Problem D. Longest Increasing Subsequence

- 2023.12.06 19:30 Update: Strengthen test cases.

Problem Description

> We assume all the set notations $S \subseteq \{0, 1, \ldots, n-1\}$ in the following context.
>
> We assume $S = \{s_0, s_1, \ldots, s_{k-1}\}$ and $s_0 < s_1 < \cdots < s_{k-1}$ in the following context.

Given an array $a = [a[0], a[1], \ldots, a[n-1]]$, please answer the following question:

- A subset S is ***increasing*** if $a[s_0] \leq a[s_1] \leq \cdots \leq a[s_{k-1}]$.
- A subset S is ***valid*** if S is *increasing* and there does not exist $S' \supsetneq S$ and S' is *increasing*.
- The score of a subset S is $a[s_0] + a[s_1] + \cdots + a[s_{k-1}]$.

Find the numbers of all valid subsets and sum of scores of all valid subsets, both modulo $1\,000\,000\,007$.

There are t test cases.

Input Format

- line 1: t

t blocks:

- line 1: n
- line 2: $a[0]$ $a[1]$ $\ldots$ $a[n-1]$

Output Format

t blocks:

- line 1: $count$ sum

Constraints

- $1 \leq t \leq 10$.
- $2 \leq n \leq 100\,000$.
  - Sum of n across all test cases $\leq 200\,000$.
- $1 \leq a[i] \leq 10^9$ for $i = 0, 1, \ldots, n-1$.

Subtasks

1. (10 points) $n \leq 15$.
2. (20 points) $n \leq 200$.
3. (35 points) $n \leq 2000$.
4. (25 points) $a[i] \leq 15$ for $i = 0, 1, \ldots, n-1$.
5. (10 points) No additional constraints.

- If you find the numbers of all valid subsets correctly, you can get 80% of a subtask's score;
- furthermore, if you output the sum of scores of all valid subsets correctly, you can get the remaining 20%.

No.	Testdata Range	Time Limit (ms)	Memory Limit (KiB)
Samples	1	1500	262144
1	1-10	1500	262144
2	1-23	1500	262144
3	1-35	1500	262144
4	1, 36-41	1500	262144
5	1-58	1500	262144

Samples

Sample Input 1

```
6
5
2 3 6 4 6
5
1 15 8 2 4
7
5 3 4 1 7 2 6
2
2 1
3
15 15 15
15
1 5 3 2 8 6 4 13 11 9 14 12 10 7 15
```

This sample input satisfies the constraints of all the subtasks.

Sample Output 1

```
2 32
3 32
6 67
2 3
1 45
53 2478
```

- In the first test case, $\{0, 1, 2, 4\}$ and $\{0, 1, 3, 4\}$ are the only valid subsets. The score for these two subsets are $17 = 2 + 3 + 6 + 6$ and $15 = 2 + 3 + 4 + 6$, respectively.
- In the second test case, $\{0, 1\}$, $\{0, 2\}$, and $\{0, 3, 4\}$ are the only valid subsets.
- In the third test case, $\{0, 4\}$, $\{0, 6\}$, $\{1, 2, 4\}$, $\{1, 2, 6\}$, $\{3, 4\}$, and $\{3, 5, 6\}$ are the only valid subsets.
- In the fourth test case, $\{0\}$ and $\{1\}$ are the only valid subsets.
- In the fifth test case, $\{0, 1, 2\}$ is the only valid subset.