Computational Physics Assignment 1

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Question 1. The Mandelbrot set: is a fractal, an infinitely ramified mathematical object that contains structure within structure within structure, as deep as we care to look.

Consider the equation,

$$z^{'} = z^2 + c \tag{1}$$

The Mandelbrot set is the set of points in the complex plane that satisfies the following definition (Computational Physics, Mark Newman): *For a given complex value of c, start with z = 0 and iterate repeatedly. If the $|z|$ the resulting value is ever greater than 2, then the point in the complex plane at position c is not in the Mandelbrot set, otherwise it is in the set.*

The algorithm is very simple, we divide the rectangle region ((x, -2, 2), (y, -2, 2)) into $N \times N$ grids. I take $N = 500$ in the code to get the high-quality image of the set. C is define in the complex plane as,

$$C_{mn} = -2 + m\frac{4}{N} + \left(-2 + n\frac{4}{N}\right) j, \tag{2}$$

Run too loops to cover the whole rectangle region. Renew the value of z each time. If the $|z|$ is larger than 2, We set $H_{ij} = 0$ (H is local number density). Otherwise, we set $H_{ij} = 1$. After the conditional statament, reset z to be zero.

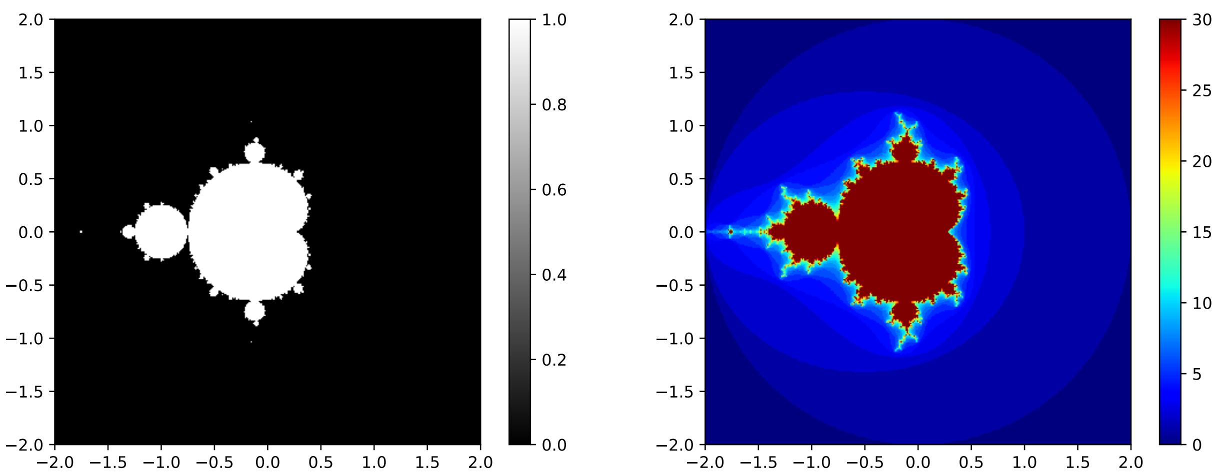

Figure 1: The Mandelbrot set. (a) Color points in black and write. The write region is the Mandelbrot set, (b) Color points using jet() function. Mapped according to the times of iterations before the modules of z becomes greater than 2.

The result is shown in Figure 1. Where different colormap has been used. Note that in fig. 1(b), we color points according to the iteration times before $|z| > 2$.

Question 2. Least-squares fitting and the photoelectric effect.

Many datas can be fitted to a straight line, $y = mx + c$. For a set of data, m, c can be calculated as following(Computational Physics, Mark Newman),

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2}, c = \frac{E_{xx} E_y - E_x E_{xy}}{E_{xx} - E_x^2}, \tag{3}$$

$$E_x = \frac{1}{N}\sum_{i=1}^{N} x_i, E_y = \frac{1}{N}\sum_{i=1}^{N} y_i, E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2, E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i. \tag{4}$$

The result is shown in Figure. 2. The data is fitted to $V = m\nu + c = 4.08823\nu - 1.73123$. The Plank constant $h = e \times m = 6.54934 \times 10^{-34}$.

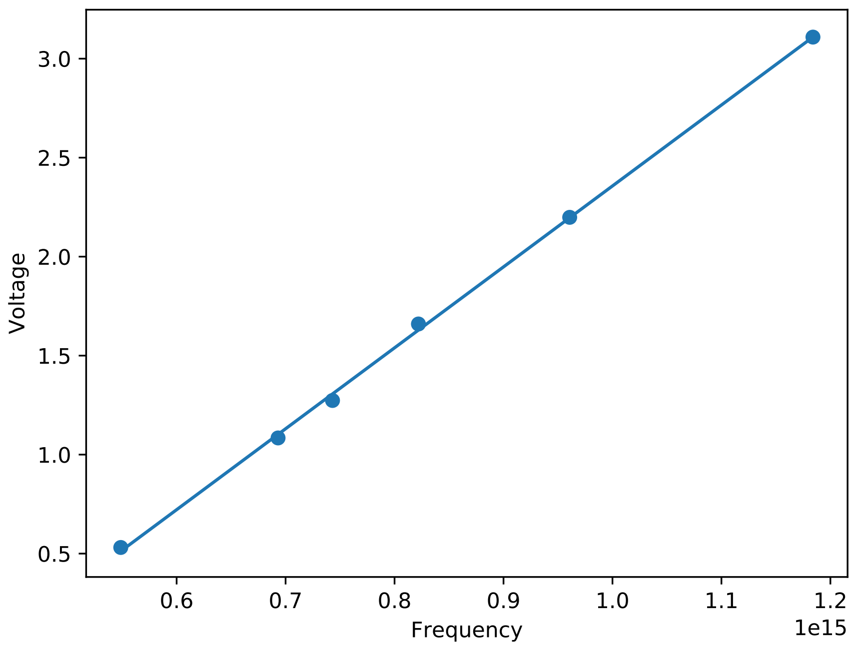

Figure 2: The least-squares fit of photoelectric effect.