Q1.

By rule induction on the judgment s lparen.

Case $\dfrac{}{\varepsilon \text{ lparen}}$ Leps where $s = \varepsilon$:

ε mparen — by Meps

Case $\dfrac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}$ Lseq where $s = (s_1)s_2$:

s_1 mparen — by inductive hypothesis on s_1 lparen

s_2 mparen — by inductive hypothesis on s_2 lparen

$(s_1)s_2$ mparen — by $\dfrac{\dfrac{s_1 \text{ mparen}}{(s_1) \text{ mparen}}\text{Mpar} \quad s_2 \text{ mparen}}{(s_1)s_2 \text{ mparen}}$ Mseq with s_1 mparen and s_2 mparen

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Q2.

By rule induction on the judgment s' tparen.

Case $\dfrac{}{\varepsilon \text{ tparen}}$ Teps where $s' = \varepsilon$

s tparen — assumption

ss' tparen — from $ss' = s\varepsilon = s$ and s tparen

Case $\dfrac{s_1 \text{ tparen} \quad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}$ Tseq where $s' = s_1(s_2)$

s tparen — assumption

$ss' = s\,s_1(s_2)$

"s tparen implies ss_1 tparen" — by induction hypothesis on s_1 tparen

ss_1 tparen — from the assumption s tparen

$ss_1(s_2)$ tparen — by the rule Tseq with ss_1 tparen and s_2 tparen.

Q3.

By rule induction on the judgment s mparen.

Case $\dfrac{}{\varepsilon \text{ mparen}}$ Meps where $s = \varepsilon$:

ε tparen — by Teps

Case $\dfrac{s' \text{ mparen}}{(s') \text{ mparen}}$ Mpar where $s = (s')$:

s' tparen by inductive hypothesis on s' mparen

(s') tparen by $\dfrac{\dfrac{}{\varepsilon \text{ tparen}}\,T_{eps} \quad s' \text{ tparen}}{\varepsilon\,(s') \text{ tparen}}\,T_{seq}$ with $\varepsilon(s') = (s')$ and s' tparen

Case $\dfrac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1 s_2 \text{ mparen}}\,M_{seq}$ where $s = s_1 s_2$:

s_1 tparen by inductive hypothesis on s_1 mparen

s_2 tparen by inductive hypothesis on s_2 mparen

$s_1 s_2$ tparen by lemma 1.2 with s_1 tparen and s_2 tparen