

# Homework 2

Due: Sept 26, 2016

Write python functions to calculate integrals using the (right-biased) Riemann Sum, Trapezoid Rule, Simpson's Rule, and Romberg integration. Test your methods using the following integrals:

$$\int_0^{4\pi} x^2 \cos(x) \, dx \quad (1)$$

$$\int_0^{100\pi} x^2 \cos(x) \, dx \quad (2)$$

$$\int_0^1 \sqrt{x} \, dx \quad (3)$$

For each function and method calculate the  $L_1$  error (absolute value of the difference between the true value and the numerical approximation) for a number of sample points ( $N$ ). Using this data, calculate the convergence rate  $p$  ( $L_1 \propto N^{-p}$ ) for each method on each integral.

Write a  $\text{\LaTeX}$  report discussing your results. It should include a short explanation of the algorithms with all relevant formula, convergence plots for each test showing the  $L_1$  error versus  $N$  for all methods, and a discussion that answers the following questions:

- Do your empirical convergence rates match theoretical expectations? If not, what properties of the integral lead to the change in behaviour?
- Do the methods behave as expected for both small and large values of  $N$ ? Make sure to try both very small and very large values of  $N$ .
- At what  $N$  do the methods become dominated by rounding error? Does this agree with theoretical predictions?
- For function (1), are the error estimates for Trapezoid Rule and Simpson's Rule correct?
- Look in detail at the convergence of Romberg Integration. Does it converge as a power law at all? Discuss.

Include the report `.tex` file and all Python files in the repo. Also include either the `.pdf` version of the report, or all figures necessary to compile it from the `.tex` file.