

# CompPhys HW 1

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**Abstract.** HW 1 consisted of two problems. The goal of the first was to plot the Mandelbrot fractal. The goal of the second was to fit data from the Millikan oil drop experiment using the method of least squares, and subsequently use the photoelectric effect equation to compare the results with the established value of Planck's constant.

## I. INTRODUCTION

### A. Problem 3.7

The Mandelbrot set consists of all of the complex numbers,  $c$ , that are not eliminated by the following algorithm: (1) Starting with  $z = 0$ , obtain  $z' = z^2 + c$ . (2) If  $|z'| > 2$ , then eliminate  $c$ , otherwise, obtain  $z'' = (z')^2 + c$  and check the magnitude of that. (3) Continue the iteration process, throwing away all  $c$ 's that give a resultant  $z^{(k)}$  magnitude greater than 2 at any point.

### B. Problem 3.8

When trying to fit a straight line to measured data, least squares fitting is often used. The least squares fit is the line with slope  $m$  and intercept  $c$  that minimizes the sum of the squared vertical distances between the data points and the line given by  $y = mx + c$ . The sum is minimized by setting the first derivative equal to zero.

For this problem in particular, the least squares fit is used on the data provided from Millikan's experiment demonstrating the photoelectric effect. From the least squares fit parameters, one can use the photoelectric effect equation,  $V = \frac{h}{e}\nu + \phi$  (where  $V$  is the stopping potential,  $h$  is Planck's constant,  $e$  is the electron charge,  $\nu$  is the frequency, and  $\phi$  is the work function), to obtain a value for Planck's constant, which can then be compared to the modern established value to determine the accuracy of the results.

## II. MATERIALS AND METHODS

Both problems are completed using code written in Python3. For more details, refer to the Jupyter notebooks (hw1-3.7.ipynb and hw1-3.8.ipynb) included in the repo.

## III. RESULTS

### A. Problem 3.7

Figure 1 is the black and white Mandelbrot fractal, with black denoting the complex numbers belonging to the set, and white denoting those outside of the set.

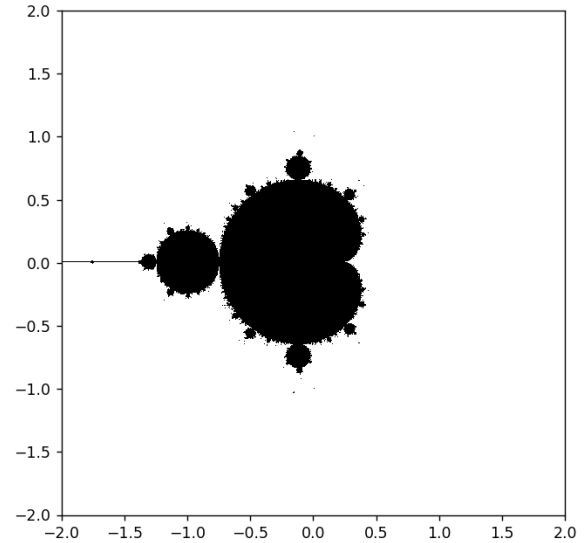


FIG. 1. Black and white Mandelbrot fractal. Resolution is 1000x1000. Number of iterations is 100. Time taken to run is 28.7 seconds.

Figure 2 is the Mandelbrot fractal with the magma colormap, scaled such that the color represents how many iterations were required to discard the number from the set.

### B. Problem 3.8

Figure 3 shows the line obtained from the least squares fit juxtaposed with the actual data points (represented as black dots) given in the millikan.txt file. The slope of the line is  $m = 4.08822735852e - 15$  and the intercept is  $c = -1.73123580398$ . The value obtained for Planck's constant is  $h = 6.54934022835e - 34$  (in units of  $J\cdot s$  or  $kgm^2/s$ ).

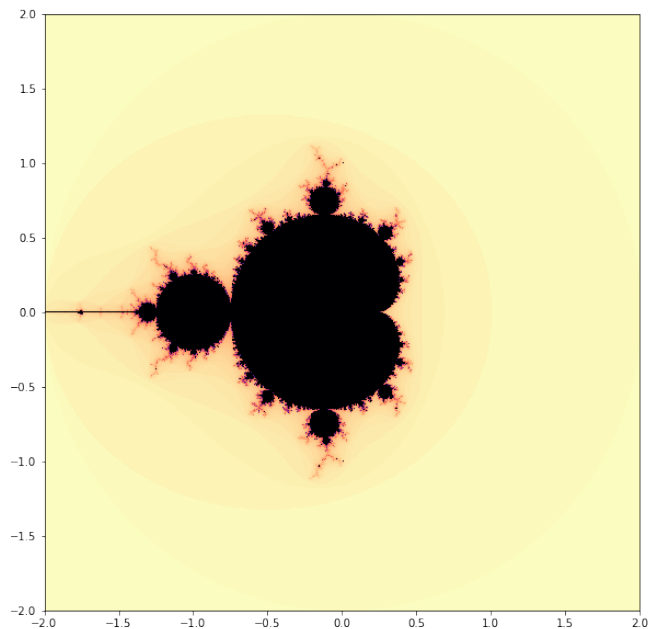


FIG. 2. Magma colormap Mandelbrot fractal. Resolution is 1000x1000. Number of iterations is 100. Time taken to run is 27.5 seconds.

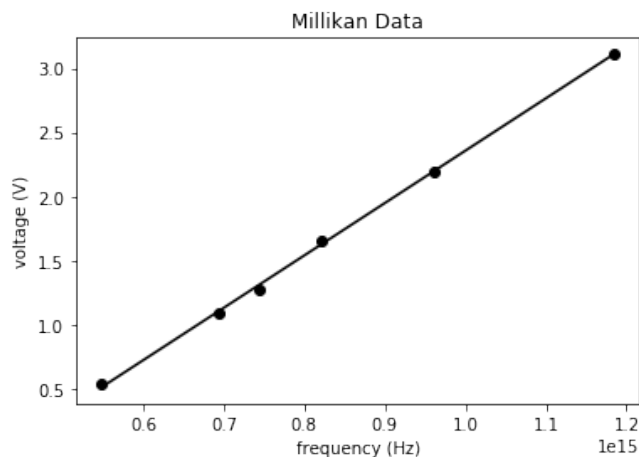


FIG. 3. Least squares fit for millikan.txt.

## IV. DISCUSSION

### A. Problem 3.7

By comparing Figures 1 and 2, it can be seen that the range for how many iterations are required to discard a

complex number is very large. Figure 2 shows that many of the white points in Figure 1 actually took quite a few iterations to eliminate. One would expect that many of the points that are currently black would be eliminated, given more iterations, because if no further points could be eliminated from the plots displayed here, then increasing the iterations should reveal no new patterns at larger zooms. The expectation is well-founded, since the Mandelbrot set has been extensively studied and many people have zoomed in with larger iterations and found patterns nested within (hence the term fractal).

### B. Problem 3.8

Visually, the least squares fit approximates the data well. Numerically, the fit also seems to be quite good, since the value obtained for Planck's constant,  $h = 6.54934022835e - 34$ , agrees with the established value of  $h = 6.62607004e - 34$  to within a couple percent. The intercept  $c = -1.73123580398$  represents the work function (or work function divided by  $e$ , depending on how you define it, irrelevant because the only difference would be whether the result is in  $eV$  or  $V$ , which differ exactly by that factor of  $e$ ) of the metal that Millikan used in his experiment. It has a negative value because it represents a binding energy for the electrons in the metal. Unfortunately, there is no way to know which metal was used to obtain this particular set of data, but the work function of sodium is 2.28 eV, which is fairly close to the magnitude of the value obtained with this data. This means the value obtained and the method used are reasonable.

## V. CONCLUSION

Note: Given that this homework is fairly straightforward, this section will be rather informal.

The results of this homework set were not too surprising. I zoomed in on the Mandelbrot fractal and it was really cool, but when I tried to make it into an animation, the rescaling of the axes became a problem. Unfortunately, I didn't have enough time to iron out the bugs, so that will have to be left as a future project.

The Millikan problem really helps one appreciate the usefulness of computers, because doing everything by hand sounds absolutely awful. Really shows how powerful and necessary they are as companions for modern research physicists!