Homework 2

Due: Wednesday Sept 27, 2017 at 12:01 am

1 Problem 1 – Integration

Write python functions to calculate integrals using Romberg integration and Gaussian Quadrature. Use the provided `gaussxy.py` code to calculate the points and weights for Gaussian Quadrature. Test your methods using the following integrals:

$$\int_0^{4\pi} x^2 \cos(x)\, \mathrm{d}x \tag{1}$$

$$\int_0^{100\pi} x^2 \cos(x)\, \mathrm{d}x \tag{2}$$

$$\int_0^{1} \sqrt{x}\, \mathrm{d}x \tag{3}$$

For each function and method calculate the L_1 error (absolute value of the difference between the true value and the numerical approximation) for a number of sample points (N). Using this data, calculate the convergence rate p ($L_1 \propto N^{-p}$) for each method on each integral.

Write a LaTeX report discussing your results. It should include a short explanation of the algorithms with all relevant formula, convergence plots for each test showing the L_1 error versus N for all methods, and a discussion that answers the following questions:

- Do your empirical convergence rates match theoretical expectations? If not, what properties of the integral lead to the change in behaviour?
- Do the methods behave as expected for both small and large values of N? Make sure to try both very small and very large values of N.
- At what N do the methods become dominated by rounding error? Does this agree with theoretical predictions?
- Look in detail at the convergence of Romberg Integration and Gaussian Quadrature. Do they converge as a power law at all? Discuss.

2 Problem 2 – Lax-Friedrichs Method

Write python functions to implement the FTCS scheme (forward in time, centered in space) method for solving a 1D scalar advection equation...

$$\frac{\partial u}{\partial t} = -v \frac{\partial u}{\partial x}$$

where u is a scalar and v is constant, specifically applied to a Gaussian Wave. You will recall from lecture that the basic approach...

$$u_j^{n+1} = u_j^n - \frac{vDt}{2\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

is unconditionally unstable. Show the instabilities growing over time in your Gaussian Wave, and compare to the corrected method invented by Peter Lax that changes the solution to be unconditionally stable.

Your tasks are as follows:

- Implement the unstable FTCS scheme and apply it to a Gaussian Wave in 1D. Show plots at multiple times that illustrate growing instabilities.
- Implement the Lax-Friedrich method to make FTCS unconditionally stable. Include plots of your Gaussian Wave at different times.
- Discuss all relevant formulas in your report.

Include the report `.tex` file and all Python files in the repo. Also include either the `.pdf` version of the report, or all figures necessary to compile it from the `.tex` file.