MATH-UA.0325-001, Analysis: Midterm Exam

Monday, November 9

Name: ______________________________

Net ID: ______________________________

This is a take home exam. No late submission will be accepted for ANY reason. Please upload the file in standard format and name it

`FirstnameLastname_midterm.pdf.`

The submission is due by Tuesday 10th of November at 11.00 New York time, please be careful with time zones.

Use a pen and not a pencil.

Please respect NYU's Code of Academic Integrity:
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Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

To any completely blank exercise will be assigned 2 points.

1. Give an example of a series $\sum_{n=1}^{\infty} a_n$ which is convergent but not absolutely convergent.

3. Let $\{x_n\}$ be recursively defined as

$$\begin{cases} x_{n+1} = x_n - x_n^2 \\ x_1 = \frac{1}{2} \end{cases}$$

a) Prove that x_n is a decreasing sequence.

b) Prove by induction that $x_n \geq 0$ for all $n \in \mathbb{N}$.

c) Prove that $\lim_n x_n$ exists (you can appeal to known theorems).

d) Find $\lim_n x_n$.

3. Let $\{x_n\}$ be a sequence such that

$$|x_{n+1} - x_n| \leq \frac{1}{n^2}.$$

Prove that $\{x_n\}$ is Cauchy.

4. Prove or disprove (by providing a counterexample) the following statements:

(i) Let $\{x_n\}$ and $\{y_n\}$ be two sequences. If $\{x_n\}$ is bounded and

$$\lim_n y_n = 0,$$

then

$$\lim_n x_n y_n = 0.$$

(ii) Let $\{x_n\}$ and $\{y_n\}$ be two sequences. If $\{x_n\}$ is bounded and

$$\lim_n y_n = +\infty,$$

then

$$\lim_n x_n y_n = +\infty.$$