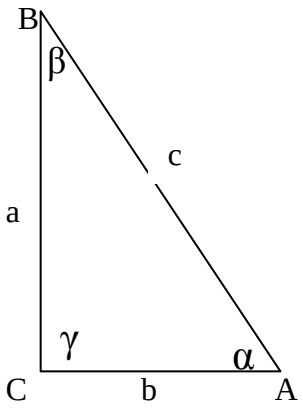
Rumus – rumus Trigonometri



$$\text{Sin } \alpha = \frac{a}{c} = \frac{\text{sisi dihadapan sudut } \alpha}{\text{hipotenusa}}$$

$$\text{Cos } \alpha = \frac{a}{c} = \frac{\text{sisi didekat sudut } \alpha}{\text{hipotenusa}}$$

$$\text{Tan } \alpha = \frac{a}{c} = \frac{\text{sisi dihadapan sudut } \alpha}{\text{sisi didekat sudut } \alpha}$$

1. JUMLAH & SELISIH DUA SUDUT

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$
$$\tan(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha\ .tg\beta}$$
$$\tan(\alpha-\beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha\ .tg\beta}$$

2. SUDUT GANDA

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\tan 2\alpha = \frac{2tg\,\alpha}{1 - tg^2\alpha}$$

3. PERKALIAN SINUS DAN COSINUS

$$2\sin\alpha.\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\cos\alpha.\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$
$$2\cos\alpha.\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$
$$2\sin\alpha.\sin\beta = -\{\cos(\alpha+\beta) - \cos(\alpha-\beta)\}$$

4. JUMLAH & SELISIH PD SINUS & COSINUS

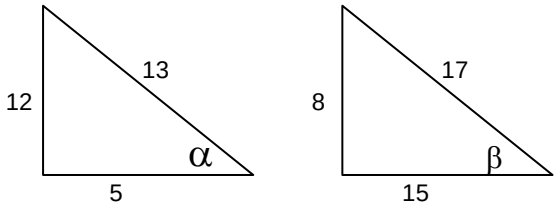
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B).\cos\tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B).\sin\tfrac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B).\cos\tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B).\sin\tfrac{1}{2}(A-B)$$

Contoh SOAL 1

Diketahui $\sin\alpha = \frac{12}{13}$ dan $\tan\beta = \frac{8}{15}$ Carilah nilai :

a. $\sin(\alpha+\beta)$
b. $\cos(\alpha+\beta)$
c. $\tan(\alpha-\beta)$

jwb



a. $\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$$\sin(\alpha+\beta) = \frac{12}{13}.\frac{15}{17} + \frac{5}{13}.\frac{8}{17} = \frac{220}{221}$$

b. $\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$$\cos(\alpha+\beta) = \frac{5}{13}.\frac{15}{17} - \frac{12}{13}.\frac{8}{17} = -\frac{21}{221}$$

c. $\tan(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha\ .tg\beta}$

$$\tan(\alpha+\beta) = \frac{\frac{12}{5}+\frac{8}{15}}{1-\frac{12}{5}.\frac{8}{15}} = \frac{\frac{44}{15}}{1-\frac{32}{25}} = \frac{\frac{44}{15}}{-\frac{7}{25}}$$

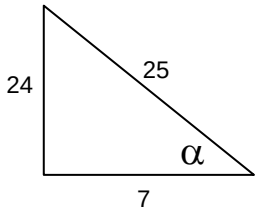
$$= \frac{44}{15}.-\frac{25}{7} = -\frac{220}{21}$$

Contoh SOAL 2

Diketahui $\cos\alpha = \frac{7}{25}$, carilah nilai :

a. $\sin 2\alpha$
b. $\cos 2\alpha$
c. $\tan 2\alpha$
d. $\sin 3\alpha$
e. $\sin 4\alpha$
f. $\cos 4\alpha$
g. $\tan 4\alpha$

jwb



$\sin\alpha = \frac{24}{25}$

$\tan\alpha = \frac{7}{25}$

a. $\sin 2\alpha = 2\sin\alpha.\cos\alpha = 2.\frac{24}{25}.\frac{7}{25} = \frac{336}{625}$

b. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{7}{25}\right)^2 - \left(\frac{24}{25}\right)^2$

$= -\frac{527}{625}$

c. $\tan 2\alpha =$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2.\frac{24}{7}}{1-\left(\frac{24}{7}\right)^2} = \frac{\frac{48}{7}}{-\frac{527}{49}} = -\frac{336}{527}$$

d. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha = 3.\frac{24}{25} - 4\left(\frac{24}{25}\right)^3 = -\frac{10.296}{15.625}$

e. $\sin 4\alpha = 2\sin 2\alpha.\cos 2\alpha = 2.\frac{336}{625}.-\frac{527}{625}$

$= -\frac{354.144}{390.625}$

f. $\cos 4\alpha = 2.\cos^2 2\alpha - 1 = 2.\left(-\frac{527}{625}\right)^2 - 1 = \frac{164.833}{390.625}$

g. $\tan 4\alpha = \frac{2\tan 2\alpha}{1-\tan^2 2\alpha} = \frac{2.\left(-\frac{336}{257}\right)}{1-\left(\frac{335}{527}\right)^2} = \frac{-\frac{672}{527}}{\frac{164.833}{277.729}}$

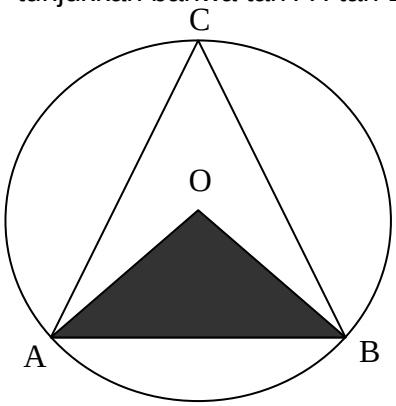
$= -\frac{354.144}{164.833}$

LATIHAN

1. Lengkapilah rumus trigonometri berikut :
   a. Cos $(\alpha+\beta)$ = …
   b. Sin $(\alpha-\beta)$ = …
   c. Tan $(\alpha+\beta)$ = …
   d. Sin 2β = …
   e. Tan 2β = …
   f. Cos $\frac{1}{2}\alpha$ = …

2. Diketahui α dan β adalah sudut lancip.
   Jika sin $\alpha = \frac{3}{5}$ dan cos $\beta = \frac{24}{25}$, hitunglah :
   a. Cos $(\alpha+\beta)$
   b. Sin $(\alpha+\beta)$
   c. Sin 2α
   d. Cos 2α
   e. Sin $\frac{1}{2}\beta$
   f. Cos $\frac{1}{2}\beta$

3. a. Hitunglah nilai dari 2 sin 75 cos 75
   b. Jika 2 cos (A+B) = cos (A – B),
   tunjukkan bahwa tan A . tan B = $\frac{1}{3}$

4. 

Pada gambar disamping, O adalah titik pusat lingkaran luar segitiga ABC. Jika Sin C = $\frac{2}{3}$, hitunglah :
   a) sin ∠AOB
   b) Cos ∠AOB
   c) Tg ∠AOB