Report of Homework 1

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1 The Mandelbrot set

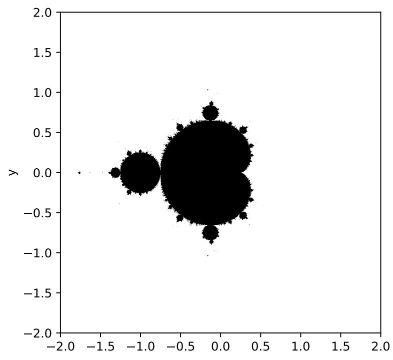
The iteration equation is

$$z' = z^2 + c \tag{1.1}$$

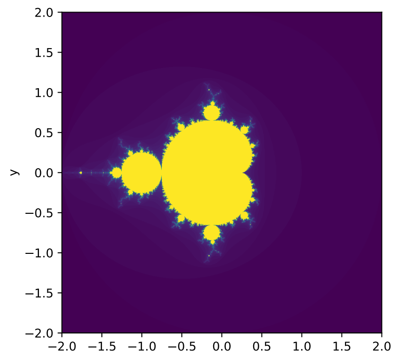
The definition of the Mandelbrot set is

> *For a given complex value of c, start with $z = 0$ and iterate repeatedly. If the magnitude $|z|$ of the resulting value is ever greater than 2, then the point in the complex plane at position c is not in the Mandelbrot set, otherwise it is in the set.*

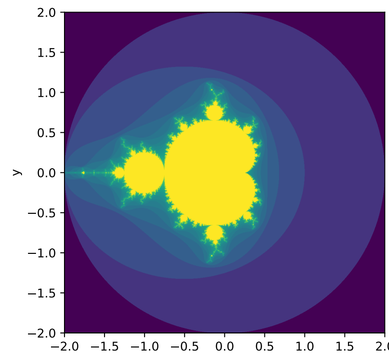
- In my simulation
  - Set the number of iterations as 100. If $|z|$ hasn't exceeded 2 by 100 times of iteration, then put c into the Mandelbrot set.
  - Evaluate $c = x + iy$ based on a 1000×1000 grid spanning the region where $-2 \leq x, y \leq 2$.
- The Result of the simulation



(a) Black points for Mandelbrot set



(b) # of iterations before ≥ 2



(c) log # of iterations

- Discussion
  - If we look into the details of the graphs, we can see that the Mandelbrot set is a fractal with self-similarity.
  - The graph is sysmetric with respect to $y = 0$. This can be easily proved since $(a \pm bi)^2 + (x \pm yi) = (a^2 - b^2 + x) \pm (2ab + y)i$.

2 Least-squares fitting and the photoelectric effect

In this program, the linear function $y = mx + c$ is used to fit the experiment data in file millikan.txt with the method of least squares. With the definition of quantities

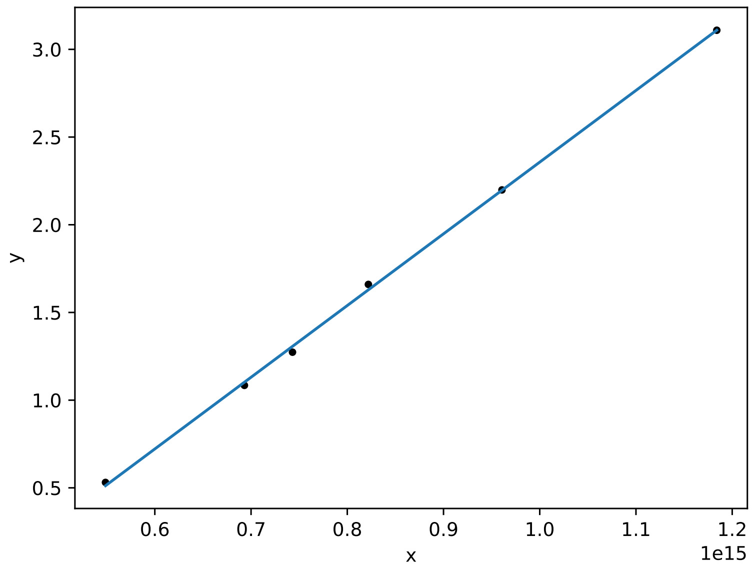
$$E_x = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad E_y = \frac{1}{N}\sum_{i=1}^{N} y_i, \quad E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2, \quad E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i \tag{2.1}$$

m and c can be given as

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2}, \qquad c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2} \tag{2.2}$$

- Result of the program

$$m = 4.0882 \times 10^{-15}, \quad c = -1.7312 \tag{2.3}$$



Given $m = \frac{h}{e}$ and $e = 1.602 \times 10^{-19}$, we can get the value of Planck's constant

$$h = 6.5493 \times 10^{-34} \tag{2.4}$$

Compared with the accepted value 6.62607×10^{-34}, the relative error is

$$\frac{|6.5493 \times 10^{-34} - 6.62607 \times 10^{-34}|}{6.62607 \times 10^{-34}} \times 100\% = 1.2\% \tag{2.5}$$

- Discussion

◦ One problem is the number of significant numbers. This has to be decided according to the original data instead of the result calculated by Python with a long chain of digits.