Homework 2

Due: Sept 27, 2016

Write python functions to calculate integrals using the (right-biased) Riemann Sum, Trapezoid Rule, Simpson's Rule, Romberg integration, and Gaussian Quadrature. Use the provided `gaussxy.py` code to calculate the points and weights for Gaussian Quadrature. Test your methods using the following integrals:

$$\int_0^{4\pi} x^2 \cos(x)\, \mathrm{d}x \tag{1}$$

$$\int_0^{100\pi} x^2 \cos(x)\, \mathrm{d}x \tag{2}$$

$$\int_0^{1} \sqrt{x}\, \mathrm{d}x \tag{3}$$

For each function and method calculate the L_1 error (absolute value of the difference between the true value and the numerical approximation) for a number of sample points (N). Using this data, calculate the convergence rate p ($L_1 \propto N^{-p}$) for each method on each integral.

Write a LaTeX report discussing your results. It should include a short explanation of the algorithms with all relevant formula, convergence plots for each test showing the L_1 error versus N for all methods, and a discussion that answers the following questions:

- Do your empirical convergence rates match theoretical expectations? If not, what properties of the integral lead to the change in behaviour?
- Do the methods behave as expected for both small and large values of N? Make sure to try both very small and very large values of N.
- At what N do the methods become dominated by rounding error? Does this agree with theoretical predictions?
- For function (1), are the error estimates for Trapezoid Rule and Simpson's Rule correct?
- Look in detail at the convergence of Romberg Integration and Gaussian Quadrature. Do they converge as a power law at all? Discuss.

Include the report `.tex` file and all Python files in the repo. Also include either the `.pdf` version of the report, or all figures necessary to compile it from the `.tex` file.