The Mandelbrot Set Image and the Photoelectric Effect.

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This short paper consists of two parts: the image of the Mandelbrot set and the least-squares fitting of the photoelectric effect. In the first part, we present the density plot of the Mandelbrot set on a grid of size N by N spanning $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. In the second part, we use Milikan's experimental data to calculate a value of Planck's constant by performing least-squares fitting.

I. THE MANDELBROT SET

A. Introduction

The Mandelbrot set is a fractal which is a mathematical object whose structure repeats itself infinitely. The elements of the Mandelbrot set can be defined as a complex number c such that the iteration of

$$z' = z^2 + c, \tag{1}$$

is convergent.

B. Methods

In practice, we follow the suggestion in [1] that we first create a grid of size $N \times N$ spanning $-2 \leq x \leq 2$ and $-2 \leq x \leq 2$. Then for a given value of $c = x + iy$ we start with $z = 0$ and iterate equation (1) for 100 times. If $|z'|$ is less than 2, then c is in the Mandelbrot set. We can also produce a more meaningful structure of the Mandelbrot set by doing a density plot with the *jet* color scheme in Python according to the number of iterations before $|z'|$ is greater than 2.

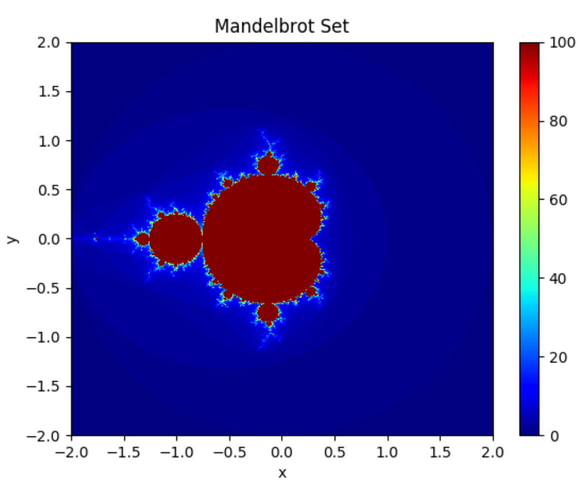

FIG. 1: The Mandelbrot set

C. Results

We perform the calculation over 2000 by 2000 grid and iterate equation (1) for 100 times. We also keep track of the number of iterations for each point c in the grid. The density plot is shown in FIG. 1. This density plot clearly shows the repeated structure of a fractal when we zoom in.

II. LEAST-SQUARES FITTING AND THE PHOTOELECTRIC EFFECT

A. Introduction

In this problem, we learn about the least-squares fitting method and apply our knowledge to the experimental data of Millikan to find the Planck's constant from the photoelectric effect.

B. Least-squares fitting

The idea of least-squares fitting is to find the closest curve which represents the underlying mathematical structure (or relations) from the data we collect. In this case, we are interested in the straight-line. We start by imagine the best fitted line with the slope m and y-intercept c. Suppose we have N data points with coordinates (x, y), then the sum of the vertical distances between the data points and the fitted line is

$$\chi^2 = \sum_{i=1}^{N} (mx_i + c - y_i)^2. \tag{2}$$

The best fitted line is then found by minimizing χ^2 which gives

$$m \sum_{i=1}^{N} x_i^2 + c \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i = 0 \tag{3}$$

$$m \sum_{i=1}^{N} x_i + cN - \sum_{i=1}^{N} y_i = 0. \tag{4}$$

m and c can be solved in terms of

$$E_x = \frac{1}{N}\sum_{i=1}^{N} x_i, \qquad E_y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{5}$$

$$E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2, \qquad E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i \tag{6}$$

and

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2}, \qquad c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2}. \tag{7}$$

This method is illustrated in FIG. (2).

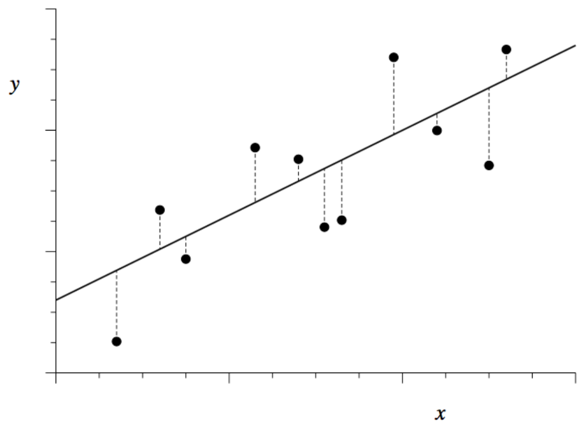

FIG. 2: The least-squares fitting method. This figure is taken from [1].

C. Photoelectric Effect

Now we apply the least-squares fitting method to the experimental data of Millikan which measured the photoelectric effect. The equation for the photoelectric effect is

$$V = \frac{h}{e}\nu - \phi, \tag{8}$$

where V is the voltage of the ejected electron from the metal, ϕ is the work function, h is the Planck's constant, e is the electric charge, and ν is the frequency of the photon.

D. Results

From the equations (7), we have the slope $m = 4.0882 \times 10^{-15}$ and y-intercept $c = -1.7312$. Comparing equation (7) to equation (8), we see that

$$\frac{h}{e} = m, \tag{9}$$

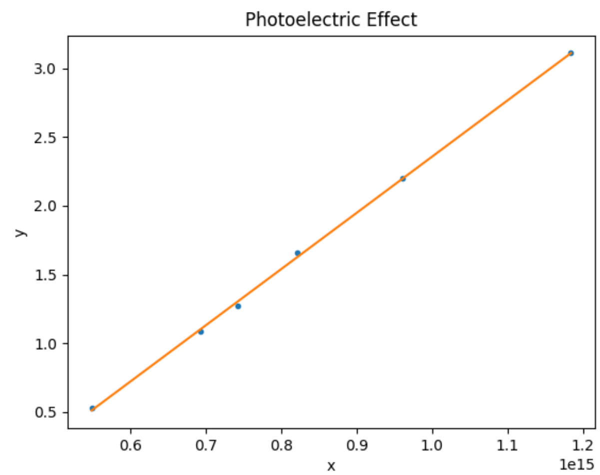

FIG. 3: The least-squares fitting plot shown in a straight line with the data points from photoelectric effect.

which gives us the experimental value of $h = 6.5493 \times 10^{-34}$, compared to the accepted value of 6.6261×10^{-34}. That is, our Planck's constant calculated from the least-squares fitting method is within 1.158% of the accepted value.

[1] Mark Newman, *Computational Physics*. CreateSpace Independent Publishing Platform, 2012.