Math 325, Analysis: Midterm Practice

Name: ______________________________

This is a closed book, closed notes exam. No study aides are allowed.

Please write the solution for each problem on the front (and back, if needed) of the page where the problem is written.

You have 65 minutes to complete the exam. To receive full credit, write legibly and in complete English sentences. All assertions must be justified with a proof.

Scratch paper will be provided for your convenience, but will not be graded.

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Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

To any completely blank exercise will be assigned 2 points.

1. Is the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n(n+1)}}$$

convergent? Is it absolutely convergent?

2. Let S be defined by

$$S = \{2\} \cup \left\{ \frac{1}{k^2} : k \in \mathbb{N} \right\}.$$

Find $\inf S$ and $\sup S$.

3. Let $\{x_n\}$ be recursively defined as

$$\begin{cases} x_{n+1} = x_n - x_n^2 \\ x_1 = \frac{1}{2} \end{cases}$$

a) Prove that x_n is a decreasing sequence.

b) Prove by induction that $x_n \geq 0$ for all $n \in \mathbb{N}$.

c) Prove that $\lim_n x_n$ exists (you can appeal to known theorems).

d) Find $\lim_n x_n$.

4. Prove or disprove (by providing a counterexample) the following statement:

let $\{x_n\}$ and $\{y_n\}$ be two sequences of strictly positive numbers such that $\{x_n\}$ converges and $\{x_n \cdot y_n\}$ converges. Then $\{y_n\}$ converegs.