

COMPX216/Y05337

Artificial Intelligence

Uninformed search

Today: Uninformed search

- Uninformed vs. heuristic search
- Generic best-first search
- The role of the queue
- Priority queues
- The generic best-first search algorithm
- An instantiation: depth-first search
- Actions with different costs: uniform-cost search

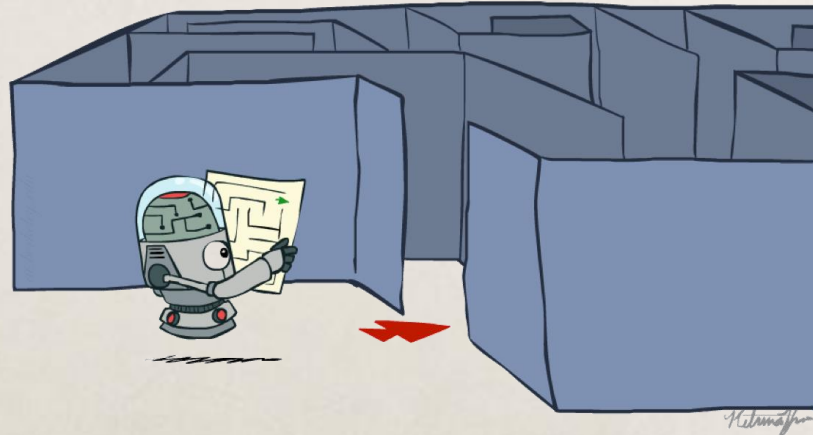
Recap: Search

■ Search problem:

- State space graph is its mathematical representation
- States (configurations of the world)
- Actions and costs
- Transition model (world dynamics)
- Start state and goal test

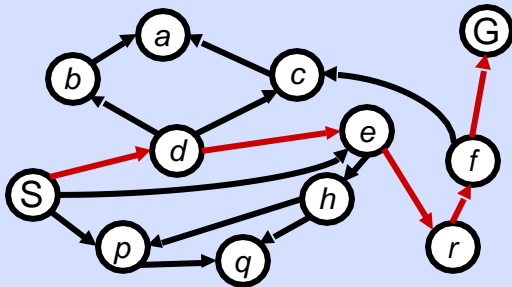
■ Search tree:

- Nodes: represent plans for reaching states
- Plans have costs (sum of action costs)



Recap: State space graphs vs Search trees

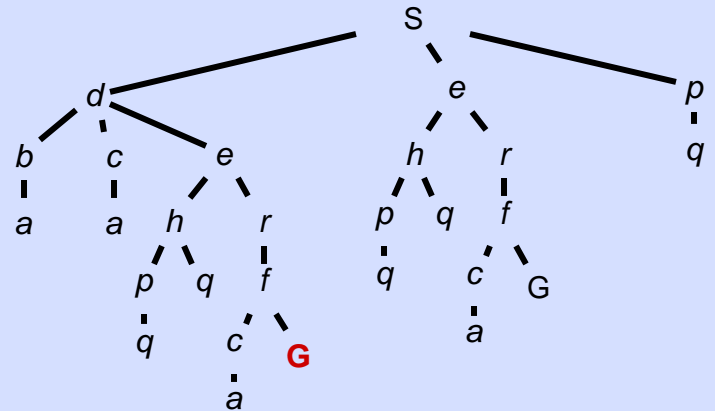
State Space Graph



Each NODE in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.

Search Tree



Uninformed vs. heuristic search

- Search algorithms can be distinguished based on whether they make use of a **heuristic function**
- For a given state, the **heuristic function** estimates the cost of the cheapest path from there to a goal state
- In some cases, it is difficult to create a useful heuristic that improves search performance
- **Uninformed search algorithms** explore the state space without the use of such a function
- Breadth-first search is an example of an uninformed search algorithm

Generic best-first search

- Many search algorithms can be cast as instances of a generic **best-first search algorithm**
- This includes uninformed search algorithms as well as heuristic ones
- The algorithms differ in how they define which node amongst the available candidate nodes is “best”
- This definition is used to decide which node in the frontier of the search tree to expand next
- For example, in breadth-first search, the “best” node is the left-most shallowest node in the frontier

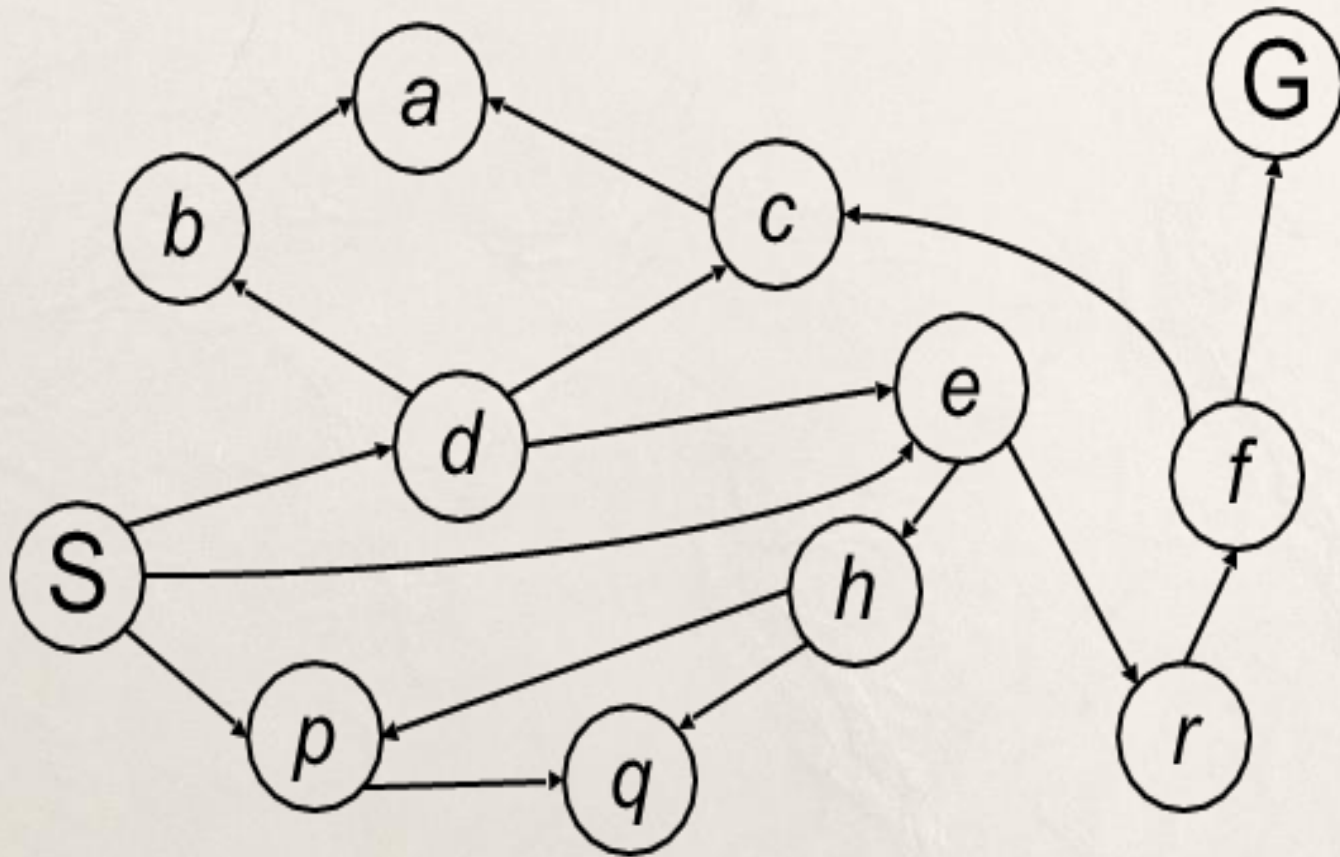
Generic best-first search

```
1  Function Tree-Search(problem, strategy)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = strategy.select_node(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

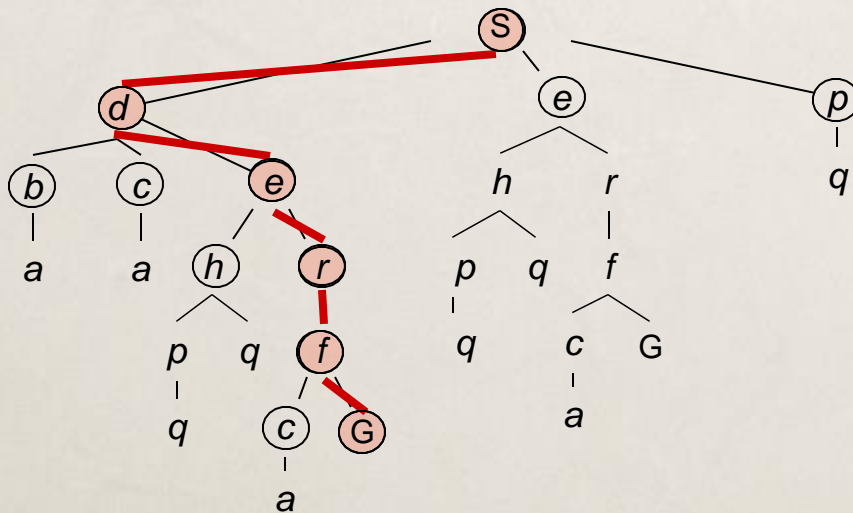
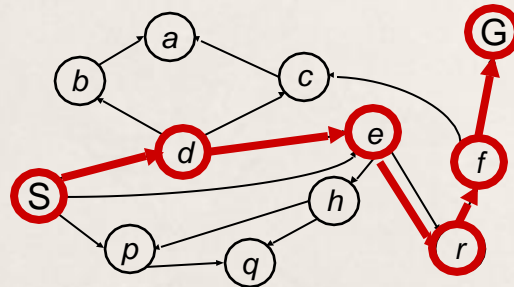

Node expansion

```
1  Function Expand(node, problem)
2      children = {}
3      for a : problem.actions(node)
4          child.state = Node(node.state.do_action(a))
5          child.parent = node
6          child.path_cost = node.path_cost + a.cost
7          child.action = a
8          child.depth = node.depth + 1
9          children.add(child)
10     return children
```


Example: Tree Search



Example: Tree Search



s
~~s → d~~
s → e
s → p
s → d → b
s → d → c
~~s → d → e~~
s → d → e → h
~~s → d → e → r~~
~~s → d → e → r → f~~
s → d → e → r → f → c
~~s → d → e → r → f → G~~

Generic best-first search

- Important ideas:
 - Frontier
 - Expansion
 - Exploration strategy
- Main question: which frontier nodes i.e., "best" to explore?

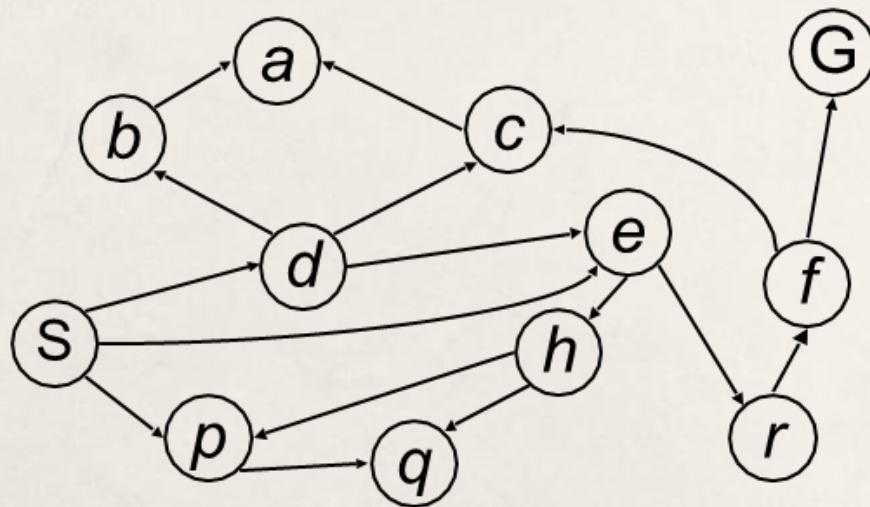
The breadth-first search (BFS) algorithm

- Root node expanded first
- Root children expanded next
- Their children next ... etc.
- BFS in general
 - All nodes at a given depth/level are expanded before any nodes in the next level
- Uses a queue (FIFO) for frontier

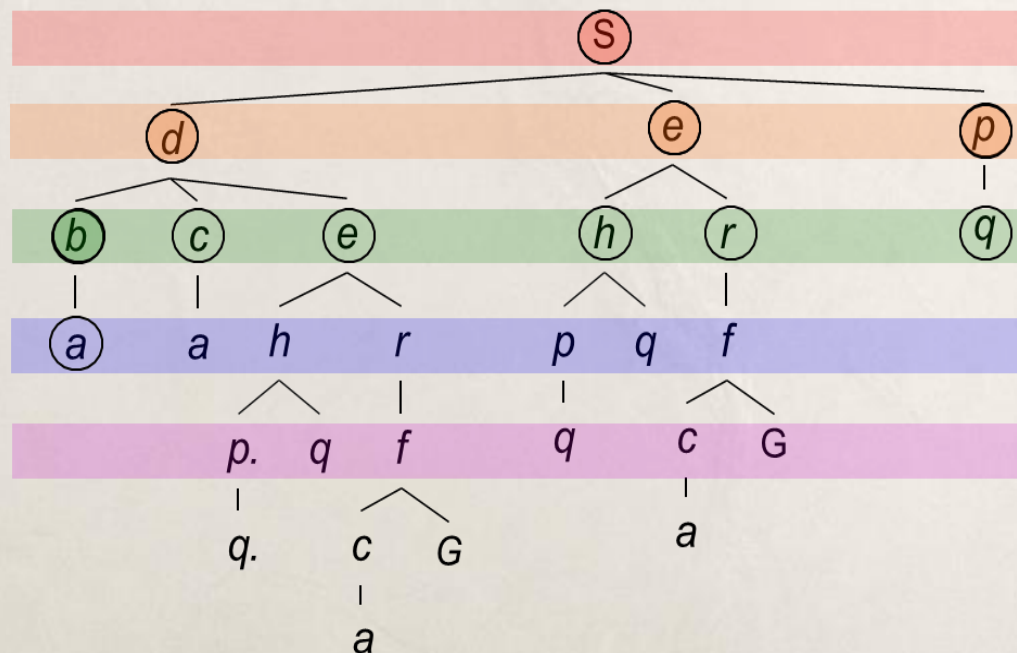
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Frontier is a FIFO queue



Search Levels/Tiers



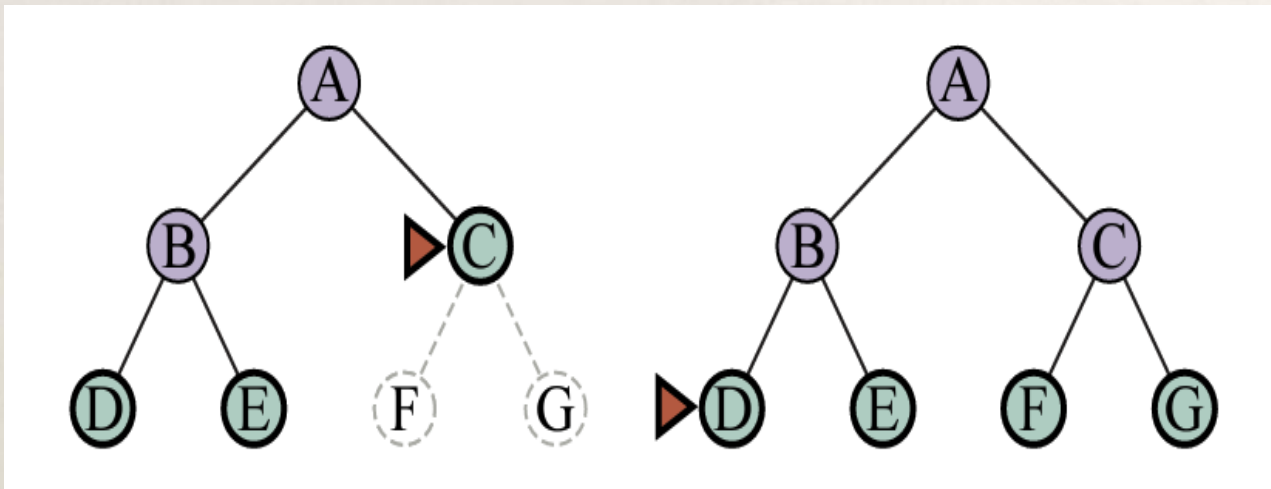
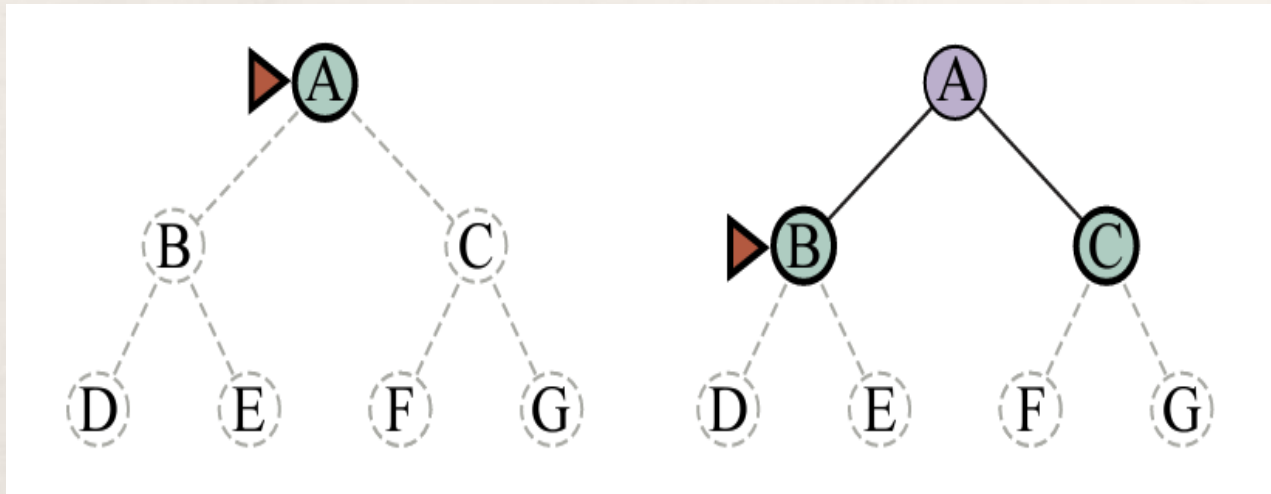
Generic best-first search

```
1  Function Tree-Search(problem, strategy)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = strategy.select_node(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

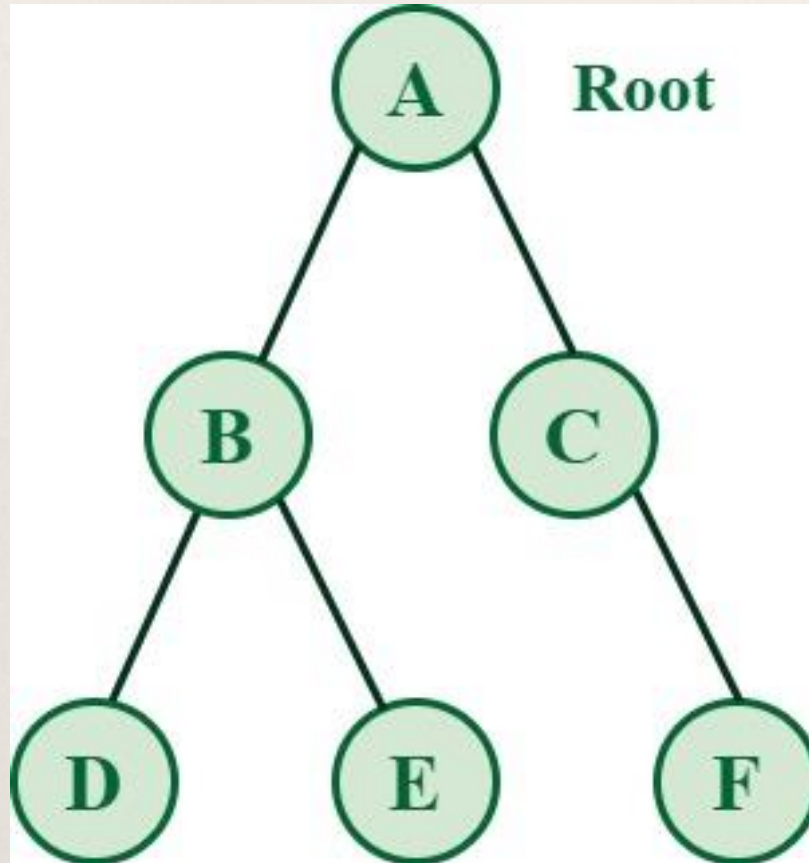
Breadth-first search (BFS)

```
1 Function Tree-Search(problem)
2   frontier = Queue{Node(problem.initial_state)}
3   while (true)
4     if (frontier.empty) return fail
5     node = frontier.pop()
6     if (node.state is goal) return solution
7     else frontier.add(Expand(node, problem))
```


The breadth-first search algorithm



Breadth-first search traversal

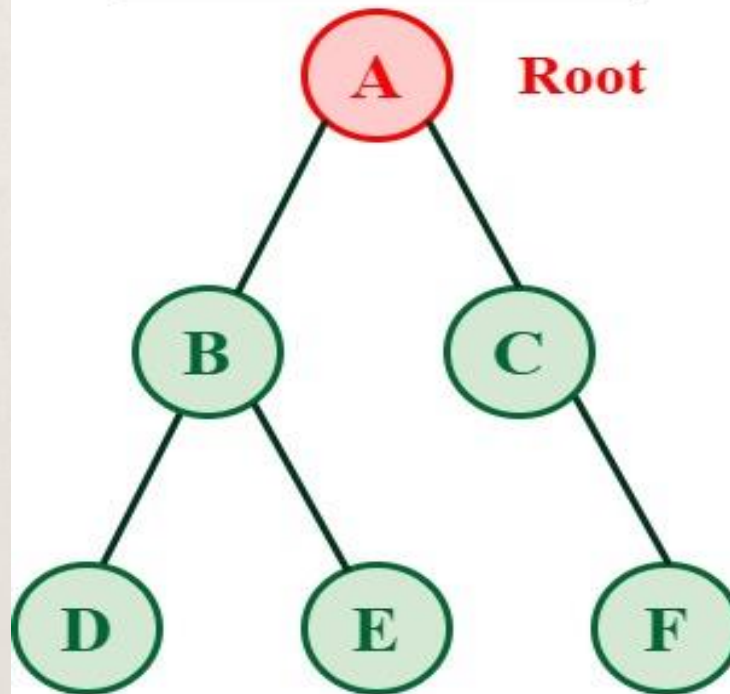


Breadth-first search traversal

Step 1: Initialization

Operation:

- **Enqueue A**



Visited: []

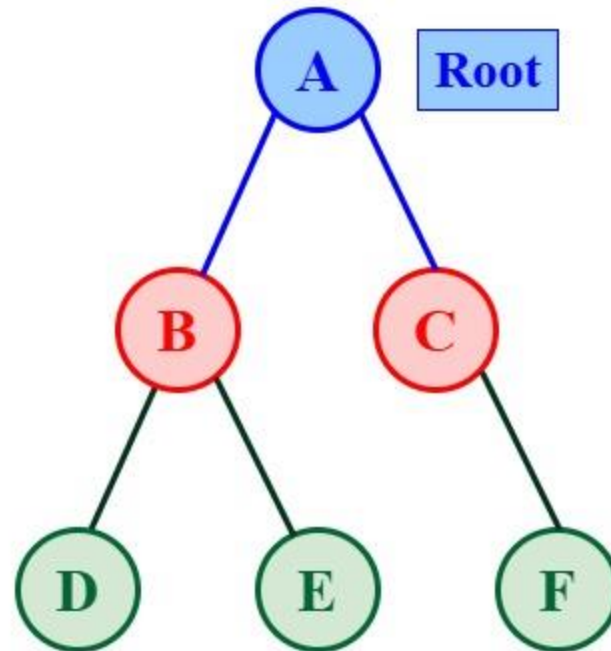
Queue (Frontier): [A]

Breadth-first search traversal

Step 2: Dequeue "A" and enqueue children

Operation:

- Dequeue A (mark as visited)
- Enqueue B and C (left to right)



Visited: [A]

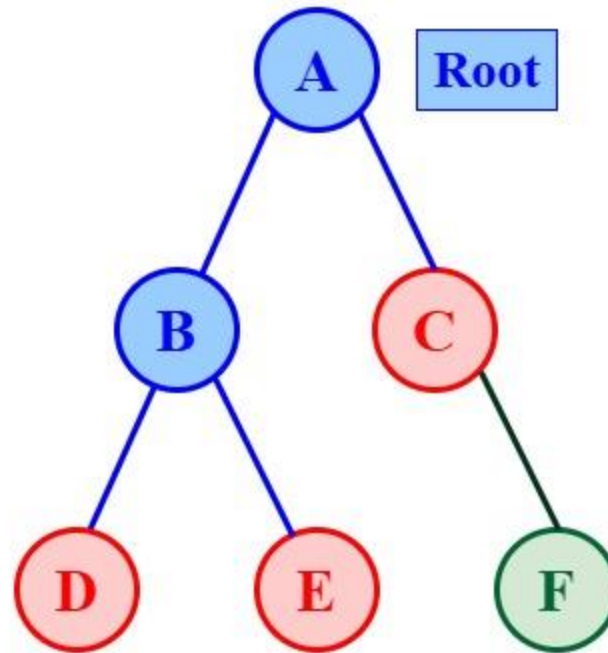
Queue (Frontier): [B, C]

Breadth-first search traversal

Step 3: Dequeue "B" and enqueue children

Operation:

- Dequeue B (mark as visited)
- Enqueue D and E (left to right)

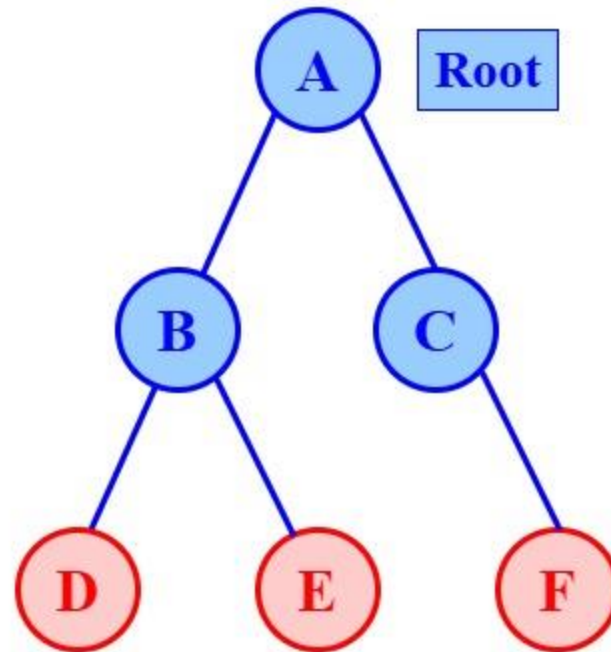


Breadth-first search traversal

Step 4: Dequeue "C" and enqueue children

Operation:

- Dequeue C (mark as visited)
- Enqueue F



Visited: [A, B, C]

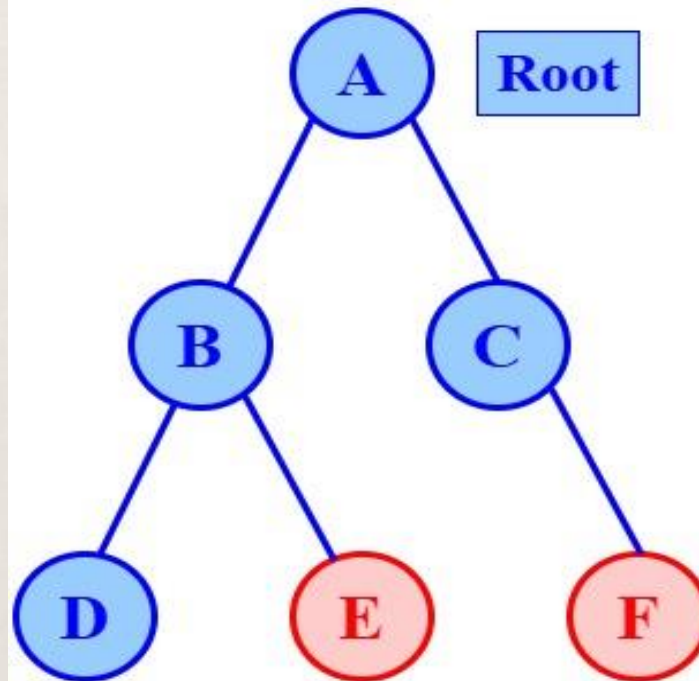
Queue (Frontier): [D, E, F]

Breadth-first search traversal

Step 5: Dequeue "D"

Operation:

- **Dequeue D (mark as visited)**
- **No children to enqueue**



Visited: [A, B, C, D]

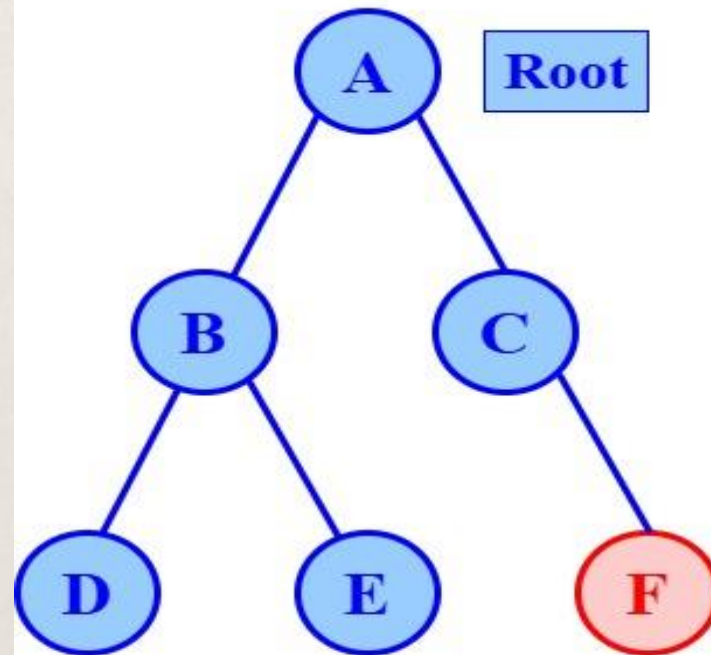
Queue (Frontier): [E, F]

Breadth-first search traversal

Step 6: Dequeue "E"

Operation:

- **Dequeue E (mark as visited)**
- **No children to enqueue**



Visited: [A, B, C, D, E]

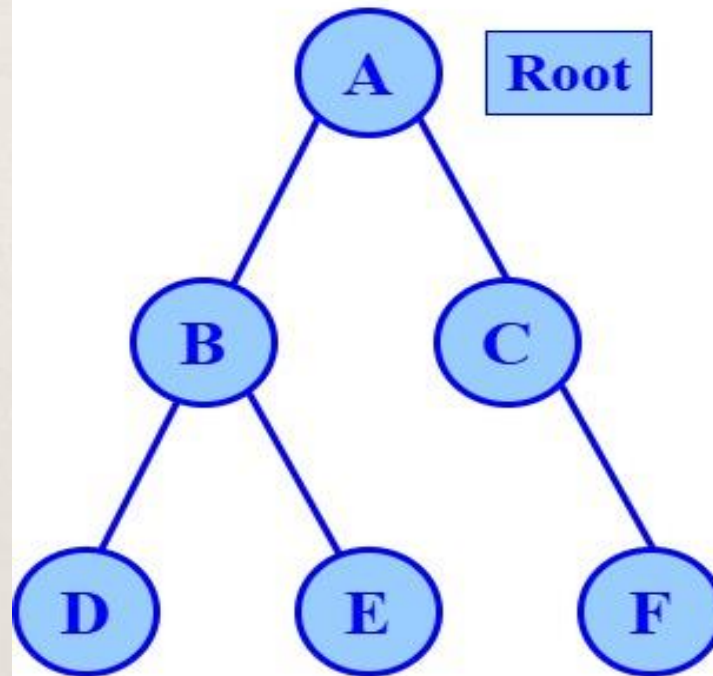
Queue (Frontier): [F]

Breadth-first search traversal

Step 7: Dequeue "F"

Operation:

- **Dequeue F (mark as visited)**
- **No children to enqueue**



Visited: [A, B, C, D, E, F]

Queue (Frontier): []

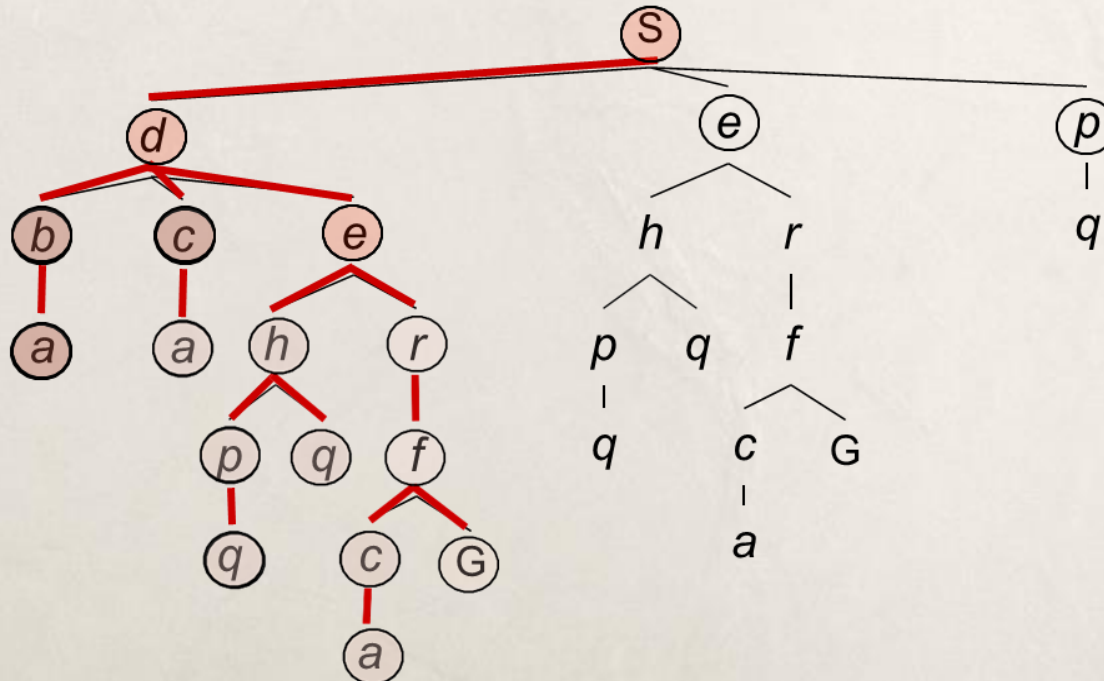
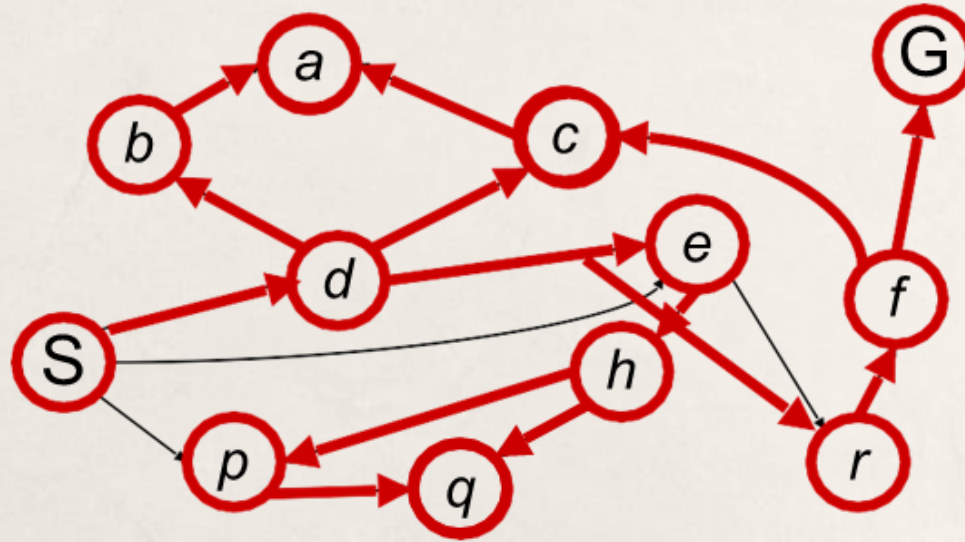
The depth-first search (DFS) algorithm

- Expands the deepest node on the frontier
- Search goes immediately to the deepest level of the search tree to a leaf node
 - Leaf node has no children
- If a leaf is reached, it is discarded and the search 'backs up' to previous depth
- Uses a stack (LIFO) for frontier

Depth-First Search

Strategy: expand a deepest node first

Implementation: Frontier is a LIFO stack



Generic best-first search

```
1  Function Tree-Search(problem, strategy)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = strategy.select_node(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

Breadth-first search (BFS)

```
1  Function Tree-Search(problem)
2      frontier = Queue{Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = frontier.pop()
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```


Depth-first search (BFS)

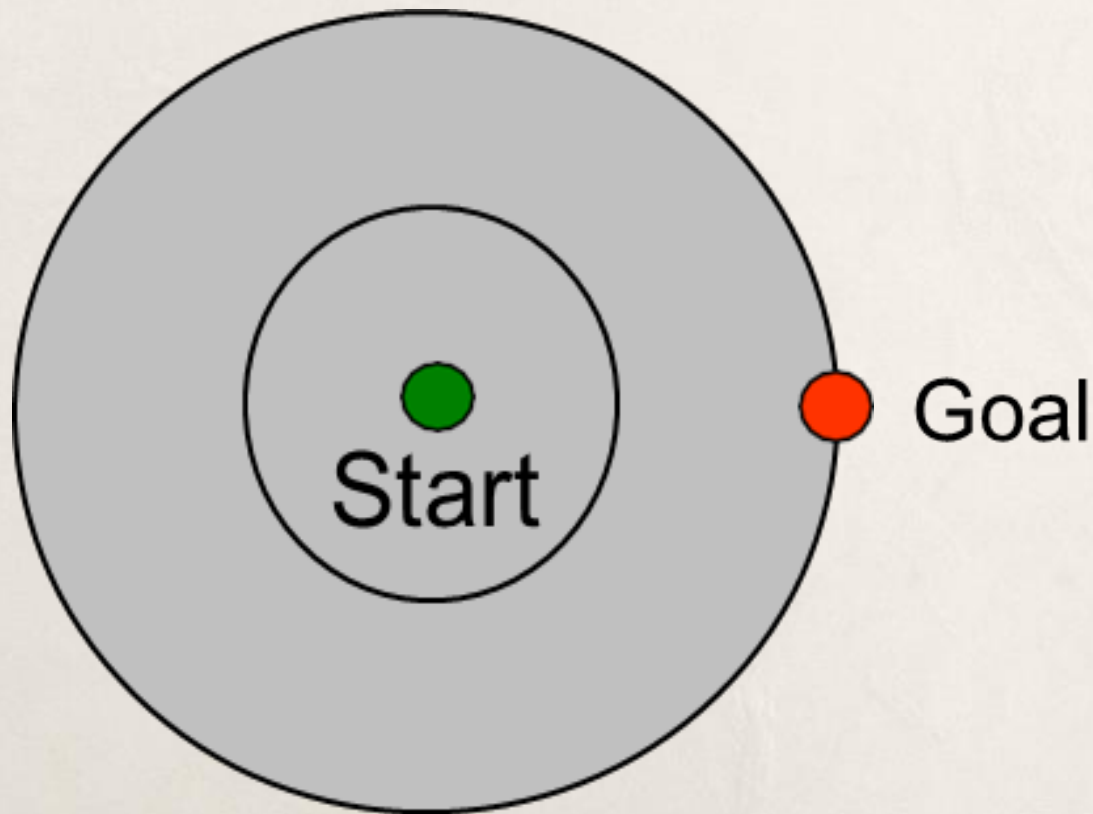
```
1  Function Tree-Search(problem)
2      frontier = Stack{Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = frontier.pop()
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```


The uniform-cost search (UCS) algorithm

- BFS is optimal when all action costs are equal because it expands the shallowest node
- Uniform-cost search is optimal for any cost
- UCS expands node with lowest path cost
- If all costs are equal, equivalent to BFS
- UCS is Dijkstra's algorithm for a single goal

The uniform-cost search (UCS) algorithm

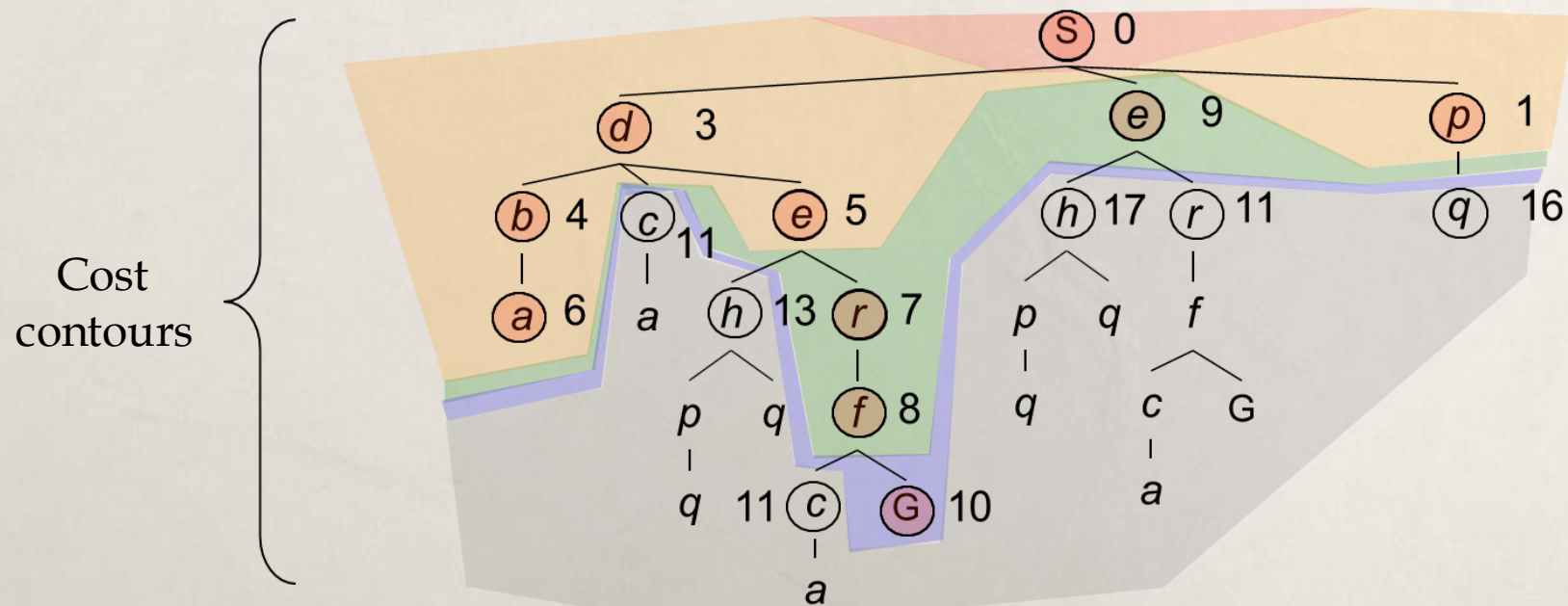
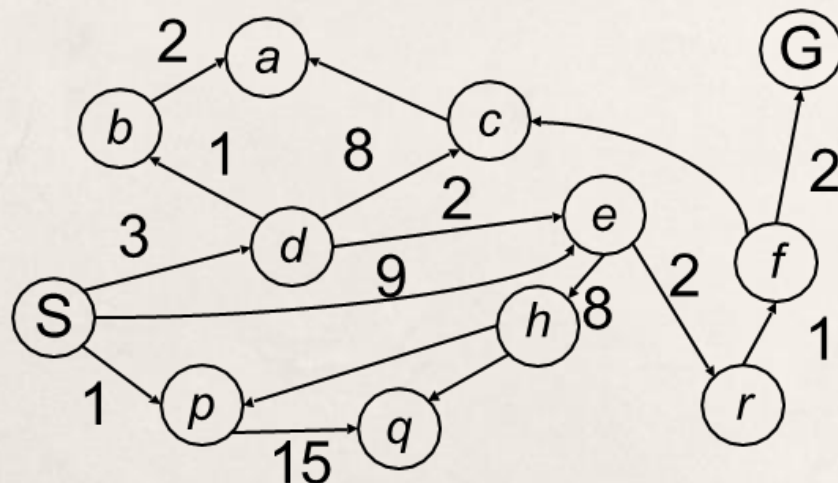
- Explores options in every "direction"



Uniform-cost search

*Strategy: expand a
cheapest node first*

*Implementation: Frontier
is a priority queue
Priority (cumulative cost)*



Generic best-first search

```
1  Function Tree-Search(problem, strategy)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = strategy.select_node(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

Breadth-first search (BFS)

```
1  Function Tree-Search(problem)
2      frontier = Queue{Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = frontier.pop()
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

Depth-first search (BFS)

```
1  Function Tree-Search(problem)
2      frontier = Stack{Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = frontier.pop()
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

Uniform-cost search (UCS)

```
1  Function Tree-Search(problem)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = min_cost_cum_path_value(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```


The magic of the queue

- A queue is used to store the nodes from the search tree that are in the frontier (i.e., that haven't been generated but not yet expanded)
- It turns out that different notions of which node is “best” can be implemented simply by using different types of queues
- The rest of the generic search algorithm can remain the same to get a variety of well-known (informed and uninformed) search algorithms
- For example, by using a first-in first-out (**FIFO**) queue, we get breadth-first search, where the shallowest nodes are expanded first
- Using a last-in first-out (**LIFO**) queue (aka **stack**), we get **depth-first search**, another uninformed search that expands the deepest nodes first
- In Python, we can use a **deque** (implemented as a doubly-linked list under the hood) to simulate FIFO and LIFO queues
 - FIFO: pop off last element in the deque for expansion
 - LIFO: pop off first element in the deque for expansion

Priority queues

- To get an even wider variety of search algorithms, we can use a **priority queue** so that we can go beyond LIFO and FIFO
 - This is implemented as a **heap** data structure under the hood
- The priority queue enables us to pop-off, at any time, the node with the highest priority in the queue
- Priority is defined by providing an evaluation function f
- In our case, the function f takes a node in the frontier as its argument and returns a numeric value that indicates its priority
- In the following, we assume that lower values returned by this function indicate greater priority
- For example, to get breadth-first search, we can use the depth of a node as the numeric value returned by the function

Avoiding repeated states

- One of the most important search ideas
- Especially important with reversible actions
- Most infinite loops/wasted time can be avoided by not returning to identical states
- We can remember nodes expanded
 - Don't re-expand them
- Can lead to exponential savings in the number of nodes generated

"reached" list

- Store states we have already expanded
- "reached" list stores expanded states
 - Often implemented as a hash table/dictionary for efficient lookup
- Check if a node is reached before expanding
- "frontier" stores unexpanded nodes
 - Often implemented as a priority queue
 - With $O(\log(n))$ insertion and extraction time instead of the faster FIFO queue, which is $O(1)$

Generic best-first search

```
1  Function Tree-Search(problem, strategy)
2      frontier = {Node(problem.initial_state)}
3      while (true)
4          if (frontier.empty) return fail
5          node = strategy.select_node(frontier)
6          if (node.state is goal) return solution
7          else frontier.add(Expand(node, problem))
```

Generic best-first search updated

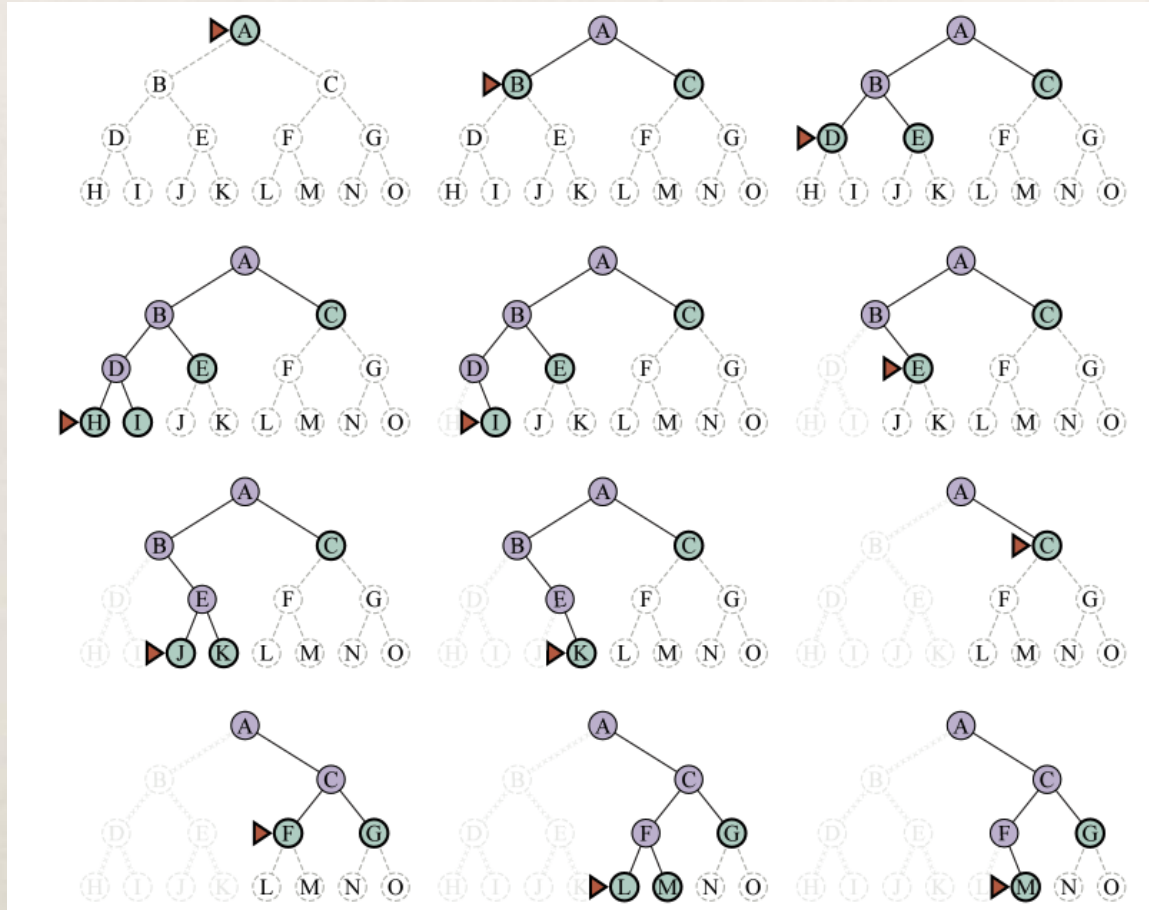
```
1  Function Tree-Search-Updated(problem, strategy)
2      reached = {} // add reached list
3      frontier = PQ{Node(problem.initial_state)}
4      while (true)
5          if (frontier.empty) return fail
6          node = frontier.pop() // based on a priority/criteria
7          if (node.state is goal) return solution
8          if (node.state in reached) continue // check if reached
9          reached.add(node.state) // mark node reached
10         frontier.add(Expand(node, problem))
```


Generic best-first search updated

```
1  Function Tree-Search-Updated(problem, strategy)
2      reached = {}
3      frontier = PQ{Node(problem.initial_state)}
4      while (true)
5          if (frontier.empty) return fail
6          node = frontier.pop()
7          if (node.state is goal) return solution
8          if (node.state in reached)
9              reached.add(node.state)
10         frontier.add(Expand(node, problem))
```

Depth-first search

- An implementation of depth-first search can be obtained using the negative of a node's depth as the value returned by f



Uniform-cost search (Dijkstra's algorithm)

- When actions have different costs, it does not make sense to make decisions based solely on the depth of nodes in the search tree
- We can use a priority queue where nodes with smaller path costs are preferred, i.e., nodes that can be reached with lower cost
- In this case, f returns the path cost of the node it is applied to

References

- Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley (ai.berkeley.edu)
- Dr. David Churchill, Department of Computer Science Memorial University of Newfoundland (<http://www.cs.mun.ca/~dchurchill>)