Computational Physics HW 1

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1 Abstract

We will be using two simple examples to demonstrate the use of recursive functions and the plot() function in pylab. The Mandelbrot set is an example of a binary density plot, obtained via an infinitely recursive function (which we approximate for a large number n). Millikan's experiment on the Photoelectric Effect makes use of a least-squares linear fit, which is plotted using a scatter plot and line graph.

2 Mandelbrot

2.1 Intro

The Mandelbrot set is a fractal, meaning that it produces a shape with infinite perimeter and fractional dimension.
It is defined based on a recursive equation, $z' = z^2 + c$ Where c is a constant in the Complex plane. When iterated over, the z' of the previous iteration becomes the z of the next iteration. This can be iterated infinitely. Suppose we start with $z_0 = 0$, and choose some value of c. If, at any point, the absolute value $|z'| >= 2$, then the iteration stops, and we determine that this point c is not in the Mandelbrot set. If we can infinitely recurse this function and $|z'|$ never exceeds 2, then the point c is in the Mandelbrot set.

2.2 Methods

We wish to create a plot of the Mandelbrot set for the domain $c = x + iy \in [-2-2i, 2+2i]$, with a given resolution N. We begin by constructing a function to compute the equation, $z' = z^2+c$, when given the inputs z and c (this is fairly straightforward). Then, we construct the recursive function. Given the inputs z, c, and now a recursion number n. First, the function checks if $|z| >= 2$, and if so, the function terminates and returns 0. If it has reached its final recursive loop and has not ever encountered a $|z| >= 2$, then the function terminates and returns 1. Otherwise, it recurses the function, inputting $z' = z^2 + c$ as our new z.

Now that we have this function, we can map it to a grid of values over the complex plane. We construct an $N \times N$ matrix with values running from $x+iy \in [-2-2i, 2+2i]$ with even step size. Then, we map our Mandelbrot function to that matrix, obtaining an $N \times N$ matrix with a 0 or 1 in each place.

2.3 Results

For each point in this $N \times N$ grid, the plot is colored yellow if the point c is in the Mandelbrot set (matrix value 1), and purple if it is not (matrix value 0). Fig. 1 is the plot. For each value, we had a recursion number of 100, determining that this would be enough to get a good sense for the Mandelbrot set. The resolution N of this plot is 1000.

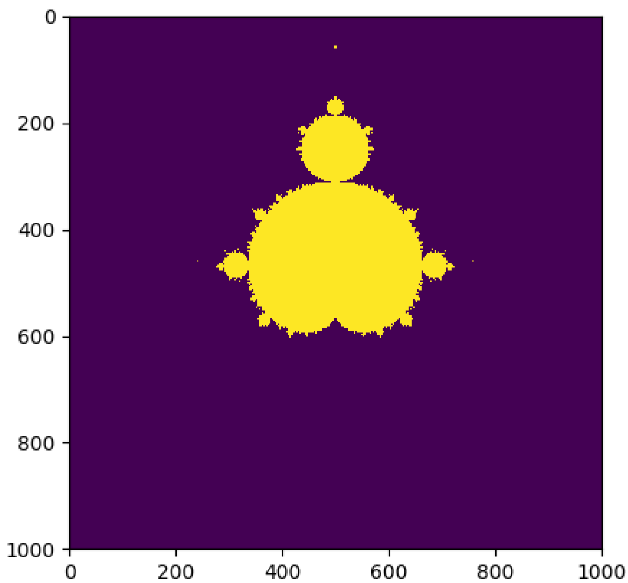

Figure 1: The Mandelbrot Set, N=1000

2.4 Conclusion

Beyond a resolution of $N = 100$, the runtime of this plot became significantly longer. It would be interesting to obtain higher resolution images of the Mandelbrot set using my code, but if I were to continue, I would want to find a more efficient way to code the recursive function to reduce runtime. I would also be interested in using a higher recursion number than $n = 100$ to see if the results of the set would differ greatly.

3 Millikan

3.1 Intro

The Photoelectric Effect is a phenomenon that occurs when light is shone on a metal surface. Photons of a certain energy can knock conduction electrons off of their atoms, thus ejecting them from the metal. The energy relationship between these photons and electrons is given by the Work function:
$V = \frac{h}{e}\nu - \phi$
Which was theorized by Einstein in 1905. Robert Millikan was able to experimentally measure Planck's constant from the Photoelectric effect using the Work function.

3.2 Methods

We will be using a least-squares fit to analyze Millikan's data. We are given a set of data points $[x_i, y_i], i = 1, ..., N$. The sum of squares of these data points is given by
$\chi^2 = \sum_{i=1}^{N}(mx_i + c - y_i)^2$
The straight line that minimized χ^2 will fit our data. If we define quantities:
$E_x = \frac{1}{N}\sum_{i=1}^{N} x_i$, $E_y = \frac{1}{N}\sum_{i=1}^{N} y_i$, $E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2$, $E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i$
Then we can use these to compute our slope m and y-intercept c:
$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2}$, $c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2}$
Our best fit line can then be applied to the Work Function, and we can compute the experimental value for Planck's constant: $h = m \times e$

3.3 Results

Using a least-squares fit of our data, we obtain the linear equation:
$V = 4.088\text{e}{-15}\nu - 1.731$
Fig. 2 is a plot of the data points with the best-fit line. Using our value of the slope, we calculate the experimental Planck's constant:
$h = m \times e = 4.088\text{e}{-15} \times 1.602\text{e}{-19}C$
$h = 6.549\text{e}{-34}\frac{m^2 kg}{s}$

3.4 Conclusion

The true value for Planck's constant is given as $h_0 = 6.626\text{e}{-34}\frac{m^2 kg}{s}$. Calculating our percent error gives:
$\%_{error} = \frac{h - h_0}{h_0} \times 100\% = \frac{6.549 - 6.602}{6.602} \times 100\%$
$\%_{error} = 0.802\%$
The error in this fitting and calculation is very small, which is a testament to the precision of Millikan's experiment.

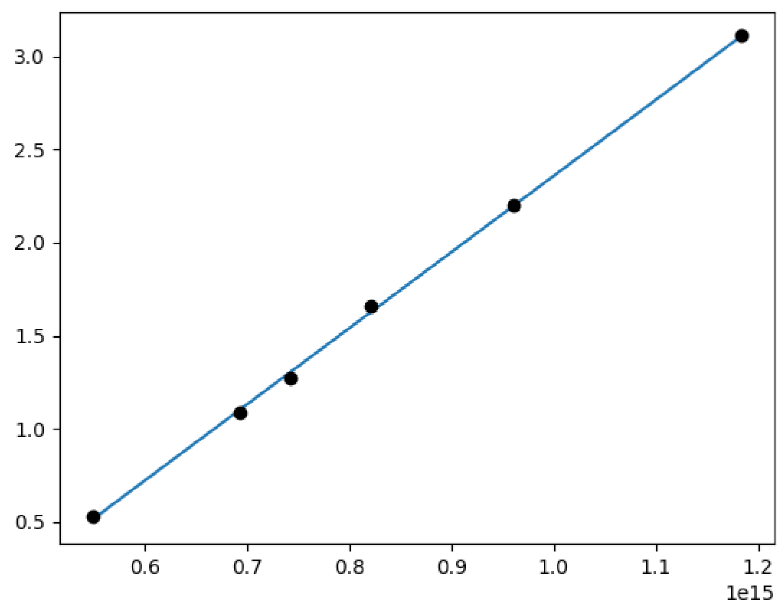

Figure 2: Best-fit line of Millikan's data on the Photoelectric Effect