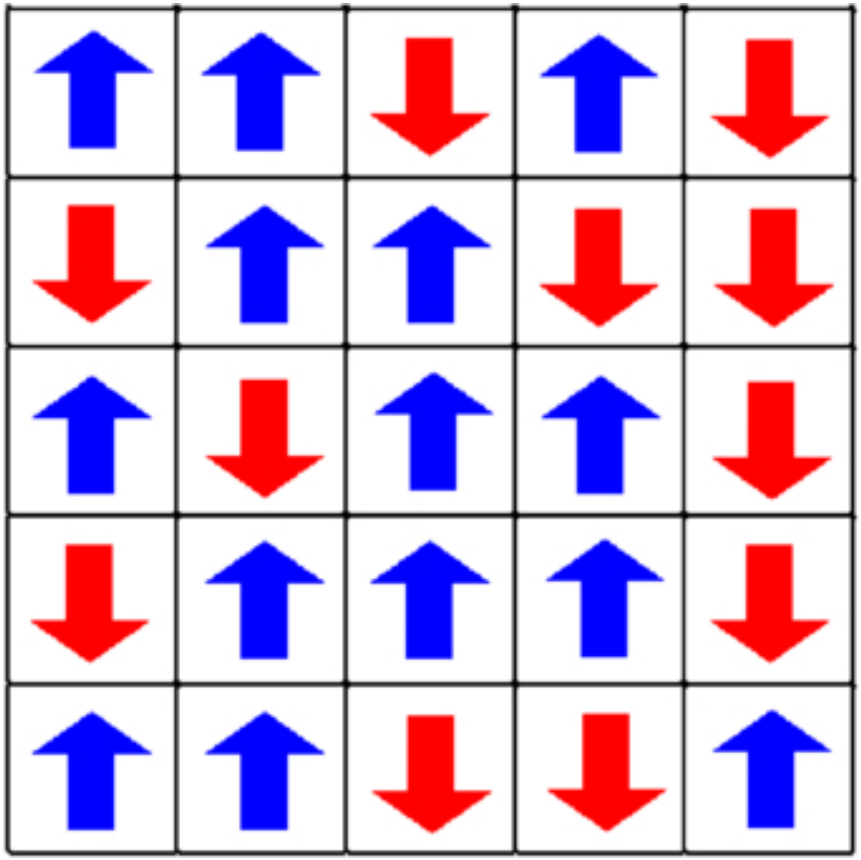
MCMC Methods for Ising Model Simulations

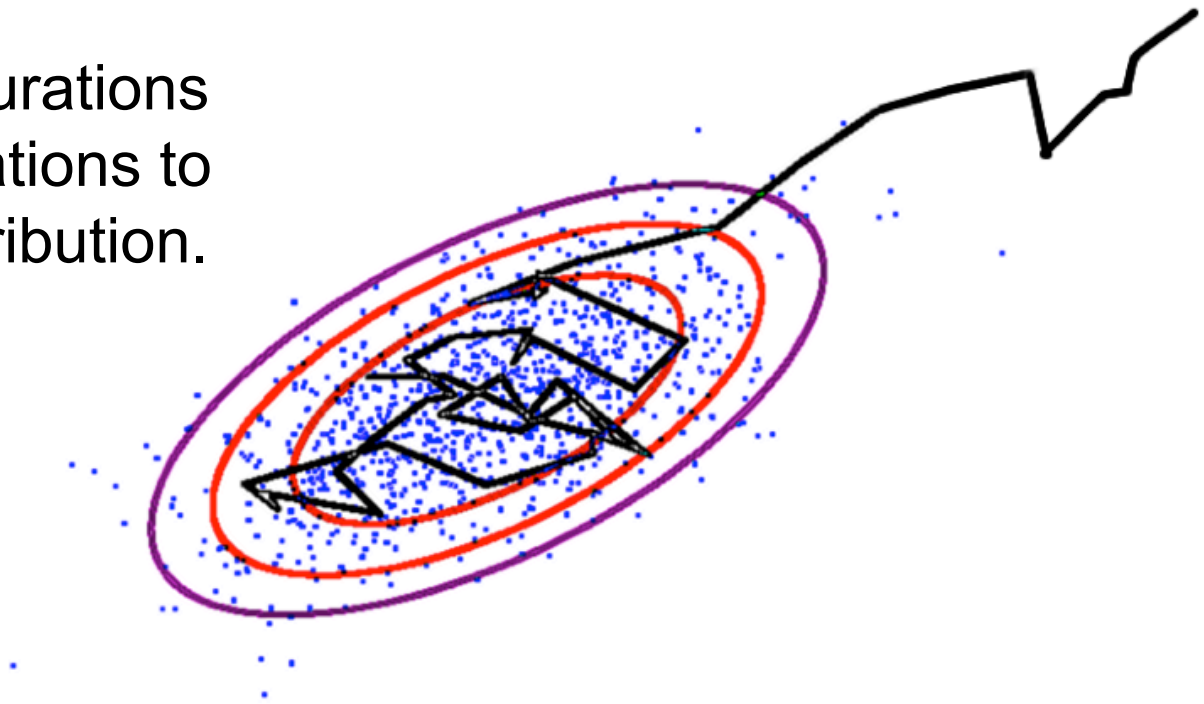
by Marc Williamson



What is MCMC?

•Efficiently estimate difficult calculations using importance sampling.

•Markov Chain configurations generated by perturbations to match underlying distribution.



Metropolis Algorithm

(1) Randomly choose a spin s_i

(2) Compute energy change ΔE due to flipping sign of s_i.

(3) If $\Delta E < 0$, accept the spin flip.

(4) If $\Delta E \geq 0$, accept with probability $P = e^{-\beta \Delta E}$

(5) Repeat

Swendsen Wang Algorithm

(1) Aligned neighboring spins form a bond
with probability $P = 1 - \exp(-2\beta)$

(2) All spins connected through bonds form a cluster.

(3) Every cluster is flipped with probability $\frac{1}{2}$.

(4) All bonds are erased.

(5) Repeat.

Wolff Algorithm

(1) Spin i is selected at random.

(2) All aligned nearest neighbors are added to spin i's cluster with $P = 1 - \exp(-2\beta)$.

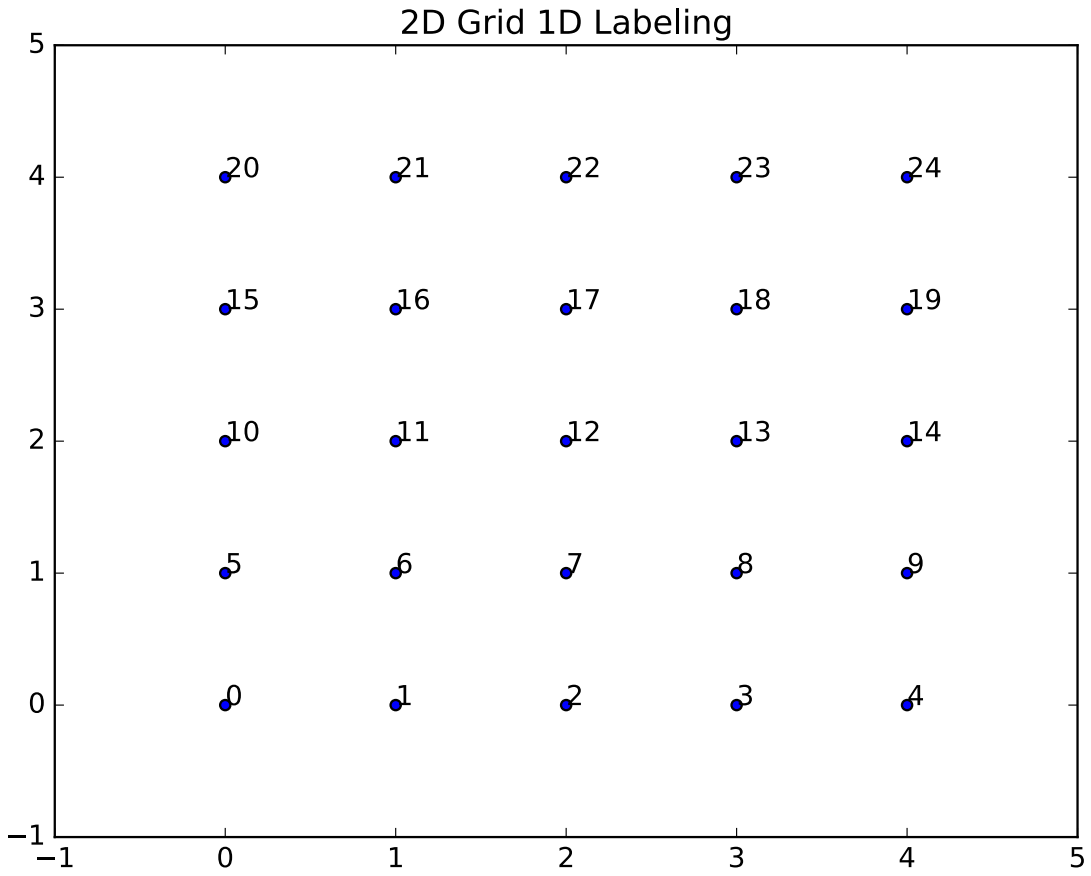
(3) Spin i cluster is grown recursively.

(4) All spins in the cluster are flipped.

(5) Repeat.

Un-**ravel**-ing Dimension Generality

•Use Numpy's unravel_index and ravel_multi_index to iterate over grid.



Critical Temperature

•Critical Temperature at inflection point.

•T=2.27 which matches theoretical value.