MATH-UA.0325-001, Analysis: Midterm Exam

Monday, November 9

Name: _____

Net ID: _____

This is a take home exam. No late submission will be accepted for ANY reason. Please upload the file in standard format and name it

`FirstnameLastname_midterm.pdf.`

The submission is due by Tuesday 10th of November at 11.00 New York time, please be careful with time zones.

Use a pen and not a pencil.

Please respect NYU's Code of Academic Integrity:
https://www.nyu.edu/about/policies-guidelines-compliance/policies-and-guidelines/academic-integrity-for-students-at-nyu.html

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

To any completely blank exercise will be assigned 2 points.

1. Is the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n(n-1)^2}}$$

absolutely convergent?

2. Let
$$S = \left\{ \frac{1-k}{k} : k \in \mathbb{N} \right\} \cup \left\{ \frac{1}{k} : k \in \mathbb{N} \right\}.$$
Find sup S and inf S. Is 0 a cluster point for S? Every statement must be proved.

3. Let x_n be a sequence such that for all $n \in \mathbb{N}$:

$$\frac{1}{\sqrt{n}} \le x_{n+1} - x_n.$$

Prove that $\lim_{n \to \infty} x_n = +\infty$.

4. Prove the following statement:

Let x_n be a sequence such that $x_n \neq 0$ and $\lim_{n\to\infty} x_n = +\infty$. Then

$$\lim_{n\to\infty} \frac{1}{x_n} = 0.$$