

1 Chapter 7: Expectation and Variance

1.1 Definition

Definition:

$$E[X] = \begin{cases} \sum_x xP(x), & X \text{ is discrete} \\ \int_x xf(x)dx, & X \text{ is continuous} \end{cases}, Var[X] = E[(X - E[X])^2]$$

1.1.1 Important expectation and variance

	Notation	$E[X]$	$Var[X]$
Discrete	$X \sim Ber(p)$	p	$p(1-p)$
	$X \sim Bin(n, p)$	np	$np(1-p)$
	$X \sim Geo(p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
	$X \sim Pois(\mu)$	μ	μ
Continuous	$X \sim Unif(a, b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
	$X \sim Exp(\lambda)$	λ^{-1}	λ^{-2}
	$X \sim Par(\alpha)$	$\frac{\alpha}{\alpha-1}$	$\frac{\alpha}{(\alpha-1)^2(\alpha-2)}, \text{ for } \alpha > 2$
	$X \sim N(\mu, \sigma^2)$	μ	σ^2

1.2 Change of variables

Exercise 7.3, 7.4

Definition: $E[g(X)] = \begin{cases} \sum_x g(x)P(X=x), & X \text{ is discrete} \\ \int_x g(x)f(x)dx, & X \text{ is continuous} \end{cases}; Var[g(X)] = E[(g(X) - E[g(X)])^2]$

Useful formulas: $Var[X] = E[X^2] - (E[X])^2; E[aX + b] = aE[X] + b; Var[aX + b] = a^2Var[X]$