PHYS-GA-2000: Computational Physics - Homework_1 Report

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Some numerical simulations are done by *Python2.7* in order to obtain a fractal figure of Mandelbrot set and numerical values of Planck constant and the m and c parameters associated with it, including a figure showing both a fitting line and scatter plot of the data from Millikan's experiment on the photoelectric effect, as well as the difference rate of h_{exp} to h_{ac}, which are Planck constants currently-accepted and experimentally-calculated, respectively.

1. INTRODUCTION

Numerical simulations by computer languages help us to understand non-analytical pictures of physical quantities as well as their analytical ones, such as the Dark Matter Bullet Cluster simulations in Ref. [1]. From this point of view, this report shows some numerical solutions to the two problems, 3.7 and 3.8 in Ref. [2], which were done by *Python2.7*. The first one is about figuring fractals called Mandelbrot set, and the other is on extracting Planck constant from Millikan's photoelectric-effect-experiment data by using least-square fitting method. Section 2 explains their methodologies and Sec. 3 shows the numerical results.

2. METHODS

2.1. Design of Mandelbrot set simulation

Mandelbrot set is defined as a set of complex numbers satisfying some particular conditions in Eqs. (1) - (3). The conditions are two things. The first is to hold a recurrsion formula in Eqs. (1) - (2) and the other is to keep its norm smaller than 2 in Eq. (3) as follows:

$$z' = z^2 + c \text{ for } c = x + iy,\ z,\ z' \in \mathbb{C} \tag{1}$$

where x and y are real numbers, and z and z' are complex numbers. The recurrsion formula can be simply represented as the following:

$$\begin{aligned} z_0 &= 0 + 0i, \\ z_1 &= c, \\ &\vdots \\ z_k &= z_{k-1} + c, \qquad k \in [0, N_{it}], \end{aligned} \tag{2}$$

where z_k means k-th iteration complex variable, and k runs over 0 to N_{it}, which is the total number of interations. Once the interations are done, we can apply a condition in Eq. (3) to the variable iterated, and then find the elements of Mandelbrot set.

$$Z^{(\alpha)} = x^{(\alpha)} + iy^{(\alpha)} \in S_{\text{Mandelbrot}} \quad \text{such that } \|z^{(\alpha)}_{N_{it}}\| \leq 2. \tag{3}$$

where α is a special index representing Mandelbrot set element, and $Z^{(\alpha)}$ is the element in the set as a complex number. Regarding the x and y, they are same as the previous ones of c.

As for recipe of the simulation, it is designed as follows. The first thing is to set $N \times N$ grids in order to establish $N \times N$ matrix consisting of only 1, which are going to represent the white background on the figure. The second is to gather special points to be black-colored by assigning each of the points to the x and y only if satisfying Mandelbrot set condition with running N_{it} iterations, which at the same time are assigned to zero to show the black points. After these steps are done, a $N \times N$ matrix having 0 and 1 as its elements can be obtained. Next, it has to be performed to figure the matrix in $N \times N$ plot size, *i.e.* x-axis range $(0, N)$ and y-axis range $(0, N)$. The last thing is to just re-scale the axes into x-axis range $(-2, 2)$ and y-axis range $(-2, 2)$, which is possible without loss of generality. For Fig. 1, it takes 1000×1000-grids points and 1000 iterations.

2.2. Least-square fitting method and its application to Millikan experiment data

According to Ref. [2], the chi-square variable to minimize is defined as

$$\chi^2 = \sum_{i=1}^{N} (mx_i + c - y_i)^2. \tag{4}$$

By minimizing the chi-square in Eq. (4), *i.e.* least-square, we can determine m and c in the following forms:

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2}, \quad c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2}. \tag{5}$$

where $E_x = \frac{1}{N}\sum_{i=1}^{N} x_i$, $E_y = \frac{1}{N}\sum_{i=1}^{N} y_i$, $E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2$, $E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i$. Then, we can also observe that the linear equation of slope m and y-intersection c can be corresponded to the photoelectric

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effect equation in Eq. (6):

$$V(\nu) = \frac{h}{e}\nu - \Phi \quad \leftrightarrow \quad y_i = mx_i + c, \tag{6}$$

where V is voltage; y_i is the i-th measured voltage; x_i is the i-th measured frequency; c is a y-intersection; h is Planck constant; e is electric charge (here, 1.602×10^{-19} C is used); ν is frequency; and Φ is work function of a surface of metal. From Eq. (6), Planck constant is calculated as

$$h_{exp} \equiv m * e, \tag{7}$$

which is what to compare with the currently-accepted Planck constant.

3. RESULTS

3.1. Results for 3.7 problem

The graph of Mandelbrot set is shown in Fig. 1 that the 3.7 problem requires.

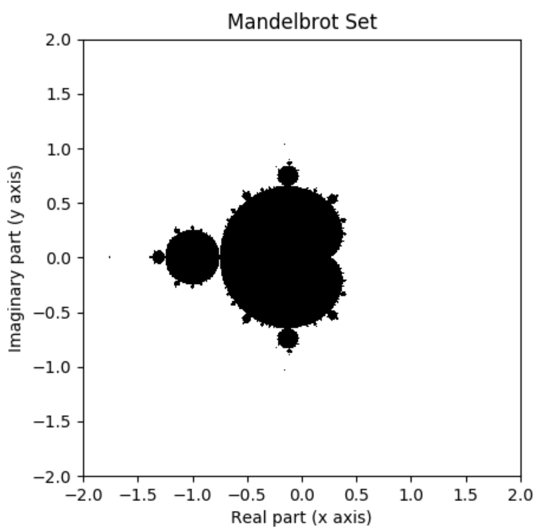

FIG. 1: Mandelbrot Set

3.2. Results for 3.8 problem

The graph of Millikan experiment on the photoelectric effect is shown in Fig. 2 that the subquestions (a) and (c) of the 3.8 problem require. In Fig. 2, the blue points are data measured by Millikan and the red line is the least-square fitting line calculated in this work. The numerical results of some parameters, m and c, in the problems are shown in Table I, which is the data that the subqustion (b) of the 3.8 problem requires. The numerical results

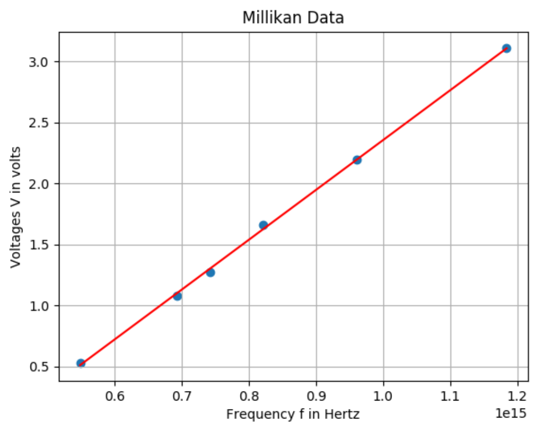

FIG. 2: Voltages-Frequency curve in Millikan experiment

TABLE I: Measurements of m and c

Parameters	m (J·s/C)	c (J/C)
Measurements	$4.08822735852 \times 10^{-15}$	-1.73123580398

TABLE II: Planck constants of h_{ac} and h_{exp} and the difference percentage

h_{ac} (J·s)	h_{exp} (J·s)	$\frac{h_{exp}-h_{ac}}{h_{ac}} \times 100$
$6.62607004 \times 10^{-34}$	$6.54934022835 \times 10^{-34}$	-1.15799880158

of Planck constants, h_{ac} and h_{exp} which are currently accepted by CODATA (`https://physics.nist.gov/cuu/Constants/index.html`) in 2014 and calculated in Eq. (7) based on Millikan experimental data, respectively, and the percentage of difference between them to the accepted are shown in Table II. This is the data that the subquestion (d) of the 3.8 problem requires. In particular, we can see that h_{exp} is slightly smaller than h_{ac} by about 1.16% of h_{ac}.

[1] Robert Thompson *et al.*, Monthly Notices of the Royal Astronomical Society **452**, 3030 (2015).

[2] Mark Newman, *Computational Physics* (Createspace, 2013), Revised and Expanded edition, Ch. 2 and Ch. 3.