NYU TANDON SCHOOL OF ENGINEERING

Three-body problem involved Jupiter

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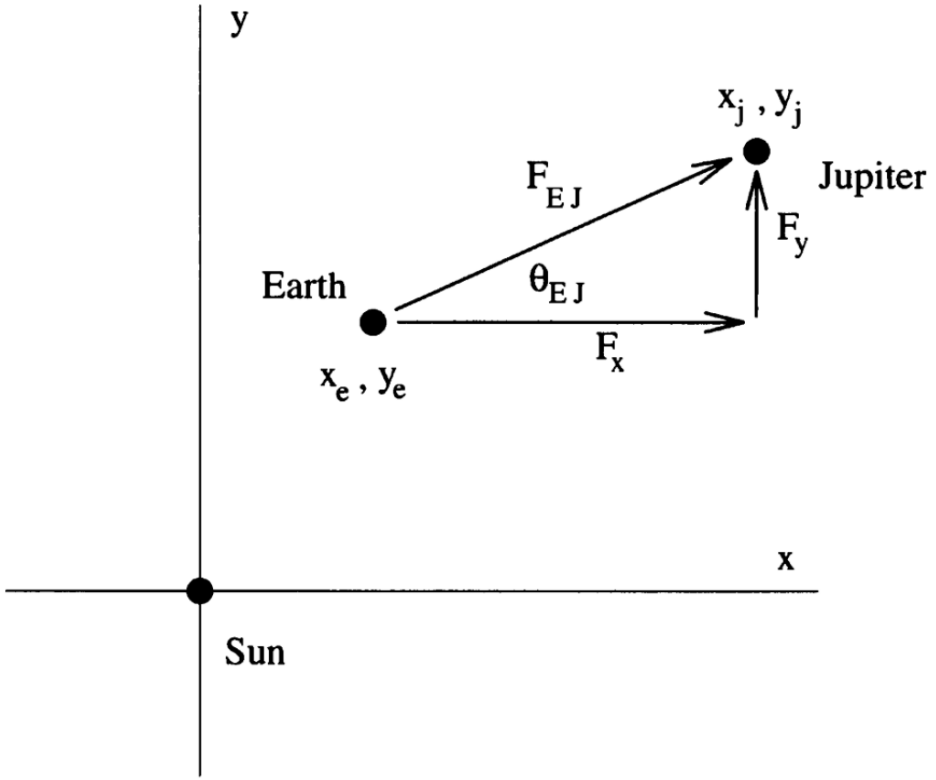
Outline

- **Sun-Earth-Jupiter**
  - The simplest three-body problem
- **Kirkwood gap**
  - Example of resonances in the solar system

- **The Sun-Earth-Jupiter**



$$F_{E,J} = \frac{G M_J M_E}{r_{EJ}^2}$$

$$F_{EJ,x} = -\frac{G M_J M_E}{r_{EJ}^2} \cos\theta_{EJ} = -\frac{G M_J M_E (x_e - x_j)}{r_{EJ}^3}$$

$$\frac{dv_{x,e}}{dt} = -\frac{G M_S x_e}{r^3} - \frac{G M_J (x_e - x_j)}{r_{EJ}^3}$$

Euler-Cromer Method (Semi-implicit Euler Method)

$$v_{n+1} = v_n + a_n \Delta t,$$

$$x_{n+1} = x_n + v_{n+1} \Delta t.$$

$$v_E = \frac{2\pi r}{T} = \frac{2\pi * 1AU}{1\ year} = 2\pi \frac{AU}{year}$$

$$\frac{M_E v_E^2}{r} = F_G = \frac{GM_S M_E}{r^2}$$

$$GM_S = v_E^2 r = 4\pi^2$$

For Earth:

$$v_{Ex,i+1} = v_{Ex,i} - \frac{4\pi^2 x_{Ei}}{r_{Ei}^3}\Delta t - \frac{4\pi^2 \frac{M_J}{M_S}(x_{Ei} - x_{Ji})}{r_{EJ,i}^3}\Delta t$$

$$x_{Ei+1} = x_{Ei} + v_{Ex,i+1}\Delta t$$

$$v_{Ey,i+1} = v_{Ey,i} - \frac{4\pi^2 y_{Ei}}{r_{Ei}^3}\Delta t - \frac{4\pi^2 \frac{M_J}{M_S}(y_{Ei} - y_{Ji})}{r_{EJ,i}^3}\Delta t$$

$$y_{Ei+1} = y_{Ei} + v_{Ey,i+1}\Delta t$$

The orbits of Jupiter and Earth

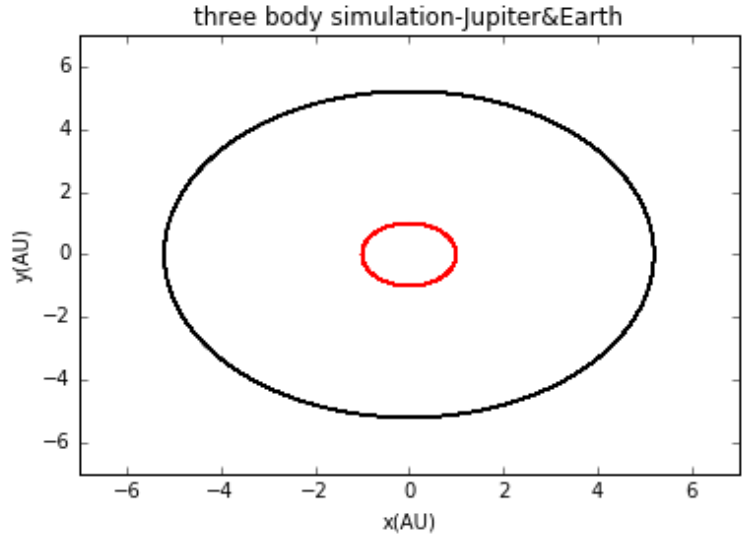


Fig a. Jupiter has its true mass

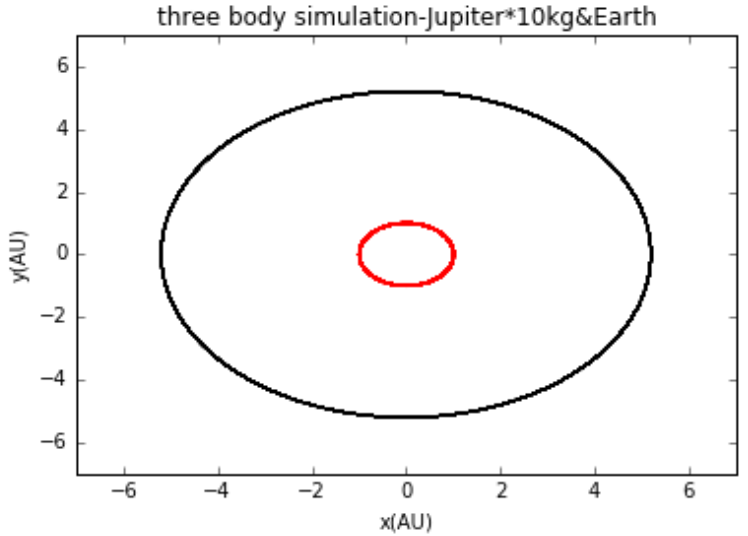


Fig b. Jupiter has 10 times its true mass

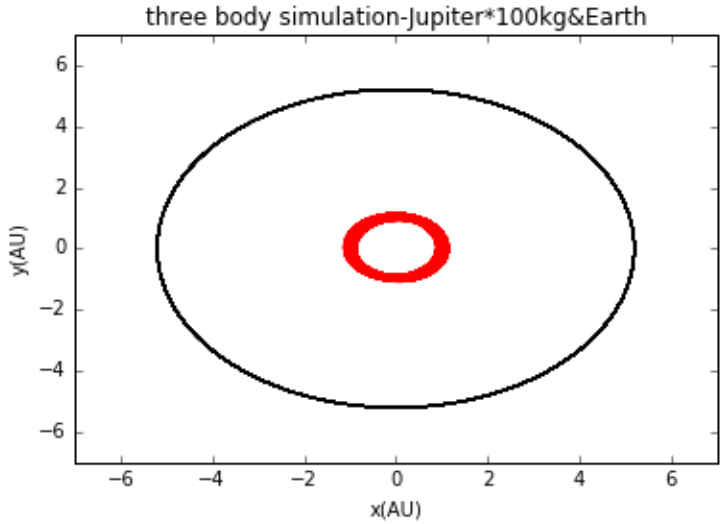
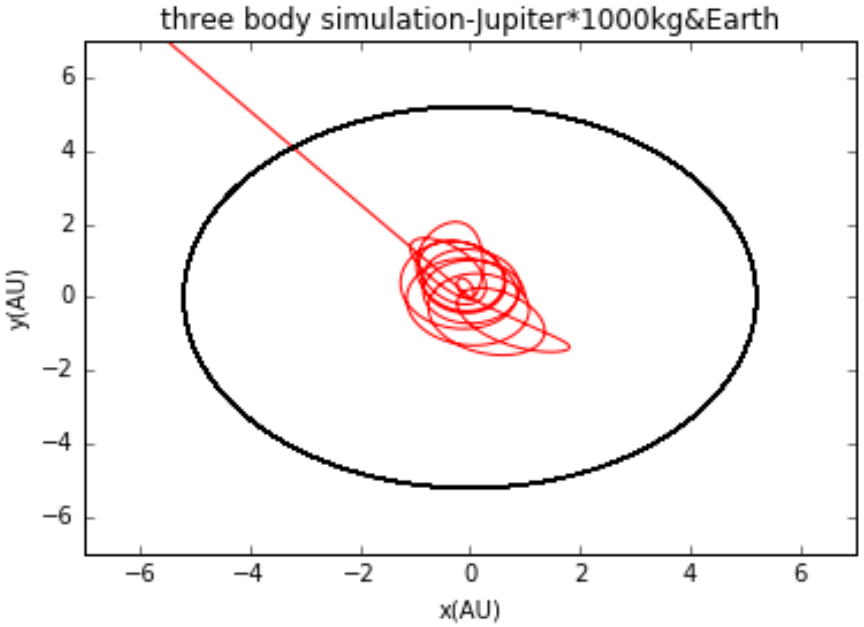
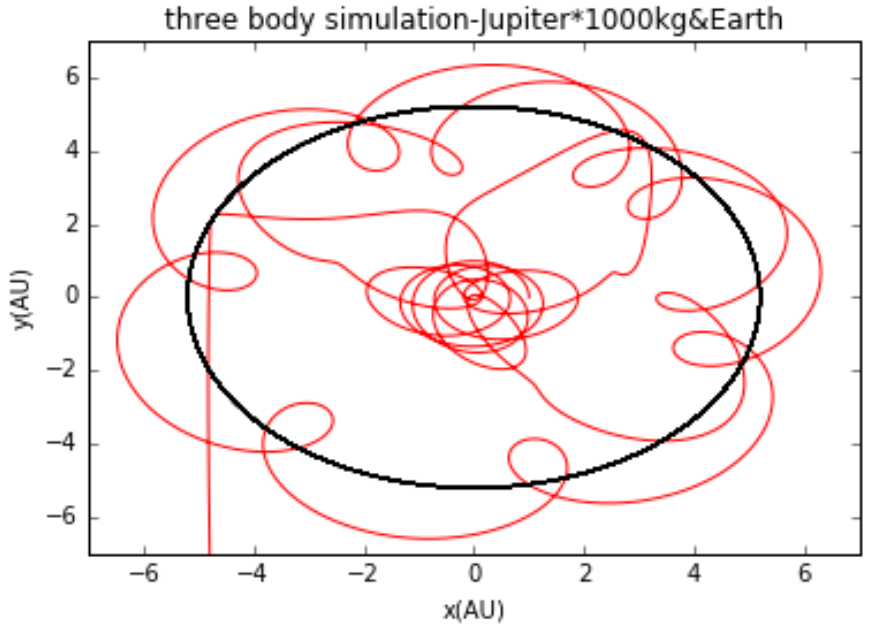


Fig c. Jupiter has 100 times its true mass

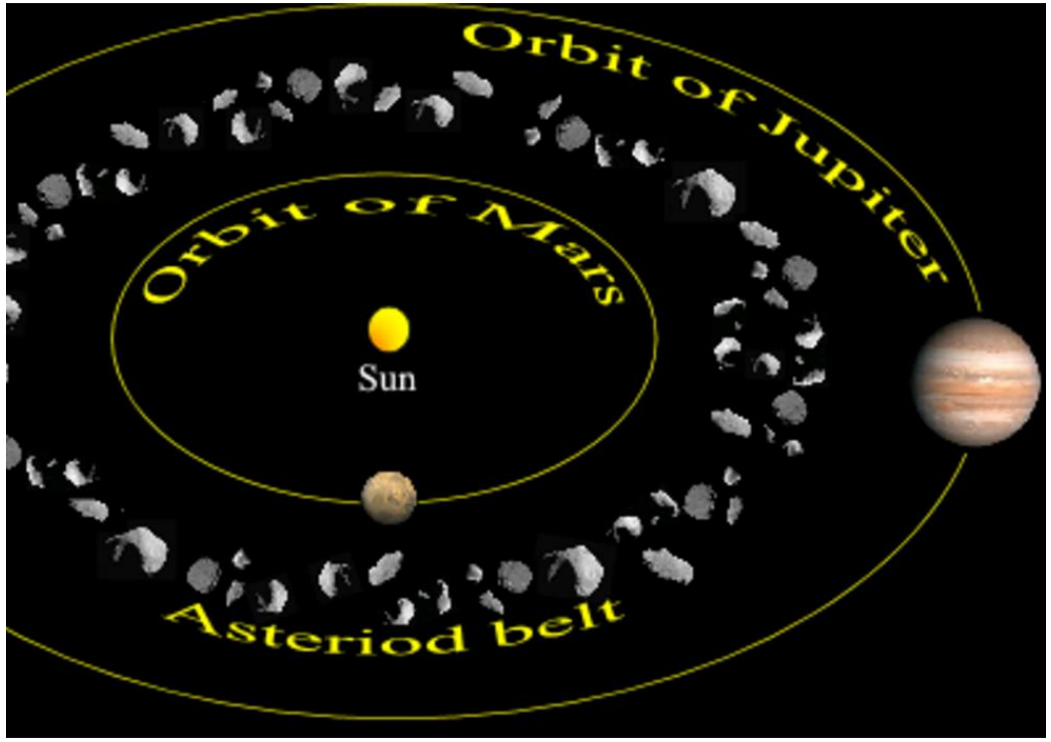
- mass of Jupiter is quite large → Earth's orbit completely unstable
- Trajectory is very sensitive to the initial conditions



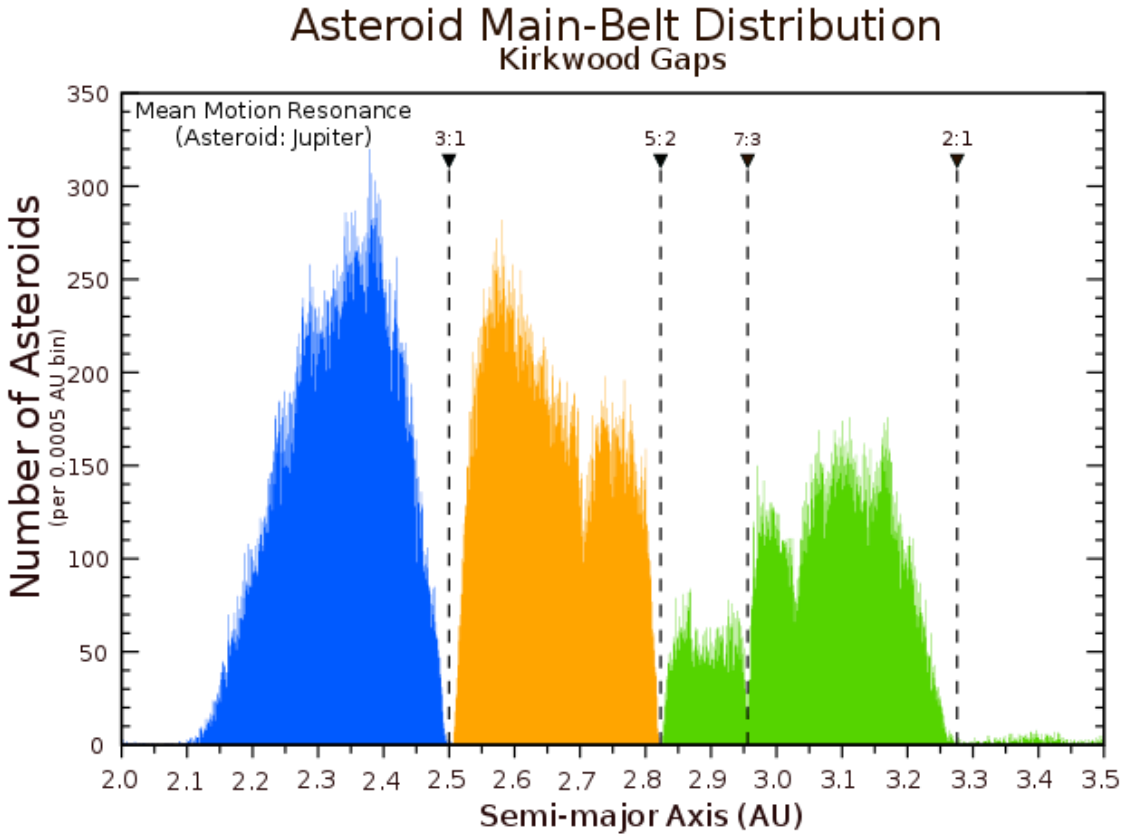


Jupiter has 1000 times its mass (different initial conditions)

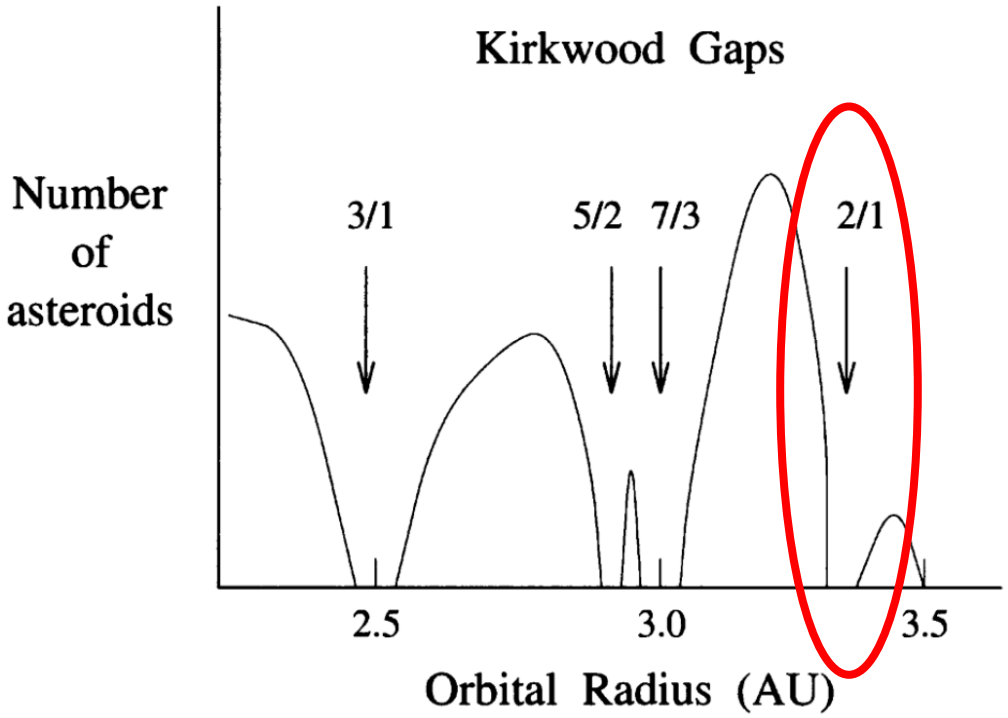
- **Kirkwood gap**



The orbit radius of gap is in resonance with Jupiter



Asteroids in the vicinity of 2/1 Kirkwood gap



Object	Radius (AU)	Velocity (AU/yr)
Asteroid number 1	3.000	3.628
Asteroid number 2	3.276	3.471
Asteroid number 3	3.700	3.267
Jupiter	5.200	2.755

Note:
The mass of asteroid is quite small compared to the mass of Jupiter, thus we can neglect the gravitational force of each asteroid when computing the motion of Jupiter.(And ignore the interactions between asteroids.)

The asteroid in the 2/1 gap is the one affected most strongly by Jupiter

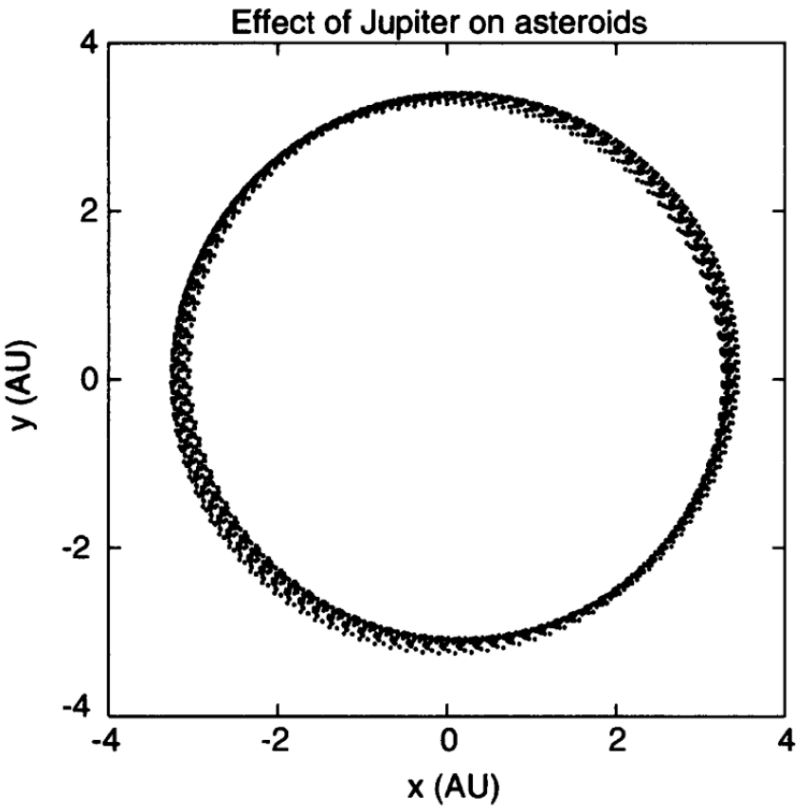


Fig a. The orbit of asteroid number 2

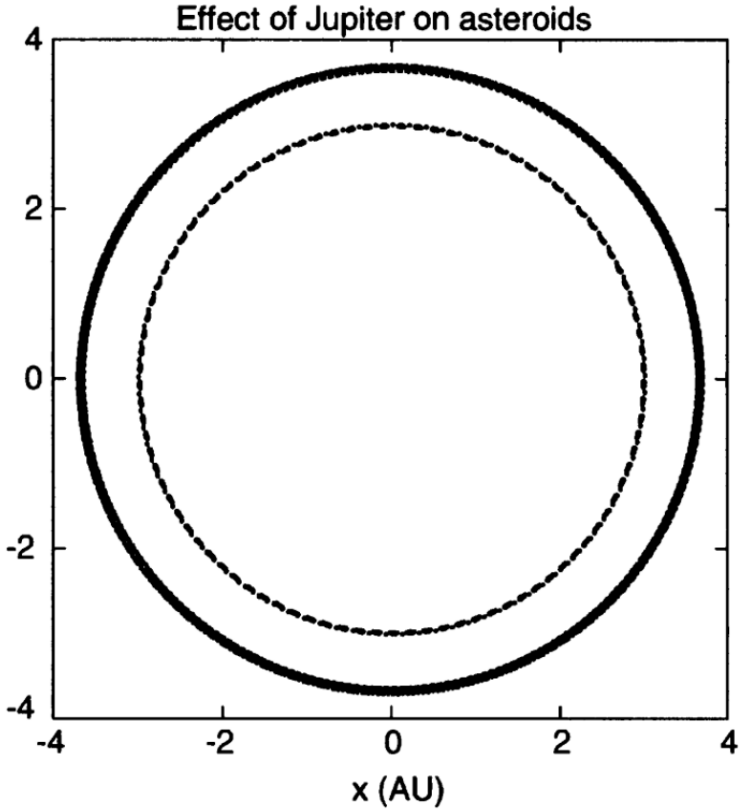
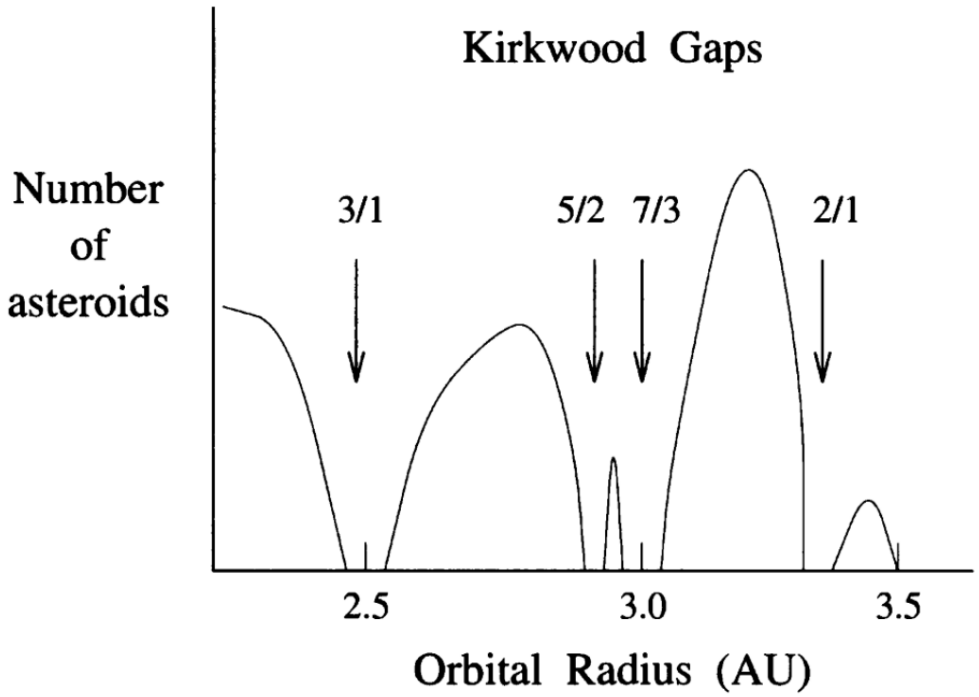


Fig b. The orbit of asteroid number 1 and 3

The table of Kirkwood gap



Resonance	Orbital Period(yr)	Orbital Radius(AU)	Orbital Velocity
3:1	3.95	2.5	3.977
5:2	4.74	2.825	3.745
7:3	5.08	2.95	3.649
2:1	5.925	3.276	3.471

The orbital period of Jupiter=11.85 year

$$v = \frac{2\pi r}{T}$$

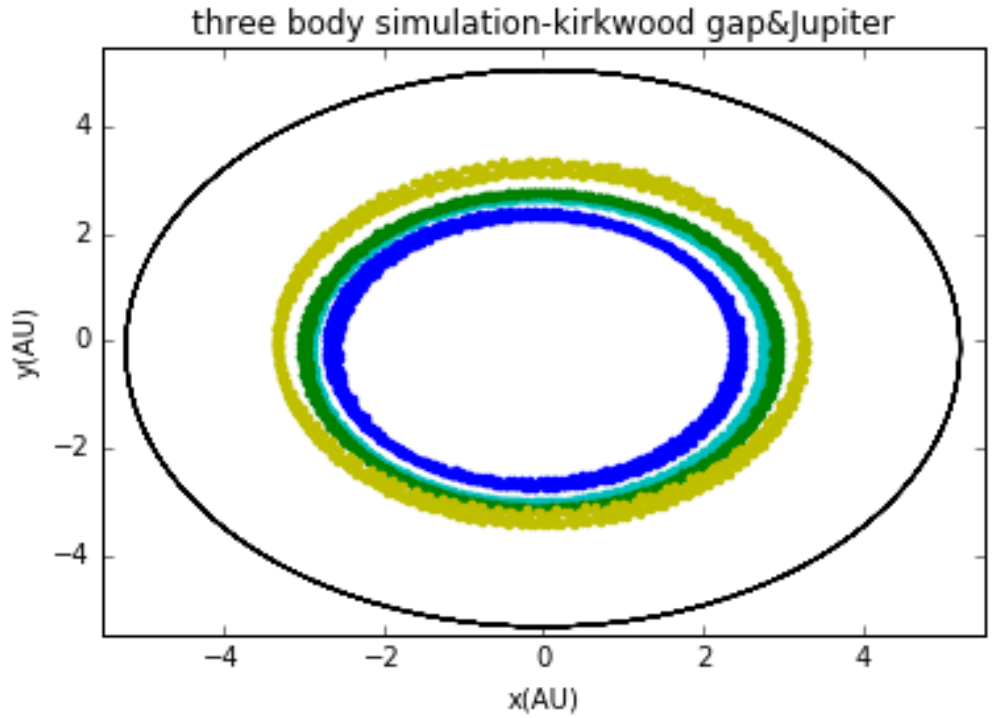


Fig a. The orbits of Jupiter and asteroids in Kirkwood gap

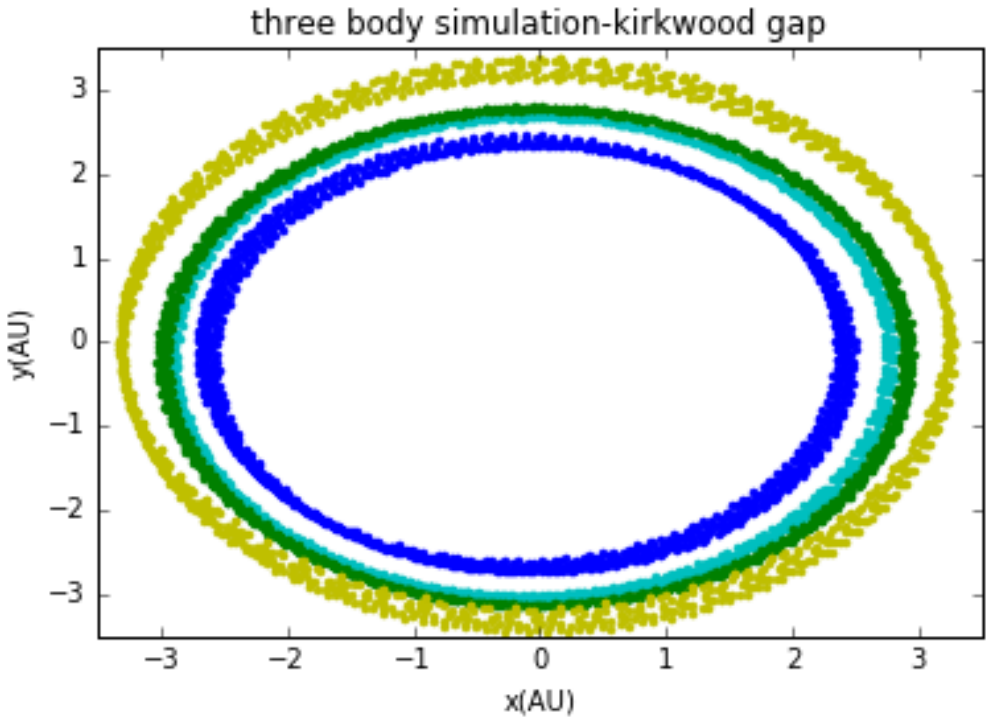


Fig b. The orbits of asteroids in Kirkwood gap

Thank you!