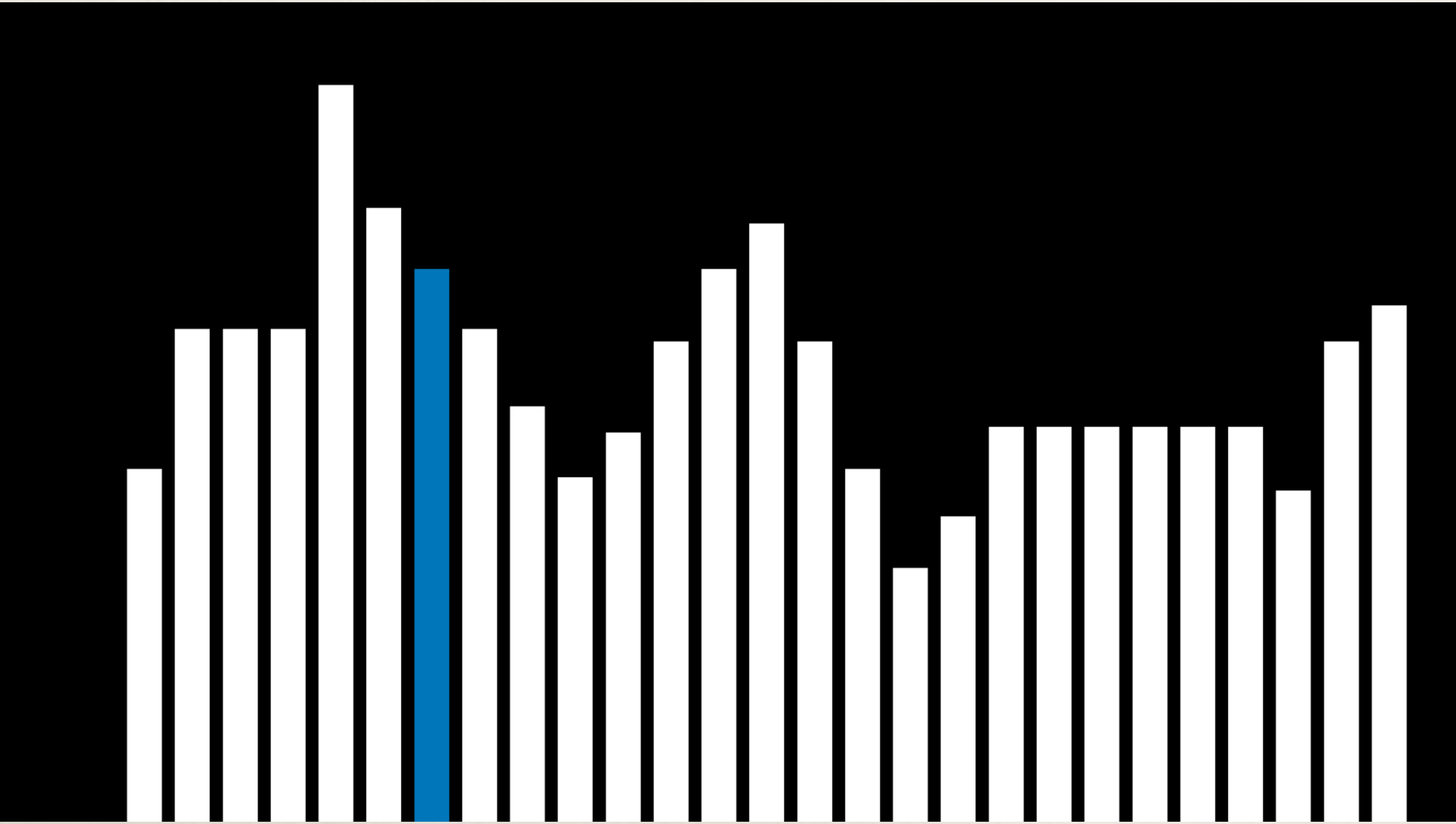

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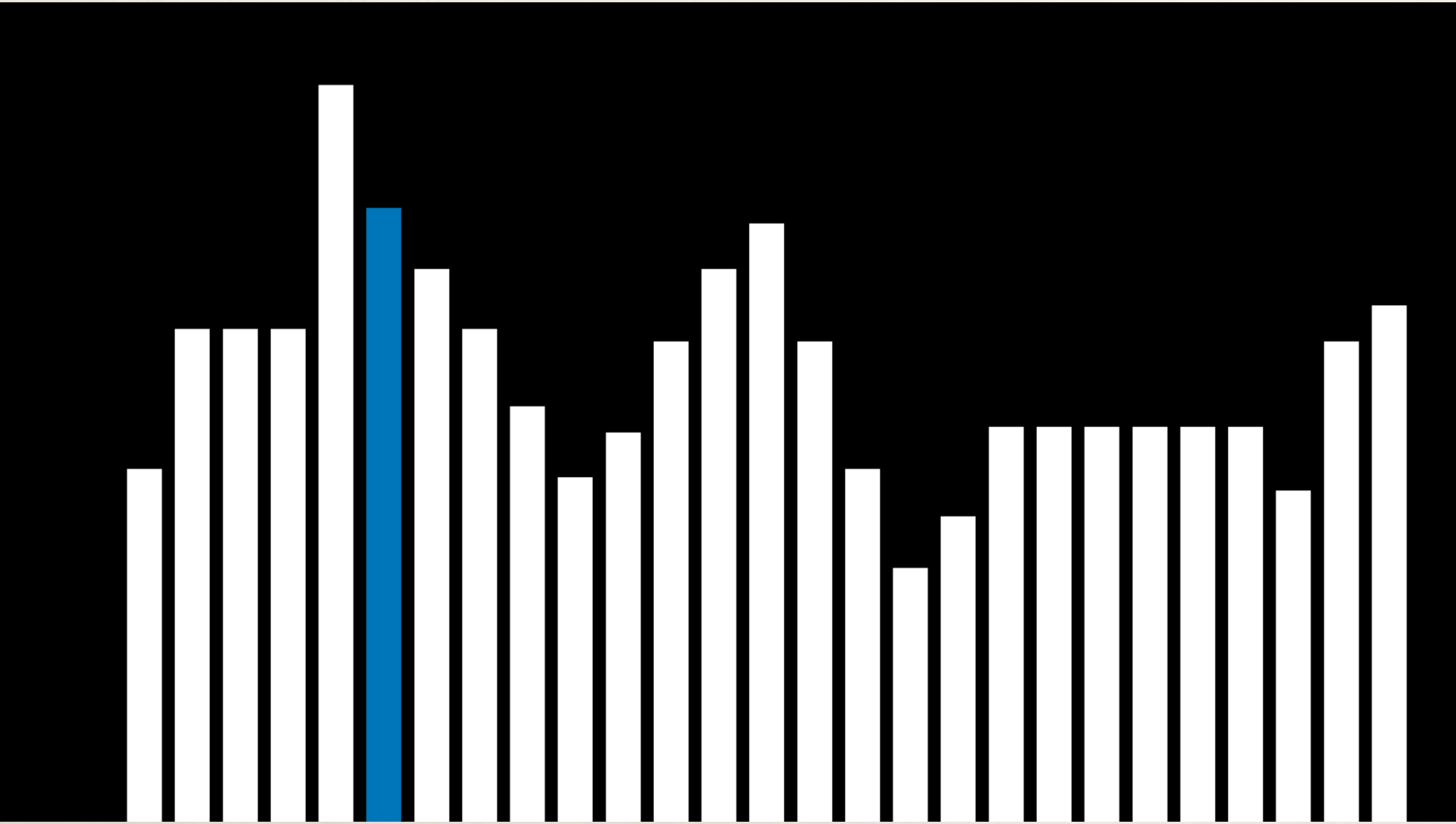

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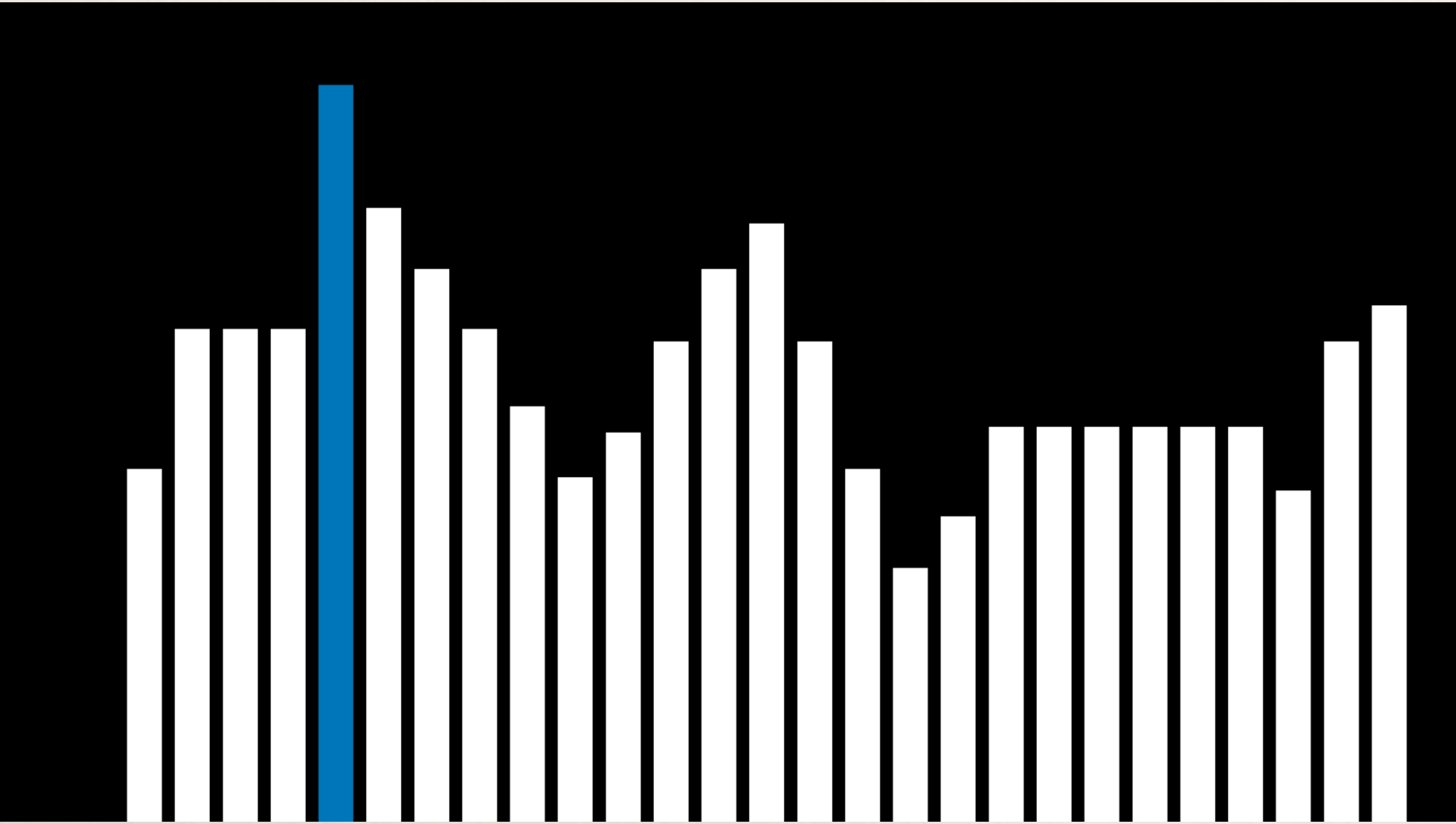

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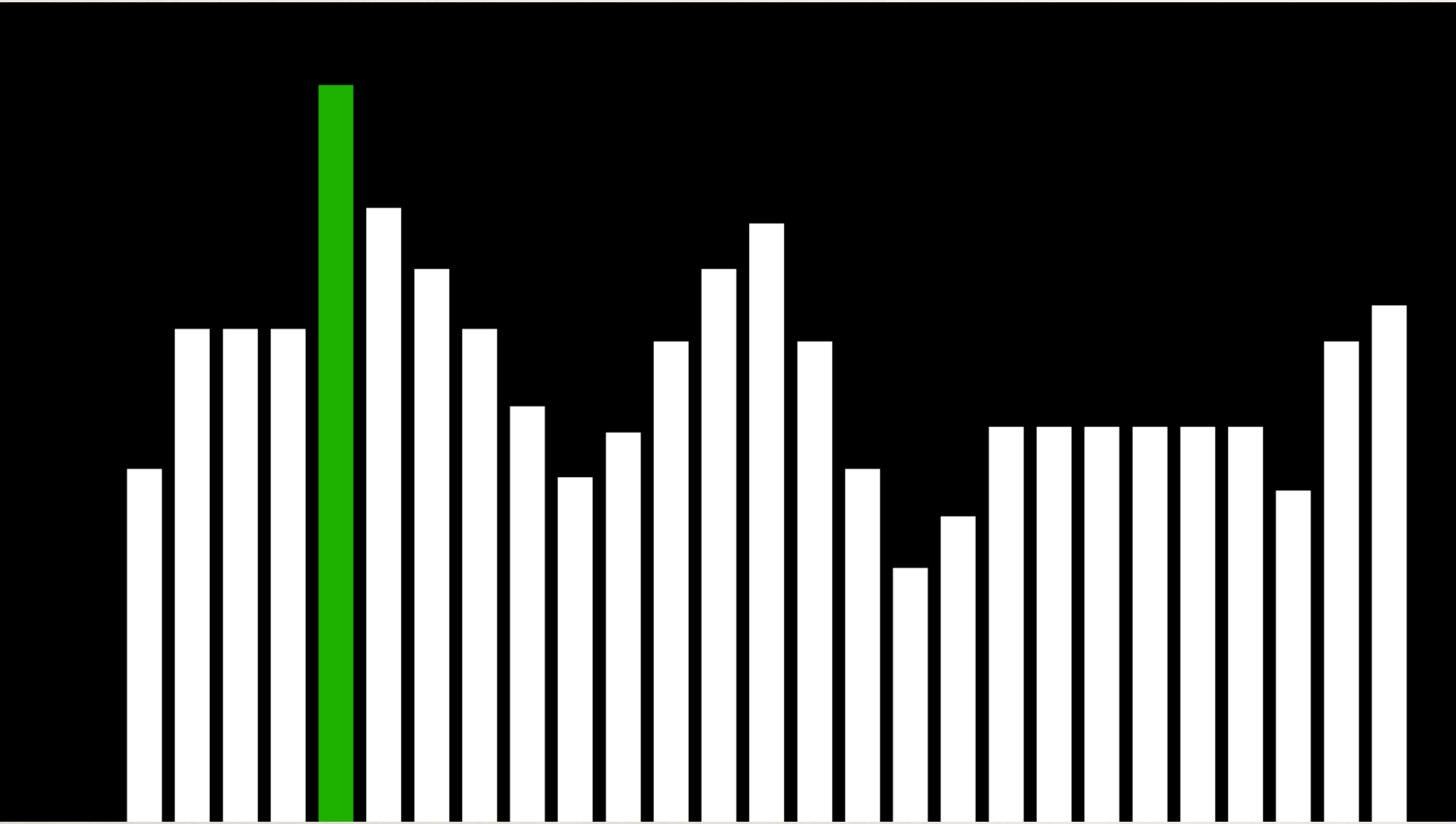

Simulated annealing

```
function SIMULATED-ANNEALING(problem, max):
    current = initial state of problem
    for t = 1 to max:
        T = TEMPERATURE(t)
        neighbor = random neighbor of current
        ΔE = how much better neighbor is than current
        if ΔE > 0:
            current = neighbor
        else
            with probability e^(ΔE/T) set current=neighbor
    return current
```

Temperature analogies

Searching for a New Apartment:

At the start, when looking for a **new apartment**, you might visit many different neighborhoods and consider various layouts (**high temperature**). Over time, you **narrow down** to a few best choices and finally commit to one (**low temperature**).

- **High temperature:** Willing to explore various locations, even ones that seem inconvenient.
- **Low temperature:** Focusing only on the best options and making a final choice.

https://propertyrecordsofmaryland.com/how-to-find-property-owner-information-in-maryland-tips-and-tools/

Temperature analogies

Decision-Making When Shopping

When you first enter a **shopping mall**, you're open to exploring multiple stores and trying out different products (**high temperature, more randomness**). As time passes and you get tired, you focus on finalizing a purchase (**low temperature, less randomness**), settling on the best option you've found.

- **Early stage (high temp):** Exploring many product options, including ones that might not seem great at first.
- **Later stage (low temp):** Narrowing down choices and making a final decision.

https://www.campaignasia.com/article/economic-hurdles-push-japanese-consumers-to-save-not-spend/452840

Local beam search

- The local search algorithms so far kept track of a single state
- **Local beam search** keeps track of *k* states instead
- In each iteration, all the successors of all *k* states are generated
- Unless a stopping condition is met, the best *k* successors are selected for the next iteration
  - Possible stopping condition: no improvement found
- This seems similar to random restart hill-climbing but is actually different: hill-climbing runs are executed *independently*
- Local beam search automatically concentrates the search on those parts of the explored search space where progress is most rapid
- We can increase exploration by adopting **stochastic beam search**
  - Chooses successors with probability proportional to their value

References

- CS50's Introduction to Artificial Intelligence with Python 2020 (https://cs50.harvard.edu/ai/2024/)