4.0 Dynamics

Newton's Laws of Motion

- A body will remain at rest or move at uniform speed along a straight line unless acted upon by an external force.
- The rate of change of linear momentum is directly proportional to the applied force and occurs in the same direction as the force.
- The forces of two interacting bodies on each other are equal and directed in opposite directions.

Mass

- Newton's first law expresses the concept of *inertia.*
  The inertia of a body is the reluctance of a body to start moving, or to stop moving once it has started.
- When you are standing on a bus, and the bus starts very quickly, your body seems to be pushed backward, and if the bus stops suddenly, then your body seems to be pushed forwards.
- Notice that when the bus turns left, you will seem to be pushed to the right, and when the bus turns right, you will seem to be pushed to the left.
- A body of large mass requires a larger force to change its speed or direction by a noticeable amount, it has a large inertia.
- The mass of a body is a measure of its inertia.
- In the SI system, the unit of mass is the kilogram (kg).

Definition of momentum

Momentum (P) is a vector quantity equal in magnitude to the product of mass and velocity.

- Note, mass (m) is a scalar quantity, while velocity (**v**) is a vector quantity.

$$\mathbf{P} = m\mathbf{v}$$

$$m\ (\mathrm{kg}) \quad \mathbf{v}(\mathrm{ms^{-1}}) \quad P\ (\mathrm{kg.\ ms^{-1}})$$

- The letter 'p' in small case is designated to represent pressure.

Theory

- If we consider a force **F** acting on a mass m with velocity **v**, the Second law may be represented by the proportionality:

But acceleration,

When F = 1N, m = 1kg and a = 1ms^{-2}

The Newton (N)

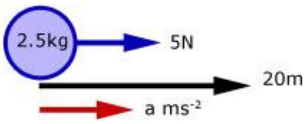
The **Newton** is the force that when applied to a **1 kg mass** will give it an **acceleration of 1 ms^{-2}**.

Linear acceleration

- Here the mass is either stationary and is accelerated by a force in a straight line or is initially moving at constant velocity before the force is applied.

Example #1

A 5N force acts on a 2.5kg mass, making it accelerate in a straight line.
i) What is the acceleration of the mass?
ii) How long will it take to move the mass through 20m?
(Answer to 2 d.p.)



i)

$F = 5\text{N} \qquad m = 2.5\text{kg} \qquad s = 20\text{m} \qquad u = 0\text{ms}^{-1}$

using $\quad F = ma$

$$\Rightarrow \qquad a = \frac{F}{m}$$

$$= \frac{5}{2.5} = 2$$

<u>Ans. acceleration is 2 ms^{-2}</u>

ii)

using $\qquad s = ut + \frac{1}{2}at^2$

$u = 0\text{ms}^{-1} \quad \Rightarrow \quad s = \frac{1}{2}at^2$

$$t^2 = \frac{2s}{a}$$

$$t = \sqrt{\frac{2s}{a}}$$

$s = 20\text{m} \qquad a = 2\text{ms}^{-2}$

$$t = \sqrt{\frac{2 \times 20}{2}} = \sqrt{20} = 4.47$$

<u>Ans. time for mass to move 20 m is 4.47 secs.</u>

<u>Linear retardation</u>

- Here the mass is already moving at constant velocity in a straight line before the force is applied, opposing the motion.

<u>Example #1</u>

A 4 kg mass travelling at constant velocity 15 ms^{-1} has a 10 N force applied to it against the direction of motion.
i) What is the deceleration produced?
ii) How long will it take before the mass is brought to rest?

i)

$$u = 15\ \text{ms}^{-1} \qquad m = 4\ \text{kg} \qquad F = 10\ \text{N}$$

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{10}{4} = 2.5$$

Ans. deceleration is 2.5 ms^{-1}

ii)

$$v = 0$$

$$v = u - at$$

$$0 = u - at$$

$$at = u$$

$$t = \frac{u}{a} = \frac{15}{2.5} = 6$$

Ans. mass brought to rest in 6 secs.

Example #2

A sky diver with mass 80kg is falling at a constant velocity of 70 ms^{-1}. When he opens his parachute he experiences a constant deceleration of 3g for 2 seconds.
i) What is the magnitude of the decelerating force?
ii) What is his rate of descent at the end of the 2 seconds deceleration?

i)

$$m = 80\ \text{kg} \qquad u = 70\ \text{ms}^{-1} \quad a = 3g$$

$$F = ma$$

$$= 80 \times 3 \times 9.8$$

$$= 2352$$

Ans. decelerating force is 2352 N

ii)

$u = 70\ \text{ms}^{-1} \quad a = 3 \times 9.8 = 29.4\ \text{ms}^{-2} \quad t = 2\ \text{secs.}$

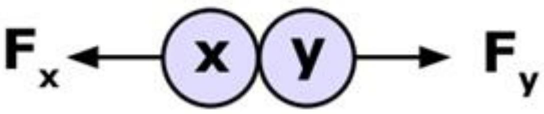
$$\begin{aligned} v &= u - at \\ &= 70 - (29.4) \times (2) \\ &= 11.2 \end{aligned}$$

<u>Ans. final rate of descent is $11.2\ \text{ms}^{-1}$</u>

<u>The Principle of Conservation of Momentum</u>

The **total linear momentum** of a system of colliding bodies, with no external forces acting, **remains constant**.

- for two perfectly elastic colliding bodies with no external forces note:
  iii) By Newton's 3rd. Law, the force on X due to Y , (F_x) is the same as the force on Y due to X , (F_y) .



$F_x = F_y$

  iv) By Newton's 2nd. Law, the rate of change of momentum is the same, since F = (rate of change of momentum)
  v) Because the directions of the momentum of the objects are opposite, (and therefore of different sign) the net change in momentum is zero.
- If we consider the speed of individual masses before and after collision, we obtain another useful equation:
  u_A = initial speed of mass A
  u_B = initial speed of mass B
  v_A = final speed of mass A
  v_B = final speed of mass B
  relative initial speed of mass A to mass B = $u_B - u_A$
  relative final speed of mass A to mass B = $v_B - v_A$

momentum before the collision equals momentum after

hence, $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

Example #1

A 5 kg mass moves at a speed of 3 ms^{-1} when it collides head on, with a 3 kg mass travelling at 4 ms^{-1}, travelling along the same line.
After the collision, the two masses move off together with a common speed.

What is the common speed of the combined masses?

$m_1 = 5$ kg $\quad v_1 = 5$ ms^{-1} $\qquad m_2 = 3$ kg $\qquad v_2 = -4$ ms^{-1}
let the combined speed after collision be v_3

then, by the law of conservation of momentum,
momentum before collision = momentum after collision

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$\Rightarrow \qquad v_3 = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

substituting for m_1 v_1 m_2 and v_2

$$v_3 = \frac{(5 \times 3) + (3 \times (-4))}{5 + 3}$$

$$= \frac{15 - 12}{8} = \frac{3}{8} = 0.375$$

<u>common speed of the two masses is 0.375 ms^{-1}</u>

Example #2

An artillery shell of mass 10 kg is fired from a field gun of mass 2000 kg.
If the speed of the shell on leaving the muzzle of the gun is 250 ms^{-1} , what is the recoil speed of the gun?

$m_1 = 10 \text{ kg} \quad v_1 = 250 \text{ ms}^{-1} \quad m_2 = 2000 \text{ kg}$

let the recoil speed of the gun after firing be v_2

then, by the law of conservation of momentum,

$$\text{momentum before firing} = \text{momentum after firing}$$

$$0 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow \quad -m_2 v_2 = m_1 v_1$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{-m_2}$$

substituting for m_1 v_1 and m_2

$$v_2 = \frac{10 \times 250}{-2000} = -\frac{25}{20} = -1.25$$

(the minus signifies gun moves in opposite direction to the shell)

the speed of recoil of the gun is 1.25 ms^{-1}

Energy changes during collisions

- If no kinetic energy is lost (K.E.= $\frac{1}{2}$ mv^2) then the collision is said to be **perfectly elastic**.
- However if kinetic energy is lost, the collision is described as **inelastic**.
- In the special case when all the kinetic energy is lost, the collision is described as **completely inelastic**.
- This is when two colliding bodies stick to one another on impact.

Newton's law of restitution

- When two bodies collide, relative velocity after collision = e × relative velocity before collision.
- e is known as the coefficient of restitution and .
- When e = 1 the collision is elastic.
- e < 1 inelastic and e = 0 then it is completely inelastic.
- For a perfectly elastic collision

Impulse of a force

- This is simply the force multiplied by the time the force acts.
- We can obtain an expression for this in terms of momentum from Newton's Second Law equation F=ma, where the force F is constant.

- Remembering that velocity, force and therefore impulse are vector quantities.
- For a mass *m* being accelerated by a constant force **F**, $\mathbf{v}_1$ is initial velocity and $\mathbf{v}_2$ is final velocity:

$$\mathbf{F}t = m(\mathbf{v}_2 - \mathbf{v}_1)$$

- Since impulse is the product of force and time, the units of impulse are (Newtons) x (seconds), or N s .

<u>Force-time graphs</u>

- The area under a force time graph represents impulse