Math 116 In-Class Worksheet: Logarithms

Directions: This worksheet is meant to be completed in groups, but you may work at your own pace. No leaving until class is done!

Last class, we explored a differential equation whose solution (I claimed) was an exponential function. To get to this, we'll go through logarithms first since it displays the fundamental theorem of calculus in all its glory.

1) a) State the Fundamental Theorem of Calculus. What kind of functions does it apply to? This is a bit of a trick question, since there is a complete answer but I don't expect you to know it! Give an incomplete answer in its stead if you don't know the complete answer.

b) Use the Fundamental Theorem to find an antiderivative for the function $f(t) = t^n$ where n is a real number. Does your solution hold for all values of n?

c) Does the expression

$$\int_1^x \frac{1}{t}\, dt$$

always make sense? Why or why not?

d) According to the fundamental theorem, what is the derivative of $f(x) = \int_1^x \frac{1}{t}\, dt$?

2) Let's set $\ln(x) = \int_1^x \frac{1}{t}\, dt$.

a) Why is $\ln(1) = 0$?

b) Draw the graph of $1/t$ from $t = 1$ to $t = 2$. Which is bigger, $\ln(2)$ or $1/2$?

c) Repeat part b) from $t = 1$ to $t = 3$. Which is bigger, $\ln(3)$ or $1/3+1/2$?

d) Finally, repeat part b) from $t = 1$ to $t = 4$. Which is bigger, $\ln(4)$ or $1/4 + 1/3 + 1/2$?

e) Convince your group members (and more importantly, convince me) that there is a number e with $1 < e < 4$ such that $\ln(e) = 1$. You can happily use whatever technology you like.

f) Now the hard part: what fantastic theorem guarantees the existence of the number e in part e) (I totally did not plan that)? Why does the theorem even apply?

3) Assuming $\ln(x) = \displaystyle\int_1^x \frac{1}{t}\, dt$, compute the following derivatives and integrals.

a) $\dfrac{d}{dx}(\ln(x^3 \tan(x^4)))$

b) $\displaystyle\int_0^{\sqrt{e-1}} \frac{t}{t^2+1}\, dt$

c) $\displaystyle\int \frac{\sin(t)\cos(t)}{1+\cos(t)}\, dt$