Contents

Preface

ရည်ရွယ်ချက်

- computer graphics and animation, computer vision, machine learning, robotics တွေအတွက် လိုအပ်တဲ့ Algebra အကြောင်းကိုလေ့လာသင်ယူရင်း အချိန်မရွေးပြန်လည်ကြည့်ရှုနိုင်ရန်။
- math အကြောင်းအရာများသည် theory အများစု ဖြစ်တဲ့အတွက် သင်ယူလေ့လာချိန်တွင် သတိထားစရာများကို မှတ်သားပြုစုရန်။
- psuedo code ဖြင့် programming ပုံစံရေးသားရန်

မူရင်းကိုးကားစာအုပ်များ

- Introduction to Linear and Matrix Algebra

Part I

fundamental

Chapter 1

vector & vector operation

1.1 vector ($\vec{v}$)

1.1.1 မှတ်စု

- vector တွေမှာ direction နှင့် magnitude နှစ်ခုလုံးရှိကြတယ်။
- v အပေါ်မှာ arrow လေးထည့်ပြီးဖော်ပြလေ့ရှိကြတယ်။
- vector ရဲ့ entry အားလုံးဟာ real numbers ထဲကသာဖြစ်ရမယ်။
- vector ရဲ့ entry အရေအတွက်ဟာ vector ရဲ့ dimension အရေအတွက်ဖြစ်တယ်။
- vector ရဲ့ မြှားပါတဲ့ဘက်ခြမ်းသည် head ဖြစ်ပြီး ဆန့်ကျင်ဘက်က tail ဖြစ်တယ်။
- (0,0) က standard position, origin ဖြစ်တယ်။

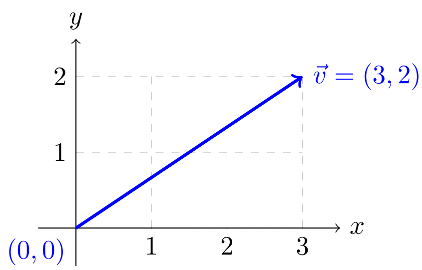

Figure 1.1: 2D vector, $\vec{v} = (3, 2) \in R^2$

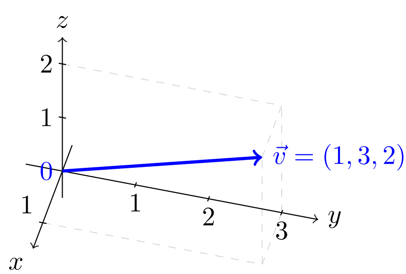

Figure 1.2: 3D vector $\vec{v} = (1, 3, 2) \in R^3$

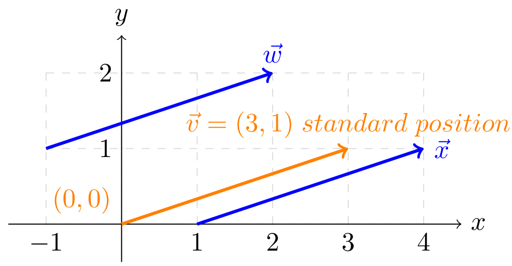

Figure 1.3: standard position vector vs non-standard position vector

1.1.2 Types of vector

1. zero vector ($\vec{0}$) - magnitude သုည, direction မရှိတဲ့ vector
   - In $\in \mathbb{R}^3$, $\vec{0}$=(0,0,0)
2. unit vector ($\hat{u}$ or u) - direction ကိုပြဖို့အတွက်ဖြစ်ပြီး ဖော်ပြချင်တဲ့ directionမှာ magnitude 1 ရှိတယ်။
   - In $\in \mathbb{R}^3$, $\hat{i}$=(1,0,0), $\hat{j}$=(0,1,0), $\hat{k}$=(0,0,1)
3. position vector - standard position ကစပြီး point တစ်ခုညွှန်ပြတဲ့ vector ကို position vector ဖြစ်တယ်။
4. standard basic vector
5. normal vector
6. dispacement vector
7. velocity vector
8. acceleration vector
9. force vector
10. tagent vector
11. grardient vector

1.1.3 position vector သို့ပြောင်းခြင်း

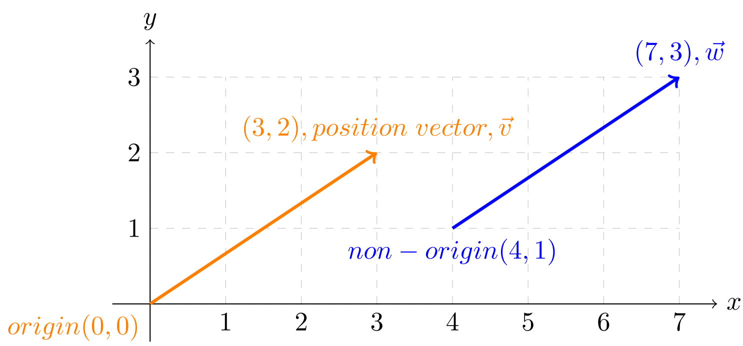

Figure 1.4: $\vec{w}$ ပုံစံကနေ $\vec{v}$ သို့ ပြောင်းလဲခြင်း

$$\begin{aligned} position\ vector &= \vec{w}_{head} - \vec{w}_{tail} \\ &= (7,3) - (4,1) \\ &= (3,2) \\ &= \vec{v} \end{aligned} \tag{1.1}$$

1.2 vector addition ($\vec{v}+\vec{w}$)

1.2.1 မှတ်စု

- vector $\vec{v}, \vec{w}$ နှစ်ခုလုံး standard position မှာရှိရမယ်။
- standard position မဟုတ်လျှင် standard position ပြောင်းပြီးမှပေါင်းရမည်။

1.2.2 Definition

$\vec{v} = (v_1, v_2, \dots, v_n) \in \mathbb{R}^n$ နှင့် $\vec{w} = (w_1, w_2, \dots, w_n) \in \mathbb{R}^n$ ဖြစ်လျှင် $\vec{v}+\vec{w}$ က

$$\vec{v}+\vec{w} \overset{def}{=} (v_1+w_1, v_2+w_2, \dots, v_n+w_n) \tag{1.2}$$

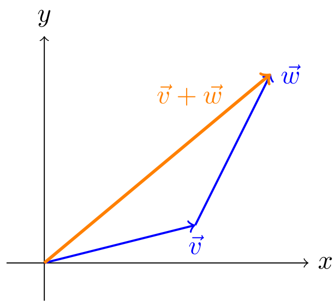

Figure 1.5: adding vector head-to-tail.

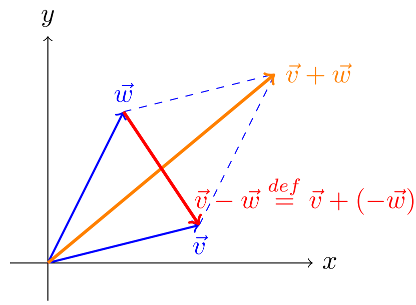

Figure 1.6: vector head-to-tail operation.

1.2.3 Theorem

$\vec{v}, \vec{w}, \vec{x} \in \mathbb{R}$ တွေဟာ vector တွေဖြစ်ခဲ့လျှင်

$$\begin{aligned} &(a) \quad \vec{v}+\vec{w} = \vec{w}+\vec{v} \quad (commutativity) \\ &(b) \quad (\vec{v}+\vec{w})+\vec{x} = \vec{v}+(\vec{w}+\vec{x}) \quad (associativity) \end{aligned} \tag{1.3}$$

1.2.4 Example

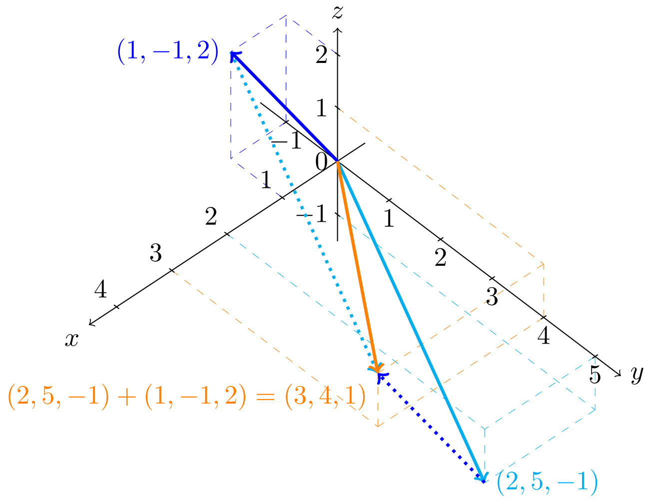

Figure 1.7: (2,5,-1)+(1,-1,2)=(3,4,1) adding head-to-tail

in 1.7,

$$\begin{aligned}
(2,5,-1)+(1,-1,2) &= (2+1, 5-1, -1+2) \\
&= (3,4,1) && theorem 1.3(a) \\
(1,-1,2)+(2,5,-1) &= (1+2, -1+5, 2-1) \\
&= (3,4,1) && theorem 1.3(a)
\end{aligned}$$

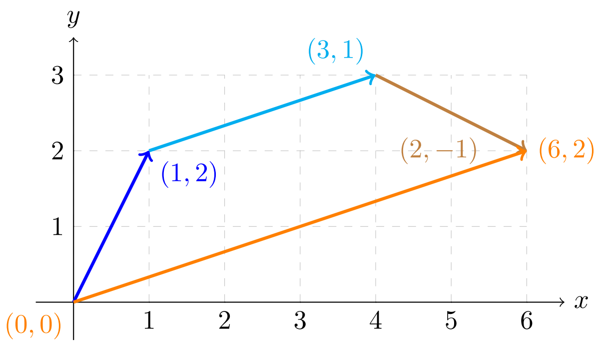

Figure 1.8: (1,2)+(3,1)+(2,-1)=(6,2)

in 1.8,

$$\begin{aligned}
(1,2)+(3,1)+(2,-1) &= (1+3+2, 2+1-1) \\
&= (6,2) \\
((1,2)+(3,1))+(2,-1) &= (1+3, 2+1)+(2,-1) \\
&= (4,3)+(2,-1) \\
&= (4+2, 3-1) \\
&= (6,2) && theorem 1.3(b) \\
(1,2)+((3,1)+(2,-1)) &= (1,2)+(3+2, 1-1) \\
&= (1,2)+(5,0) \\
&= (1+5, 2+0) \\
&= (6,2) && theorem 1.3(b)
\end{aligned}$$

1.3 scalar multiplication (c$\vec{v}$)

1.3.1 မှတ်စု

- $|c| > 1$ ဖြစ်လျှင် $\vec{v}$ သည် stretch ဖြစ်မည်။
- $|c| < 1$ ဖြစ်လျှင် $\vec{v}$ သည် shrink ဖြစ်မည်။
- $c < 0$ ဖြစ်လျှင် $\vec{v}$ ရဲ့ direction ကပြောင်းပြန်ဖြစ်သွားမည်။

1.3.2 Definition

$$c\vec{v} \stackrel{def}{=} (cv_1, cv_2, \ldots, cv_n) \tag{1.4}$$

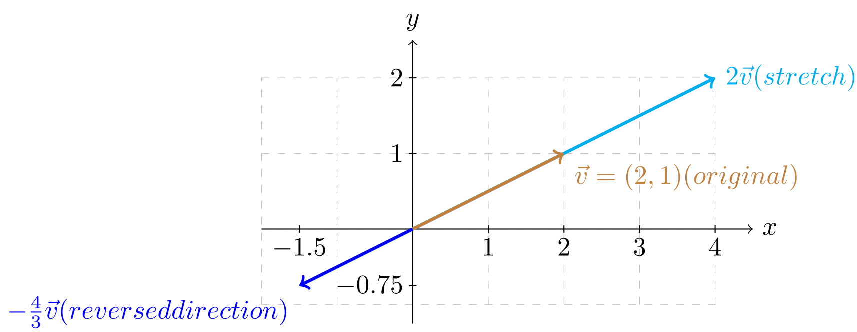

Figure 1.9: scalar multiplication

1.3.3 Theorem

$\vec{v}, \vec{w} \in \mathbb{R}^n$ များသည် vectors, $c, d \in \mathbb{R}$ များသည် scalars များဖြစ်သည်။

$$\begin{aligned} (a)\quad & c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w} \\ (b)\quad & (c + d)\vec{v} = c\vec{v} + d\vec{v} \\ (c)\quad & c(d\vec{v}) = (cd)\vec{v} \end{aligned} \tag{1.5}$$

1.3.4 Example

In 1.10, $\vec{v} = (2, 1, -1)$, $\vec{w} = (-1, 0, 3)$, $3\vec{v} - 2\vec{w} =?$

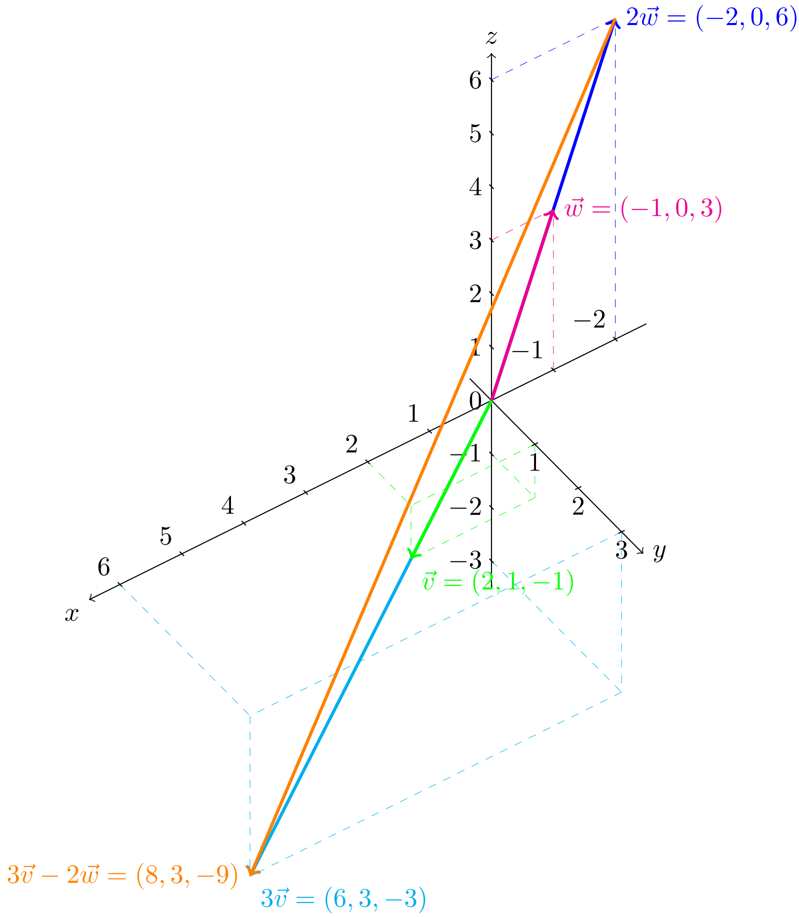

Figure 1.10: $3\vec{v} - 2\vec{w}$

$$\begin{aligned} 3\vec{v} - 2\vec{w} &= 3(2,1,-1) - 2(-1,0,3) \\ &= (6,3,-3) - (-2,0,6) \\ &= (6+2, 3-0, -3-6) \\ &= (8,3,-9) \end{aligned}$$

ပုံ1.11 မှာ,hexagon ရဲ့ (0,0) ကနေ သူရဲ့ထောင့်တွေဆီကို သွားတဲ့ vecotr 6 ခု ကိုပေါင်းရင်, vector တစ်ခုကတော့ (1,0)

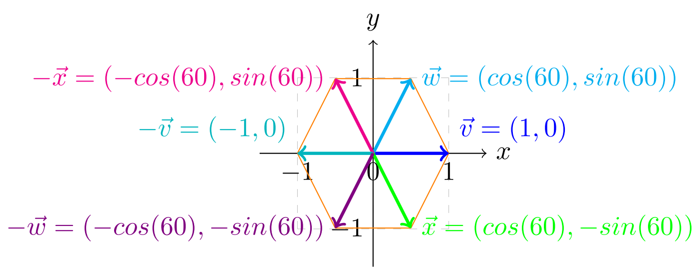

Figure 1.11: vector to corners of hexagon

$$\vec{v} + \vec{w} + \vec{x} + (-\vec{v}) + (-\vec{w}) + (-\vec{x}) = 0$$

equation တွေကနေ$\vec{x}$ ကိုရှာခြင်း,

$$
\begin{aligned}
(a) \qquad \vec{x} - (3,2,1) &= (1,2,3) - 3\vec{x} \\
\vec{x} &= (3,2,1) + (1,2,3) - 3\vec{x} \\
\vec{x} + 3\vec{x} &= (3+1, 2+2, 1+3) \\
4\vec{x} &= (4,4,4) \\
\vec{x} &= \frac{1}{4}(4,4,4) \\
\vec{x} &= (1,1,1)
\end{aligned}
$$

$$
\begin{aligned}
(b) \qquad \vec{x} + 2(\vec{v} + \vec{w}) &= -\vec{v} - 3(\vec{x} - \vec{w}) \\
x + 2\vec{v} + 2\vec{w} &= -\vec{v} - 3\vec{x} + 3\vec{w} \\
4\vec{x} &= -3\vec{v} + \vec{w} \\
\vec{x} &= \frac{1}{4}(3\vec{v} + \vec{w})
\end{aligned}
$$

1.4 linear combination

1.4.1 မှတ်စု

- $\vec{v} = (v_1, v_2, \dots, v_n)$ ထဲက $\vec{v}_1, \vec{v}_2, \vec{v}_n$ တွေက တစ်ခုနဲ့တစ်ခု သီးသန့်ဖြစ်နေတဲ့ vector space များဖြစ်တယ်။

1.4.2 Definition

$c_1, c_2, \dots, c_k \in \mathbb{R}$ ဖြစ်ပြီး $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k \in \mathbb{R}^n$ ပုံစံရှိတဲ့ vector ရဲ့ linear combination ကိုရှာချင်လျှင်,

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k \tag{1.6}$$

1.4.3 Example

$\vec{v} = (1,2,3)$ ဟာ $\vec{v}_1 = (1,1,1)$ နဲ့ $\vec{v}_2 = (-1,0,1)$ တို့ linear combination လုပ်ထားတဲ့ vector ဖြစ်လားတွက်ချင်ရင်, definition အရ $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2$ ဖြစ်တယ်။

$$
\begin{aligned}
(1,2,3) &= c_1(1,1,1) + c_2(-1,0,1) \\
(1,2,3) &= (c_1 - c_2, c_1 + 0, c_1 + c_2) \\
(1,2,3) &= (c_1 - c_2, c_1, c_1 + c_2)
\end{aligned}
$$

vector space တစ်ခုချင်းစီကိုညီလိုက်မယ်ဆိုလျှင်

$$
\begin{aligned}
(eq1) \qquad c_1 - c_2 &= 1 \\
(eq2) \qquad c_1 &= 2 \\
(eq3) \qquad c_1 + c_2 &= 3
\end{aligned}
$$

eq2 အရ c_1သည် 2 ဖြစ်ပြီး eq1 မှ c_2 ကိုရှာသော $2-c_2 = 1, c_2 = 1$ ဖြစ်တယ်။ eq3 တွင် အစားသွင်းကြည့်လျှင် $2 + 1 = 3$ သည်မှန်ကန်သည်။ ထို့ကြောင့် (1,1,1) နှင့် (-1,0,1) တို့၏ linear combination သည် (1,2,3) ဖြစ်သည်။

(1,2,3) ဟာ (1,1,0) နှင့် (2,1,0) တို့ရဲ့ linear combination ဖြစ်သလား။

$$\begin{aligned} (eq1) \quad & c_1 + 2c_2 = 1 \\ (eq2) \quad & c_1 + c_2 = 2 \\ (eq3) \quad & 0 \neq 3 \end{aligned}$$

eq3 အရ linear combination မဖြစ်နိုင်ပါ။

1.4.4 Standard Basic Vector

entries တွေက entry တစ်ခုသာ 1ဖြစ်နေပြီး ကျန် entries တွေက သုညဖြစ်နေလျှင် အောက်ပါအတိုင်းဖော်ပြလို့ရတယ်။ $j = 1, 2, \ldots, n$ ဖြစ်ပြီး $e_j \in \mathbb{R}_n$ ကိုဖော်ပြလျှင်

$$e_j \overset{def}{=} (0, 0, \ldots, 0, \underbrace{1}_{j-th \text{ entry}}, 0, \ldots, 0) \tag{1.7}$$

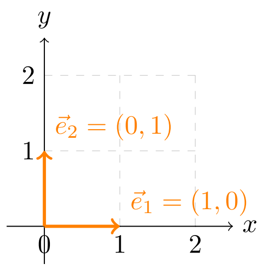

Figure 1.12: $\mathbb{R}^2$ vector

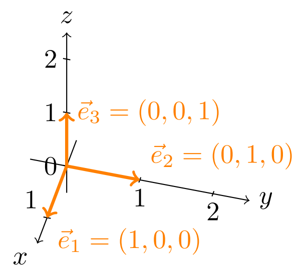

Figure 1.13: $\mathbb{R}_3$ vector

standard basic vector ရဲ့ linear combination ကို 1.6 အတိုင်းတွက်လျှင်, $\vec{v} \in \mathbb{R}^n, \vec{v} = (\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n)$ဖြစ်လျှင်

$$\vec{v} = \vec{v}_1 e_1 + \vec{v}_2 e_2 + \cdots + \vec{v}_n e_n \tag{1.8}$$

1.4.5 Example

$3\vec{e}_1 - 2\vec{e}_2 + \vec{e}_3 \in \mathbb{R}^3$ ၏ linear combination ကိုရှာလျှင်, 3D standard basic vector ဖြစ်တဲ့ အတွက် $\vec{e}_1 = (1, 0, 0), \vec{e}_2 = (0, 1, 0), \vec{e}_3 = (0, 0, 1)$ ဖြစ်တယ်။

$$\begin{aligned} 3\vec{e}_1 - 2\vec{e}_2 + \vec{e}_3 &= 3(1, 0, 0) - 2(0, 1, 0) + (0, 0, 1) \\ &= (3, 0, 0) - (0, 2, 0) + (0, 0, 1) \\ &= (3 + 0 + 0, 0 - 2 + 0, 0 - 0 + 1) \\ &= (3, -2, 1) \end{aligned}$$

$(3, 5 - 2, -1)$ ကို $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4 \in \mathbb{R}^4$ ၏ linear combination ပုံစံဖြင့် $(3, 5, -2, -1) = 3\vec{e}_1 + 5\vec{e}_2 - 2\vec{e}_3 - \vec{e}_4$ အတိုင်း ရေးသည်။

1.5 Dot Product $\vec{v} \cdot \vec{w}$

1.5.1 မှတ်စု

- dimension တူတဲ့ vector အချင်းချင်းသာ dot product ရှာလို့ရတယ်။
- dot product ရဲ့ရလဒ်က vector မဟုတ်ပဲ number ဖြစ်တယ်။
- vector နှစ်ခုရဲ့ dot product သာရှိနိုင်တယ်။
- $\vec{v} \cdot (\vec{w} \cdot \vec{x})$ တွင် $\vec{w} \cdot \vec{x}$ သည် number ဖြစ်တဲ့အတွက် $\vec{v}\cdot$number ကို dot produt လုပ်မရပါ။

1.5.2 Definition

$$\vec{v}\cdot\vec{w} \overset{def}{=} \vec{v}_1\vec{w}_1 + \vec{v}_2\vec{w}_2 + \cdots + \vec{v}_n\vec{w}_n \tag{1.9}$$

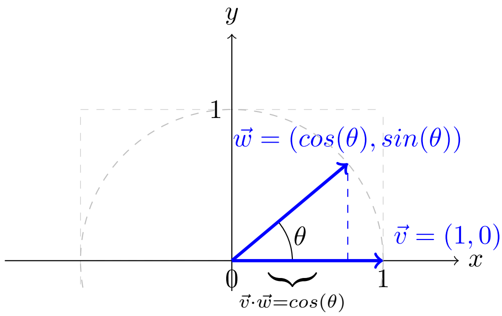

Figure 1.14: $\vec{v}\cdot\vec{w} = 1cos(\theta) + 0sin(\theta)$

1.5.3 Theorem

$$\begin{aligned} (a) \quad & \vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{v} \\ (b) \quad & \vec{v}\cdot(\vec{w}+\vec{x}) = \vec{v}\cdot\vec{w} + \vec{v}\cdot\vec{x} \\ (c) \quad & \vec{v}\cdot(c\vec{w}) = c(\vec{v}\cdot\vec{w}) \end{aligned} \tag{1.10}$$

1.5.4 Example

$(1, 2, 3)$နှင့်$(4, -3, 2)$ တို့၏ dot product ကိုရှာလျှင်

$$\begin{aligned} (1,2,3)\cdot(4,-3,2) &= 1\cdot 4 + 2\cdot(-3) + 3\cdot 2 \\ &= 4 - 6 + 6 \\ &= 4 \end{aligned}$$

$(1, 2) \in \mathbb{R}^2$နှင့်$(2, 5, 3) \in \mathbb{R}^3$တို့သည် dimension မတူသောကြောင့် dot product ရှာလို့မရပါ။

1.6 Vector Length $\|v\|$

1.6.1 မှတ်စု

- vector ရဲ့ tail က standard basic point ဖြစ်နေမှသာတွက်လို့ရမယ်။
- vector က origin ကမစလျှင် eq1.1 အတိုင်းပြောင်းပါ။
- (v_1, v_2) ရဲ့ တစ်စိတ်တစ်ပိုင်း component v_1 ကို $|v_1|$ ပုံစံဖော်ပြနိုင်တယ်။

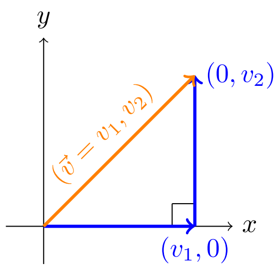

Figure 1.15: a vector $\vec{v} \in \mathbb{R}^2$

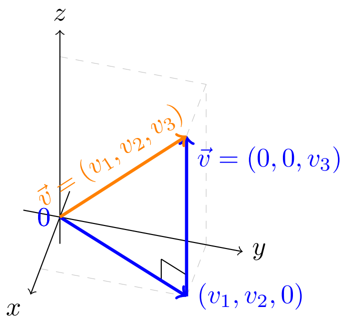

Figure 1.16: a vector $\vec{v} \in \mathbb{R}^3$

In Fig 1.15,$\vec{v} = (v_1, v_2) \in \mathbb{R}^2$ ကို $\vec{v} = (v_1, 0) + (0, v_2)$ ကနေတွက်ထုတ်ထားတာဖြစ်တယ်။

$$\begin{aligned}
\|\vec{v}\| &= \sqrt{\|(v_1, 0)\|^2 + \|(0, v_2)\|^2} \\
&= \sqrt{|v_1|^2 + \|v_2|^2} \\
&= \sqrt{v_1^2 + v_2^2} \\
&= \sqrt{(v_1, v_2) \cdot (v_1, v_2)} \\
&= \sqrt{\vec{v} \cdot \vec{v}}
\end{aligned}$$

In Fig 1.16,$\vec{v} = (v_1, v_2, v_3) \in \mathbb{R}^3$ ကို $\vec{v} = (v_1, v_2, 0) + (0, 0, v_3)$ ကနေတွက်ထုတ်ထားတာဖြစ်တယ်။

$$\begin{aligned}
\|\vec{v}\| &= \sqrt{\|(v_1, v_2, 0)\|^2 + \|(0, 0, v_3)\|^2} \\
&= \sqrt{(\sqrt{v_1^2 + v_2^2})^2 + \|v_3\|^2} \\
&= \sqrt{v_1^2 + v_2^2 + v_3^2} \\
&= \sqrt{(v_1, v_2, v_3) \cdot (v_1, v_2, v_3)} \\
&= \sqrt{\vec{v} \cdot \vec{v}}
\end{aligned}$$

1.6.2 Definition

$\vec{v} = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$ ၏ length ကိုရှာချင်ရင်

$$\|\vec{v}\| \overset{def}{=} \sqrt{\vec{v} \cdots \vec{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2} \tag{1.11}$$

1.6.3 Example

(2,-5,4,6) ၏ length ကိုရှာလျှင်

$$\begin{aligned}
\|(2, -5, 4, 6)\| &= \sqrt{2^2 + (-5)^2 + 4^2 + 6^2} \\
&= \sqrt{81} \\
&= 9
\end{aligned}$$

$(cos(\theta), sin(\theta))$ ၏ length ကိုရှာလျှင်

$$\begin{aligned}
\|(cos(\theta), sin(\theta))\| &= \sqrt{cos^2(\theta) + sin^2(\theta)} \\
&= \sqrt{1} \\
&= 1
\end{aligned}$$

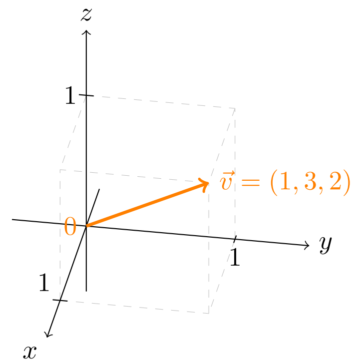

Figure 1.17: 3D vector $\vec{v} = (1, 3, 2) \in R^3$

$$\|(1,1,1)\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

1.6.4 Theorem

$\vec{v} \in \mathbb{R}^n$, $c \in \mathbb{R}$ ဖြစ်လျှင်

$$\begin{array}{ll} (a) & \|c\vec{v}\| = |c| \cdot \|\vec{v}\| \\ (b) & \|\vec{v}\| > 0, \text{with equality if and only if } \vec{v} = 0 \end{array} \tag{1.12}$$

1.6.5 Unit Vector ($\hat{u} = 1$)

unit vector $\hat{u}$ ရဲ့ length ကတော့ 1ရှိပါတယ်။ တစ်ခြား vector တွေကနေ unit vector ဖြစ်အောင် scaling လုပ်တာကို normalization လုပ်ခြင်းလို့ခေါ်ပါတယ်။ direction က မူရင်း vector ၏ direction အတိုင်းသာဖြစ်တယ်။ $\hat{u} \in \mathbb{R}^2$ ကတော့ unit circle ဖြစ်ပြီး $\hat{u} \in \mathbb{R}^3$ ကတော့ unit sphere ဖြစ်တယ်။ $\vec{v} \in \mathbb{R}^n$ဖြစ်လျှင်

$$\begin{array}{ll} (a) & \hat{u} = \dfrac{\vec{v}}{\|\vec{v}\|} \\ (b) & \|\hat{u}\| = 1 \end{array} \tag{1.13}$$

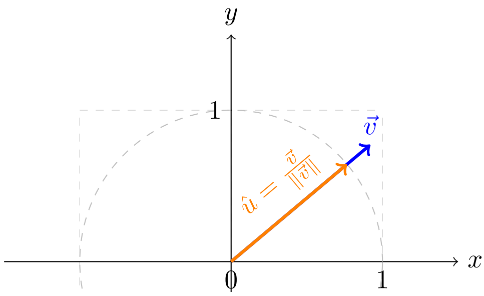

Figure 1.18: renormalizing a vector $\vec{v} \in \mathbb{R}^2$

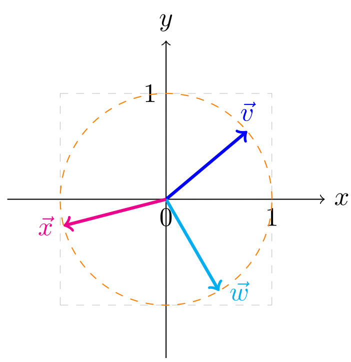

Figure 1.19: $(\|\vec{v}\| = \|\vec{w}\| = \|\vec{x}\| = 1) \in \mathbb{R}^2$

$\vec{v} = (3, 4)$ ကို unit vector သို့ scaling လုပ်လျှင်,

$$\begin{aligned}
\hat{u} \quad of \quad \vec{v} &= \frac{\vec{v}}{\|\vec{v}\|} \\
&= \frac{(3,4)}{\sqrt{3^2+4^2}} \\
&= \frac{(3,4)}{5} \\
&= (\frac{3}{5}, \frac{4}{5}) \\
\|\hat{u}\| \quad of \quad \vec{v} &= \sqrt{(\frac{3}{5})^2 + (\frac{4}{5})^2} \\
&= \sqrt{\frac{9}{25} + \frac{16}{25}} \\
&= \sqrt{\frac{25}{25}} \\
&= \sqrt{1} \\
&= 1
\end{aligned}$$

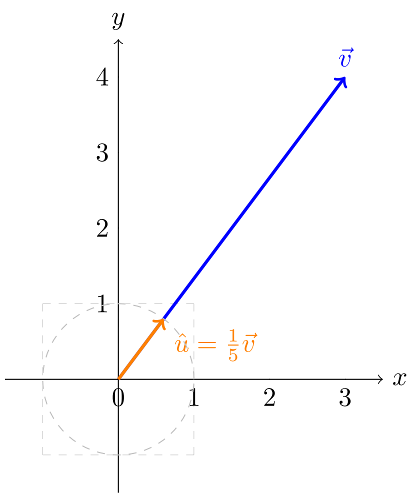

Figure 1.20: renormalizing a vector $\vec{v} \in \mathbb{R}^2$

1.6.6 Cauchy-Schwarz Inequality

$\vec{v} \in \mathbb{R}^n$ နှင့် $\vec{w} \in \mathbb{R}^n$ တို့သည် linearly dependent ဖြစ်နေမှသာလျှင် Cauchy Inequality ကသုံးလို့ရတယ်။ $\vec{v}$ ရဲ့ head က $\vec{w}$ ၏ tail မှာဆက်နေလျှင်, or, $\vec{w}$ ရဲ့ head က $\vec{v}$ ၏ tail မှာဆက်နေမှသာ Cauchy Inequality ကမှန်ကန်တယ်။

$$|\vec{v} \cdot \vec{w}| \leq \|\vec{v}\| \|\vec{w}\| \tag{1.14}$$

$\vec{v} = (1,2), \vec{w} = (3,4),$

$$\begin{aligned}
\vec{v} \cdot \vec{w} &= (1,2) \cdot (3,4) \\
&= 3 + 8 \\
&= 11 \\
|\vec{v} \cdot \vec{w}| &= |11| \\
&= 11 \\
\|\vec{v}\| &= \sqrt{1^2 + 2^2} \\
&= \sqrt{5} \\
\|\vec{w}\| &= \sqrt{3^2 + 4^2} \\
&= \sqrt{25} \\
&= 5 \\
\|\vec{v}\|\|\vec{w}\| &= 5\sqrt{5} \\
&\approx 11.028 \\
|\vec{v} \cdot \vec{w}| &\leq \|\vec{v}\|\|\vec{w}\|
\end{aligned}$$

1.6.7 Triangle Inequality

တြိဂံတစ်ခု၏ အနားတစ်ခုသည် ကျန်အနားနှစ်ခု ပေါင်းခြင်းထက်ငယ် သို့မဟုတ် ပေါင်းခြင်းနှင့်ညီနိုင်သည်။

$$\begin{aligned}
\vec{x} &= \vec{v} \cdot \vec{w} \\
\|\vec{x}\| &\leq \|\vec{v}\| + \|\vec{w}\| \qquad (1.15) \\
\|\vec{v} \cdot \vec{w}\| &\leq \|\vec{v}\| + \|\vec{w}\|
\end{aligned}$$

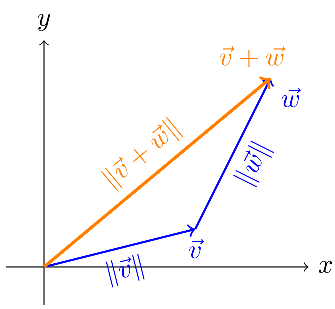

Figure 1.21: $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$

1.6.8 The Angle between Vector

$\vec{v}, \vec{w} \in \mathbb{R}^n$ နှစ်ခုကြားက angle θ ကိုရှာချင်လျှင်, Figure1.6အရ $\vec{v}$ နှင့် $\vec{w}$ ကိုဆက်ထားတဲ့ vector လိုအပ်တဲ့အတွက် vector substraction လုပ်မှသာ ရနိုင်တယ်။ ချိတ်ဆက်ထားတဲ့ vector သည် $\vec{v} - \vec{w}$ ဖြစ်လာသည်။

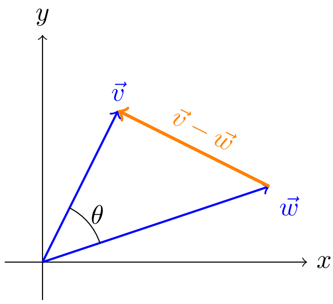

Figure 1.22: $\vec{v}, \vec{w}, and\ \vec{v}-\vec{w} \in \mathbb{R}^2$

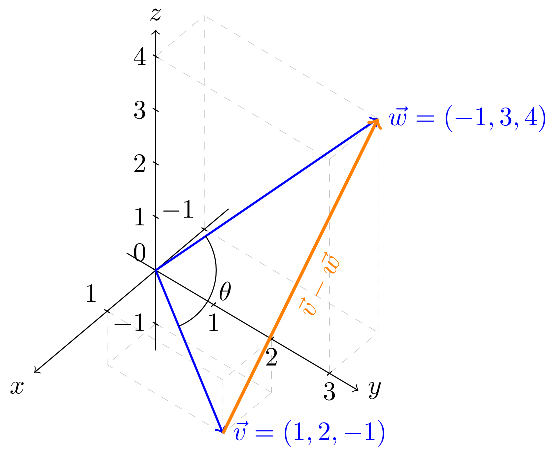

Figure 1.23: $\vec{v}, \vec{w}, and\ \vec{v}-\vec{w} \in \mathbb{R}^3$

law of cosines အရ $\theta, \vec{v}, \vec{w}, \vec{v}-\vec{w}$ တို့ကို

1.6.9 Definition

$$
\begin{aligned}
\|\vec{v}-\vec{w}\|^2 &= \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\| \cos(\theta) \\
\|\vec{v}-\vec{w}\|^2 &= (\sqrt{(\vec{v}-\vec{w})^2})^2 \\
&= (\vec{v}-\vec{w})^2 \\
&= (\vec{v}-\vec{w}) \cdot (\vec{v}-\vec{w}) \\
&= \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{w} - \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{w} \\
&= \vec{v}^2 - 2(\vec{v}\cdot\vec{w}) + \vec{w}^2 \\
&= \|\vec{v}\|^2 - 2(\vec{v}\cdot\vec{w}) + \|\vec{w}\|^2 \\
\|\vec{v}\|^2 - 2(\vec{v}\cdot\vec{w}) + \|\vec{w}\|^2 &= \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\| \cos(\theta) \\
\vec{v}\cdot\vec{w} &= \|\vec{v}\|\|\vec{w}\| \cos(\theta) \\
cos(\theta) &= \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|} \\
\theta &= cos^{-1}(\frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|})
\end{aligned}
\tag{1.16}
$$

1.6.10 Example

$\vec{v} = (1, 2), \vec{w} = (3, 4)$ တို့ကြားက angle θ ကိုရှာလျှင်

$$
\begin{aligned}
\theta &= cos^{-1}(\frac{(1,2)\cdot(3,4)}{\|(1,2)\|\|(3,4)\|}) \\
&= cos^{-1}(\frac{9}{5\sqrt{5}}) \\
&\approx 0.1799 radian \\
&\approx 10.30^\circ
\end{aligned}
$$

$\vec{v} = (0,1), \vec{w} = (3,0)$ တို့ကြားက angle θ ကိုရှာလျှင်

$$\begin{aligned}
\theta &= cos^{-1}(\frac{(0,1)\cdot(3,0)}{\|(0,1)\|\|(3,0)\|}) \\
&= cos^{-1}(\frac{0}{3}) \\
&= cos^{-1}(0) \\
&= 1.570796326794897 radian \\
&= 90.00000000000004^{\circ}
\end{aligned}$$

1.7 Cross Product $\vec{v} \times \vec{w}$

$\vec{v} = (v_1, v_2, v_3), \vec{w} = (w_1, w_2, w_3)$

$$\vec{v} \times \vec{w} = (v_2 w_3 - v_3 w_2 + v_3 w_1 - v_1 w_3 + v_1 w_2 - v_2 w_1) \tag{1.17}$$

Chapter 2

Matrix & Matrix operation

2.1 Matrix ($\mathbb{A}$)

2.1.1 မှတ်စု

- matrix ရဲ့ row တိုင်းသည်တူညီသော entry အရေအတွက်ရှိရမည်။
- matrix ရဲ့ column တိုင်းသည်တူညီသော entry အရေအတွက်ရှိရမည်။
- matrix $\mathbb{A}$ ၏ entry တွေကို $a_{i,j}$ ပုံစံဖြင့်ဖော်ပြတယ်။
- row ကို m ဖြင့်ဖော်ပြပြီး dimension $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^n$ ကိုကိုယ်စားပြုတယ်။
- column ကို n ဖြင့်ဖော်ပြပြီး coordiante တစ်ခုချင်းစီကို ကိုယ်စားပြုသည်။

$\mathbb{A} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ နှင့် $\mathbb{B} = \begin{bmatrix} 3 & 0 & 2 \\ 0 & -1 & 1 \end{bmatrix}$ မှာ $\mathbb{A}$ ကတော့ 2×2 matrix၊ $\mathbb{B}$ ကတော့ 2×3 ဖြစ်တယ်။

$\mathbb{A} \in \mathbb{M}_2$ ကို $\mathbb{A} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \in \mathbb{M}_n$ ပုံစံဖြင့်ဖော်ပြတယ်

$\mathbb{B} \in \mathbb{M}_{2,3}$ ကို $\mathbb{B} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} \in \mathbb{M}_{m,n}$ ပုံစံဖြင့်ဖော်ပြတယ်

2.2 Matrix addition ($\mathbb{A} + \mathbb{B}$)

2.2.1 Definition

$\mathbb{A}, \mathbb{B} \in \mathbb{M}_{m,n}$သည် matrix, $c \in \mathbb{R}$သည် scalar, $1 \leq i \leq m, 1 \leq j \leq n$ ဖြစ်လျှင်

$$\begin{aligned} (a) \quad & [\mathbb{A} + \mathbb{B}]_{i,j} = a_{i,j} + b_{i,j} \\ (b) \quad & [cA]_{i,j} = ca_{i,j} \end{aligned} \tag{2.1}$$

$\mathbb{A}, \mathbb{B} \in \mathbb{M}_{m,n}$ ဖြစ်လျှင်,

$$\mathbb{A}+\mathbb{B} = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \dots & a_{m,n} \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{1,2} & \dots & b_{1,n} \\ b_{2,1} & b_{2,2} & \dots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m,1} & b_{m,2} & \dots & b_{m,n} \end{bmatrix}$$
$$= \begin{bmatrix} a_{1,1}+b_{1,1} & a_{1,2}+b_{1,2} & \dots & a_{1,n}+b_{1,n} \\ a_{2,1}+b_{2,1} & a_{2,2}+b_{2,2} & \dots & a_{2,n}+b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1}+b_{m,1} & a_{m,2}+b_{m,2} & \dots & a_{m,n}+b_{m,n} \end{bmatrix}$$

2.2.2 Theorem

$\mathbb{A}, \mathbb{B}, \mathbb{C} \in \mathbb{M}_{m,n}$ နှင့် $c, d \in \mathbb{R}$ ဖြစ်လျှင်

$$\begin{aligned} &(a) & \mathbb{A}+\mathbb{B} &= \mathbb{B}+\mathbb{A} \\ &(b) & (\mathbb{A}+\mathbb{B})+\mathbb{C} &= \mathbb{A}+(\mathbb{B}+\mathbb{C}) \\ &(c) & c(\mathbb{A}+\mathbb{B}) &= c\mathbb{A}+c\mathbb{B} \\ &(d) & (c+d)\mathbb{A} &= c\mathbb{A}+d\mathbb{A} \\ &(e) & c(d\mathbb{A}) &= (cd)\mathbb{A} \end{aligned} \tag{2.2}$$

2.2.3 Example

$\mathbb{A} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}, \mathbb{B} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbb{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ ဖြစ်လျှင်,

$\mathbb{A}+\mathbb{B}$ ကိုရှာလျှင်

$$\mathbb{A}+\mathbb{B} = \begin{bmatrix} 1+2 & 3+1 \\ 2+0 & -1+1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 4 \\ 2 & 0 \end{bmatrix}$$

$2\mathbb{A}-3\mathbb{B}$ ကိုရှာလျှင်

$$2\mathbb{A}-3\mathbb{B} = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 2-6 & 6-3 \\ 4-0 & -2-3 \end{bmatrix}$$
$$= \begin{bmatrix} -4 & 3 \\ 4 & -5 \end{bmatrix}$$

2.3 Matrix Multiplication ($\mathbb{A}\mathbb{B}$)

2.3.1 မှတ်စု

- $dimension^m$ နှင့် $coordinate^n$ အတွက် $m \times n$ ဖြစ်တဲ့ $\mathbb{A} \in \mathbb{M}_{m,n}$
- $dimension^n$ နှင့် $coordinate^p$ အတွက် $n \times p$ ဖြစ်တဲ့ $\mathbb{B} \in \mathbb{M}_{n,p}$
- $m \times n$ နှင့် $n \times p$ တွင် n နှင့် n တူနေမှသာ product ရှာလို့ရတယ်။

2.3.2 Definition

$\mathbb{A} \in \mathbb{M}_{m,n}$ နှင့် $\mathbb{B} \in \mathbb{M}_{n,p}$ ၏ product $\mathbb{AB}$ သည် $m \times p$ ဖြစ်လာတယ်။ $m \times p$ matrix ၏ (i,j) entry တွေသည် $1 \leq i \leq m$ ဖြစ်ပြီး $1 \leq j \leq p$ ဖြစ်ပြီး $[\mathbb{AB}]_{i,j}$နေရာမှာရှိတဲ့ တန်ဖိုးကိုရှာလိုလျှင်,

$$[AB]_{i,j} \stackrel{def}{=} a_{i,1}b_{1,j} + a_{i,2}b_{2,j} + \cdots + a_{i,n}b_{n,j} \tag{2.3}$$

$$\mathbb{A} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} \in \mathbb{M}_{3,2} \text{ နှင့် } \mathbb{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \end{bmatrix} \in \mathbb{M}_{2,4} \text{ ဖြစ်လျှင်,}$$

$$\begin{aligned} \mathbb{AB} &= \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix}_{3\times \overset{\text{တူ}}{2}} \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \end{bmatrix}_{\overset{\text{တူ}}{2}\times 4} \\ &= \begin{bmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} & a_{1,1}b_{1,3} + a_{1,2}b_{2,3} & a_{1,1}b_{1,4} + a_{1,2}b_{2,4} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} & a_{2,1}b_{1,3} + a_{2,2}b_{2,3} & a_{2,1}b_{1,4} + a_{1,2}b_{2,4} \\ a_{3,1}b_{1,1} + a_{3,2}b_{2,1} & a_{3,1}b_{1,2} + a_{3,2}b_{2,2} & a_{3,1}b_{1,3} + a_{3,2}b_{2,3} & a_{3,1}b_{1,4} + a_{1,2}b_{2,4} \end{bmatrix}_{3\times 4} \end{aligned}$$

2.3.3 Theorem

$\mathbb{A}, \mathbb{B}, \mathbb{C}$ တွေက matrix တွေဖြစ်ပြီး $c \in \mathbb{R}$ ဖြစ်လျှင်,

$$\begin{aligned} (a) &\quad (\mathbb{AB})\mathbb{C} = \mathbb{A}(\mathbb{BC}) \\ (b) &\quad \mathbb{A}(\mathbb{B} + \mathbb{C}) = \mathbb{AB} + \mathbb{AC} \\ (c) &\quad (\mathbb{A} + \mathbb{B})\mathbb{C} = \mathbb{AC} + \mathbb{BC} \\ (d) &\quad c(\mathbb{A} + \mathbb{B}) = c\mathbb{A} + c\mathbb{B} \end{aligned} \tag{2.4}$$

2.3.4 Example

$$\mathbb{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}, \text{ and } \mathbb{C} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & -1 \end{bmatrix}$$

$\mathbb{AB}$ကိုရှာလျှင်,

$$
\begin{aligned}
\mathbb{AB} &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times \overset{\text{တူ}}{2}} \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}_{\overset{\text{တူ}}{2}\times 3} \\
&= \begin{bmatrix} 1\times 5+2\times 8 & 1\times 6+2\times 9 & 1\times 7+2\times 10 \\ 3\times 5+4\times 8 & 3\times 6+4\times 9 & 3\times 7+4\times 10 \end{bmatrix}_{2\times 3} \\
&= \begin{bmatrix} 21 & 24 & 27 \\ 47 & 54 & 61 \end{bmatrix}
\end{aligned}
$$

$\mathbb{AC}$ကိုရှာလျှင်,

$$
\mathbb{AC} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times \overset{\text{မတူ}}{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & -1 \end{bmatrix}_{\overset{\text{မတူ}}{3}\times 2} = \text{2 နှင့် 3 မတူသောကြောင့်ရှာလို့မရပါ}
$$

$\mathbb{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \mathbb{B} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ဖြစ်လျှင်

$$
\begin{array}{ccc}
\mathbb{AB} & and & \mathbb{BA} \\
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & and & \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\
\begin{bmatrix} 1\times 1+1\times 1 & 1\times 0+1\times 1 \\ 0\times 1+1\times 1 & 0\times 0+1\times 1 \end{bmatrix} & and & \begin{bmatrix} 1\times 1+0\times 0 & 0\times 1+1\times 1 \\ 1\times 1+1\times 0 & 1\times 1+1\times 1 \end{bmatrix} \\
\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} & \neq & \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \\
\mathbb{AB} & \neq & \mathbb{BA}
\end{array}
$$

2.3.5 Square & Identity Matrix

row နှင့် column တူနေရင် square matrix ဖြစ်တယ်။ Identity Matrix နှင့်မြှောက်ရင် မြှောက်မယ့် Matrix ဘာမှမပြောင်းလဲသွားဘူး။ Identity matrix သည်main diagonal သည် 1ဖြစ်တဲ့ square matrix ဖြစ်တယ်။

$$
\mathbb{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbb{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

2.3.6 Theorem

$\mathbb{A} \in \mathbb{M}_{m,n}$ဖြစ်လျှင်

$$
\begin{aligned}
&(a) \quad \mathbb{AI}_n = \mathbb{A} = \mathbb{I}_m\mathbb{A} \\
&\qquad \mathbb{A}^2 = \mathbb{AA} \\
&\qquad \mathbb{A}^3 = \mathbb{AAA} \\
&(b) \quad \mathbb{A}^k \overset{def}{=} \underbrace{\mathbb{AA}\ldots\mathbb{A}}_{\text{k copies}}
\end{aligned} \tag{2.5}
$$

$\mathbb{A} = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$ ဖြစ်ပြီး $\mathbb{A}\mathbb{I}, \mathbb{I}\mathbb{A}$ ကိုရှာလျှင်

$$
\begin{aligned}
\mathbb{A}\mathbb{I} &= \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 3 \times 1 + 5 \times 0 & 3 \times 0 + 5 \times 1 \\ 1 \times 1 + 4 \times 0 & 1 \times 0 + 4 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \\
\mathbb{I}\mathbb{A} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 1x3 + 0x1 & 1x5 + 0x4 \\ 0x3 + 1x1 & 0x5 + 1x4 \end{bmatrix} \\
&= \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \\
\mathbb{A}\mathbb{I} &= \mathbb{I}\mathbb{A} \\
\mathbb{I}^2 &= \mathbb{I} \\
\mathbb{I}^7 &= \mathbb{I}
\end{aligned}
$$

$\mathbb{A}^2, \mathbb{A}^4$ ကိုရှာလျှင်

$$
\begin{aligned}
\mathbb{A}^2 &= \mathbb{A}\mathbb{A} \\
&= \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 3 \times 3 + 5 \times 1 & 3 \times 5 + 5 \times 4 \\ 1 \times 3 + 4 \times 1 & 1 \times 5 + 4 \times 4 \end{bmatrix} \\
&= \begin{bmatrix} 14 & 35 \\ 7 & 21 \end{bmatrix} \\
\mathbb{A}^4 &= (\mathbb{A}^2)^2 \\
&= \mathbb{A}^2\mathbb{A}^2 \\
&= \begin{bmatrix} 14 & 35 \\ 7 & 21 \end{bmatrix} \begin{bmatrix} 14 & 35 \\ 7 & 21 \end{bmatrix} \\
&= \begin{bmatrix} 441 & 1225 \\ 245 & 686 \end{bmatrix}
\end{aligned}
$$

2.4 Row Vector and Column Vector

row တစ်ခုပဲရှိတဲ့ $1 \times n$ matrix $\mathbb{A} = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$ ကို row vector $\vec{v}$ လို့ခေါ်သည်။ column တစ်ခုပဲရှိတဲ့ $m \times 1$ matrix $\mathbb{A} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ ကို column vector $\vec{w}$ လို့ခေါ်သည်။

2.5 Matrix Transpose $\mathbb{A}^T$

$\mathbb{A} \in \mathbb{M}_{m,n}$ သည် $m \times n$ matrix ဖြစ်ရင် $\mathbb{A}$ ၏ transpose ဖြစ်တဲ့ $\mathbb{A}^T$ သည် $n \times m$ ဖြစ်တယ်။ $\mathbb{A}$ ၏ entry $a_{i,j}$ သည် $\mathbb{A}^T$ ၏ entry $a_{j,i}$ ဖြစ်လာမည်။

2.5.1 Definition

$$\mathbb{A} = \begin{bmatrix} a_{i_1,j_1} & a_{i_1,j_2} & a_{i_1,j_3} \\ a_{i_2,j_1} & a_{i_2,j_2} & a_{i_2,j_3} \end{bmatrix} \Rightarrow \mathbb{A}^T = \begin{bmatrix} a_{j_1,i_1} & a_{j_1,i_2} \\ a_{j_2,i_1} & a_{j_2,i_2} \\ a_{j_3,i_1} & a_{j_3,i_2} \end{bmatrix} \tag{2.6}$$

2.5.2 Theorem

$$\begin{aligned} (a) &\quad (\mathbb{A}^T)^T = \mathbb{A} \\ (b) &\quad (\mathbb{A}+\mathbb{B})^T = \mathbb{A}^T + \mathbb{B}^T \\ (c) &\quad (\mathbb{A}\mathbb{B})^T = \mathbb{B}^T\mathbb{A}^T \\ (d) &\quad (c\mathbb{A})^T = c\mathbb{A}^T \end{aligned} \tag{2.7}$$

2.5.3 Example

$\mathbb{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \mathbb{B} = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ဖြစ်လျှင်

$$\begin{aligned} \mathbb{A}^T &= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \\ \mathbb{B}^T &= \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \\ (\mathbb{A}\mathbb{B})^T &= \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right)^T \\ &= \begin{bmatrix} -1 & 3 & 1 \\ -3 & 7 & 3 \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & -3 \\ 3 & 7 \\ 1 & 3 \end{bmatrix} \\ \mathbb{B}^T\mathbb{A}^T &= \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -3 \\ 3 & 7 \\ 1 & 3 \end{bmatrix} \end{aligned}$$

$$(\vec{v})^T\vec{w} = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = v_1w_1 + v_2w_2 + \cdots + v_nw_n = \vec{v}\cdot\vec{w} \tag{2.8}$$

2.6 Block Matrix

2.6.1 မှတ်စု

- large matrix တွေကို matrix operation လုပ်ဖို့လွယ်ကူအောင်သုံးတယ်။
- large matrix ကြီးကို block လေးတွေအလိုက်ခွဲရင် submatrices တွေရလာတယ်။

2.6.2 Example

$$\mathbb{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -2 & 3 \end{bmatrix} \text{ and } \mathbb{B} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\mathbb{A} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & -2 & 3 \end{array}\right] \text{ and } \mathbb{B} = \left[\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

$$\mathbb{C}, \mathbb{D} \text{ will be } \mathbb{C} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix} \text{ and } \mathbb{D} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbb{A} = \begin{bmatrix} \mathbb{I}_3 & \mathbb{I}_3 \\ 0 & \mathbb{C} \end{bmatrix} \text{ and } \mathbb{B} = \begin{bmatrix} \mathbb{D} & 0 \\ 0 & \mathbb{D} \end{bmatrix}$$

$$\mathbb{AB} = \begin{bmatrix} \mathbb{I}_3\mathbb{D} & \mathbb{I}_3\mathbb{D} \\ 0 & \mathbb{CD} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbb{D} & \mathbb{D} \\ 0 & \mathbb{CD} \end{bmatrix}$$

$$\mathbb{CD} = \begin{bmatrix} 5 & 4 \\ -7 & 1 \end{bmatrix}$$

$$\mathbb{AB} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ -1 & 1 & -1 & 1 \\ \hline 0 & 0 & 5 & 4 \\ 0 & 0 & -7 & 1 \end{array}\right]$$

2.6.3 Theorem

$\mathbb{A} \in \mathbb{M}_{m,n}$ သည် $a_1, a_2, \ldots, a_n$ ဆိုတဲ့ column တွေရှိပြီး $\vec{v} \in \mathbb{R}^n$ သည် column vector ဖြစ်လျှင်

$$\mathbb{A}\vec{v} = \left[\begin{array}{c|c|c|c} a_1 & a_2 & \ldots & a_n \end{array}\right] \begin{bmatrix} v1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = v_1 a_1 + v_2 a_2 + \cdots + v_n a_n \tag{2.9}$$

2.7 Linear Transformation $T(\vec{v})$

2.7.1 မှတ်စု

- linear transformation မှာ rotate, stretch, shrink, reflect ဖြစ်တာတွေပါဝင်တယ်။
- vector နှင့် matrix တွေကို linear transformation လုပ်လို့ရတယ်။

Drawing pic Drawing pic

2.7.2 Definition

$e_1, e_2, \ldots, e_n$ သည် standard basic vector ဖြစ်လျှင်

$$\begin{aligned} (a) \quad & \vec{v} = v_1 e_1 + v_2 e_2 + \cdots + v_n e_n \\ (b) \quad & T(\vec{v}) = T(v_1 e_1 + v_2 e_2 + \cdots + v_n e_n) \\ & \quad = v_1 T(e_1) + v_2 T(e_2) + +v_n T(e_n) \end{aligned} \tag{2.10}$$

2.7.3 Theorem

$T : \mathbb{R}^n \to \mathbb{R}^m$ ဖြစ်တဲ့ function နှင့် linear transformation ဖြစ်လျှင်

$$\begin{aligned} (a) \quad & T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w}) \\ (b) \quad & T(c\vec{v}) = cT(\vec{v}) \end{aligned} \tag{2.11}$$

2.7.4 Example

function တွေသည် linear transformation ဖြစ်သည်ကိုရှာလျှင်,

$T(v_1, v_2) = (1 + v_1, 2 + v_2)$ function သည် linear transformation ဖြစ်သည်ကိုရှာလျှင် eq2.11 (b) အရ $v_1 = 0, v_2 = 0, c = 2$ ကို အစားထိုးကြည့်လျှင်,

$$\begin{aligned} T(c\vec{v}) &= T(c(v_1, v_2)) \\ &= T(2(0, 0)) \\ &= T(0, 0) \\ &= (1 + 0, 2 + 0) \\ &= (12, 2) \\ cT(\vec{v}) &= cT(v_1, v_2) \\ &= 2(1, 2) \\ &= (2, 4) \end{aligned}$$

$T(c\vec{v}) \neq cT(\vec{v})$ (linear transformation function မဟုတ်ပါ)

$T(v_1, v_2) = (v_1 - v_2, v_1 + v_2)$ function သည် linear transformation ဖြစ်သည်ကိုရှာလျှင် eq2.11 (b) အရ $v_1 = 1, v_2 = 1, c = 2$ ကို အစားထိုးကြည့်လျှင်,

$$\begin{aligned}
T(c\vec{v}) &= T(c(v_1 - v_2, v_1 + v_2)) \\
&= T(2(1,1)) \\
&= T(2,2) \\
&= (2-2, 2+2) \\
&= (0,4) \\
cT(\vec{v}) &= cT(v_1 - v_2, v_1 + v_2) \\
&= 2(1-1, 1+1) \\
&= 2(0,2) \\
&= (0,4) \\
T(c\vec{v}) &\neq cT(\vec{v}) \text{ (linear transformation function ဖြစ်သည်)}
\end{aligned}$$

$\mathbb{R}^2 \to \mathbb{R}^2$ linear transformation $T(e_1) = (1,1), T(e_2) = (-1,1)$ ဖြစ်ပြီး $T(2,3)$ ကိုရှာလျှင်

$$\begin{aligned}
(2,3) &= 2e_1 + 3e_2 \\
T(\vec{v}) &= T(v_1 e_1 + v_2 e_2) \\
&= T(2e_1 + 3e_2) \\
&= 2T(e_1) + 3T(e_2) \\
&= 2(1,1) + 3(-1,1) \\
&= (2,2) + (-3,3) \\
&= (-1,5)
\end{aligned}$$

2.8 Linear Transformation of Matrix

$\vec{v} = (v_1, v_2, \ldots, v_n)$ ကို Matrix ပုံစံပြောင်းရေးလျှင် column vector $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ ဖြစ်လာမည်။

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\mathbb{A}} \begin{bmatrix} 2.1 & 0.3 \\ 0.2 & 1.2 \end{bmatrix}$$

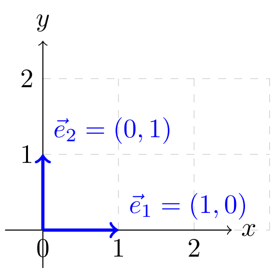

Figure 2.1: $\vec{e}$

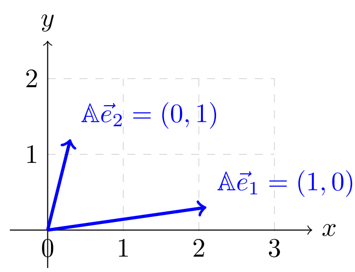

Figure 2.2: $\mathbb{A}\vec{e}$

2.8.1 projection အရိပ်ကျခြင်း $P(\hat{u})\vec{v}$

$$
\begin{aligned}
\hat{u} &= \frac{\vec{v}}{\|\vec{v}\|} \\
P_u(\hat{u}) &= \hat{u}\hat{u}^T \\
P_u(v) &= P_u(\hat{u})\vec{v}
\end{aligned}
\tag{2.12}
$$

project a vector onto a line defined by its unit vector. $\vec{v} = (1,2,3)$ ဖြစ်ပြီး unit vector ၏ line ပေါ် projection ကျတာကို ရှာလျှင် $\vec{v}$ ၏ unit vector $\hat{u}$ ကိုရှာလျှင်

$$
\begin{aligned}
\|\vec{v}\| &= \sqrt{1^2 + 2^2 + 3^2} \\
&= \sqrt{14} \\
\hat{u} &= \frac{1}{\sqrt{14}}(1,2,3) \\
P_u(\hat{u}) &= \hat{u}\hat{u}^T \\
&= (\frac{1}{\sqrt{14}}\begin{bmatrix}1\\2\\3\end{bmatrix})(\frac{1}{\sqrt{14}}\begin{bmatrix}1 & 2 & 3\end{bmatrix}) \\
&= \frac{1}{14}\begin{bmatrix}1\times1 & 1\times2 & 1\times3\\ 2\times1 & 2\times2 & 2\times3\\ 3\times1 & 3\times2 & 3\times3\end{bmatrix} \\
&= \frac{1}{14}\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{bmatrix} \\
P_u(\vec{v}) &= P_u(\hat{u})\vec{v} \\
&= \frac{1}{14}\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} \\
&= \frac{1}{14}\begin{bmatrix}1\times1+2\times2+3\times3\\ 2\times1+4\times2+6\times3\\ 3\times1+6\times2+9\times3\end{bmatrix} \\
&= \frac{1}{14}\begin{bmatrix}14\\28\\52\end{bmatrix} \\
&= \begin{bmatrix}1\\2\\3\end{bmatrix} \text{ (သူ့ရဲ့} \hat{u} \text{အပေါ်သူ projection ဖြစ်တော့ မပြောင်းလဲ)}
\end{aligned}
$$

project a vector onto a line defined by unit vector of another vector. $\vec{v} = (1,2,3)$ ဖြစ်ပြီး $\vec{w} = (1,3,2)$ ၏ line ပေါ် projection ကျတာကို ရှာလျှင် $\vec{w}$ ၏ unit vector $\hat{u}$ ကိုရှာလျှင်

$$\begin{aligned}
\|\vec{w}\| &= \sqrt{1^2+3^2+2^2} \\
&= \sqrt{14} \\
\hat{u}_{\vec{w}} &= \frac{1}{\sqrt{14}}(1,3,2) \\
P_u(\vec{w})\vec{v} &= \frac{1}{14}\begin{bmatrix}1\\3\\2\end{bmatrix}\begin{bmatrix}1 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} \\
&= \frac{1}{14}\begin{bmatrix}1 & 3 & 2\\3 & 9 & 6\\2 & 6 & 4\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} \\
&= \frac{1}{14}\begin{bmatrix}13\\39\\26\end{bmatrix} \\
&= \begin{bmatrix}\frac{13}{14}\\\frac{39}{14}\\\frac{26}{14}\end{bmatrix}
\end{aligned}$$

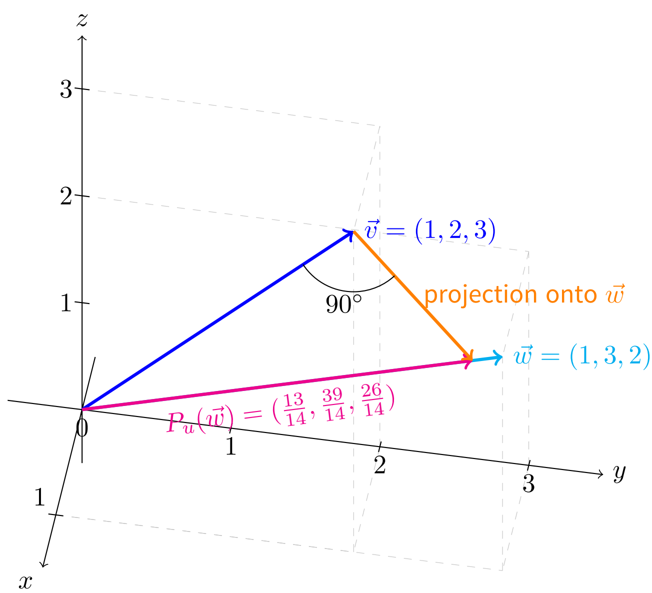

Figure 2.3: 3D vector $\vec{v} = (1,3,2) \in R^3$

2.8.2 Reflection အလင်းပြန်ခြင်း $F(\vec{v})$

$$
\begin{aligned}
\hat{u} &= \frac{\vec{v}}{\|\vec{v}\|} \\
F(\hat{u}) &= 2\hat{u}\hat{u}^T - \mathbb{I} \\
F(\vec{v}) &= F(\hat{u})\vec{v}
\end{aligned} \tag{2.13}
$$

$\vec{v} = (1, 2, 3)$ ဖြစ်ပြီး $\vec{w} = (1, 3, 2)$ ၏ line တလျှောက် reflectiion ဖြစ်တာကို ရှာလျှင်

$\vec{w}$ ၏ unit vector $\hat{u}$ သည် $\frac{1}{\sqrt{14}}(1, 3, 2)$ ဖြစ်သည်။

$$
\begin{aligned}
F(\hat{u}_{\vec{w}}) &= \frac{2}{14}\begin{bmatrix}1\\3\\2\end{bmatrix}\begin{bmatrix}1 & 3 & 2\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \\
&= \frac{2}{14}\begin{bmatrix}1 & 3 & 2\\3 & 9 & 6\\2 & 6 & 4\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \\
&= \begin{bmatrix}\frac{2}{14} & \frac{6}{14} & \frac{4}{14}\\\frac{6}{14} & \frac{18}{14} & \frac{12}{14}\\\frac{4}{14} & \frac{12}{14} & \frac{8}{14}\end{bmatrix} - \begin{bmatrix}\frac{14}{14} & 0 & 0\\0 & \frac{14}{14} & 0\\0 & 0 & \frac{14}{14}\end{bmatrix} \\
&= \begin{bmatrix}-\frac{12}{14} & \frac{6}{14} & \frac{4}{14}\\\frac{6}{14} & \frac{4}{14} & \frac{12}{14}\\\frac{4}{14} & \frac{12}{14} & -\frac{6}{14}\end{bmatrix} \\
&= \begin{bmatrix}-\frac{6}{7} & \frac{3}{7} & \frac{2}{7}\\\frac{3}{7} & \frac{2}{7} & \frac{6}{7}\\\frac{2}{7} & \frac{6}{7} & -\frac{3}{7}\end{bmatrix} \\
F(\vec{v}) &= \begin{bmatrix}-\frac{6}{7} & \frac{3}{7} & \frac{2}{7}\\\frac{3}{7} & \frac{2}{7} & \frac{6}{7}\\\frac{2}{7} & \frac{6}{7} & -\frac{3}{7}\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} \\
&= \begin{bmatrix}\frac{6}{7}\\\frac{25}{7}\\\frac{5}{7}\end{bmatrix} \\
&\approx \begin{bmatrix}0.857\\3.571\\0.714\end{bmatrix}
\end{aligned}
$$

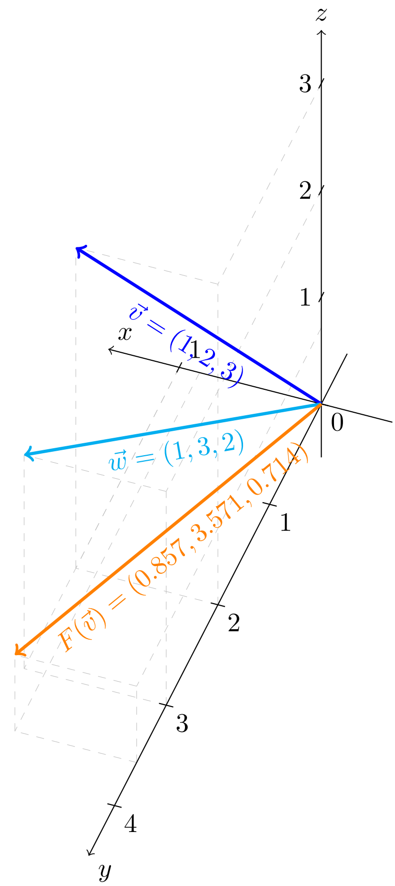

Figure 2.4: Reflection of $\vec{v}$ by unit vector of $\vec{w}$

2.8.3 2D rotation $R_2^\theta(\vec{v})$

$$
\begin{aligned}
[R_2^\theta] &= \left[\begin{array}{c|c} R_2^\theta(\vec{e}_1) & R_2^\theta(\vec{e}_2) \end{array}\right] \\
&= \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \\
R_2^\theta(\vec{v}) &= [R_2^\theta]\vec{v} \\
&= \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}
\end{aligned} \tag{2.14}
$$

$\vec{v} = (3, 1)$ ကို $\frac{\pi}{6}$ ဖြင့် counter clockwise အတိုင်းလှည့်လျှင်,

$$\begin{aligned}
R_2^{\frac{\pi}{6}} &= \begin{bmatrix} cos(\frac{\pi}{6}) & -sin(\frac{\pi}{6}) \\ sin(\frac{\pi}{6}) & cos(\frac{\pi}{6}) \end{bmatrix} \\
&= \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \\
R_2^{\frac{\pi}{6}}(\vec{v}) &= \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 2.098 \\ 2.366 \end{bmatrix}
\end{aligned}$$

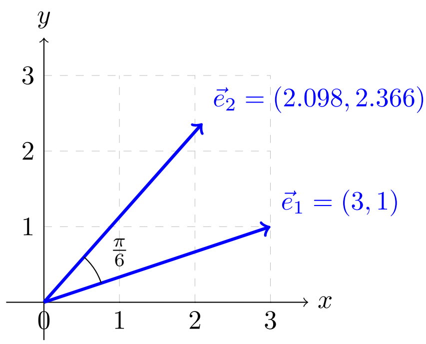

Figure 2.5: 2D rotation of $\vec{v}$ by $\frac{\pi}{6}$ counter clockwise

$\vec{v} = (3, 1)$ ကို $\frac{\pi}{6}$ ဖြင့် clockwise အတိုင်းလှည့်လျှင်,

$$\begin{aligned}
R_2^{-\frac{\pi}{6}} &= \begin{bmatrix} cos(-\frac{\pi}{6}) & -sin(-\frac{\pi}{6}) \\ sin(-\frac{\pi}{6}) & cos(-\frac{\pi}{6}) \end{bmatrix} \\
&= \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \\
R_2^{-\frac{\pi}{6}}(\vec{v}) &= \begin{bmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 3.098 \\ -0.634 \end{bmatrix}
\end{aligned}$$

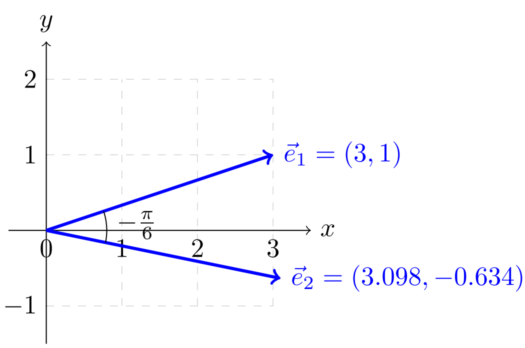

Figure 2.6: 2D rotation of $\vec{v}$ by $\frac{\pi}{6}$ clockwise

2.8.4 Rotation in higher dimension $R^{\theta}_{xyz}(\vec{v})$

$$
\begin{aligned}
[R^{\theta}_{yz}] &= \left[\ R^{\theta}_{yz}(e_1) \mid R^{\theta}_{yz}(e_2) \mid R^{\theta}_{yz}(e_3)\ \right] \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{bmatrix} \\
[R^{\theta}_{zx}] &= \begin{bmatrix} cos(\theta) & 0 & -sin(\theta) \\ 0 & 1 & 0 \\ sin(\theta) & 0 & cos(\theta) \end{bmatrix} \\
[R^{\theta}_{xy}] &= \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{2.15}
$$

$\vec{v} = (3, -1, 2)$ သည် z-axis အတိုင်း $\theta = \frac{2\pi}{3}$ ဖြင့် counter clockwise အတိုင်း rotate လုပ်လျှင်,

$$
\begin{aligned}
[R^{\frac{2\pi}{3}}_{xy}] &= \begin{bmatrix} cos(\frac{2\pi}{3}) & -sin(\frac{2\pi}{3}) & 0 \\ sin(\frac{2\pi}{3}) & cos(\frac{2\pi}{3}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -0.5 & -0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
[R^{\frac{2\pi}{3}}_{xy}]\vec{v} &= \begin{bmatrix} -0.5 & -0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \\
&\approx \begin{bmatrix} -0.6340 \\ 3.0981 \\ 2 \end{bmatrix}
\end{aligned}
$$

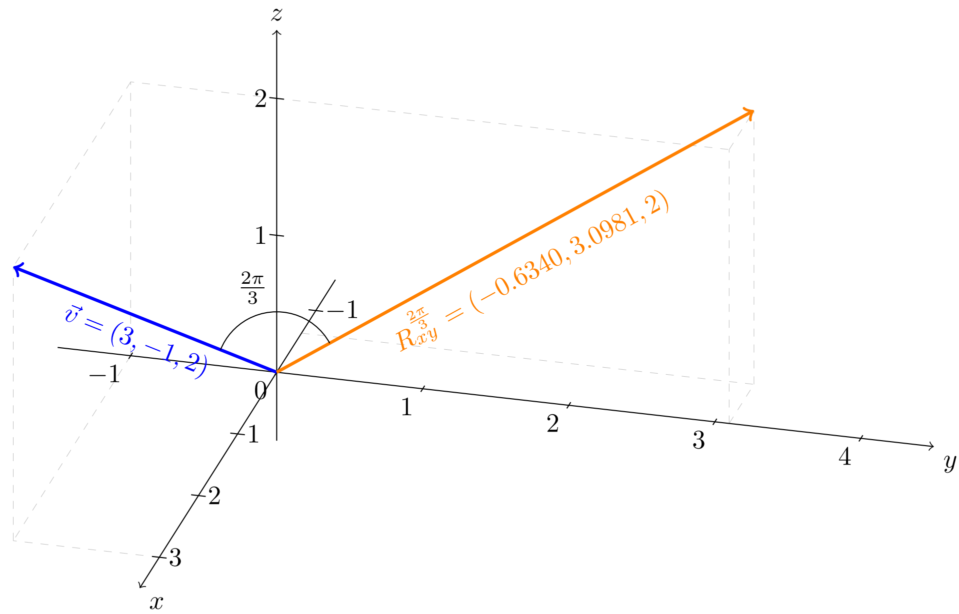

Figure 2.7: rotation in 3d of $\vec{v}$ in z-axis

2.8.5 Combination of Linear Transformation $(S \cdot T)(\vec{v})$

$$\begin{aligned}(S \circ T)(\vec{v}) &= S(T(\vec{v})) \\ &= [S]([T]\vec{v}) \\ &= ([S][T])\vec{v}\end{aligned} \tag{2.16}$$

$\vec{v}$ သည် $\vec{w}$ အပေါ် projection ဖြစ်ပြီး 2d rotation $\frac{\pi}{3}$ counter clockwise အတိုင်းလည်လျှင် $\vec{w}$ ၏ unit vector ကိုအရင်ရှာပြီး $P_{\vec{w}}$ ကိုရှာရမယ်။ $R^{\frac{\pi}{3}}$ ကိုရှာရမယ်။

$$(P_{\vec{w}} \circ R^{\frac{\pi}{3}})(\vec{v}) = ([P_{\vec{w}}][R^{\frac{\pi}{3}}])(\vec{v})$$

2.9 Area

$\vec{v}, \vec{w} \in \mathbb{R}^3$ ဖြစ်လျှင်,

$$Area = \|\vec{v} \times \vec{w}\| = \sqrt{\|\vec{v}\|^2 \|\vec{w}\|^2 - (\vec{v} \cdot \vec{w})^2} = \|\vec{v}\| \|\vec{w}\| sin(\theta) \tag{2.17}$$

$\vec{v} = (1, 2, -1), \vec{w} = (2, 1, 2)$ အနားတွေရှိတဲ့ paprallelogram ၏ area ကိုရှာလျှင်,

$$\begin{aligned}\|\vec{v} \times \vec{w}\| &= \|(1, 2, -1) \times (2, 1, 2)\| \\ &= \|(4 + 1, -2 - 2, 1 - 4)\| \\ &= \sqrt{5^2 + (-4)^2 + (-3)^2} \\ &= \sqrt{50} \\ &= \sqrt{25x2} \\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}
\|\vec{v}\| &= \sqrt{1^2 + 2^2 + (-1)^2} \\
&= \sqrt{6} \\
\|\vec{w}\| &= \sqrt{2^2 + 1^2 + 2^2} \\
&= 3 \\
\vec{v} \cdot \vec{w} &= 1 \times 2 + 2 \times 1 + (-1) \times 2 \\
&= 2 \\
\sqrt{\|\vec{v}\|^2 \|\vec{w}\|^2 - (\vec{v} \cdot \vec{w})^2} &= \sqrt{(\sqrt{6})^2 3^2 - 2^2} \\
&= \sqrt{54 - 4} \\
&= \sqrt{50} \\
&= 5\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
\theta &= cos^{-1}(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\|\vec{w}\|}) \\
&= cos^{-1}(\frac{2}{3\sqrt{6}}) \\
&\approx 74.207^\circ \\
\|\vec{v}\|\|\vec{w}\| sin(\theta) &= \sqrt{6} \times 3 \times \frac{5\sqrt{3}}{9} \\
&= \frac{5\sqrt{18}}{3} \\
&= 5\sqrt{2}
\end{aligned}$$

2.10 volume

$\vec{v}, \vec{w}, \vec{x} \in \mathbb{R}^3$ တို့သည် parallelpiped ၏ အနားတွေဖြစ်လျှင်,

$$volume = |\vec{v} \cdot (\vec{w} \times \vec{x})| = \|\vec{v}\|\|\vec{w} \times \vec{x}\||cos(\theta)| \quad (2.18)$$

$(1, 0, 1), (-1, 2, 2), (3, 2, 1)$ အနားများရှိသော parallelpiped ၏ volume ကိုရှာလျှင်,

$$\begin{aligned}
|(1, 0, 1) \cdot ((-1, 2, 2) \times (3, 2, 1))| &= |(1, 0, 1) \cdot (-2, 7, -8)| \\
&= | - 2 + 0 - 8| \\
&= 10
\end{aligned}$$

Chapter 3

Linear System and Subspaces

One solution

$x+2y=4, -x+y=-1$ linear system တွေ၏ solution(intersection) ကိုရှာရန် $y=1$ ကို အစားထိုးကြည့်လျှင်

$$\begin{aligned} x+2y &= 4 \\ x &= 4-2y \\ &= 4-2 \\ &= 2 \qquad (2,1) \text{ from } x+2y=4 \\ -x+y &= -1 \\ x &= y+1 \\ &= 1+1 \\ &= 2 \qquad (2,1) \text{ from } -x+y=-1 \end{aligned}$$

linear system နှစ်ခု၏ solution (intersection) တစ်ခုသာရှိပြီး (2,1) ဖြစ်သည်။

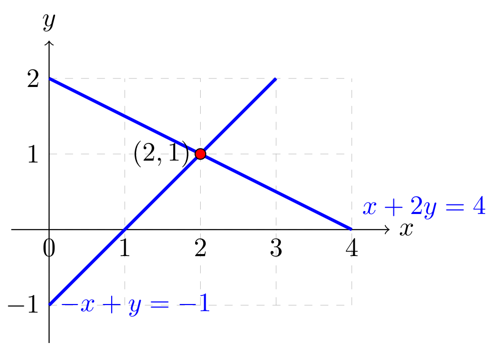

Figure 3.1: 2D vector, $\vec{v}=(3,2)\in R^2$

unlimited solution

$x + 2y = 4, 2x + 4y = 8$ linear system တွေ၏ solution(intersection) ကိုရှာရန် $y = 1$ ကို အစားထိုးကြည့်လျှင်

$$\begin{aligned} x + 2y &= 4 \\ x &= 4 - 2y \\ &= 4 - 2 \\ &= 2 \qquad (2,1) \\ 2x + 4y &= 8 \\ 2x &= 8 - 4y \\ x &= 4 - 2y \\ &= 4 - 2 \\ &= 2 \qquad (2,1) \end{aligned}$$

linear system နှစ်ခု ၏ coordinate တွေသည် တစ်ထပ်တည်းကျနေသည့်အတွက် line နှစ်ကြောင်းလုံးသည် တစ်ထပ်တည်းကျနေသည်။ solution (intersection) တွေအများကြီးရှိသည်။

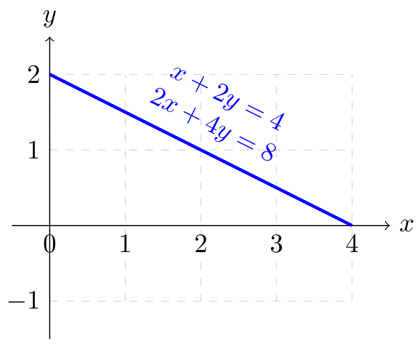

Figure 3.2: solutions are too many

no solution

$x + 2y = 4, x + 2y = 3$ linear system တွေ၏ solution(intersection) ကိုရှာရန် $y = 1$ ကို အစားထိုးကြည့်လျှင်

$$\begin{aligned} x + 2y &= 4 \\ x &= 4 - 2y \\ &= 4 - 2 \\ &= 2 \qquad (2,1) \\ x + 2y &= 3 \\ x &= 3 - 2y \\ &= 3 - 2 \\ &= 1 \qquad (1,1) \end{aligned}$$

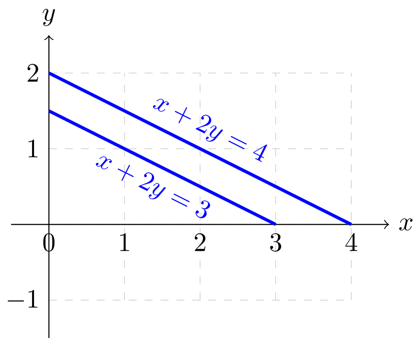

Figure 3.3: 2D vector, $\vec{v} = (3, 2) \in R^2$