Computational Physics
Assignment 1

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This report will utilize basic computational methods to visualize The Mandelbrot Set and to graph a linear fit of Millikan's data on the Photoelectric Effect. In the colored map of the Mandelbrot Set, we observe a chaotic behavior near the boundary of points that avoid divergence of the complex function. From the Photoelectric data, we extrapolated a Planck constant of $6.5493 * 10^{-34} J \cdot s$.

BACKGROUND AND THEORY

The Mandelbrot Set

The Mandelbrot Set is typically depicted as an intricate fractal structure on the complex plane. A point c on the complex plane is described by a real (x) and an imaginary coordinate (y):

$$c = x + iy \tag{1}$$

In mathematics, iteration is the repeated process of converting an output of a function into the input of another function or itself. Such problems are best approached computationally. For example, Equation 2 is an iterative function with a complex constant c and a variable z that takes the value of the last z' calculated. Interestingly, for certain values of c and an initial z of 0, the function z does not diverge.

$$z' = z^2 + c \tag{2}$$

For a point c to be considered in the Mandelbrot Set, a finite amount of iterations must always satisfy $|z| \leq 2$. A circle of radius 2 on the complex plane forms a boundary containing all possible c's but only c's that avoid divergence will be mapped. In practice, about 100 successful iterations per value of c is satisfactory and the resolution will mainly depend on the number of c's sampled.

Least-Squares Fit

In 1905, Albert Einstein postulated the existence of quantized packets of light or photons. Soon after and for nearly a decade, Robert Millikan conducted experiments to verify Einstein's prediction. By shining light on the surface of a metal, Millikan was able to detect free conducting electrons ejected from the material. Coined the photoelectric effect, it not only demonstrated a minimum energy requirement to release an electron from a metallic surface but it also hinted at a linear relationship between the frequency of the incident light and the voltage detected.

$$V = \frac{h}{e}\nu - \phi \tag{3}$$

In Equation 3, ν is the frequency of the incident light, h is Planck's constant, e is the charge of the electron, and ϕ is the minimum energy requirement or the work function. With a scatter plot of Millikan's Voltage vs. Frequency data and the charge of the electron, Planck's constant can be worked out. It is common practice to base the fit of N number of points by minimization of χ^2, Equation 4. For a straight line of the form $y = mx + c$, where m is the slope and c is the y-intercept:

$$\chi^2 = \sum_{i=i}^{N} (mx_i + c - y_i)^2 \tag{4}$$

The minimum value of χ^2 is obtained from differentiation with respect to m and c, and setting these two equations to zero.

$$m \sum_{i=1}^{N} x_i^2 + c \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i = 0 \tag{5}$$

$$m \sum_{i=1}^{N} x_i + cN - \sum_{i=1}^{N} y_i = 0 \tag{6}$$

For convenience, the following new terms are defined to simplify Equations 5.

$$\begin{aligned} E_x &= \frac{1}{N} \sum_{i=1}^{N} x_i \quad & E_y &= \frac{1}{N} \sum_{i=1}^{N} y_i \\ E_{xx} &= \frac{1}{N} \sum_{i=1}^{N} x_i^2 \quad & E_{xy} &= \frac{1}{N} \sum_{i=1}^{N} x_i y_i \end{aligned} \tag{7}$$

After some rearranging, below are the slope and y-intercept equations for a best fit for a given set of data points.

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2} \quad c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2} \tag{8}$$

RESULTS AND DISCUSSION

The Mandelbrot Set

Figure 1 depicts the famous Mandelbrot beetle (the red area) on the complex plane. The color gradient bar on the right helps map the number of iterations for each c before failing $|z| \leq 2$. Notice the function fails to iterate outside of the circle of radius 2 and notice c's of higher iterations are clustered on the perimeter of the beetle. The beetle itself encompassed c's that pass 100 iterations and are likely to continue iterating.

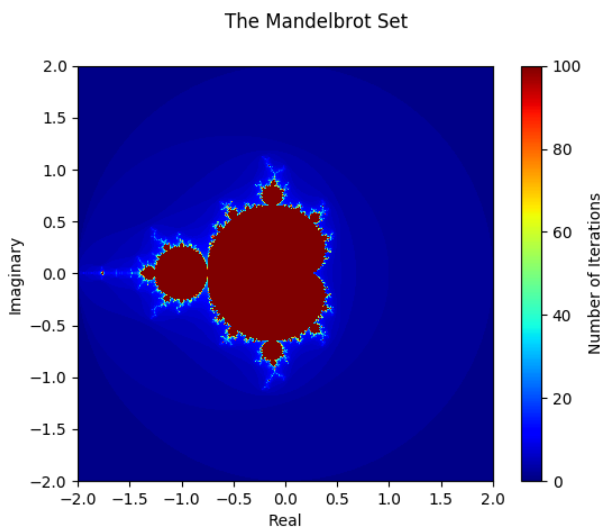

FIG. 1. A colored map of the diverging behavior arising from a complex constant c in an iterative function z.

Below is a magnified area of Figure 1. It is meant to illustrate the infinitely complicated boundary and the fractal details.

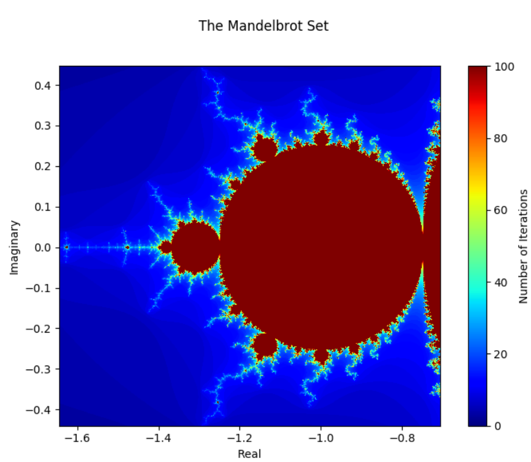

FIG. 2. Magnified area of the Mandelbrot Set

Least-Squares Fit

Millikan's photoelectric measurements are depicted in Figure 3. The light source frequency was measured in Peta-Hertz and the voltage reading of the ejected electrons was measured in Volts. The six data points are plotted in Figure 4 and a linear regression based on Equations 8 was superimposed.

F (PHz)	V (V)
0.5487	0.5309
0.6931	1.0842
0.7431	1.2734
0.8219	1.6598
0.9607	2.1986
1.1840	3.1089

FIG. 3. Millikan's photoelectric effect data. Voltage readings for corresponding light source frequencies.

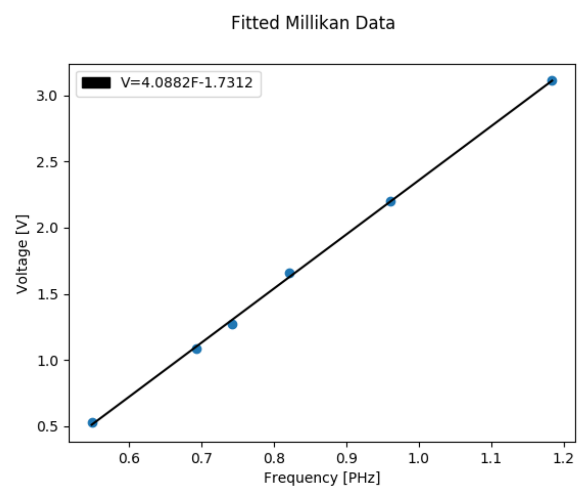

FIG. 4. Millikan's original data (blue markers) graphed along with a calculated least-squares fit (black). The legend contains the equation for the fit.

From the slope of the fit, $4.0882 * 10^{-15}\ V \cdot s$, and the accepted value of electron charge, $1.602 * 10^{-19}\ C$, the experimental Planck's constant was found to be $6.5493 * 10^{-34} J \cdot s$. This value is about 1.16% off from the accepted value of $6.6261 * 10^{-34} J \cdot s$. Therefore, Millikan's data is in good agreement with theory.

CONCLUSION

Computational methods are an essential staple of modern physics research. In the first half, function iteration and visualization of The Mandelbrot Set through coding proved effective. In the second half, linear regression and extrapolation of existing data also proved successful.

REFERENCES

[1] Newman, M. (2013). *Computational physics* (pp. 122-125). Createspace.