Problem F. Path of the Mind

- 2023.10.06 15:00 Update: Fixed typo in Sample Output 4.

Problem Description

You opened Chapter 75 of "*Neko-chan Chronicle*," where the Brave Neko-chan Squad reached the final trial, the "*Path of the Mind*."

In the Path of the Mind, each squad member must confront their inner demons, strengthen their resolve, and overcome them to successfully reach the other side.

The Path of the Mind consists of k stages. While these stages may appear identical to observers, those within will sense subtle differences. Through the Brave Neko-chan Squad's pre-trial investigation, they found that each stage can be quantified by three values: *damage* $sd[i]$, *endurance* $se[i]$, and *property* $sp[i]$ $(0 \leq i \leq k-1)$. The "property" is the reason for the diverse nature of the Path of the Mind.

Simultaneously, the n brave warriors possess distinct attributes: *damage* $wd[j]$, *endurance* $we[j]$, and *property* $wp[j]$ $(0 \leq j \leq n-1)$. Battle with inner demons is akin to a debate: starting with a brave warrior, both sides take turns speaking, inflicting damage and depleting the opponent's endurance until one admits defeat (endurance ≤ 0).

The Brave Neko-chan Squad has its own interpretation of attributes: they believe attributes are the result of different personalities combined! Each personality has a "*prime*" influence, and moving towards the extreme of that personality increases the "*power*" of that prime. In other words, decomposing the quantified attributes into "*prime factorization*" reveals a person's character! When a brave warrior's attributes align with a stage's property, it indicates a profound impact, making the challenge more difficult. When a warrior with property p_w faces a stage with property p_s, they create an affinity of $f = \gcd(p_w, p_s)$, and both damage and endurance of that stage increase by f.

Since the trial grounds won't present insurmountable challenges, assistance is offered for the enigmatic Path of the Mind. Each member of the Brave Neko-chan Squad receives a shield with an energy level of a. When damaged, the shield automatically consumes energy to resist some of the damage (see below). This is both help and a test, examining the warriors' self-awareness of their abilities. Warriors can discuss and choose the level of assistance a they wish for their squad. While selecting $a = 10^{10^{100}}$ would ensure an easy passage, choosing non-optimal a could also lead to success without any rewards.

The shield will **automatically resist** attack damage as long as it has energy. If the opponent's damage is d_s and the current shield energy is a', the damage received becomes $\max(d_s - a', 0)$, and the shield's energy becomes $\max(a' - 1, 0)$.

As the leader of the Brave Neko-chan Squad, Neko-chan shoulders the mission of choosing the minimum assistance level a. Although her legendary story continues, the exact answer is not written, leaving you curious about how much help the Brave Neko-chan Squad ultimately received. You investigated the information about each warrior and each stage at that time. Now, find out the minimum value of a!

Oh, by the way, because Neko-chan's charisma is extraordinary, the squad consisted of tens of thousands of members at that time!

Input Format

- line 1: n k
- line 2: $wd[0]$ $wd[1]$ $\ldots$ $wd[n-1]$
- line 3: $we[0]$ $we[1]$ $\ldots$ $we[n-1]$
- line 4: $wp[0]$ $wp[1]$ $\ldots$ $wp[n-1]$
- line 5: $sd[0]$ $sd[1]$ $\ldots$ $sd[k-1]$
- line 6: $se[0]$ $se[1]$ $\ldots$ $se[k-1]$
- line 7: $sp[0]$ $sp[1]$ $\ldots$ $sp[k-1]$

Output Format

- line 1: the minimum possible value of a.

Constraints

- $1 \leq n \leq 100\,000$.
- $1 \leq k \leq 500$.
- $1 \leq wd[i], we[i] \leq 10^6$ for $i = 0, 1, \ldots, n-1$.
- $1 \leq wp[i] \leq 1000$ for $i = 0, 1, \ldots, n-1$.
- $1 \leq sd[i], se[i] \leq 10^6$ for $i = 0, 1, \ldots, k-1$.
- $1 \leq sp[i] \leq 1000$ for $i = 0, 1, \ldots, k-1$.
- All input values are integers.

Subtasks

1. (10 points) $k = 1$.
2. (10 points) $n = 1$.
3. (75 points) $n \leq 2000$.
4. (5 points) No additional constraints.

No.	Testdata Range	Time Limit (ms)	Memory Limit (KiB)
Samples	1-4	2500	262144
1	5-17	2500	262144
2	18-30	2500	262144
3	18-42	2500	262144
4	1-58	2500	262144

Samples

Sample Input 1

```
3 4
7 4 1000000
3 29 1
6 8 1
9 1 3 1000000
5 9 5 1
8 12 7 997
```

This sample input satisfies the constraints of Subtasks 3, 4.

Sample Output 1

```
10
```

Let us simulate how the warrior 1 with $(d_w, e_w, p_w) = (4, 29, 8)$ fight through their inner demons with assistance level 10:

(d_w, e_w, p_w)	Shield	Stage	(d_s, e_s, p_s)	Notes
$(4, 29, 8)$	10	0	$(9, 5, 8)$	The warrior enters stage 0.
$(4, 29, 8)$	10	0	$(17, 13, 8)$	Stage 0 gains $8 = \gcd(8, 8)$ damage and endurance.
$(4, 29, 8)$	10	0	$(17, 9, 8)$	Warrior attacks for 4 damage.
$(4, 22, 8)$	9	0	$(17, 9, 8)$	Stage 0 attacks for $7 = \max(17 - 10, 0)$ damage.
$(4, 22, 8)$	9	0	$(17, 5, 8)$	Warrior attacks for 4 damage.
$(4, 14, 8)$	8	0	$(17, 5, 8)$	Stage 0 attacks for $8 = \max(17 - 9, 0)$ damage.
$(4, 14, 8)$	8	0	$(17, 1, 8)$	Warrior attacks for 4 damage.

$(4, 5, 8)$	7	0	$(17, 1, 8)$	Stage 0 attacks for $9 = \max(17 - 8, 0)$ damage.
$(4, 5, 8)$	7	0	$(17, -3, 8)$	Warrior attacks for 4 damage. **Warrior wins!**
$(4, 5, 8)$	7	1	$(1, 9, 12)$	The warrior enters stage 1.
$(4, 5, 8)$	7	1	$(5, 13, 12)$	Stage 1 gains $4 = \gcd(8, 12)$ damage and endurance.
$(4, 5, 8)$	7	1	$(5, 9, 12)$	Warrior attacks for 4 damage.
$(4, 5, 8)$	6	1	$(5, 9, 12)$	Stage 1 attacks for $0 = \max(5 - 7, 0)$ damage.
$(4, 5, 8)$	6	1	$(5, 5, 12)$	Warrior attacks for 4 damage.
$(4, 5, 8)$	5	1	$(5, 5, 12)$	Stage 1 attacks for $0 = \max(5 - 6, 0)$ damage.
$(4, 5, 8)$	5	1	$(5, 1, 12)$	Warrior attacks for 4 damage.
$(4, 5, 8)$	4	1	$(5, 1, 12)$	Stage 1 attacks for $0 = \max(5 - 5, 0)$ damage.
$(4, 5, 8)$	4	1	$(5, -3, 12)$	Warrior attacks for 4 damage. **Warrior wins!**
$(4, 5, 8)$	4	2	$(3, 5, 7)$	The warrior enters stage 2.
$(4, 5, 8)$	4	2	$(4, 6, 7)$	Stage 2 gains $1 = \gcd(8, 7)$ damage and endurance.
$(4, 5, 8)$	4	2	$(4, 2, 7)$	Warrior attacks for 4 damage.
$(4, 5, 8)$	3	2	$(4, 2, 7)$	Stage 2 attacks for $0 = \max(4 - 4, 0)$ damage.
$(4, 5, 8)$	3	2	$(4, -2, 7)$	Warrior attacks for 4 damage. **Warrior wins!**
$(4, 5, 8)$	3	3	$(1\,000\,000, 1, 997)$	The warrior enters stage 3.
$(4, 5, 8)$	3	3	$(1\,000\,001, 2, 997)$	Stage 3 gains $1 = \gcd(8, 997)$ damage and endurance.
$(4, 5, 8)$	3	3	$(1\,000\,001, -2, 997)$	Warrior attacks for 4 damage. **Warrior wins!**

It can be shown that with an assistance level of 9, warrior 1 will lose to the "Path of the Mind." Neko-chan had to choose an assistance level of at least 10 to ensure the survival of the Brave Neko-chan Squad.

Sample Input 2

```
1 4
5
17
1
3 8 12 6
8 7 10 14
1 1 1 1
```

This sample input satisfies the constraints of Subtasks 2, 3, 4.

Sample Output 2

```
10
```

Sample Input 3

```
4 1
3 8 1 7
20 3 7 16
1 1 1 1
6
8
1
```

This sample input satisfies the constraints of Subtasks 1, 3, 4.

Sample Output 3

```
11
```

Sample Input 4

```
1 1
1
1
1
100000
100000
1000
```

This sample input satisfies the constraints of all the subtasks.

Sample Output 4

```
200000
```

The warrior cannot take any damage. Since the battle lasts for $100\,000$ rounds and the damage for stage 0 is $100\,001$, Neko-chan had to choose an assistance level of at least $200\,000$ for the warrior to survive.