Reinforcement Learning

Introduction and Model-Free Learning

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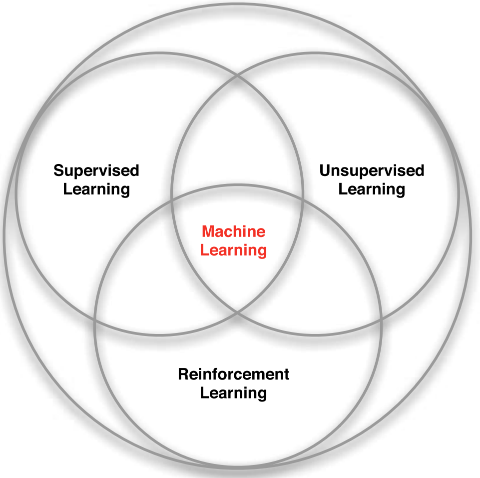
Disclaimer

All this material is a free re-arrangement of David Silver's UCL Course on RL.

You are also encouraged to take a look to his Youtube lectures.

What is reinforcement learning?

Branches of Machine Learning



RL Characteristics

What makes reinforcement learning different from other machine learning paradigms?

- There is no supervisor, only a *reward* signal.
- Feedback is delayed, not instantaneous
- Time really matters (sequential, non i.i.d. data)
- Agent is *active*: its actions affect the environment he lives in.

Rewards

- A reward R_t is a scalar feedback signal
- Indicates how well agent is doing at step t
- The agent's job is to maximise cumulative reward over an episode

Rewards: examples

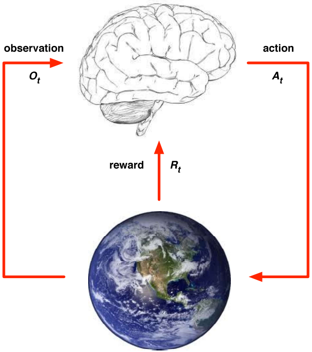
- Fly stunt manoeuvres in a helicopter
    - +ve reward for following desired trajectory
    - -ve reward for crashing
- Defeat the world champion at Go
    - +ve/-ve reward for winning/losing a game
- Make a humanoid robot walk
    - +ve reward for forward motion
    - -ve reward for falling over
- Play Atari games better than humans
    - +ve reward for increasing/decreasing score

Inside a RL agent

Sequential Decision Making

- Goal: *select actions to maximise total future reward*
- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward.
  - A financial investment may take months to mature
  - Refuelling a helicopter now might prevent a crash in several hours
  - Blocking opponent moves might help winning chances many moves from now

Agent and environment



- At each step t the agent:
  - Receives observation O_t
  - Receives scalar reward R_t
  - Executes action A_t
- The environment:
  - Receives action A_t
  - Emits observation O_{t+1}
  - Emits scalar reward R_{t+1}
- t increments at env. step

State

- The history is the sequence of observations, actions, rewards

$$H_t = O_1, R_1, A_1, \ldots, A_{t-1}, O_t, R_t$$

- The state is the information used to determine what happens next.
  - It is a function of the history:

$$S_t = f(H_t)$$

Agent and environment states

Agent state S_t^a	Environment state S_t^e
whatever information the agent uses to pick the next action	whatever data the environment uses to pick the next observation/reward
it is the information used by RL algorithms	usually not visible by the agent

- Full observability: agent directly observes environment state
- Partial observability: agent indirectly observes environment state

Inside a reinforcement learning agent

- An agent may include one or more of these components:
  - Policy: agent's behaviour function
  - Value function: how good is each state and/or action
  - Model: representation of the environment

Policy

- A policy is the agent's behaviour
- It is a map from state to action
- Deterministic policy: $a = \pi(s)$
- Stochastic policy: $\pi(a|s) = \mathbb{P}[A_t = s | S_t = s]$

Definition

The return G_t is the total discounted reward from time-step t.

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The discount $\gamma \in [0, 1]$ is the present value of future rewards
- The value of receiving reward R after $k + 1$ time-steps is $\gamma^k R$.
- This values immediate reward above delayed reward.
- γ close to 0 leads to *myopic* evaluation
- γ close to 1 leads to *far-sighted* evaluation

Value function

- Value function is a prediction of future reward
- Used to evaluate goodness/badness of states
- And therefore to select between actions

Definition

The *state-value function* $v_\pi(s)$ is the expected return starting from state s, and then following policy π

$$v_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

Definition

The *action-value function* $q_\pi(s, a)$ is the expected return starting from state s, taking action a, and then following policy π

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a]$$

Bellman expectation equation

The value function can be decomposed into two parts:

- immediate reward R_{t+1}
- discounted value of successor state $\gamma v(S_{t+1})$

$$\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi[G_t|S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots|S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \dots)|S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1}|S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1})|S_t = s]
\end{aligned}$$

Bellman Expectation Equation

The state-value function can again be decomposed into immediate reward plus discounted value of successor state,

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s]$$

The action-value function can similarly be decomposed,

$$q_\pi(s, a) = \mathbb{E}_\pi[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) | S_t = s, A_t = a]$$

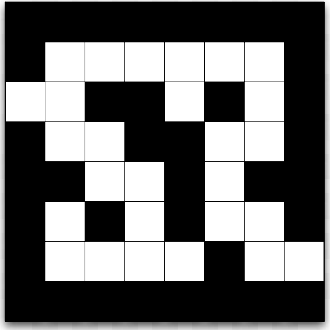
Model

- A model predicts what the environment will do next
- $\mathcal{P}$ predicts the next state
- $\mathcal{R}$ predicts the next (immediate) reward

$$\mathcal{P}^a ss' = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$$

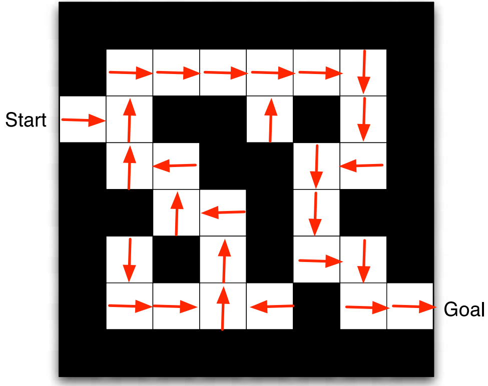
$$\mathcal{R}_s^a = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$$

Maze example



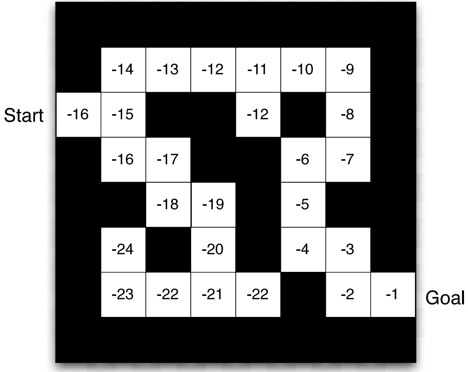
- Rewards: -1 per time-step
- Actions: N, S, W, E
- States: Agent's location

Maze example: policy



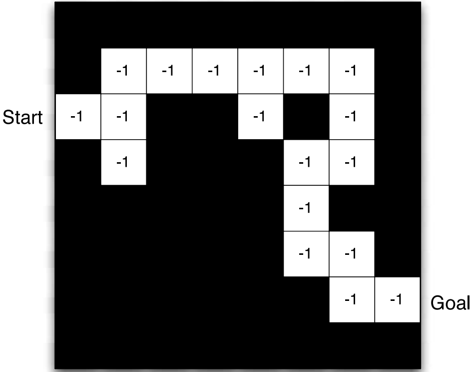
- Arrows represent policy $\pi(s)$ for each state s

Maze example: value function



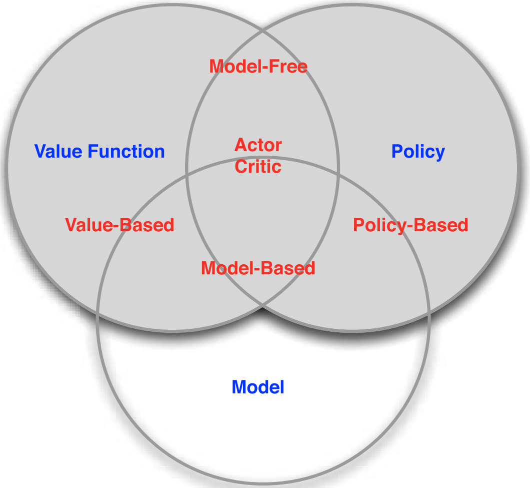
- Numbers represent policy $v_\pi(s)$ for each state s

Maze example: model



- Grid layout represent transition model $\mathcal{P}^a_{ss'}$
- Numbers represent immediate reward R^a_s from each state s (same for all a)

RL taxonomy



Model-Free
Value Function
Actor
Critic
Policy
Value-Based
Policy-Based
Model-Based
Model

Model-free prediction

Model-Free prediction

- Model-free prediction
  - estimate the value function given a policy in a non-observable environment
    - Monte-Carlo Learning
    - Temporal-Difference Learning

- MC methods learn directly from episodes of experience
- MC is *model-free*: no explicit knowledge of environment mechanisms
- MC learns from complete episodes
  - Caveat: can only apply to *episodic* environments (all episodes must terminate).
- MC uses the simpliest possible idea: value = mean return

Monte-Carlo Policy Evaluation

- Goal: learn v_π from episodes of experience under policy π

$$S_1, A_1, R_2, \ldots, S_k \sim \pi$$

- Recall that the *return* is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

- Recall that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

- Monte-Carlo policy evaluation uses *empirical mean* return instead of *expected* return

Every-Visit Monte-Carlo Policy Evaluation

- To evaluate state s
- Every time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + Gt$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Incremental Monte-Carlo Updates

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, ..., S_T$
- Compute return G_t
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$
$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

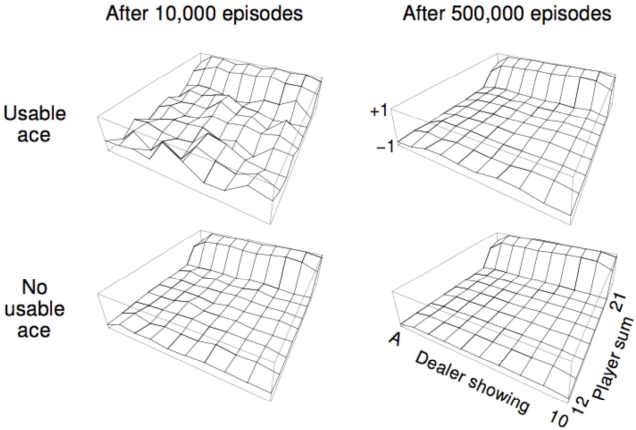
- Usually a running mean is employed, i.e. forget old episodes

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

Blackjack Example

- States (200 of them):
  - Current sum (12-21)
  - Dealer's showing card (ace-10)
  - Do I have a useable ace? (yes-no)
- Action stick: Stop receiving cards (and terminate)
- Action twist: Take another card (no replacement)
- Reward for stick:
  - $+1$ if sum of cards $>$ sum of dealer cards
  - 0 if sum of cards $=$ sum of dealer cards
  - -1 if sum of cards $<$ sum of dealer cards
- Reward for twist:
  - -1 if sum of cards $>$ 21 (and terminate)
  - 0 otherwise
- Transitions: automatically twist if sum of cards $<$ 12

Blackjack Value Function after Monte-Carlo Learning



Policy: stick if sum of cards ≤ 20, otherwise twist

MC and TD

- Goal: learn v_π online from experience under policy π
- Incremental every-visit Monte-Carlo
  - Update value $V(S_t)$ toward actual return G_t

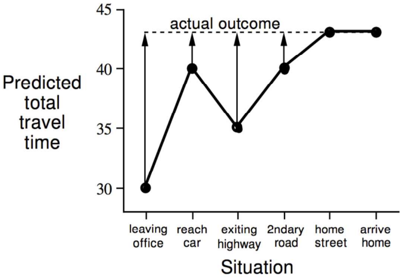
$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

- Simplest temporal-difference learning algorithm: TD(0)
  - Update value $V(S_t)$ toward estimated return $R_{t+1} + \gamma V(S_{t+1})$

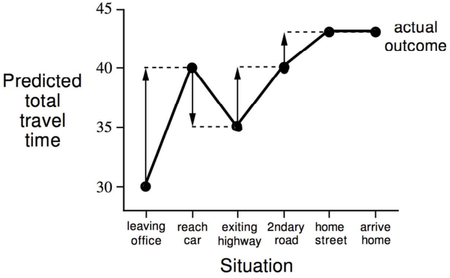
$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

  - $R_{t+1} + \gamma V(S_{t+1})$ is called the TD target
  - $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the TD error

Changes recommended by Monte Carlo methods (α=1)



Changes recommended by TD methods (α=1)



Advantages and disadvantages of MC vs. TD

- TD can learn *before* knowing the final outcome
  - TD can learn online after every step
  - MC must wait until end of episode before return is known
- TD can learn without the final outcome
  - TD can learn from incomplete sequences
  - MC can only learn from complete sequences
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments

Bias/variance trade-off

- Return $G_t = R_{t+1} + R_{t+2} + \ldots + \gamma^{T-1} R_T$ is *unbiased* estimate of $v_\pi(S_t)$
- True TD target $R_{t+1} + v_\pi(S_{t+1})$ is unbiased estimate of $v_\pi(S_t)$
- TD target $R_{t+1} + v(S_{t+1})$ is biased estimate of $v_\pi(S_t)$
- TD target is much lower variance than the return:
  - Return depends on many random actions, transitions, rewards
  - TD target depends on one random action, transition, reward

Model-free control

Model-Free control

- Model-free prediction
  - estimate the value function given a policy in a non-observable environment
    - Monte-Carlo Learning
    - Temporal-Difference Learning
- Model-free control
  - find a good policy in a non-observable environment
    - On-Policy Monte-Carlo Control
    - On-Policy Temporal-Difference Learning
    - Off-Policy Learning

How to improve a policy

- Given a policy π
- Evaluate the policy π

$$v_\pi(s) = \mathbb{E}[R_{t+1} + \gamma R_{t+2} + \ldots | S_t = s]$$

$$q_\pi(s, a) = \mathbb{E}[R_{t+1} + \gamma R_{t+2} + \ldots | S_t = s, A_t = a]$$

- Improve the policy by acting greedily with respect to v_π

$$\pi' = \text{greedy}(v_\pi)$$

$$\pi' = \text{greedy}(q_\pi)$$

- In general, need more iterations of improvement / evaluation
- In model-free contexts, action-value function $Q(s, a)$ is our only option

Example of greedy action selection

"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
- You open the left door and get reward 0
  $V(\text{left}) = 0$
- You open the right door and get reward $+1$
  $V(\text{right}) = +1$
- You open the right door and get reward $+3$
  $V(\text{right}) = +2$
- You open the right door and get reward $+2$
  $V(\text{right}) = +2$

$$\vdots$$

- Are you sure you've chosen the best door?

ϵ-greedy exploration

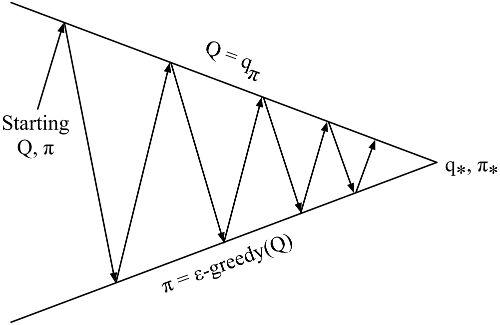
- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- With probability $1 - \epsilon$ choose the greedy action
- With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon, & \text{if } a^* = \arg\max_{a \in \mathcal{A}} Q(s, a) \\ \epsilon/m, & \text{otherwise} \end{cases}$$

On and off policy learning

- On-policy learning
  - "Learn on the job"
  - Learn about policy π from experience sampled from π
- Off-policy learning
  - "Look over someone's shoulder"
  - Learn about policy π from experience sampled from μ

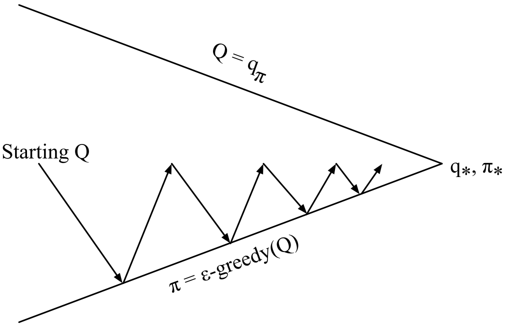
Monte-Carlo policy iteration



Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$

Policy improvement ϵ-greedy policy improvement

Monte-Carlo control



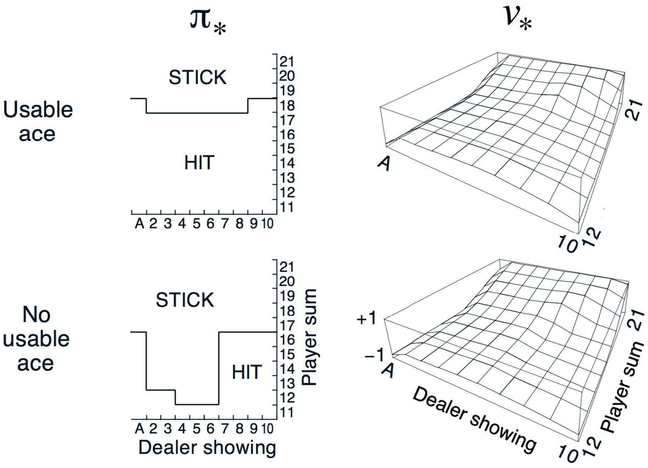
Every episode:

Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$

Policy improvement ϵ-greedy policy improvement

Back to the blackjack example

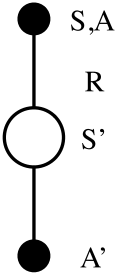
Monte-Carlo control in blackjack



MC vs TD control

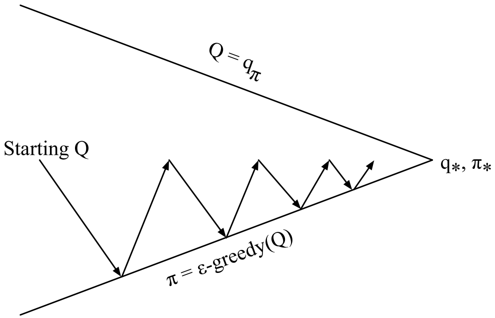
- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
  - Lower variance
  - Online
  - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
  - Apply TD to $Q(S, A)$
  - Use ϵ-greedy policy improvement
  - Update every time-step

Updating action-value functions with SARSA



$$Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma Q(S',A') - Q(S,A))$$

On-policy control with SARSA



Every time-step:

Policy evaluation with Sarsa, $Q \approx q_\pi$

Policy improvement with ϵ-greedy policy improvement.

SARSA algorithm for on-policy control

Initialize $Q(s,a), \forall s \in S, a \in \mathcal{A}(s)$, arbitrarily, and $Q(terminal - state, \dot{}) = 0$

for each episode **do**

  Intialise S

  Choose A from S using policy derived from Q (e.g., ϵ-greedy)

  for each step of episode **do**

    Take action A, observe R, S'

    Choose A' from S' using policy derived from Q (e.g., ϵ-greedy)

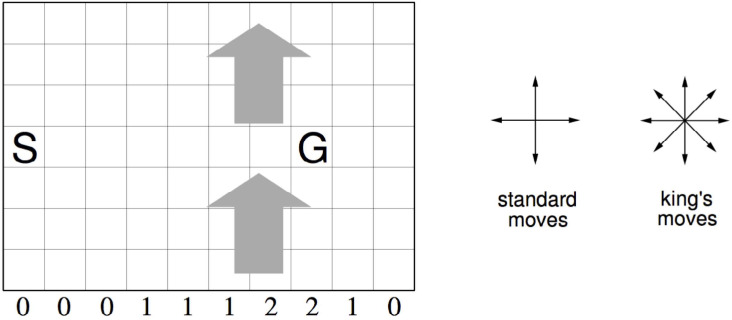
    $Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma Q(S',A') - Q(S,A))$

    $S \leftarrow S'; A \leftarrow A'$

  end for

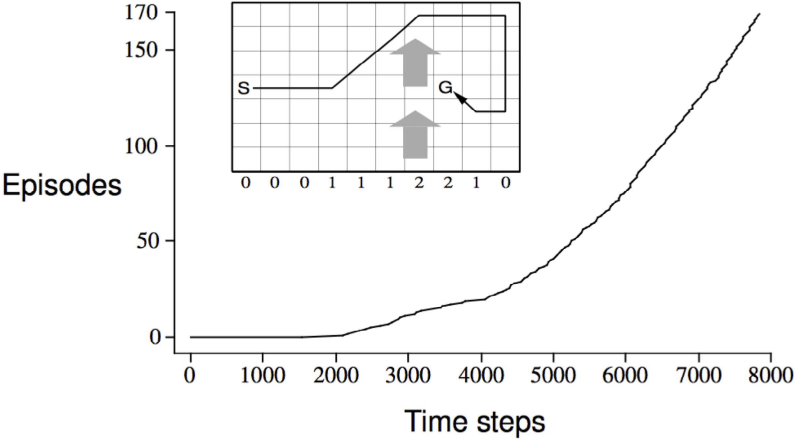
end for

Windy gridworld example



- Reward = -1 per time step until reaching goal
- Undiscounted

SARSA on the Windy Gridworld



Off-policy learning

- Evaluate target policy $\pi(a, s)$ to compute $v_\pi(s)$ or $q_\pi(s, a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, \ldots, S_T\} \sim \mu$$

- Why is this important?
- Learn from observing humans or other agents
- Re-use experience generated from old policies $\pi_1, \pi_2, \ldots, \pi_{t-1}$
- Learn about *optimal* policy while following *exploratory* policy
- Learn about *multiple* policies while following *one* policy

Q-learning

- We now consider off-policy learning of action-values $Q(s, a)$
- Next action is chosen using behaviour policy $A_{t+1} \sim \mu(\cdot|S_t)$
- But we consider alternative successor action $A' \sim \pi(\cdot|S_t)$
- And update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + Q(S_{t+1}, A') - Q(S_t, A_t))$$

Off-policy control with Q-learning

- We now allow both behaviour and target policies to improve
- The target policy π is greedy w.r.t. $Q(s, a)$

$$\pi(S_{t+1}) = \arg\max_{a'} Q(S_{t+1}, a')$$

- The behaviour policy μ is ϵ-greedy w.r.t. $Q(s, a)$
- The Q-learning target then simplifies:

$$\begin{aligned} & R_{t+1} + \gamma Q(S_{t+1}, A') \\ =& R_{t+1} + \gamma Q(S_{t+1}, \arg\max_{a'} Q(S_{t+1}, a')) \\ =& R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') \end{aligned}$$

Q-learning algorithm for off-policy control

Initialize $Q(s, a), \forall s \in S, a \in \mathcal{A}(s)$, arbitrarily, and $Q(terminal - state, \dot{}) = 0$

for each episode **do**

  Intialise S

  for each step of episode **do**

    Choose A from S using policy derived from Q (e.g., ϵ-greedy)

    Take action A, observe R, S'

    $Q(S, A) \leftarrow Q(S, A) + \alpha(R + \gamma \max_{a'} Q(S', a') - Q(S, A))$

    $S \leftarrow S'$

  end for

end for