Computational Physics HW 1

Maaz ul Haq
Department of Physics, New York University
(Dated: September 15, 2017)

In this article we test two python codes. One for generating the Mandelbrot plot and the other to apply least squares method to Millikan's experimental data on photoelectric effect. 3 plots for the former are displayed on a 3000×3000 pixel resolution and $[-2, 2] \times [-2, 2]$ grid. For the latter, the data points are displayed on a graph and a least squares line is fitted on to the graph from which the slope and the intercept is inferred.

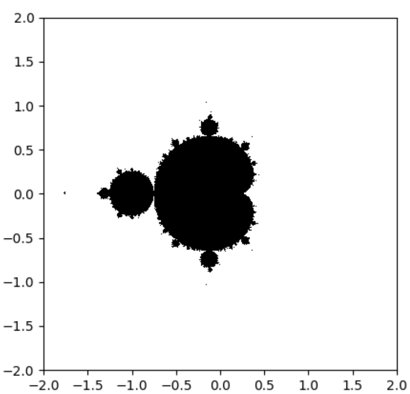
I. THE MANDELBROT SET

The Mandelbrot Set is defined as the set of all $c \in \mathbb{C}$ such that $|z_n| \leq 2 \quad \forall n \in \mathbb{N}_0$ where $z_{n+1} = {z_n}^2 + c$ and $z_0 = 0$. The plot of the set exhibits a fractal structure where a self-similar structures repeats at all scales.

The intent of this exercise is to:

1. Plot the Mandelbrot set in black (for points in Mandelbrot set) and white (otherwise) on a $[-2, 2] \times [-2, 2]$ grid at a 3000×3000 pixel resolution.

FIG. 1. Mandelbrot plot as mentioned in 1.



2. Plot the points outside of the Mandelbrot set in Jet color scheme based on the number of iterations it takes for $|z| > 2$ on a $[-2, 2] \times [-2, 2]$ grid at a 3000×3000 pixel resolution.

3. Plot the points outside of the Mandelbrot set in Hot color scheme based on the logarithm of the number of iterations it takes for $|z| > 2$ on a $[-2, 2] \times [-2, 2]$ grid at a 3000×3000 pixel resolution.

The exercise is straightforward to code. A nested for loop was used in all 3 cases which was set to break once $|z| > 2$ condition was achieved.

FIG. 2. Mandelbrot plot as mentioned in 2.

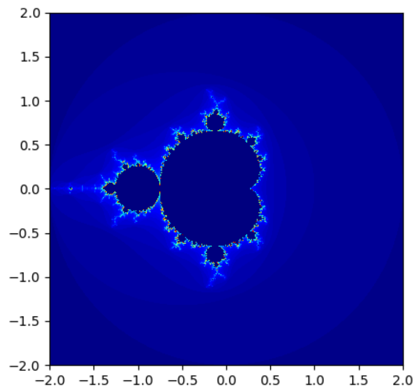
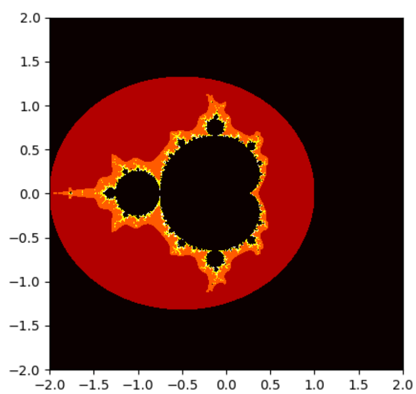


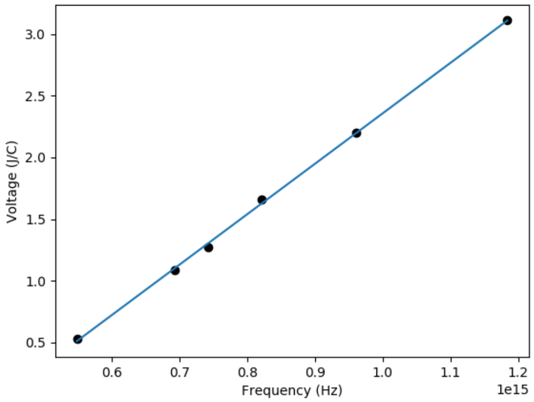
FIG. 3. Mandelbrot plot as mentioned in 3.



II. LEAST SQUARES LINE FIT ON MILLIKAN'S DATA ON PHOTOELECTRIC EFFECT

The method of least squares is a regression method to fit a straight line onto a given set of points where the sum of the squares of the errors between the data and the line is minimized. Following the convention of the book, suppose we have N data points and we set the line to be $y_i = mx_i + c$. Then, the sum of the squares of the

FIG. 4. Millikan's data on Photoelectric Effect and Least Squares line fit



errors between the i-th data point and the line is

$$\chi^2 = \sum_{i=1}^{N} \left(m{x_i}^2 + c - y_i \right)$$

Differentiating with respect to m and c and equating to zero, we get the optimization equations for minimizing error.

$$m \sum_{i=1}^{N} {x_i}^2 + c \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i = 0$$

$$m \sum_{i=1}^{N} x_i + cN - \sum_{i=1}^{N} y_i = 0$$

For convenience, we set

$$E_x = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad E_y = \frac{1}{N} \sum_{i=1}^{N} y_i$$

$$E_{xx} = \frac{1}{N} \sum_{i=1}^{N} {x_i}^2 \qquad E_{xy} = \frac{1}{N} \sum_{i=1}^{N} x_i y_i$$

Then, solving for the optimization equations for m and c, we get

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - {E_x}^2} \qquad c = \frac{E_{xx} E_y - E_x E_x y}{E_{xx} - {E_x}^2}$$

Using these equations and variables in the code, it is easy to plot a least squares line fit on a graph of N data points.

The data points we are provided with are the ones obtained by Robert Millikan in his experiment to measure the Photoelectric effect where he measured the stopping potential for the electron against the frequency of light shown on the metal surface. The relation between the stopping potential V and the frequency ν, due to Einstein in 1905, is

$$V = \frac{h}{e} \nu - \phi$$

where h is Planck's constant, e is the charge of an electron and ϕ is the work function.

Using the code and the list of given data points, we plot the points on a graph and draw the line of least squares as required.

The slope of the least squares line corresponds to h/e in the equation for Photoelectric effect. The slope of the line is found to be $m = 4.088 \times 10^{-15}$ and it is given that $e = 1.602 \times 10^{-19}$ C. This implies that $h_{Millikan} = 6.549 \times 10^{-34}$ kg m^2 s^{-1} which deviates 1.157% from today's accepted value of $h = 6.626 \times 10^{-34}$ kg m^2 s^{-1}.